A DYNAMIC STOCHASTIC MODEL OF CORPORATE BEHAVIOR OVER THE BUSINESS CYCLE WITH A SPECIAL APPLICATION TO THE MAJOR U.S. MILITARY AIRFRAME BUILDERS

John Dudley Finnerty

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A Dynamic Stochastic Model of
Corporate Behavior Over the Business Cycle
With A Special Application To
The Major U.S. Military Airframe Builders

by

John Dudley Finnerty

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This thesis contains a formulation of a dynamic stochastic model of corporate behavior over the business cycle and applies the basic model to firms in the U.S. airframe industry. The literature dealing with the theory of the firm is surveyed and a taxonomy is developed within which the major contributions to the literature are appraised.

The basic model is formulated as an optimal control problem.

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The model is used to study the behavior of the firm over the business cycle and to suggest a possible reconciliation of the traditional and managerial theories of the firm. Financial considerations are incorporated into the model and the relationship between the firm's optimal operating decisions and its optimal financial decisions is examined. Organizational factors are introduced and some of the consequences of decentralized decision-making for the loss of control and X-efficiency are studied.

The basic model is extended to the major airframe builders by incorporating factors specific to that industry's institutional milieu. A model of a representative airframe builder is formulated as a stochastic optimal control problem and is used to study the impact of the government's progress payments policy and the likely impact of making interest expense an allowable cost under government contracts.

A Dynamic Stochastic Model of Corporate Behavior Over the Business Cycle With A Special Application To The Major U.S. Military Airframe Builders

by

John Dudley Finnerty
A.B., Williams College, 1971
B.A., Cambridge University, 1973
M.A., Cambridge University, 1977

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I. INTRODUCTION AND OVERVIEW OF THE THESIS

A. THE MODERN BUSINESS ENTERPRISE

The modern corporation operates within a business environment that is unlike the textbook world of perfect competition. According to Galbraith and others, the economic system in the United States is dominated by large, diversified corporations in which stock ownership is widely dispersed and effective control is exercised by corporate managers, rather than by the shareholder-owners of the firm.[1] This separation of ownership from control, it is argued, permits a considerable degree of managerial discretion, and it is largely for this reason that researchers, particularly economists, have recently devoted a great deal of effort to reformulating the theory of the firm. Their research has resulted in a variety of models, each of which attempts to provide a more realistic explanation of the behavior of the firm than that provided by the traditional theory. While these modern revisions have gone a long way toward improving the theory — by incorporating managerial objectives, by giving financial considerations an important role, by dealing with the growth of the firm over time, and by treating uncertainty — the task of theory building is, in the opinion of this writer, not yet complete. The purpose of this paper is to survey the literature dealing with the theory of the firm. This is done in the second chapter. The main purpose of this thesis is first, to extend the theory of the firm by developing a model of the firm that can be used to study the behavior of the firm within a multiperiod stochastic environment; and, second, to apply the model to firms in the U.S. airframe industry in order to study the behavior of these firms in the context of that industry's institutional milieu.

ABSTRACT

This thesis contains a formulation of a dynamic stochastic model of corporate behavior over the business cycle and applies the basic model to firms in the U.S. airframe industry. The literature dealing with the theory of the firm is surveyed and a taxonomy is developed within which the major contributions to the literature are appraised.

The basic model is formulated as an optimal control problem. The model is used to study the behavior of the firm over the business cycle and to suggest a possible reconciliation of the traditional and managerial theories of the firm. Financial considerations are incorporated into the model and the relationship between the firm's optimal operating decisions and its optimal financial decisions is examined. Organizational factors are introduced and some of the consequences of decentralized decision-making for the loss of control and X-efficiency are studied.

The basic model is extended to the major airframe builders by incorporating factors specific to that industry's institutional milieu. A model of a representative airframe builder is formulated as a stochastic optimal control problem and is used to study the impact of the government's progress payments policy and the likely impact of making interest expense an allowable cost under government contracts.

Practical men, who believe themselves to be quite exempt from any intellectual influences, are usually the slaves of some defunct economist. Madmen in authority, who hear voices in the air, are distilling their frenzy from some academic scribbler of a few years back.

— John Maynard Keynes
General Theory of Employment, Interest, and Money

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I. INTRODUCTION AND OVERVIEW OF THE PAPER

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The modern corporation operates within a business environment that is unlike the textbook world of perfect competition. According to Galbraith and others, the economic system in the United States is dominated by large, diversified corporations in which stock ownership is widely dispersed and effective control is exercised by corporate managers, rather than by the shareholder-owners of the firm.[1] This separation of ownership from control, it is argued, permits a considerable degree of managerial discretion, and it is largely for this reason that researchers, particularly economists, have recently devoted a great deal of effort to reformulating the theory of the firm. Their research has resulted in a variety of models, each of which attempts to provide a more realistic explanation of the behavior of the firm than that provided by the traditional theory. While these modern revisions have gone a long way toward improving the theory - by incorporating managerial objectives, by giving financial considerations an important role, by dealing with the growth of the firm over time, and by treating uncertainty - the task of theory building is, in the opinion of this writer, not yet complete. The purpose of this thesis is to model the behavior of the modern corporate enterprise over the business cycle and to apply the basic model to firms in the U.S. airframe industry. The principal purpose of this chapter is to characterize the modern business enterprise.

The traditional view of the individual firm being buffeted about by market forces beyond its control is, in the opinion of this writer, The traditional view of the individual firm's being buffeted about by market forces beyond its control is, in the opinion of this writer, inappropriate in the context of the modern corporate economy. In 1975 there were 169 manufacturing corporations each with assets over $1 billion. Collectively these firms controlled 67 percent of all manufacturing assets and earned 61 percent of all sales of manufacturing corporations. In the same year there were more than 750 manufacturing corporations with assets in excess of $100 million, and collectively these firms controlled roughly 90 percent of all manufacturing assets and earned roughly 86 percent of all manufacturing sales.[2] These large firms enjoy considerable degrees of market power, and in many cases they are capable of dominating the markets in which they participate. As a consequence, they play a crucial role in determining how efficiently the economy functions. For these reasons it is the behavior of the large dominant corporations that the modern theory of the firm seeks to explain.

The purpose of this chapter is to prepare the reader for the next chapter's survey of models of the firm. The present section characterizes the modern business enterprise and its operating environment and discusses in broad terms some of the important differences between the traditional view and the modern view(s) of the firm. Sections B and C discuss the use of mathematical models in the theory of the firm and define the important economic terms that are used throughout the thesis. Section D describes the differences between models of the firm, which are the subject of this thesis, and models of industry and general equilibrium models, which are not. Some interesting features of these latter types of models, and their relationship to models of the firm, are also discussed in section D. The final section summarizes the chapter and provides an overview of the thesis.

1. The Objectives of the Firm

A crucial aspect of any theory of firm behavior is the nature of the firm's objectives. In the opinion of this writer, in order to be able to understand how firms behave and to be able to use this understanding to indicate how firms should be expected to behave under differing sets of circumstances, it is first necessary to determine the firm's objectives — i.e. what it is trying to achieve.[3] The traditional view is that the corporation is owned by the stockholders. This subsection discusses the objectives of the firm, taking as its point of departure the traditional view that the firm seeks to maximize the economic welfare of the firm's owners. The remainder of the subsection deals with challenges to this view, with attempts to defend it, and with the alternative objectives that have been suggested.

According to the traditional view, the managers of the firm act as the agents of the owners, and in this capacity, try to make as much money for the owners as they possibly can. However, when control is exercised by the managers, rather than by the owners, there is no guarantee that the objectives of the firm will be those of the owners. Robert J. Larner carried out a study of corporate control in which he classified firms as management-controlled unless 10 percent or more of the firm's voting stock was held by an individual, family, corporation, or group of business associates.[4] He estimated that in 1963, 84 percent of the largest 200 nonfinancial corporations and 75 percent of the largest 500 nonfinancial corporations were management-controlled.[5] In addition, he found that management control had increased substantially since 1929 when an earlier study had been carried out by Adolph A. Berle, Jr., and Gardner C. Means.[6] The main implication of Larner's study is that, to an increasing extent, effective control of the largest firms is exercised by professional managers,[7] rather than by the firm's shareholder-owners.[8,9] This has led economists to study whether the goals of these two groups — the shareholders and the managers — conflict. It is generally accepted among economists that shareholders derive utility (or satisfaction) from the dividends they receive and the market value of the shares they hold.[10] It is also widely held that managers derive utility from the size and rate of growth of the firms they manage, the amount of compensation they receive, and the size of the staffs they control.[11] It follows that, to the extent that faster growth, larger executive salaries, and larger staffs mean lower dividends and lower share values, the separation of ownership from control will permit the firm's managers to pursue their own goals at the expense of shareholders' goals, and further, that the degree of management control will determine the relative weights attached to each set of goals when the firm sets its policy. When share ownership is widely dispersed, as it is in modern large corporations, the firm's managers may be free to pursue their own goals to the detriment of the goals of shareholders, with the only restraints on their activities being those imposed by the product markets, the financial markets, and the government.

Though the apparent separation of ownership from control gives corporate managers the freedom to pursue their own goals, economists are not in agreement as to whether corporate managers take advantage of this freedom. The traditional (or neoclassical) view, which will be considered in more detail in sections B through E of chapter two, maintains that the firm seeks to maximize total profit. One generally held interpretation of the traditional view is that the firm is run by an owner-entrepeneur who maximizes total profit in order to optimize his own welfare. In the context of a multiperiod model that takes into account the separation of ownership from control, neoclassical economists have substituted the goal of maximizing the stock market value of the firm's equity shares for the goal of maximizing profits, arguing that the overriding objective of the firm's managers is to do the best they possibly can for the firm's shareholder-owners. In each case the firm is assumed to conduct its business in the manner most consistent with the objectives of its owners.[12]

The neoclassical view has been widely criticized. In light of the apparent separation of ownership from control, it is difficult for many economists to see why corporate managers would feel compelled to maximize the return to shareholders. As Edith Penrose has argued, "providers of capital, like providers of labor services, must be remunerated, sometimes handsomely, but a desire to remunerate them as handsomely as possible is not a plausible explanation of the behavior of modern corporations."[13] Might the managers of the firm be forced by external market forces to maximize profits (or the market value of the firm's shares)?[14] Friedman and others have argued that there is an economic natural selection mechanism at work that forces firms to maximize profits.[15] For example, in the competition for investment funds in the capital markets, the competitive mechanism operating within these markets will ensure that only those firms that maximize profits will survive. Yet, Sidney Winter has carefully shown that this argument may not always hold and that, in some states of the environment, firms that seek only a satisfactory level of profits or firms that seek to maximize some other quantity, such as the rate of growth of the firm, are more likely than the profit maximizers to survive.[16] Moreover, Gordon Donaldson has shown that large firms raise a very large portion of their investment funds internally, which gives them a high degree of independence from the capital markets and the discipline these markets might otherwise be able to impose on the firm's behavior.[17]

A second argument put forward in defense of the profit maximization hypothesis maintains that, unless the firm maximizes profits, it will find itself vulnerable to a takeover raid.[18] That is, some other firm, once it recognizes that profits are not being maximized, will purchase a controlling interest in the non-profit maximizer, and then, by operating the company more efficiently, increase the profits of the taken over firm, and thereby increase its own profits as well. This argument assumes, of course, that sufficient information is available to enable firms to determine how large another firm's profits would be if that firm were operated at maximum efficiency. Unfortunately, the bulk of the financial information released to the public relates to actual operating results and provides little indication as to what might have happened to profits under some alternative operating policy. Also in contrast to the takeover argument, two recent studies by Singh have suggested that, over fairly wide ranges of profitability, the probability that a firm will be taken over does not decrease as the firm's profitability increases.[19] In addition, two other recent studies have shown that acquired firms earned profits that were only slightly below average for their industries and that many of the firms acquired by conglomerate firms were among the more profitable firms in their industries.[20] These studies imply that a firm's relative profitability may have little effect on its probability of being taken over, and hence, that the threat of takeover may not be strong enough to force firms to maximize profits.[21] Moreover, it may be size rather than profits that provides the firm the greater protection against a takeover, by making it more difficult for a potential takeover raider to raise sufficient capital with which to purchase a controlling interest. Marris takes this counter-argument a step further by arguing that the rise of management-controlled corporations has brought about a fundamental change in the way the corporate sector of the economy functions and has altered the selection process in such a way that only firms that maximize growth can survive.[22] Thus, according to Marris, it is growth maximization, rather than profit maximization, that gives the greater probability of survival.

A third line of argument offered in support of the hypothesis that corporate managers seek to maximize profits focuses on executive compensation. Wilbur Lewellen carried out an extensive study of top executive compensation for fifty very large manufacturing firms over the period 1940 to 1963. He found that their ownership-oriented compensation — primarily from profit-sharing and stock option plans — was the only source of growth of total executive after-tax compensation during the period 1940 - 1963, and that it formed approximately one-half of total executive after-tax compensation during the period 1955 - 1963. Over the period 1940 - 1963 salary and bonus averaged somewhat less than one-quarter of total executive before-tax compensation.[23] In a subsequent study he found that executives' total compensation was more closely related to the profitability and to the market value of the firm than to the firm's level of sales.[24] In view of these results, it would appear that the interests of corporate managers are closely allied with the interests of shareholders, but there exists evidence to the contrary. D. R. Roberts and J. R. McGuire et al. carried out cross-section regression studies of executive compensation which revealed that measures of size explained a larger portion of the inter-firm variation in executive compensation than measures of profitability.[25] The best way to obtain high compensation is to become a high-ranking executive of a large corporation. In addition, Robin Marris and others argue that large firms show a general preference for internal promotion and that professional managers are willing to sacrifice some of the firm's profits in order to achieve a faster rate of growth and greater size because more rapid growth and larger size mean more higher-paying jobs near the top of the corporate ladder and more opportunities for promotion.[26] According to this line of argument, managers' salaries are more closely correlated with factors other than the firm's profitability, such as size, and, to the extent permitted by the separation of ownership from control, managers will sacrifice profits in order to pursue these other objectives.

Within the last two decades economists' efforts have intensified to formulate alternative models of the firm that the modelers believe reflect more accurately the objectives of corporate managers.[27] These alternative theories have been labeled 'managerial' because of their identification of the firm's objectives with the objectives of the firm's managers (rather than its owners). William J. Baumol has developed a sales maximization model.[28] In describing his model Baumol argued that sales were more amenable than profits to objective measurement and that the salaries of top executives were more closely related to the size of the firm than to its profitability.[29] Robin Marris and Oliver Williamson have developed models in which managers act so as to maximize their own utility. In the Marris model the manager's utility function has two arguments, the rate of growth of the firm's productive assets and what he calls the valuation ratio, which is the ratio of the stock market value of the firm to the book value of the firm's net assets.[30] In the Oliver Williamson model the manager's utility function has two arguments: first, staff expenditures, which include outlays on advertising, research and development, and managerial emoluments; and second, discretionary profits, which are profits that are spent at the discretion of the firm's managers.[31] These three models will be discussed more fully in section G of chapter two, where it is argued that the Baumol model can be regarded as a utility maximization model in which the utility function has, as its lone argument, the level of the firm's sales. It is also pointed out that, in contrast to the neoclassical models, the level of the firm's overall (as opposed to discretionary) profits enters each of the three managerial models as a constraint rather than as an argument of the objective function.

The foregoing has indicated the range of views concerning the appropriate objective function for a model of the modern business enterprise. By way of summary, the debate is, in essence, over whether the utility function of the firm's owners (the traditional view) or the utility function of the firm's managers (the managerial view) is more appropriate as the objective function of the firm. Stated somewhat differently, the debate concerns the extent to which the apparent separation of ownership from control has permitted corporate managers to pursue their own goals to the detriment of those of the owners.

Before concluding this subsection it should be noted that there is a third view concerning the objectives of the firm. According to this view the modern corporation is a diversified and complex bureaucratic organization comprised of various social groupings, each with its own set of objectives, and the behavior of such an organization is the result of a bargaining process among the various social groupings.[32] The main consequence of this interactive process, so the argument goes, is that the firm has no single, all-embracing objective, such as profit maximization or growth maximization, and the objectives of the firm cannot be identified strictly with those of any one grouping, such as the shareholders or top management. The observed behavior of the firm is unlikely to be optimal with respect to the objectives of any one grouping or with respect to any one goal. Rather, in order to accommodate the conflicting interests of top executives, middle managers, shareholders, etc., the firm's managers aim toward a satisfactory level of profits, a satisfactory level of sales, a satisfactory rate of growth, and satisfactory values for the other variables of interest, where in each case the 'satisfactory' value is established through the bargaining process. The behavioral models of the firm, as they are called, are discussed in section H of chapter two, where it is pointed out that, while the behavioral models have performed well in terms of explaining certain types of behavior, such as price setting behavior, of particular firms, they have yielded little in the way of general results. In particular, due to the absence of optimizing behavior on the part of the firm, it is virtually impossible to determine how the firm will, in general, react to external stimuli, such as an increase in the corporate tax rate.[33] In the opinion of this writer, while the descriptive features of the behavioral models are not without merit, a model of the firm is more likely to yield useful analytical implications if it is of the optimization variety. With the exception of the behavioral models discussed in section H, all the other models of the firm discussed in chapter two assume optimizing behavior on the part of the firm.

It is the author's opinion that a more general treatment than the ones described above can be given the firm's objectives. Moreover, this more general treatment could draw on the behavioral, as well as the traditional and managerial, views of the firm. It is the author's opinion, which is based in part on his own field research, that corporate objectives are set largely by top management, but that the goals of shareholders and other groups within the firm can have a direct impact on the firm's behavior. Hence, the objective function of the firm should reflect the goals of managers as well as those of shareholders and other groups within the firm — but as interpreted by top management. The corporate utility function, then, should not be that of any single group, but rather, should at least reflect both traditional and managerial sources of utility.

A model of the firm incorporating the corporate utility function just discussed in its objective function could, in contrast to the behavioral models, be of the optimization variety. One of the uses to which such a model could be put is the study of the trade offs between traditional and managerial sources of satisfaction. Another of the possible uses is the study of the behavior of the modern corporate enterprise over the business cycle. If the corporate utility function has several arguments — or loosely, several goals — it might be shown that the relative weights attached to these goals may vary over the business cycle in response to changes in the firm's operating environment and the resulting changes in the severity of the constraints within which the firm must operate. One of the important implications of such a systematic shift in goals may be that in certain time periods it appears that a firm is maximizing profits (or owner utility), while during other periods it appears that the same firm is maximizing growth (or managerial utility).

In this subsection it was suggested that the growth rate of the firm is a variable that is of interest to corporate managers. The next subsection examines the corporate growth process and an important closely related phenomenon, diversification.

2. Corporate Growth and Diversification

One of the phenomena of the modern corporate economy that has most impressed the proponents of the managerial theories of the firm is the rapid growth of firms that are already large.[34] Such rapid growth, and in particular, the apparent failure of growth to slow noticeably when the firm becomes large, is at variance with the traditional theory of the firm.

According to the traditional view, for each industry there is an optimum size for the firm. If a firm expands beyond this optimum size, it will suffer an unavoidable decrease in profits. Since, according to the traditional theory, the firm must maximize profits in order to survive, the firm must stop growing once it achieves the optimum size. A modern version of the same argument maintains that firms may grow very rapidly — they may even maximize the growth rate at the expense of short run profits — until they reach this optimum, but once the optimum has been reached, the growth rate falls to zero.[35] The existence of an optimum size was initially argued on the grounds that production functions exhibit decreasing returns to scale, if not over the entire range of output, then at least at high rates of ourput.[36] When empirical studies revealed that returns to scale were either constant or increasing in most industries, proponents of the traditional theory argued that even with constant or increasing returns to scale there would still be an optimum size because, as the firm grows, managers find it increasingly difficult to control the organization.[37] The loss of control would gradually erode operating efficiency and would eventually cause the long run average total cost curve to turn upward. Oliver E. Williamson has described how the multidivision structure of the modern corporate enterprise, with responsibility for the day-to-day operations vested in individual profit centers and with the head office responsible for long-range planning, coordination of the activities of the divisions, and corporate policy, has enabled corporations to achieve large size without suffering the serious losses of internal efficiency predicted by the traditional theorists.[38] Moreover, the empirical evidence indicates that "firms which are at present small or medium-sized do not in general tend to display either higher or lower growth rates than firms which are already large."[39] Thus, empirical support is lacking both for the optimum size hypothesis and for the natural life cycle hypothesis. In the opinion of this writer, any theory that attempts to explain the behavior of modern corporations should be consistent with the empirical finding that firms, no matter how large they become, continue to grow.

The growth process proceeds along two avenues. Along the first, a firm can grow internally by expanding its capital stock to support increasing sales in existing product lines or by undertaking investment and developing new product lines, i.e., by diversifying. As Galbraith has pointed out, the large modern corporation can spend tremendous sums on advertising in order to foster demand for its products.[40] A firm is not restricted to the markets in which it currently operates and in both markets for new products and markets for existing products, market demand — both the shapes and the positions of the demand curves facing the firm — can be influenced through advertising. As Marris has pointed out, the modern corporation also expends huge sums on research and development in the hopes of developing new products in the sale of each of which it can exploit a temporary monopoly.[41] Through its outlays for advertising and research and development the firm is able to promote its own growth from within.

Firms also grow externally by taking over[42] — i.e., by purchasing a controlling interest in — other firms.[43] This was the primary vehicle for corporate growth during the 1960's. Between 1966 and 1970 conglomerate mergers were responsible for the disappearance of 72 industrial firms from Fortune magazine's list of The 500 Largest Industrial Corporations (hereafter shortened to FORTUNE 500).[44,45] Through a succession of mergers some companies were able to grow at phenomenal rates. Getty Oil, which had previously been too small to make the FORTUNE 500, took over Skelly Oil and Tidewater Oil and made the list in 1967 as No. 79. Through a series of mergers I.T.T. rose from No. 28 in 1966 to No. 8 in 1970.[46] Indeed, the period from 1954 to 1974 was one of sustained growth for many companies. One of the most remarkable is Occidental Petroleum, which experienced an average annual rate of growth of sales of 106 percent — an increase of 191 million percent over the two decades.[47] Nearly as remarkable is Beatrice Foods, which has acquired nearly 400 companies over the past quarter-century — probably more than any other company in the FORTUNE 500 — and has, largely due to these acquisitions, increased annual sales from $200 million to $4.6 billion over this period.[48]

Due to the conglomerate merger movement and to the internal development of new products, the modern corporation is not only large, but it is also highly diversified, producing hundreds, and sometimes even thousands, of different products. Beatrice Foods, for example, has established 397 individual profit centers that are responsible for the production and sale of 8000 different products.[49] Such diversity can, however, have disadvantages as well as advantages. Controlling so diverse a company requires a highly efficient internal organization that can ensure that poor coordination does not dissipate the gains that accrue from spreading the production and financial risks over many product lines.

With the exception of the work of Monsen and Downs and the important contribution of Oliver Williamson, economists have been very slow to apply organizational theory to the theory of the firm.[50] According to Williamson, this failure is the result of the "common tendency to invoke the standard behavioral assumption (profit maximization) without regard for circumstances."[51] Yet firms expend large sums in an effort to determine how the efficiency of their internal operations can be improved.[52] In addition, over the past fifty years large firms have replaced what Williamson calls the unitary form of organization and its highly centralized decision-making with what he calls the multidivision form and its more decentralized decision-making.[53] According to Williamson, the multidivision form of organization enables these large, diversified firms to achieve greater operating efficiencies than would be attainable under the unitary form.[54]

Both diversification and the rise of the multidivision form of organization have facilitated rapid and continued growth. If a firm chose to produce only a single output, its expansion would be limited by the growth of market demand for that good and by the retaliatory actions of its market rivals should it try to expand its market share. If a firm chose to maintain a unitary form of organization, its growth would be limited by the decision-making capacity of top management. In the opinion of this writer, increasing diversification and the development and further refinement of the multidivision organizational structure are crucial aspects of the process of sustained growth. It is also the opinion of this writer that the formulation of models of the firm that reflect the importance of the firm's internal organization — in terms of both operating efficiency and growth — is a direction in which further research could fruitfully proceed.

3. Corporate Planning under Uncertainty

The top management of the typical large firm spends a significant portion of its time engaged in corporate planning. This is partly in response to the need to coordinate the activities of the various operating divisions and to ensure that the different parts of the organization act in a manner consistent with the objectives of the whole. But the need for effective corporate planning can be attributed primarily to the existence of uncertainty.[55] Because the state of the environment at some future point in time cannot be known with certainty, the firm must develop a set of contingency plans. A firm can set various targets, for example, a target level of profits and a target rate of growth of sales, but the actual level of profits and rate of growth of sales will depend on the state of the environment as well as on the operating policy of the firm. In establishing its operating policy — the prices of the products it sells, the amounts to be spent on advertising and on research and development, and so on — the firm must take into account the possible future states of the environment, the relative likelihood of each, and the effectiveness of each of the alternative sets of operating policies in terms of meeting the firm's objectives.

The uncertainty a firm faces can be categorized as being of two types: production (or business) uncertainty, which is associated with the production and investment decisions of the firm, and financial uncertainty, which is associated mainly with the financial decisions of the firm. Both types of uncertainty and the relationship between them are discussed at greater length in section I of chapter two, where they are treated in connection with Vickers's model of the firm and Arzac's extension of that model.[56] For the purposes of this discussion, it is important to note just that there are uncertainties (or risks) associated with both types of decisions.

A second aspect of the firm's financial and business decisions that is explored further in chapter two should also be mentioned. It concerns the issue of whether the firm's financing decisions can be separated from its investment decisions. This question is examined in sections I and K of chapter two, where it is shown that, under certain restrictive conditions, the firm's financial decisions — i.e. its decision concerning what portion of profits to pay out as dividends and its decision concerning the relative proportions of debt and equity to have in its capital structure — can be made independently of its investment decisions. It is also shown that, in general, under uncertainty these decisions are not separable. Leland's model of the firm in the context of stock market equilibrium, which is presented in section K, illustrates this last point.

In terms of modeling the behavior of the firm under uncertainty, either of two general approaches may be adopted. One approach is to employ a mean-variance framework.[57] An example of a model that uses this approach is the Lintner model[58] discussed in section J of chapter two. The second approach — which can be shown to be inclusive of the first — is the time-state-preference approach,[59] which is discussed and illustrated by two models developed by Leland in section K of chapter two.

4. The Constraints on the Firm

In the foregoing discussion much was said concerning the objectives of the firm and the extent to which the firm's objectives reflect the managers', rather than the owners', objectives, and much was also said concerning the complexities that are due to the existence of uncertainty. But very little was said concerning the constraints the modern corporation faces. The discussion of constraints has been left until now because there is general agreement among economists and businessmen as to the nature of these constraints, whereas the nature of the firm's objectives is a subject on which there has been, and still is, considerable disagreement.

In spite of its tremendous economic power, the modern corporation must plan and carry out its activities subject to the following constraints:

— The government (sometimes with the help of various special interest groups that suggest new legislation) establishes the legal framework within which the firm must operate. It protects private property and the sanctity of contracts, sets the minimum wage, protects workers' collective bargaining rights, establishes standards of product quality, and so on. By lobbying the firm can work through the political process, and in so doing, it can exercise influence, though not control.

— The stockholders reelect the board of directors each year.[60] If profits are judged by shareholders to be inadequate, or if the firm engages in questionable overseas activities, the shareholders could vote the current directors out of office and elect new directors who would see to it that the firm's behavior is more in line with shareholders' interests.[61] When share ownership is widely dispersed, however, it is very expensive and very time-consuming for shareholders to organize and to agree on a new slate of directors.[62] Shareholder reaction is normally felt only when the situation has become serious.[63] In addition, low profits may depress the stock market value of the firm's shares and thereby increase the threat of takeover. This threat intensifies if the lower share value arouses the firm's shareholders and if other firms attribute the low profits and low share value to ineffective management of the firm's assets. The reason why top management is so fearful of a takeover is that the acquiring firm often replaces top management soon after the takeover.[64] It is largely for this reason that the managerial models of the firm developed by Baumol, Marris, and O.E. Williamson each impose a constraint either on minimum total profit or on the minimum stock market value of the firm.

— Another serious danger that may arise when profits are low is the increased likelihood that the firm will not be able to cover its bond interest obligations, in which case the firm's bond creditors can order that the firm be liquidated. Even if bond interest can be covered, if the surplus of profits over bond interest obligations is small, it may be very costly, or even impossible, for the firm to raise additional finance from external sources.

— The current state of technology, as embodied in the firm's production function, limits the amount of output that can be obtained from any given quantity of inputs. The firm can try to alter the state of technology by spending funds on research and development, but it is normally assumed that such spending is subject to diminishing returns (at least beyond some point).[65]

— There are market-imposed constraints. Customers can refuse to purchase the firm's product if quality is judged to be poor or price is judged to be excessive (or both). While the firm can influence the state of demand for its products through advertising, it is normally assumed that such expenditures are subject to diminishing returns. Moreover, the price a firm can charge for each of its products is limited to some extent by the prices charged by its competitors.[66] If the firm purchases inputs from another large firm or hires workers that belong to large powerful unions, then the ability of the firm to influence the cost of its inputs may be restricted.

— The rate at which the firm can grow is limited by the need to bring new managers into the organization and by the adverse effect these new managers will have on the firm's internal operating efficiency if too many are brought into the organization too fast.[67] If a firm continues to increase its growth rate, then this 'Penrose effect' will eventually cause the firm's profit rate to fall.[68] If carried to excess through the acquisition of marginally profitable, or possibly even unprofitable, enterprises, the acquiring firm damages its long run profitability and increases its risk of encountering serious financial difficulties, as the recent experience of the high growth conglomerates of the 1960's attests.

— The unpredictability of acts of nature, such as floods, earthquakes, and lightning, constitutes a constraint, though of a sort different from those constraints mentioned previously. In particular, the potentially adverse impact of this constraint can often be mitigated through the judicious selection of plant locations and through the purchase of insurance.

— The lack of perfect knowledge of future market conditions, future technologies, future actions and reactions of competitors, etc., also imposes a constraint on the firm's policy choices. Indeed, this lack of perfect knowledge is one of the major sources of uncertainty. Unlike the constraints mentioned earlier in this subsection, which are normally incorporated in models of the firm in the form of constraining equations or inequalities, the uncertainties resulting from the unpredictability of acts of nature and from the lack of perfect knowledge, normally necessitate a fundamental change in the form of the firm's objective.[69]

If the reader will permit an analogy to be drawn between the firm's selection of its 'best' operating and financial policies and a mathematical programming problem the significance of the above constraints can be explained in the following manner. As the firm seeks the set of operating and financial policies that will optimize its performance with respect to its objective, it must ensure that all the constraints are satisfied. That is, the constraints delimit the set of feasible policies from which the optimal policy is to be chosen.

5. Summary

The managers of modern large diversified corporations possess what amounts in many cases to a considerable degree of discretion that permits them to pursue their own objectives, possibly even to the detriment of shareholders' objectives. Yet each firm's managers must always be mindful of the limits of their power and of what lies beyond their control.

In setting corporate policy, the managers of each firm must make difficult policy choices under uncertainty. Top management must balance its own objectives against those of the firm's shareholders and those of middle managers and other groups within the firm, subject to a multiplicity of constraints and subject also to the complexities attributable to uncertainty.

To make the necessary policy choices under these conditions requires careful planning. To coordinate the activities of the divserse operating components of the corporation and to guide the firm toward the attainment of its objectives require a decentralized form of organization that still permits top management to exercise effective control. In the opinion of this writer, if economists are to be successful in their efforts to understand how the modern corporate enterprise behaves, they must first construct models that incorporate the internal planning and decision processes of these firms.

B. THE ROLE OF MATHEMATICAL MODELS IN THE THEORY OF THE FIRM

What economists call the theory of the firm is actually a collection of models, each of which purports to explain the economic behavior of business firms. The modern revisions to the theory of the firm, in particular, attempt to explain the behavior of the large corporate enterprises that dominate the economy. The purpose of this section is to describe in general terms the use of mathematical models in the theory of the firm and to indicate the modeling techniques most frequently employed.

An economic model is a simplified analytical framework; it is the representation of, as well as the embodiment of, an economic theory. While there is no inherent reason why economic models must be mathematical, it is the approach normally adopted, and the use of mathematics in formulating economic models has become increasingly sophisticated in recent years. One need only compare current issues of the major economic journals with issues published five or ten years ago to recognize this trend.

One of the branches of economic theory most affected by the increasingly sophisticated use of mathematics is the theory of the firm. Both the traditional theory and the modern revisions may be conveniently expressed as mathematical models, and once expressed in this form, the models can be used to study the implications of each of the alternative theories concerning the behavior of the firm. This mathematical approach has a number of advantages. The use of mathematics facilitates a precise and more concise statement of each theory. It requires that assumptions be made explicit and that the firm's objectives and the constraints it faces be specified clearly. It also permits the modeler to draw on a variety of mathematical techniques, such as mathematical programming and the calculus of variations, to assist him in discovering the implications of his assumptions. In recent years the development of new models has often been accompanied by the use of more sophisticated analytical techniques as economists have sought to capture in their models intertemporal dependencies, the impact of uncertainty, and the interrelationship between the firm's financial decisions and its operating decisions. The discussion in chapter two, which parallels the evolution of the theory of the firm, illustrates this tendency.

1. The Static Optimization Problem

Models of the firm are typically expressed as mathematical programming problems. The general mathematical programming problem is:

$$\underset{\{\bar{x}\}}{\text{maximize}} \quad f(\bar{x}) \qquad \text{subject to} \quad \bar{x} \in \Omega , \tag{1}$$

where $\bar{x}$ is a n-component vector and Ω is a subspace of Euclidean n-space called the feasible region. Additionally, it is usually assumed that f is twice continuously differentiable with respect to all of its arguments.

Problem (1) is a static optimization problem. The solution to (1) is the vector $\bar{x}^* \in \Omega$ for which $f(\bar{x}^*) \geq f(\bar{x})$ holds for all $\bar{x} \in \Omega$. The method of Lagrange multipliers can be used to obtain a characterization of the solution to (1) provided Ω is defined by a set of equality constraints. If Ω is delimited, by a set of constraints that includes one or more inequality constraints, then the method of generalized Lagrange multipliers (i.e. the application of the Kuhn-Tucker necessary conditions) must be used to obtain the characterization. If $f(\bar{x})$ and the constraints defining Ω are all linear, then linear programming can be applied, although this technique is generally more useful when a numerical solution, as opposed to a characterization,[70] is desired.[71]

The objective function $f(\overline{x})$ in problem (1) is the firm's objective function; it reflects the goals and objectives of the firm. The objective function varies according to whether the objective of the firm is to maximize one of the following:

- size of profits, sales, capital stock, or staff
- growth of profits, sales, or capital stock
- stock market valuation of the firm
- market share (of sales)
- total compensation of top executives,

or possibly some other variable or some function of the above variables. In the traditional models of the firm, which are discussed in sections B through E of chapter two, $f(\overline{x})$ is of the form $\pi(q)$, where π is total profits and q is the rate of output. When the traditional model is modified to allow for uncertainty, $f(\overline{x})$ becomes the mathematical expectation of the utility function of (risk averse) shareholders.[72] A model possessing such an objective function is discussed in section K of chapter two. Recognizing the separation of ownership from control in modern corporations, many economists have suggested that the behavior of these firms is determined by the objectives of the firm's managers, in which case $f(\overline{x})$ becomes the utility function of managers and $\overline{x}$ is the set of arguments of the managerial utility function.[73] This treatment of $f(\overline{x})$ is explored more fully in section G of chapter two.

The feasible region Ω is determined by the constraints the firm faces. The nature of these constraints was discussed in the previous section. In formulating a model of the firm it is necessary for the modeler to incorporate each of the constraints he believes to be material. Normally this involves expressing each (economic) constraint as an appropriate equation or inequality that must be satisfied.

It should be noted that in the actual statement of a model it is often the case that one or more constraints are either left implicit or omitted altogether. There are a number of reasons for this. First, the nonnegativity constraints are not always stated explicitly. For example, a firm's rate of output must always be nonnegative, but in statements of the traditional models this constraint is usually left implicit. Second, some of the constraints may be assumed away, as, for example, when it is assumed that uncertainty does not exist. This is usually done in order to simplify the model by omitting constraints that the modeler believes to be incidental to the questions the model is designed to answer.

Third, it is often possible to use one or more of the original equality constraints to reformulate the problem with fewer decision variables.[74] In the extreme case, it is possible to convert a constrained problem

$$\begin{aligned} &\underset{\{x_1,x_2,\ldots,x_n\}}{\text{maximize}} && f(x_1,x_2,\ldots,x_n) \\ &\text{subject to} && g_i(x_1,x_2,\ldots,x_n) = b_i\,, \quad i=1,\ldots,m \quad (m<n) \end{aligned} \tag{2}$$

into an equivalent unconstrained problem

$$\underset{\{x_1,x_2,\ldots,x_{n-m}\}}{\text{maximize}} \quad f(x_1,x_2,\ldots,x_{n-m}) \tag{3}$$

by appealing to the implicit function theorem.[75] However, such a simplification requires that information concerning the nature of the solution — information that is provided by the values of the Lagrange multipliers — be sacrificed. Thus a model may be expressible in more than one form, although these mathematically equivalent forms may not be equivalent in terms of the amount of useful economic information that is obtainable. In this particular case, formulation (3) would be preferred to formulation (2) in terms of computational simplicity, though (2) might be preferred to (3) in terms of more fully characterizing the economic nature of the solution to the problem. The modeler will select the formulation that best balances the advantages of computational simplicity and the informational requirements of the economic problem at hand.

Fourth, sometimes it is simpler to remove a constraint and compensate for this by adding a variable to the objective function, even though this procedure does not necessarily produce an equivalent problem. For example, a firm's managers may feel constrained by the need to maintain good employee relations. Rather than trying to formulate this constraint on the basis of some measure of employee satisfaction, such as the number of days work lost due to strikes, and having to specify some minimum number b to go into the constraint $g(\bar{x}) \geq b$, it might prove easier to treat the state of employee relations — the number of days work lost due to strikes — as an argument of the managerial utility function (i.e. the objective function). This approach was adopted by Marris, who suggested two possible forms for the valuation constraint in his model.[76] According to the strong form, there is a market-determined minimum valuation ratio below which the firm's valuation ratio cannot fall without the firm being taken over. According to the weak form of the constraint, the firm's valuation ratio becomes an argument of the managerial utility function (whereas previously under the strong form managerial utility had been a function of the firm's growth rate only). As discussed in section G of chapter two, incorporating the valuation ratio in the managerial utility function implicitly involves the assumptions that both the growth rate and the valuation ratio contribute directly to managerial utility and, in view of the standard differentiability assumptions, that a continuous rate of trade off exists between the two (at least over some range of growth rates). For such a continuous trade off to exist it is necessary that the Lagrange multiplier that measures the trade off between the growth rate and the minimum valuation ratio, when the strong form of the constraint is hypothesized, be a continuous function of the minimum valuation ratio.

Most of the models of the firm examined in chapter two have been expressed in the form of mathematical programming problems. The models expressed in this form are of two general types. In the first case, the model involves a single time period within which a single set of simultaneous decisions concerning prices, input levels, output levels, will be made. Once determined, the values of these variables remain fixed for the duration of the period. The models discussed in sections B through E of chapter two are of this type. In the second case, the model involves more than one time period, but decisions are made in the initial period that lock the firm onto a steady state growth path in which all quantities, such as total revenue, total assets, the stock market value of the firm, etc., grow at the same constant rate forever. Once this steady state growth rate has been determined, the values of these quantities remain in fixed relation to one another throughout subsequent time periods. The Marris model in section G and the Lintner model in section J of chapter two are of this type. In each case, the nature of the economic model makes it amenable to formulation as a mathematical programming problem.

2. The Dynamic Optimization Problem

The mathematical programming problem (1) is a static optimization problem. Characterizing its solution involves the determination of the optimal values of the decision variables for an economic problem that is stationary either with respect to the variables themselves or with respect to the relationships among the variables over time. Many of the modern revisions of the traditional theory of the firm discussed in chapter two are multiperiod models in which the relationships among variables such as total assets, total revenue, and total profit are free to change over time.[77] In other words, the firm is not assumed to be on a steady state growth path. In such models, where the values of variables such as the firm's capital stock may be temporally interdependent, the model normally cannot be expressed adequately as a static optimization problem. For example, in the Arrow model, which is discussed in section L of chapter two, investment is irreversible in the sense that imperfections in the market for capital goods makes it impossible for the firm to disgorge unneeded fixed assets instantaneously. Therefore, the current investment decision must take into account the possibility that demand for the firm's products may fall sharply several periods into the future. While the firm might, on the basis of strong current demand, want to purchase additional fixed assets, the prospect of sharply falling demand and its inability to instantaneously adjust its capital stock when that happens may cause it to postpone its investment plans. A more general treatment of the firm in a multiperiod context that allows for factors such as the irreversibility of investment requires the use of a dynamic optimization technique.

In building multiperiod non-steady state models of the firm, most economists have chosen to express their models as optimal control problems.[78] The general optimal control problem may be written:

$$\left.\begin{array}{l} \underset{\{\bar{u}(t)\}}{\text{maximize}} \quad J = \displaystyle\int_{t_o}^{t_1} I(\bar{x}(t),\bar{u}(t),t)\,dt \\ \\ \text{subject to:} \\ \qquad \dfrac{dx_i}{dt}(t) = f_i(\bar{x}(t),\bar{u}(t),t) \quad , \quad i=1,\ldots,n \\ \qquad \{\bar{u}(t)\} \in U \\ \qquad (\bar{x}(t),t) \in S \quad \text{at} \quad t = t_1 \\ \qquad t_o \quad \text{and} \quad \bar{x}(t_o) = \bar{x}_o \quad \text{given} \quad , \end{array}\right\} \qquad (4)$$

where t denotes time, which is measured in continuous units over the interval $t_o \leq t \leq t_1$; $\bar{x}(t)$ is a n-component vector of state variables; $\bar{u}(t)$ is a m-component vector of control variables;

U is the set of all admissible control trajectories; and $S \subset E^{n+1}$ is the terminal surface. In words, the vector $\overline{x}(t)$ characterizes the state of the firm, as for example, the size of its capital stock, at each time t. The vector $\overline{u}(t)$ contains the firm's policy variables, as for example, current output, current outlays for advertising, and the current investment decision. Typically, $t_o = 0$, and, if there is a finite planning period of length T,[79] $t_1 = T$, while, if the firm's time horizon is assumed to be infinite,[80] then t_1 is replaced by ∞ in (4). For the finite horizon problem, the terminal state of the system, $\overline{x}(T)$, must satisfy the equation of the terminal surface, S, as for example, a required terminal capital stock.

There exist the following three approaches to solving problem (4): the calculus of variations, dynamic programming, and the maximum principle of Pontryagin.[81] The analysis of the Jorgenson model in subsection 1 of section L in chapter two demonstrates the application of the calculus of variations. The other models discussed in section L of chapter two illustrate applications of Pontryagin's maximum principle.

The objective functional $J = \int_{t_o}^{t_1} I(\overline{x}(t),\overline{u}(t),t)\, dt$

in problem (4) reflects the objectives of the firm, just as the objective function $f(\overline{x})$ in problem (1) represents the objectives of the firm. Also analogous to problem (1), the constraint set in (4) reflects the constraints the firm faces. That is, problem (4) can be viewed as the dynamic analogue to problem (1), with the control variables $\overline{u}(t)$ in (4) playing the same role as the decision variables $\overline{x}$ in (1).[82] The difference between the two problems lies in the control problem's treatment of temporal interdependencies, which are captured both in the objective functional and in the n first order differential equations that form part of the constraint set.

3. Optimization vs. Non-Optimization

Problems (1) and (4) are optimization problems. Models of the firm expressed in either of these forms assume that the firm exhibits some sort of maximizing behavior, whether it be the maximization of profits, the maximization of the stock market value of the firm, the maximization of its rate of growth or of managerial utility, or the maximization of some other quantity. Such models have been criticized on the grounds that modern corporations are bureaucratic organizations in which the information required to make important decisions is imperfect and in which major decisions are arrived at through a bargaining process that involves the various special interest groups that comprise the organization. As a result, business firms do not exhibit optimizing behavior, but rather, as the economists of the behavioral school of thought argue, they exhibit a satisficing behavior — aiming toward a level of profits, a rate of growth, etc., that are satisfactory to the competing interest groups.

In spite of the arguments of the behavioralists, the great majority of economists conducting research on the theory of the firm prefer to work with optimization models. As a more careful examination of the behavioral models that are discussed in section H of chapter two will bear out, these models yield very little in the way of meaningful predictions as to how the firm will behave in response to various external stimuli such as a change in the corporate tax rate. The behavioralists hypothesize the firm's use of rules of thumb for such activities as price-setting — indeed, several behavioralist studies have provided strong empirical support for such behavior — but these theories generally fail to explain how these rules of thumb are determined or why they may change over time.[83] Often, the behavioral models are simply attempts to explain specific patterns of observed behavior (e.g. how a particular department store establishes its prices), rather than attempts to develop a more general analytical framework that can be used to predict changes in behavior (e.g. how the department store's mark-up will change in response to a change in its overhead). As regards the objectives of the firm, the behavioral models leave essentially unanswered the question: what are satisfactory values for total profits, the rate of growth, etc.? While the behavioral studies have led to interesting results, the deficiencies just discussed limit the ability of these models to yield useful policy implications.[84]

It is the opinion of this writer that a model of the firm, if it is to yield meaningful policy implications, should hypothesize some form of optimizing behavior. The problem of imperfect information to which the behavioralists refer is a realistic one, but the impact of imperfect information can be incorporated in an optimization model that allows for uncertainty.[85] For example, instead of attempting to maximize utility, under uncertainty managers aim to maximize expected utility — where the group whose (expected)utility is being maximized will depend on whether the model is of the traditional or of the managerial type. Moreover, a comparison of the certainty and the uncertainty versions of such an optimization model would indicate the impact of uncertainty on the behavior of the firm.[86]

As indicated earlier in this chapter, it is this writer's opinion that the objective function in a model of the firm should reflect both traditional and managerial sources of utility and that the model should also reflect the importance of factors internal to the firm. The development of such a model, if it allowed for uncertainty, could be used to study the impact of uncertainty on the firm's decision-making processes and on its resulting behavior. Such a model could have the advantage of capturing many of the important factors that influence the behavior of the firm, as observed by the behavioralists, while retaining the optimizing quality that is so useful analytically.

4. Characterizing a Solution

Thus far in this section, the discussion has concentrated on the nature of the modeling techniques employed in the theory of the firm, and very little has been said concerning the solution of the mathematical programming problem (or the optimal control problem), in which form the model was expressed. In this regard, a careful distinction needs to be drawn between the 'characterization' of a solution and the 'computation' of a solution.

In chapter two various models of the firm, each of which is expressed in the form of either a mathematical programming problem or an optimal control problem, are studied with the aid of the standard mathematical techniques for solving these problems. In each case, one of the primary objectives of the analysis is to characterize the solution to a mathematical programming problem (or optimal control problem) that is representative of a typical firm and to interpret this solution's general economic implications, rather than to compute the optimal value of the objective function in a problem that is specific to a particular firm. While a mathematical programming problem in which the objective function and constraints were specific to a particular firm in a particular period of time could be used to compute specific values for the decision variables, such as price, output, and input levels, and from these the resulting values of total revenue, total profit, etc. for the firm,[87] the purpose of the models discussed in chapter two is more general. Actual numerical values are not important to these modelers, but rather, it is how the firm determines these optimal values that is their primary concern. For example, in the traditional models of the firm it is the decision rules, such as the marginal revenue equals marginal cost rule for profit maximization, that characterize the optimal solution to the mathematical programming formulation of the model that are the end toward which the analysis is directed. A specific numerical problem and its solution might also be of interest, but generally only as an illustration.

C. DEFINITIONS

In the introductory discussions of the modern business enterprise and of the role of mathematical modeling in the theory of the firm there were several economic terms, such as capital, profit, and risk, that were used repeatedly. These terms, many of which have specific economic meanings that differ from their meanings in other forms of usage, are used throughout the paper. The purpose of this section is to give each of these important terms a specific economic meaning.

1. Firms, Plants, and Industries

The subject with which this paper deals is the theory of the firm. A *firm* is a business enterprise that purchases economic inputs, such as land, labor, and capital, that it uses to produce goods and services that it sells to consumers, other firms, and the government, all taking place under the direction and control of a single management group.[88] A firm may produce a single product, as often assumed in the traditional models, or it may produce a range of products. The term conglomerate is often applied to a firm that produces a widely varied set of products, although this notion is not precise. This paper is concerned with a particular subset of the set of all firms: the large multiproduct corporation. What distinguishes the corporation from the other two forms of business organization — the proprietorship and the partnership — is that the corporation is itself a legal entity that can raise investment funds by selling shares, or certificates of ownership. This characteristic bestows upon the firm certain advantages not shared by the other forms of business organization.[89] In this paper the terms corporation, corporate enterprise, business enterprise, and firm are used interchangeably.

The term firm needs to be clearly distinguished from the terms *plant* and *industry*. The former refers to the numerous different production facilities operated by a firm, while the latter, in principle at least, refers to a set of firms that either (i) all produce the same product or (ii) all use a production process that can produce the same set of products.[90] For example, the United States aerospace industry consists of makers of military aircraft as well as makers of commercial aircraft (and some firms that currently produce both types of aircraft), and also consists of builders of missiles and space vehicles. It should be noted, however, that defining an industry may involve rather difficult practical problems since, in practice, it often proves difficult to delimit products and production processes sharply.[91] For this reason several different classification schemes may exist for an industry. For example, the United States aerospace industry contains the United States military airframe industry,[92] which may be defined to include those firms that work as prime contractors for the U.S. government and produce the technically sophisticated attack, fighter, bomber, cargo and tanker aircraft used by the Department of Defense. This definition excludes producers of missile airframes, helicopters, and commercial aircraft that do not also produce the above-listed military aircraft, though, under a wider definition, one, or possibly all, of these might be included.[93] How broadly one defines a product and the boundaries of the industry associated with that product generally depends on the nature and scope of the study requiring the definition.

2. The Production Function, Capital, and Investment

The ability of the firm to combine inputs to produce goods and services is restricted by the state of technology. The firm's *production function* is a technological relationship between the various inputs it purchases and the various outputs it produces and may be expressed mathematically as

$$F(q_1,\ldots,q_i,\ldots,q_n, x_1,\ldots,x_j,\ldots,x_m) = 0 \quad , \qquad (5)$$

where there are n products and q_i stands for the output of product i and where there are m inputs and x_j represents the amount used of input j. The economic significance of the production function (5) is twofold: (i) at any point in time the state of technology, and hence the firm's ability to combine inputs to form outputs, is constrained, but (ii) over time the firm can alter the state of technology by increasing its expenditure for research and development (and thereby trading off present profit for future profit).[94]

One of the arguments of the production function (5) that is of particular interest to economists is the input *capital*, which is the stock of goods that are used in production and that have themselves been produced.[95] The term capital has several meanings, and the notion of capital as a productive resource is also referred to as *producer's capital* to distinguish it from other forms of capital.[96] Producer's capital consists of buildings, plant, and equipment, which are referred to collectively as *fixed capital*, and inventories of raw materials, components, and semi-finished goods, which are collectively called *working capital*. In terms of the production function (5), when economists speak of the input capital they almost always mean fixed capital, rather than the more inclusive producer's capital. Fixed capital is singled out for special treatment because of its 'fixity', i.e. because the process of adding to a firm's stock of fixed capital is relatively more time-consuming than adding to its stock of working capital, to its labor force, or to its supply of any other 'variable input'.

The concepts of fixed capital and working capital refer to physical goods, or what is commonly called real capital. A distinction must be made between real capital, which is a productive resource, and *money capital*, which is not.[97] This distinction will become important when the role of financial capital in the theory of the firm is discussed in chapter two.[98]

When a firm increases its producer's capital by exchanging money capital for physical goods it is said to invest. *Investment* involves the purchase of additional plant and equipment and the building up of inventories. A portion of total investment is intended to replace worn out machinery and the remainder represents *net investment*. The reduction in the firm's capital stock through wear and tear is called *depreciation*, and net investment, which represents the net expansion of the firm's productive base, is equal to total (or gross) investment minus depreciation.

3. Profit

In the traditional models of the firm the goal of the enterprise is to maximize total profit. *Profit* is the difference between the total revenue accruing from the sale of the various products the firm produces and the total cost incurred in producing those goods, measured over some specific time period. It is important to distinguish clearly between the accounting notions of profit and the economic notions of profit.[99] The distinction arises essentially from the manner in which accountants and economists define *costs*. In arriving at figures for income from operations, pretax income, income before extraordinary items, or net income, any one of which may be loosely referred to as profits, the accountant subtracts money outlays and depreciation from sales revenue.[100] In contrast, an economist would subtract the opportunity cost — the maximum amount the input would earn in its best alternative use — of each input rather than its money cost. The economist would also deduct the opportunity cost of inputs, such as the firm's capital, that the firm owns and uses in production. When such inputs have alternative uses, the firm incurs an opportunity cost by not renting the services of these inputs to other producers who would be willing to pay for them, and for this reason, economists impute a cost to the firm's use of these inputs.[101] For example, if a firm owns the land on which its production facilities rest, the rent the firm would have had to pay, if it had leased the land, would not be deducted by the accountant, whereas the imputed rent would be deducted by the economist, Therefore, the economic concept of profit would agree with the accounting concept only in special cases, as for example, when the firm actually rents all the capital services it uses; and when this is not the case, a firm may be earning a positive profit in the accounting sense, but experiencing a negative profit (i.e. a loss) in the economic sense.[102]

Another distinction that should be made with regard to profit is the difference between reported profit and discretionary profit. In this paper *reported profit* is the net income (before extraordinary items) reported in the firm's income statement[103] and *discretionary profit* is reported profit plus discretionary expenses in excess of what would be required for profit maximization.[104] Discretionary expenses include the firm's outlays for advertising, research and development, training, and staff — all of which are made at the discretion of management.[105] This distinction is important for two reasons. First, to the extent that the firm desires to smooth out fluctuations in reported profit, it can do so by varying discretionary expenses. Second, the managerialists argue that a portion of what would otherwise be reported profit, which would in that case either be paid out as dividends or else added to retained earnings, will be used by the firm's managers to pay for larger staffs or to finance additional advertising and growth-promoting research and development. That is, discretionary profit is the vehicle by which managers can promote their own goals.

In this paper both the economic definition and the accounting definition of profit are important. While it would be desirable to work with the economist's definition of profit exclusively, since it reflects the opportunity cost of all inputs, it is the accountant's definition of profit that is reflected in the annual income statements each firm furnishes its shareholders. When the role of finance is considered in section I of chapter two, the importance of the accountant's definition of profit will become clearer. To preview that discussion, fluctuations in the firm's reported profit — and in particular, fluctuations in its net income — will play a major role in determining the stock market value of the firm.

4. Utility

In contrast to the traditional models of the firm in which the goal of the business enterprise is to maximize total profit, in managerial models the goal is to maximize the *utility* of the firm's managers. Managers derive utility, or satisfaction, from a number of sources. In the O.E. Williamson model these sources include staff expenditure, managerial emoluments, and discretionary profit.[106] In the Marris model the sources of utility are the firm's growth rate and its valuation ratio.[107] In these models the level of utility, U, is expressed in fuctional form as

$$U = U(q_1,\ldots,q_i,\ldots,q_n) \quad , \tag{6}$$

where q_i is the quantity of the i^{th} source of utility and where $\frac{\partial U}{\partial q_i} > 0$ for all i.

There are two important theoretical issues connected with utility functions that should be noted. The first concerns the nonmeasurability of utility. Utility, or satisfaction, is a subjective concept, and the utility index embodied in (6) is ordinal rather than cardinal. In contrast to traditional economic theory, in which total utility U and marginal utility $\partial U/\partial q_i$ each were given a quantitative significance, modern economic theory does not assume such measurability.[108] But while the numerical value of $\partial U/\partial q_i$ is no longer important, its sign is. The condition $\partial U/\partial q_i > 0$ implies that the firm's managers never become satiated. For example, for the Marris model's utility function, this nonsatiability means that, no matter how fast the firm is growing, faster growth will always increase the level of managerial utility.

The second important issue concerns revealed preference.[109] Provided that certain 'axioms of revealed preference' are satisfied, the existence and nature of the managers' *indifference map* — a collection of surfaces, along each of which all combinations $(q_1,\ldots,q_n)$ yield the same level of utility[110] — can be inferred from the managers' policy choices (i.e., from the preferences managers reveal as they select from among the alternative business strategies). The significance of this result is that, while the Marris and O.E. Williamson models are based on the assumed existence of managerial utility functions exhibiting certain specified properties, it is possible, in principle at least, to carry this analysis a step further and infer these properties from the axioms of revealed preference. At the very least, the existence of utility functions of these types might be tested empirically via revealed preference theory.[111]

5. Size and Growth Rate

One of the arguments of Marris's managerial utility function is the firm's rate of growth. Measurement of the firm's rate of growth requires a measure of the firm's size. At present there is no single measure of size, and hence no single measure of growth, that is free of conceptual difficulties.[112] In the economic literature a wide variety of measures of size have been employed.[113] It is important to note that, as Marris argues, capital, profits, and output grow together in moving equilibrium so that "maximizing the long run growth rate of any one indicator can reasonably be assumed equivalent to maximizing the growth rate of most others."[114] Moreover, Bates has demonstrated empirically the high degree of correlation among alternative measures of size.[115] For the purpose of consistency in discussing the size and the growth rate of the firm, the *size* of a firm is defined to be the book value of its net assets, where net assets are the sum of fixed assets, plus inventory, plus current assets net of current liabilities, and where assets are valued at historic cost net of depreciation.[116] The firm's *growth rate* is the annual growth rate of its net assets. In view of Bates's empirical results, the implications to be derived from models of the firm should be relatively insensitive to these particular measures of size and the growth rate.

6. Perfect Competition, Monopoly, Oligopoly, and Monopolistic Competition

The firm's freedom to pursue its own objectives, whether that involves greater profits, faster growth, or an increase in some other quantity that contributes to managerial utility, is constrained by the environment within which the firm operates. One of the more influential factors is the structure of the industries to which the firm belongs. The behavior of the traditional firm under different industrial structures is discussed in sections B through E of chapter two. The purpose of this subsection is merely to characterize the main types of industrial structure identified in the economic literature.

Economists identify four types of industrial structure: perfect competition, monopoly, oligopoly, and monopolistic competition.[117] Under *perfect competition* all firms produce a homogeneous product and each firm acts as a price-taker. One practical explanation for this type of behavior is that there are so many firms, and every firm is so small in relation to the size of the market, that no single firm is capable of altering the market price through its own actions. Under *monopoly* the firm takes the market demand curve for the product as the demand curve for its output and determines both price and the quantity of output in light of market demand and the costs of production. The practical explanation for this sort of behavior is that there exists just one producer.

Under *oligopoly* there is more than one producer. Each has some influence over market price and each has to weigh carefully the potential reactions of its competitors when it changes its operating policies. One practical interpretation of this is that there are only a few large producers, such as in the U.S. automobile industry, who recognize the interdependence of their actions.

Under *monopolistic competition* each producer's output is slightly differentiated (either in its physical characteristics or in the minds of consumers) from what other producers in the industry are selling. In addition, each producer is able to act like a monopolist in the sale of its own product. One practical explanation of this is that there is a very large number of producers of slightly heterogeneous goods.[118]

Empirical studies have revealed that oligopoly is a prevalent form of industrial structure in the U.S. economy.[119] In particular, the U.S. military airframe industry, which was referred to earlier in this chapter, can be described as an oligopoly.

7. Uncertainty and Risk

A second quality of the firm's environment with which it must contend is uncertainty. Some economic theorists distinguish between *risk*, a situation in which all possible outcomes and the probability of occurrence associated with each are known, and *uncertainty*, a situation in which the possible outcomes or the probabilities (or both) are unknown.[120] In what follows such a distinction will not be employed. Following Hirshleifer, the term uncertainty will be used as a synonym for risk.[121] That is, *risk* and *uncertainty* will both refer to a situation in which the possible future states of the economic environment are known and a probability (possibly subjective) can be attached to each.[122]

In an uncertain world the attitude of managers toward risk can have a significant impact on the firm's behavior. It is normally assumed that managers are *risk averse*.[123] Given the choice between a fair gamble — one in which the mathematical expectation of return just equals its price — and the expected return with certainty, the risk averse individual will take the certain amount.[124] Put differently, a risk averse individual will demand more than a fair gamble before agreeing to play. The implication is that a risk averse manager might be willing to sacrifice some of his expected return in exchange for a reduction in risk.

The concept of risk aversion can be illustrated with the aid of a risk-expected return indifference map, such as the one shown in Figure I-1. Each curve, called an indifference curve, shows those risk-expected return combinations that yield the same level of expected utility. Expected utility increases in the northwest direction due to decreasing risk or increasing expected return, or both. Under risk aversion, the indifference curves are drawn upward sloping, as in Figure I-1, to reflect the fact that an increase in risk necessitates an increase in the risk averse individual's expected return in order to leave him equally well-off.[125]

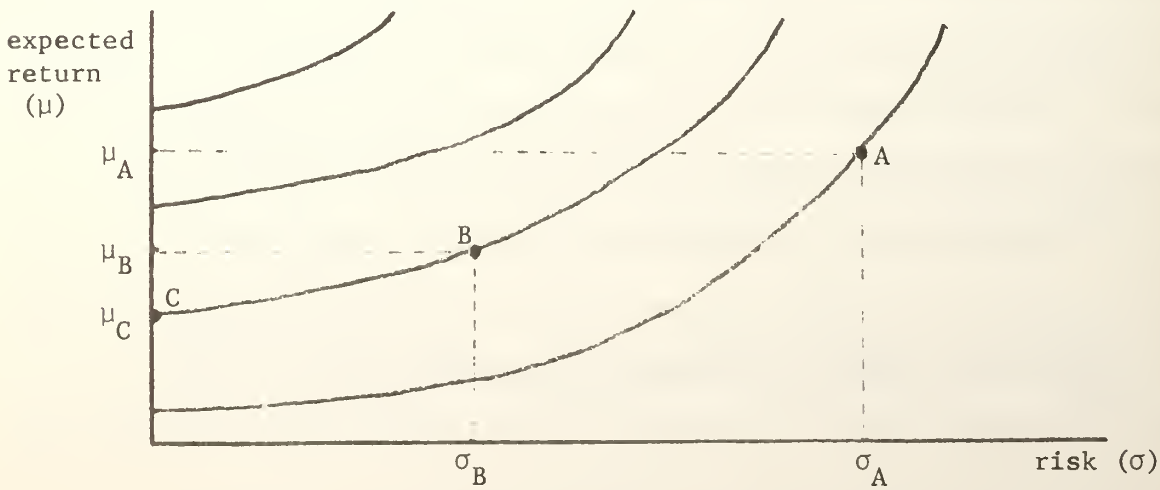


Figure I-1 Indifference Curves under Risk Aversion

For the hypothetical manager having indifference curves like those shown in Figure I-1, the risk-return combination (σ_A, μ_A) would be preferred — in the sense of being on a higher indifference curve — over the combination (σ_B, μ_B) , even though the latter involves a lower expected return than the former. If (σ_B, μ_B) is attainable, say, by adopting more 'conservative' operating policies or by using part of the expected returns to purchase additional information and thereby reduce risk, the risk averse manager will do so. What this implies for models of the firm that allow for uncertainty is that the objective function must be formulated in terms of expected utility maximization (rather than in terms of expected returns maximization).[126]

Another useful concept, which is illustrated in Figure I-1 and which will be needed in the discussion of the Lintner model in section J of chapter two, is that of a *certainty equivalent return*. In Figure I-1, the risk-expected return combinations (σ_B, μ_B) and $(0, \mu_C)$ lie on the same indifference curve and therefore yield the same level of expected utility. Since the combination $(0, \mu_c)$ is riskless, or certain, and since it is equivalent to (σ_B, μ_B) in terms of expected utility, it is called the certainty equivalent return of the combination (σ_B, μ_B) . Moreover, $(0, \mu_C)$ is the certainty equivalent return for each of the risk-expected return combinations that lies on the same indifference curve.

8. Takeovers and Mergers

A third factor affecting the firm's choice of operating policies is the takeover mechanism. A *takeover* is said to occur when firm X acquires more than 50% of the equity of firm Y .[127] Firm X is called the *acquiring firm* (or more colorfully, the *takeover raider*) and firm Y is termed the *acquired firm*. Singh and Kuehn distinguish between a takeover, in which the acquired firm becomes a part of (normally an operating division of) the acquiring firm, and a *merger*, in which the firms X and Y amalgamate to form a new legal entity, firm Z .[128] This distinction is important in that mergers usually take place under the mutual agreement of the parties involved, often for financial convenience,[129] whereas takeovers are frequently contested affairs that result in the dismissal of the acquired firm's managers.[130]

The economic significance of takeovers is twofold in nature. For the acquiring firm, takeover is a means of rapid growth and is also a means of rapidly diversifying into new product lines and thereby diminishing the risk associated with profit fluctuations that occur over the business cycle.[131] For the acquired firm, takeover carries with it the danger of dismissal of the firm's managers. Therefore, according to Marris, the threat of involuntary takeover imposes a constraint on the firm's managers' policy choices.[132] If policies are adopted that lead to poor performance — as judged by the firm's shareholders — then the firm's shareholders will be more willing to sell their shares to a takeover raider. Such a constraint plays an important role in each of the managerial models of the firm discussed in section G of chapter two.

9. The Business Cycle

A fourth aspect of the firm's environment, and one that will be discussed in connection with the Arrow model presented in section L, is the *business* (or *trade*) *cycle*, which refers to the more or less regular oscillations in the level of aggregate economic activity that occur over time. While the judicious use of demand management policies in the western industrial economies appears to have sharply reduced the possibility of a depression, the recent pattern of economic activity bears witness to the fact that the possibility of recession has not been eliminated.[133] The amplitude of the fluctuations has been diminished, though not all the way to zero.

The importance of the business cycle as it affects the behavior of the firm is, in the opinion of this writer, lost in static theories. The level of business activity — the state of the business cycle — is, however, one of the fundamental determinants of what Marris calls the 'super-environment' of the firm.[134] In the opinion of this writer, the business cycle can exert a significant impact on the behavior of the individual firm, and the nature of this impact is examined in chapters three and five of this thesis.

D. MODELS OF THE FIRM, MODELS OF INDUSTRIAL STRUCTURE, MICRO-MACRO MODELS, AND GENERAL EQUILIBRIUM MODELS

Models of the firm can be developed for any one of a number of purposes. One use, which is examined in chapter two of this paper, is the study of the individual economic unit — the firm — in isolation. Such models of the individual firm have been used to study the behavior of the firm under alternative objectives and to learn how that behavior changes in response to external stimuli such as an increase in the corporate tax rate. In addition, models of the firm can be incorporated in models of industrial structure in order to study how the interaction of firms affects prices, output levels, etc.; in micro-macro models in order to study the interaction between decisions at the microeconomic level (e.g. pricing decisions) and observed macroeconomic phenomena (e.g. the rate of inflation); and in general equilibrium models in order to study how the interaction of all economic agents (firms, consumers, etc.) determine prices, output levels, etc., throughout the economy.

The purpose of this section is to describe briefly models of industrial structure, micro-macro models, and general equilibrium models. It is this writer's view that important developments in these three classes of models will follow closely on the development of more meaningful models of the individual firm. Particularly useful in this regard would be models of the firm that gave some role to the firm's internal decision-making processes and that could be used to study the firm's reactions to such external stimuli as a key rival's change in behavior.

1. The Need for a Theory of the Firm

To quote Robin Marris: "The 'firm' is the unit of delegated authority in a decentralized productive system."[135] The individual firm plays a crucial role in deciding what goods will be produced, by what methods and in what quantities they will be produced, and in what geographical markets and at what prices they will be sold.

The theory of the firm is directly concerned with this unit of delegated authority. It is concerned only indirectly with how the interaction of firms in an industry may give rise to a particular industrial structure; or with how the growth of the individual firms that comprise the economy affects the growth rate of the economy; or with how the interaction of producers, consumers, workers, and the government determines the allocation of the nation's resources and the distribution of national income. The first is the concern of models of industrial structure; the second is the concern of micro-macro models; and the third is the concern of general equilibrium models.

Traditionally, economists have been more interested in explaining how the economic system functions than in describing how the individual firm behaves. In the traditional theory, the model of the firm is designed to explain and predict changes in prices in the market place rather than the behavior of real firms.[136] Along with the increasing awareness that firms are no longer so small, so numerous, and so competitive — as the traditional theory suggests — that individually their influence on the economy is hardly significant, have come increasing efforts directed toward developing a theory that can explain the process of corporate growth and diversification described in section A. In short, it has become increasingly evident to economists that:

> the analysis of a decentralized economic system will depend on a specific theory of the behavior of the [productive] units to which decisions are delegated, in other words on some kind of theory of the firm.[137]

This heightened interest in the firm is due not only to the interesting theoretical questions that have been raised, but also to the important policy implications that answers to these questions are likely to provide. Such questions include the following:

- What effect does a motive other than profit maximization have on the efficiency with which resources are allocated within the firm, both at a point in time and over time?[138]

- Is a conglomerate better able (and if so, is it also more likely) to achieve a more efficient allocation of resources than a population of independent single-product firms?[139]
- To what extent is the large diversified corporation better able than smaller, less-diversified firms to cope with uncertainty?[140]

Such questions are concerned mainly with the efficiency of markets internal to the firm relative to markets external to it, as for example, differences in efficiency that may be implied by the differences in transactions costs — either explicit or implicit — in these two sets of markets. Answers to the above questions would, to the extent that they made clearer the costs and benefits associated with breaking up large firms, be likely to have important implications for the future course of antitrust policy.[141]

In addition to policy questions that could be answered directly, a more meaningful theory of the firm would, by contributing to models of industrial structure, micro-macro models, and general equilibrium models, also help provide answers to policy-related questions such as the following:

- What is the link between the individual firm's ability to set prices and the overall rate of inflation?[142]
- What is the link between the growth of the individual firm and the growth of the economy?[143]

The remainder of this section contains a survey of these other three classes of models.

2. Models of Industrial Structure

Models of industrial structure fall into three classes: (i) analytical models using discrete size, (ii) analytical models using continuous size, and (iii) simulation models.[144] Models of industrial structure are chiefly concerned with the size distribution of firms within an industry and with explaining and predicting changes in this distribution.[145] In particular, economists have used these models to help explain the process of increasing industrial concentration that has been observed throughout the postwar period.[146] Such models are of interest because of the relationship between industrial structure and the conduct and performance of firms belonging to that industry.[147]

Models of industrial structure are directly concerned with whole industries. They are not directly concerned with the behavior of the individual firms that comprise the industry, and as a result, the individual firm has no significant role to play. All firms in an industry are considered identical, except for size. These models generally are also stochastic. Any differences that exist among the firms in an industry, say, differences in the quality of management, differences in policy choices, etc., and the effects of these differences on the growth of the firm, are captured within the random character of the model. As discussed below, this randomness assumption requires, at the very least, that all firms within the same size class face identical growth opportunities. This simplistic treatment of the individual firm is one of the major weaknesses of these models.[148]

a. Models Using Discrete Size[149]

In models using discrete size the growth pattern of firms in an industry is modeled as a finite state Markov chain. The population of firms is divided into M size classes. For convenience, the classes are normally arranged in ascending order, so that class 1 contains the smallest firms and class M contains the largest firms. In addition, there is a class 0, the purpose of which is to allow for births (entry of new firms into the industry) and deaths (exit of firms from the industry, say due to bankruptcy or takeover).

Let $S^{(t)}$ denote the size class to which an arbitrarily selected firm belongs in period t. By the definition of a Markov process, the probability of a change in size — i.e. of a movement from one size class to any other size class — in the future is conditional on the present size of the firm only. Additional information concerning the earlier pattern of the firm's growth can not alter this probability. In symbols,

$$P\left(S^{(t+1)} = j_{t+1} \middle| S^{(t)} = j_t, S^{(t-1)} = j_{t-1}, \ldots, S^{(0)} = j_0\right) = P\left(S^{(t+1)} = j_{t+1} \middle| S^{(t)} = j_t\right) \tag{7}$$

where $j_{t+1}, \ldots, j_0$ are integers between 0 and M inclusive. Equation (7) states that the conditional probability that the firm belongs to class j_{t+1} in period $t+1$, given that it belonged to class j_t in period t, to class j_{t-1} in period $t-1$, ..., and to class j_0 in period 0, is just the conditional probability that the firm belongs to class j_{t+1} in period $t+1$, given that it belonged to class j_t in period t. If the conditional probabilities (7) are independent of the time variable t, then the process is said to have stationary transition probabilities, and these are defined by

$$p_{ij} = P\left(S^{(t+1)} = j \middle| S^{(t)} = i\right) , \quad \text{for all } t .$$ [150]

The transition probabilities are arranged in a transition probability matrix, denoted by P, in Figure I-2. For classes 1 to M the diagonal probabilities, $p_{11}, p_{22}, \cdots, p_{MM}$, give the probability that the firm will remain in its present size class.

$$\begin{array}{c c} & \begin{array}{ccccccc} 0 & 1 & 2 & \cdot & \cdot & \cdot & M \end{array} \\ \begin{array}{c} 0 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ M \end{array} & \left[\begin{array}{ccccccc} p_{00} & p_{01} & p_{02} & \cdot & \cdot & \cdot & p_{0M} \\ p_{10} & p_{11} & p_{12} & \cdot & \cdot & \cdot & p_{1M} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ p_{M0} & p_{M1} & p_{M2} & \cdot & \cdot & \cdot & p_{MM} \end{array}\right] \end{array} = P$$

Figure I-2 Transition Probability Matrix

The probabilities above the diagonal, p_{ij}, $0 < i < j$, give the probability that the firm will grow, i.e. move to a higher size class, while the probabilities below the diagonal, p_{ij}, $i > j > 0$, give the probability that the firm will decrease in size, i.e. fall to a lower size class. The zeroth class, which can be interpreted as a pool containing both potential members of the industry and firms that have failed during previous periods, is not a size class. The transition probabilities p_{0j}, $j > 0$, represent the probability that a new firm will enter the industry, starting out in class j. The transition probabilities p_{j0}, $j > 0$, give the probability that a firm in class j will leave the industry.[151]

Since there is nothing, in theory at least, that prevents a firm in one size class from reaching any of the other $M-1$ size classes or from failing (i.e. transitioning to the zeroth class), all states of the Markov chain communicate and the chain is said to be irreducible.[152] Since the chain is finite and irreducible, all states must be persistent and non-null. If, in addition, it is assumed that the states of the Markov chain are aperiodic, and there is certainly nothing in the nature of the growth pattern of firms that would lead one to think otherwise, then, by a well-known result, the size distribution of firms approaches asymptotically the stationary probability distribution

$$\pi = (\pi_0, \pi_1, \ldots, \pi_M) \tag{8}$$

satisfying the equations

$$\pi = \pi P$$

$$\sum_{i=0}^{M} \pi_i = 1 \quad ,$$

where P is the transition probability matrix. Moreover, the stationary distribution (8) is independent of the initial size distribution of firms.[153] The economic interpretation of π is the following: the growth process of firms in the industry eventually reaches a steady state in which π_i is the proportion of firms in class i. In steady state firms may still increase or decrease in size, but entries into and exits from each state cancel each other out.

Discrete models are of two types, depending on whether the law of proportionate effect is assumed to hold. The law of proportionate effect, which was introduced to economics by Gibrat, states that the distribution of growth rates facing each firm is independent of the firm's size.[154] According to this law, a firm having sales of \$1 million has the same probability of growing by g percent as a firm that has sales of \$1 billion. In terms of the transition probability matrix in Figure I-2, when the size classes $1, \ldots, M$ are defined so that firm size increases in a geometric progression as firms move from lower classes to higher classes, the law of proportionate effect requires that

$$p_{11} = p_{22} = p_{33} = \cdots = p_{MM} ; \quad p_{12} = p_{23} = p_{34} = \cdots = p_{M-1,M} ;$$
$$p_{21} = p_{32} = p_{43} = \cdots = p_{M,M-1} ; \text{ and so on.}$$

Eatwell randomly selected 290 U.S. manufacturing corporations that existed continuously from 1961 to 1967 from among the 2837 firms quoted in Moody's Manual of Industrials. He found that firms in all size groups have the same mean proportionate growth rate, which is in agreement with the findings of earlier studies.[155] However, he also found that the variance of the growth rate was not the same for all size classes, which is also consistent with earlier findings.[156] Most studies indicate that the variance decreases with size; that is, that, as the firm increases in size, its growth pattern becomes more stable. Also linear correlation and rank correlation studies indicate that corporate growth rates are serially correlated — growth tends to persist between periods — which appears to contradict the law of proportionate effect.[157] However, Ijiri and Simon have proposed a model in which the law of proportionate effect holds and in which some serial correlation is permitted.[158] When the model was tested in a simulation study, it was found that it produced size distributions that closely fitted real world grouped size data. Nevertheless, the empirical results concerning the variances of the growth rates for different size classes have brought into serious question the validity of the law of proportionate effect.

A model that assumes the law of proportionate effect is valid has been proposed by Simon and Bonini.[159] The model, which also assumes that new firms enter the industry through the lowest size class at a constant rate, is a continuation of earlier work by Simon, in which it was shown that under the two assumptions just stated, the growth process approaches a steady state.[160] In steady state the size distribution of firms can be approximated by the Yule distribution, and if the firms are large, it can also be approximated by the Pareto distribution.[161] Simon and Bonini tested their model against data on the 500 largest industrial firms in the United States in 1955. Their results support the validity of the law of proportionate effect, although they were hesitant to draw any conclusions concerning the goodness of fit of the Pareto distribution.[162] Engwall used the same data and concluded that the Pareto distribution fit the data only if firms with sales in excess of $398 million were included in the sample.[163] Steindl has reported similar findings: the Pareto distribution is valid only for the largest firms.[164] Quandt tested the goodness of fit of the Pareto distribution to data on the 500 largest industrial firms in the United States in 1955 and 1960.[165] He found that the

Pareto distribution fit the aggregated data reasonably well, but that when the 500 largest firms were divided by SIC code into thirty industries, the Pareto distribution fit the size distributions of firms within industries very poorly. A possible explanation for this has been offered by Steindl, who argues that, if the number of firms with sales above the cut-off ($398 million) is too small, then the fit of the Pareto distribution will be unacceptable.[166]

One of the reasons the above studies have produced mixed results is the fact that the steady state distribution holds only in the limit. The observed distribution at any point in time would, at best, only approximate the Pareto distribution. For industries that were expanding rapidly, say as the result of technical progress that led to new uses for or new variations of the industry's basic products, the approximation might be rather poor, even if only the largest firms were included in the sample. For this reason, it is the author's opinion that studies should consider how the industry's structure evolves over time. The model discussed next is designed to accomplish this.

A model more general than the one just discussed has been developed by Adelman. Her model does not require that the law of proportionate effect hold.[167] Adelman estimated the transition probabilities for the United States steel industry for the years 1929-39 and 1945-56. Only steel producers having assets exceeding one million dollars were considered, and these firms were divided into six size classes, with a zeroth class added to allow for births and deaths. Adelman's estimated transition probability matrix is given in Figure I-3.[168]

$$\begin{array}{c|ccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
0 & 0.99942 & 0.00040 & 0.00016 & 0.00001 & 0.00001 & 0 & 0 \\
1 & 0.021 & 0.911 & 0.068 & 0 & 0 & 0 & 0 \\
2 & 0.024 & 0.039 & 0.908 & 0.028 & 0.001 & 0 & 0 \\
3 & 0 & 0 & 0.076 & 0.872 & 0.052 & 0 & 0 \\
4 & 0.008 & 0 & 0 & 0.016 & 0.947 & 0.028 & 0 \\
5 & 0 & 0 & 0 & 0 & 0.037 & 0.926 & 0.037 \\
6 & 0 & 0 & 0 & 0 & 0 & 0.024 & 0.976
\end{array}$$

Figure I-3 Transition Probability Matrix for U.S. Steel Industry, 1929-39 and 1945-56

From Figure I-3 it can be seen that

(i) The probability of remaining in the same class is many times greater than the probability of leaving the class, and for all classes but one, this probability of remaining exceeds .9 .

(ii) The probability of jumping more than one class is either zero or very near zero.

(iii) The probability of growth is slightly greater than the probability of decline.

Similar results have been reported by Engwall and by Archer and McGuire.[169]

From the transition probability matrix in Figure I-3 Adelman obtained the following stationary probability distribution for firms in the U.S. steel industry:[170]

$$\pi = (0.948,\ 0.00938,\ 0.01169,\ 0.00376,\ 0.00903,\ 0.00708,\ 0.01091) \quad .$$

This distribution includes the zeroth class. It is more meaningful to consider the relative distribution of firms in classes 1 through 6 — those firms that are active in the industry. This can be accomplished by normalizing the results so that $\sum_{i=1}^{6} \pi_i = 1$. The equilibrium size distribution of these firms is shown in Table I-1. Adelman showed that this equilibrium distribution is independent of the number of firms in the zeroth class.[171]

Table I-1 Equilibrium Size Distribution of Firms in the U.S. Steel Industry

	1929		1956		Equilibrium	
size class	% firms	% assets	% firms	% assets	% firms	% assets
1	25.00	1.15	27.68	0.98	18.09	0.11
2	43.47	8.64	39.29	6.79	22.55	0.68
3	16.30	9.49	8.93	4.36	7.25	0.61
4	11.96	24.70	16.96	21.90	17.42	3.92
5	1.09	5.42	5.36	26.86	13.65	12.54
6	2.18	50.60	1.79	39.11	21.04	82.14
	100.00	100.00	100.00	100.00	100.00	100.00

Source: Adelman, op. cit., p. 901.

Table I-1 indicates that considerable growth in the median size firm is to be expected, on the basis of this model at least. In both 1929 and 1956 the median size firm belonged to the second size class, but in equilibrium the median size firm will belong to the fourth class.

The data in Table I-1 also indicate that a lessening in the degree of concentration in the U.S. steel industry is to be expected. In 1929 roughly 15 percent of the steel firms controlled nearly 81 percent of the industry's assets. By interpolation, in 1956 the top 15 percent controlled roughly 75 percent of the assets, while in equilibrium the top 15 percent of the steel firms would control only about 60 percent of the industry's assets.

Adelman also used the steel industry data to construct an index of industrial mobility based on the index of social mobility devised by Prais.[172] The index measures industrial mobility as the ratio of the average number of years spent in a size class in a perfectly mobile industry — one in which p_{ij} is independent of i — to the average number of years a firm in any size class in the industry and year in question could expect to remain in that size class.[173] Her conclusion was that the mobility of firms in the steel industry would decline as the industry moves toward the steady state.[174]

Adelman's model represents an improvement over the discrete models discussed earlier in this subsection. Unlike those models, Adelman's model considers the evolution of industrial structure over time, rather than just the equilibrium size distribution toward which current size distributions appear to be converging. Her model is general enough to be applied to other industries to determine trends in industrial structure or to be reapplied to more recent data for the U.S. steel industry to test whether the trend in its structure has altered. However, like the models considered earlier, the role of the individual firm has been subsumed in the random character of the model.

b. Models Using Continuous Size[175]

The discussion of Adelman's model in the previous subsection pointed out the major applications of the discrete models, namely, first, to study trends in industrial concentration and in industrial mobility as firms in an industry grow, and second, to identify the stationary probability distribution toward which the size distribution at any point in time is moving. Continuous models, which are discussed in this subsection, are applied similarly to the study of real world industries.

Gibrat formulated a continuous model of industrial structure based on the following two assumptions: (i) the law of proportionate effect is valid and (ii) the number of firms is constant.[176] Under these assumptions the size distribution of firms is asymptotically lognormal.[177] To see why this should be so, note first that the change in a firm's size from one period to the next can be written as

$$S_t - S_{t-1} = g_t \cdot S_{t-1} ,$$

where S_t is the size of the firm in period t and g_t is the percentage increase in the size of the firm between periods $t-1$ and t. Then

$$S_t = (1 + g_t)\, S_{t-1} ,$$

which leads recursively to the expression

$$S_t = S_0 (1+g_1)(1+g_2) \cdots (1+g_t) \quad . \tag{9}$$

If the percentage increase g_i is small, then $g_i \cong \log (1+g_i)$, so that taking the logarithm of both sides of (9) leads to the approximation

$$\log S_t = \log S_0 + g_1 + g_2 + \ldots + g_t \quad .$$

From the (strong version of) the law of proportionate effect, the random variables g_i are independent and identically distributed. Then, by the central limit theorem, $\log S_t$ is asymptotically normally distributed with mean $t \cdot \bar{g}$ and variance $t \cdot \sigma^2$, where $\bar{g}$ is the mean and σ^2 is the variance of the distribution of g . Thus, the size distribution of firms will be approximately lognormal for large t .

The asymptotic lognormality of the size distribution of firms is dependent on assumptions (i) and (ii). Reasons for doubting the validity of the law of proportionate effect were cited in the previous subsection. As regards the second assumption, a large turnover of firms tends to disrupt the limiting process, particularly when large numbers of small firms enter the industry and large numbers of large firms leave the industry.[178] The less valid are the two assumptions for a particular industry, the further that industry's size distribution of firms is likely to deviate from lognormality.[179] In spite of these objections to the model, Hart and Prais, Simon and Bonini, and Engwall present empirical evidence that size distributions of firms in various industries and in various countries are approximately lognormal.[180]

Engwall's results are particularly noteworthy because part of his study is devoted to socialist countries. The evidence of lognormal size distributions of firms in those countries implies that the observed stochastic growth process is not exclusively a capitalist phenomenon.

One consequence of the stochastic growth process described by Gibrat is that concentration will tend to increase over time. Large firms have the same probability as small firms of growing at any particular rate, and as a result, the logarithmic variance, $t \cdot \sigma^2$, increases without bound as t increases. If the stochastic process is a reasonable description of the actual growth process of firms, and the empirical evidence just cited would suggest that it is, then it should be expected that industrial concentration would increase. This too is consistent with the findings of several studies.[181]

A model that does not require the logarithmic variance to increase steadily with time has been devised by Hart and Prais.[182] In their model the logarithmic variances in two successive periods are related according to the equation[183]

$$\sigma^2_{t+1} = \beta^2 \cdot \sigma^2_t + \sigma^2_\varepsilon , \qquad (10)$$

where

σ^2_t = var[log (S_t)]

β = regression coefficient

σ^2_ε = residual variance.

If $\hat{\beta}$, the estimator for β, is less than one and σ^2_ε is small, then the variance will tend to decrease, i.e. the size distribution of firms will regress toward the mean. If $\hat{\beta} = 1$, growth is purely stochastic and the validity of the law of proportionate effect cannot be rejected. If $\hat{\beta} > 1$, then large firms grow proportionately faster and concentration will increase faster than predicted by Gibrat's model.

In proposing their model, Hart and Prais suggest that the size distribution of firms tends to regress toward the mean as firms attempt to achieve some optimum size (determined, say, by financial factors). This implies that $\beta < 1$. Their model has been criticized on two grounds. Their assumption that there exists an optimum size requires that firms' long run average total cost curves be U-shaped, which makes their model incompatible with Bain's findings that firms' average cost curves are L-shaped.[184] Second, Eatwell summarizes evidence provided by a number of empirical studies that implies that there has been a recent tendency for $\hat{\beta} > 1$.[185] This also suggests that the law of proportionate effect is invalid, although as Eatwell points out, this is not necessarily true for all industries. For the five recent (running up to 1960) studies cited by Eatwell, in one case $\hat{\beta}$ was significantly less than one and in three cases it was significantly greater than one.[186]

Thus, on the basis of the evidence accumulated thus far, there does not appear to be much support for Hart's and Prais's model. There does appear to be support for Gibrat's model, although there are several studies, including Eatwell's, that indicate that the law of proportionate effect does not hold universally. Possibly a somewhat more general model, one that allows for some serial correlation in the growth rates of firms, is needed. Models that permit this, and in particular the model of Ijiri and Simon, are the subject of the next subsection.

c. Simulation Models[187]

The two previous subsections described analytical models of industrial structure. In those models all firms were treated identically and the stochastic processes were assumed to be stationary. The growth process of each firm in an industry, regardless of the firm's size, the quality of its management, its previous pattern of growth, or any other distinguishing features, was explained by the same stochastic process. Moreover, the nature of the process, as embodied in the transition probability matrix or the distribution of the growth rate, remained time invariant. Such models are of considerable value in showing that industrial concentration can increase steadily through the operation of a chance mechanism — without concerted efforts by producers either to take over their rivals or to drive them from the industry. The empirical validation of such models also lends support to Bain's findings that there is no optimum size of the firm. But the models' implication that growth proceeds by historical accident independent of the financial and productive characteristics of the individual firm and independent of the influence of the firm's managers is difficult for many economists to accept. Difficulties in obtaining meaningful results from analytical models are often encountered, however, when the models are designed so as to treat the individual firm in what the modeler regards as a 'realistic' fashion. For this reason several economists have turned to simulation.

Simulation makes it possible to relax the restrictive assumptions required in analytical modeling. For example, it is no longer necessary to assume that the law of proportionate effect is valid. In a simulation model the growth process of firms is determined by the set of rules written into the computer program that governs the simulation. The mechanism that determines the size of the firm in any period can be designed to take into account (i) the firm's growth history, e.g. by permitting firms that experience above-average growth in one period to enjoy above-average growth probabilities the following period;[188] (ii) special characteristics of the firm, such as its propensity to conduct research and development or the expertise of its management; (iii) variations in the rate at which firms enter the industry and in the rate at which firms leave the industry (the latter as a result, say, of a decreasing risk of failure as the firm becomes older), both of which cause the number of firms to vary over time; and (iv) a growth mechanism that is not constant but that varies, say, over the business cycle. All that is required is a set of rules to specify the operation of the growth mechanism. If one hopes to model a specific industry, however, this requires that one have in mind a valid model of the individual firm in order that the set of rules be realistic.

To demonstrate the use of a simulation model, the results of ten runs of a continuous size simulation model, which was developed by the author, have been provided in Table I-2. Output is in the form of four-firm concentration ratios computed at 20-year intervals for 100 years. Initially there were 100 firms of equal size. Growth rates were assumed to be normally distributed, with mean growth of five percent per annum and a standard deviation of ten percent, for each firm in each year. To reflect the impact of the business cycle, it was assumed that births occurred according to the following five-year cycle: three, one, none, one, two (per year). It was assumed that any firm whose size had fallen below zero had gone bankrupt. In order to keep the model simple, takeovers were not permitted and all firms were assumed to be indistinguishable except for differences in size.

Table I-2 Four-Firm Concentration Ratios Resulting from 10 Runs of a Simulation Model Using Continuous Size

	Four-Firm Concentration Ratio at Year:					
Run	0	20	40	60	80	100
1	4.0	9.7	10.1	13.0	16.3	21.1
2	4.0	7.6	9.7	11.9	13.0	16.0
3	4.0	7.3	10.3	11.9	13.8	15.0
4	4.0	7.5	9.2	8.9	9.6	12.0
5	4.0	7.1	10.1	11.1	11.3	9.8
6	4.0	6.5	7.5	8.9	11.1	12.1
7	4.0	7.7	8.9	12.7	13.8	15.8
8	4.0	6.8	8.2	10.9	12.3	16.0
9	4.0	7.7	10.7	13.5	17.0	16.1
10	4.0	8.5	10.2	12.4	14.9	14.6
Avg	4.0	7.6	9.5	11.5	13.3	14.9

Even though the growth prospects of all firms are identical, the firms do not remain of equal size for very long. Within 20 years concentration has nearly doubled, with the industry leader (not shown) having nearly tripled his market share. The simulation results display a clear tendency toward increasing concentration, a tendency that is even more marked if no new firms are allowed to enter the industry. A second set of simulation runs, this time with no new entries permitted, yielded an average concentration ratio of 19.9 at the century mark, and in one of the runs concentration reached 29.2 after 100 years. If some sort of takeover mechanism had been built in, concentration would have increased even more rapidly. In addition, any of the factors (i) - (iv) listed above could be added to the model to determine what effect they would have, either individually or collectively, on industrial concentration.

d. Models of Market Organization

The stochastic analysis of changes in industrial structure has led to some interesting results. Yet, in assessing the true economic contributions of these models, one must recognize that, by their very nature, these models can say very little about the individual firm. For example, one cannot see what connections exist among the individual firm's policy choices, the firm's interactions with its rivals, and the performance of the markets it serves. The simulation technique offers some hope that the models can be made to reflect better the role of the individual firm. In the opinion of this writer, it would be more valuable in terms of policy implications to determine how decisions by the individual firm and how the interaction of the firm with its rivals affect the level of industrial concentration than it would be to merely identify the probability distribution — Yule, Pareto, lognormal, or whatever — that comes closest to fitting actual size distributions of firms. For while models based on the law of proportionate effect have been found to be appropriate for describing actual size distributions of firms, they say nothing concerning the internal dynamics of corporate growth.

Recently a number of models of market organization that attempt to explain the connection between market equilibrium price and quantity and the objectives and price-setting behavior of firms in the industry have been formulated.[189] Winter has devised a markup pricing model in which each firm each period sets the value of two parameters: its productive capacity and the price of its product — the latter as a percentage markup over average variable cost.[190] Firms do not optimize, they satisfice. If profits prove unsatisfactory, the firm alters its markup or its size. Winter's model allows firms to enter the industry, and this is accomplished by new entrants charging a lower price than existing producers. Winter permits prices, capacity, and demand to assume only a finite number of values, and shows that under the above assumptions, the development of the industry over time can be modeled as a finite state Markov chain. In the persistent states are firms that employ the same percentage markup that is sufficient to discourage entry; that earn satisfactory profits at a level commensurate with profit rates in other industries; and that experience the same degree of capacity utilization. What is distinctive about Winter's model, as the survey in chapter two will make clear, is that producers do not optimize anything, and yet, their market behavior converges to the competitive equilibrium with probability one.

A more recent model is due to Maccini. His model, while it is not stochastic in nature, is concerned with dynamic adjustments in market price and output.[191] Two aspects of Maccini's approach are noteworthy. First he develops a model of the representative firm, in which the firm sets the price of its product and alters its market share by varying its price in relation to the ruling market average price. Then he collects these firms into an aggregate product market model. However, he does assume that the firm behaves as if any price changes it makes will not affect the market average price (i.e. he assumes that monopolistic competition prevails) and that the firm maximizes the present value of its expected profit stream (i.e. he assumes that the firm's shareholders and managers are risk neutral).

The models of Winter and Maccini demonstrate one of the uses to which a model of the firm can be put. Two other uses will be discussed in the next subsection. What models such as the two just discussed also make clear is the critical role models of the firm can play in models of industrial structure and in models of market organization. Similarly, results obtainable from models of the types discussed in the next subsection are also affected by the treatment given the individual firm.

3. Micro-Macro Models and General Equilibrium Models

Microeconomics deals with economics in the small — the behavior of individual economic decision-making units and individual markets — while macroeconomics deals with economics in the large — the behavior of broad economic aggregates, such as total investment, and their role in determining the general levels of prices and employment. There has always been a rather uneasy coexistence of the two, for while it is clear that the decisions made by individual economic units determine, through the system of markets that tie the economy together, the values of the aggregates, no one has been successful in explaining precisely how this is accomplished. Moreover, it is not assured that existing macroeconomic models and microeconomic models are logically consistent. In view of first the power that many firms have to set prices and then to maintain them in the face of falling demand, and second, the recent period of high, persistent inflation in the face of high unemployment, the need for a synthesis of microeconomics and macroeconomics appears to many economists to be a pressing one.[192] Such a synthesis would also be likely to reveal the relationship between the growth-promoting activities of individual firms and the overall growth of the economy. Such knowledge would be of particular value to economies whose industrial sectors are only just beginning to develop.[193]

Such a synthesis could be brought about presumably through the development of general equilibrium models, which determine the simultaneous equilibrium of all markets and which also yield the values of economic aggregates. A somewhat less ambitious approach would be one that sought to link the relevant microeconomic decisions with a particular economic aggregate, i.e. a micro-macro model. In models of both types the individual firm would play a critical role.

Marris has developed a micro-macro model that links the growth of firms with the growth of the economy, his underlying premise being that "aggregate behavior must be seen as the specific result of the behavior of the firms."[194] In his model the growth rate of the economy is the weighted average of the growth rates of individual firms. Each firm selects a steady state growth path, as in Marris's microeconomic model of the firm,[195] but actual growth rates may vary due to the disturbances in the firm's environment. Also as in the microeconomic model, each firm's optimum growth rate depends on its expected profit rate, which at the macroeconomic level makes the average (planned) growth rate of the economy dependent on the macroeconomic average profit rate. In equilibrium, growth rates, profit rates, and valuation ratios are determined at both the microeconomic and the macroeconomic levels.

General equilibrium models are more complex than micro-macro models, encompassing simultaneously all micro-macro relationships, rather than just some small subset (as in a micro-macro model). Consequently, their analysis generally requires the use of more sophisticated mathematical techniques.[196] It is at least partly for this reason that general equilibrium models have been slow to adapt to recent developments in the theory of the firm. The majority of general equilibrium models have assumed a manufacturing sector throughout which perfect competition prevails.[197] Two notable exceptions are the models of Negishi and Arrow, both of which assume monopolistic competition.[198] Both models assume that the firm maximizes profits. Neither model takes into account either the interaction among the monopolistic firms or the influence of factors internal to these firms. As in the case of models of industrial structure, the behavior of the individual firm has remained an unexplained phenomenon.

E. SUMMARY OF CHAPTER ONE AND OVERVIEW OF THE THESIS

The modern business enterprise is large and diversified, and in many cases, management-controlled. In the opinion of this writer, there is little resemblance between the modern corporate enterprise and the business firm depicted by traditional economic theory. Various models have been suggested as replacements for the traditional model, and the majority of these have been most conveniently expressed in mathematical form. The models that have been proposed are of both direct and indirect interest; in the first case, because of recent important policy debates, such as one concerning the breaking up of the major oil companies and another concerning the importance of large size to risk-bearing in defense contracting;[199] and in the second case, because of the need for more meaningful models of the firm to be incorporated in models of industrial structure, micro-macro models, and general equilibrium models.

The remainder of this thesis is concerned exclusively with the theory of the firm. Chapter two surveys the literature dealing with the theory of the firm and presents a representative collection of models of the firm. Each model is carefully set out and analyzed, and the contributions of each to the theory of the firm are evaluated. The models are presented in more or less chronological order, beginning with the traditional models in sections B through E, and then following the evolution of the theory of the firm through the more sophisticated treatments given to the objectives of the firm, to the links between the firm's financial decisions and its operating decisions, to the behavior of the firm under uncertainty, and to the behavior of the firm in a multiperiod context. Also discussed is the relationship between each of these models and similar models that have appeared in the literature, but which are not presented in detail in chapter two.

It is this writer's intention in providing a comprehensive survey of the literature to indicate, not only the evolution of the theory of the firm to date, but also the directions in which further research might fruitfully proceed.

The author's basic theoretical model is developed in chapters three through five. The model is initially formulated in chapter three, where it is used to study the behavior of the firm over the business cycle. The firm is modeled as a discounted collective utility maximizer - with collective utility reflecting both shareholder and managerial sources of satisfaction. It is shown that the traditional and managerial theories of the firm may be reconciled by interpreting these classes of models in terms of the behavior of the firm over the business cycle. Financial considerations are incorporated into the model in chapter four. The modified model, which is formulated as a stochastic optimal control problem utilizing the time-state-preference approach to modeling uncertainty, is used to examine the relationship between the firm's optimal operating decisions and its optimal financial decisions. Organizational factors are introduced in chapter five, and some of the consequences of decentralized decision-making for the loss of control and X-efficiency are suggested.

The extension of the basic model to firms in the U.S. airframe industry is carried out in chapters six and seven. The institutional milieu within which these firms operate is characterized and their planning processes are described in chapter six. A model of the representative airframe builder is formulated and is used to study several procurement policy issues in chapter seven.

CHAPTER ONE FOOTNOTES

1. J. K. Galbraith, The New Industrial State, 1st ed. (Houghton Mifflin; Boston; 1967). This view is shared by the proponents of the managerial theories of the firm. The managerial viewpoint is discussed below and several of the managerial models are set out in section G of chapter two.

2. "The Fortune Directory of the 500 Largest Industrial Corporations," Fortune (May 1976); "The Fortune Directory of the Second 500 Largest Industrial Corporations," Fortune (June 1976); and Federal Trade Commission, Quarterly Financial Report for Manufacturing Corporations Washington, D.C.; 4th Quarter, 1975).

3. The importance from a modeling standpoint of determining what the typical company is trying to achieve, as well as the difficulties associated with trying to determine the objectives of actual firms, are discussed in W. J. Baumol, "Models of Economic Competition," in P. Langhoff, ed., Models, Measurement and Marketing (Prentice-Hall; Englewood Cliffs, N. J.; 1965), pp. 143-168.

4. R. J. Larner, Management Control and the Large Corporation (Dunellen; New York; 1970). The classification scheme adopted by Larner is somewhat arbitrary. Berle and Means had required stock ownership of 20 percent or more for ownership control, but Larner argued that 20 percent seemed too high a lower limit in view of the increases in size and the widening dispersion of share ownership characterizing the 200 largest nonfinancial corporations between 1929 (the period studied by Berle and Means) and 1963 (the period studied by Larner). Ibid., pp. 10-11. This still leaves unanswered the question as to why 10 percent is better than 15 percent or some other figure. Indeed, Larner made two exceptions to his classification scheme, classifying May Department Stores Co. as ownership-controlled, even though the May family held a stock interest of less than four percent, because the family had five representatives on the board of directors, and classifying Transcontinental Gas Pipe Line Corp. as management-controlled even though Stone & Webster Company held 11 percent of the outstanding shares, because the latter had no representatives on Transcontinental's board of directors. Ibid., pp. 78-79, 84-85.

5. Ibid., p. 16.

6. Ibid., p. 16. The results of the Berle and Means study are reported in A. A. Berle, Jr., and G. C. Means, The Modern Corporation and Private Property, rev. ed. (Harcourt, Brace and World; New York; 1968), p. 66.

7. Larner's findings have not gone unchallenged, however, and there has been a continuing debate in the economic and business literature as to the extent of the separation of ownership from control. Robert Sheehan argues that family ownership and control is still significant among the 500 largest corporations. Sheehan reported that in roughly 150 of the 500 largest industrial corporations (as ranked by sales by Fortune magazine) controlling ownership rested in the hands of either a single individual or members of a single family. See R. Sheehan, "Proprietors in the World of Big Business," Fortune (June 1967), p. 178. Philip Burch challenges the separation of ownership from control hypothesis more strongly than Sheehan. Burch studied ownership and control in 450 large companies and concluded that 42% of the largest publicly held corporations are controlled by one person or by a single family and that another 17% are in the "possible family control" category. See P. H. Burch, Jr., The Managerial Revolution Reassessed (D. C. Heath; Lexington, Mass.; 1972). Peter Drucker argues that management control is diminishing due to the growing influence of pension funds that own large blocks of company shares. See P. F. Drucker, The Unseen Revolution (Harper & Row; New York; 1976). For a contrary view see M. R. Darby's review of Drucker's book. M. R. Darby, "Should pension funds be cause for concern?," Business Week (July 19, 1976). Evidence in support of the Berle and Means study and the Larner study is provided by John Palmer, whose study of the 500 largest industrial corporations indicates that 161 firms in 1965 and 143 firms in 1969 were owner-controlled. See J. P. Palmer, "The Separation of Ownership from Control in Large U.S. Industrial Corporations," Quarterly Review of Economics & Business (vol. 12; no. 3; Autumn 1972), pp. 55-62. He concludes that the separation of ownership from control is widespread, that there is a strong negative correlation between the size of the firm and the degree of owner control (i.e. the largest firms have the smallest percentage of owner control), and that his study is consistent with those of Berle and Means and Larner and shows that "the frequency of owner control of large corporations seems to have declined from 1929 through 1969." Ibid., p. 61. To summarize the foregoing, the results of the various studies, when taken collectively, remain inconclusive. This is due at least in part to the differing criteria adopted for distinguishing between owner control and manager control. Thus, the issue as to whether management-controlled firms are predominant among the largest nonfinancial firms in the American economy remains unsettled.

8. Introductory economics textbooks typically refer to the shareholders of a firm as the firm's owners, but strictly speaking this is untrue. A corporation is a separate legal entity. All corporate property is owned by the corporation. Individual shareholders have no specific claim against any of the corporation's assets. The shares he owns gives the individual shareholder voting rights, rather than ownership rights. His shares entitle him to a share of the profits, which he receives in the form of dividends, and in the event the corporation fails, to a claim against the residual assets of the corporation after all creditors have been paid. Thus, while the traditional view is that the stockholders are the owners of the corporation, this is only true in a strict sense for those corporations in which the shareholders are also the firm's managers.

9. This does not mean that management has complete freedom to ignore the interests of the company's shareholders or their elected representatives, the board of directors. Evidence of this is provided by the Gulf payoff scandal. Management had been given virtually complete freedom to run the company, and in fact, had been able to conceal the magnitude of the payments from its board for a period of years. Once the deception became apparent, top management was removed. See B. E. Calame, "Gulf Officers' Ouster Was Boldly Engineered by Mellon Interests," Wall Street Journal (January 15, 1976), and W. Robertson, "The Directors Woke Up Too Late at Gulf," Fortune (June 1976). A second example is the recent resignation of the president of United Brands Co. after a dispute with the company's major stockholders. See L. R. Gallese, "United Brands President Quits in Rift with Chairman; Milstein Is Successor," Wall Street Journal (January 31, 1977) and "United Brands shifts to a shirtsleeve boss," Business Week (February 14, 1977). There are indications that such difficulties have made corporate boards of directors increasingly assertive. See B. E. Calame and E. Morgenthaler, "Outside Directors Get More Careful, Tougher After Payoff Scandals," Wall Street Journal (March 24, 1976).

10. As discussed in sections F and I of chapter two, under the appropriate assumptions the stock market value of a share of equity is equal to the present value of the flow of dividends accruing to the holder of that share and maximizing shareholder utility is perfectly equivalent to setting dividend policy so as to maximize the stock market value of the firm's equity. The Vickers, Lintner, and Jorgenson models discussed in chapter two are examples of models that employ the stock market value of the firm's equity as a measure of shareholder utility (or satisfaction). It should be emphasized that the proponents of the managerial and behavioral theories, whose views and models of the firm are explored in depth in chapter two, do not dispute the view that shareholders are mainly concerned with the stock market value of their shares. What they dispute is the notion that the objective of the firm is that of its shareholders.

11. These sources of managerial satisfaction are cited by Baumol, Marris, O. E. Williamson, and other proponents of the managerial theories. For references see footnotes 28, 30, and 31 below.

12. See, for example, F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," American Economic Review (vol. 48; no. 3; June 1958), p. 262.

13. E. T. Penrose, The Theory of the Growth of the Firm (Basil Blackwell; Oxford; 1959), p. 28.

14. In the remainder of this chapter the neoclassical objective of the firm will be referred to simply as profit maximization and the qualification 'or market value of the firm's shares' will not be added to 'profit'. This is the convention followed in the literature. One of the main justifications for this convention is the fact that in the absence of both uncertainty and capital market imperfections, profit maximization is equivalent to stock market value maximization.

See Modigliani and Miller, op. cit., pp. 262-263. This equivalence will be demonstrated clearly in the discussion of the Jorgenson model in section L of chapter two. It should be noted that the equivalence between profit maximization and stock market value maximization vanishes under uncertainty and does not necessarily hold when capital markets are imperfect.

15. M. Friedman, Essays in Positive Economics (University of Chicago Press; Chicago; 1953), pp. 3-43; T. C. Koopmans, Three Essays on the State of Economic Science (McGraw-Hill; New York; 1957), pp. 132-146. A summary of the theoretical arguments both for and against the profit maximization hypothesis can be found in A. Singh, Take-overs: Their Relevance to the Stock Market and the Theory of the Firm (Cambridge University Press; Cambridge; 1971), pp. 2-13.

16. S. G. Winter, Jr., "Economic 'Natural Selection' and the Theory of the Firm," Yale Economic Essays (vol. 4; Spring 1964), pp. 225-272.

17. G. Donaldson, Corporate Debt Capacity (Division of Research, Graduate School of Business Administration, Harvard University; Boston; 1961). Over the last decade, however, the pattern of financing observed by Donaldson has changed. For example, in 1965, over 90 percent of capital outlays were financed internally, but by 1974, the proportion of internally financed capital outlays had fallen to approximately 65 percent. Moreover, much of this increase in external financing took the form of long term bank borrowing and the issuance of debt instruments, as evidenced by an increase of the ratio of total debt to equity from 65 percent in 1965 to 97 percent in 1974. J. F. Weston, review of A. Wood, A Theory of Profits, in Journal of Economic Literature (vol. 14; no. 4; December 1976), pp. 1280-1281, and "Recent Developments in Corporate Finance," Federal Reserve Bulletin (vol. 61; no. 8; August 1975), pp. 463-471.

18. A. A. Alchian and R. A. Kessel, "Competition, Monopoly, and the Pursuit of Pecuniary Gain," in Aspects of Labor Economics (Conference of the Universities — National Bureau Committee for Economic Research; Princeton; 1962), pp. 156-175, and H. Manne, "Mergers and the Market for Corporate Control," Journal of Political Economy (vol. 73; no. 2; April 1965), pp. 110-120, reprinted under the same title in M. Gilbert, ed., The Modern Business Enterprise (Penguin; Middlesex, England; 1972), pp. 153-169.

19. Singh, op. cit., p. 139, and A. Singh, "Take-overs, 'Natural Selection' and the Theory of the Firm," Economic Journal (vol. 85; no. 339; September 1975), pp. 497-515. Though Singh's studies were carried out on a restricted sample of industries in the United Kingdom, in the opinion of this writer the similar structure and organization of the manufacturing sectors of the British and American economies do permit inferences to be drawn from his results.

20. S. E. Boyle, "Pre-merger Growth and Profit Characteristics of Large Conglomerate Mergers in the United States 1948-1968," St. John's Law Review (vol. 44; special edition; Spring 1970), pp. 152-170, and Federal Trade Commission, Economic Report on Conglomerate Merger Performance, An Empirical Analysis of Nine Corporations (Washington, D.C.; November 1972).

21. Indeed, there is evidence that a firm's relatively poor performance may cause it to seek acquisition candidates in order to increase its profitability to the average for industry generally. H. I. Ansoff, et al., Acquisition Behavior of U.S. Manufacturing Firms, 1946-1965 (Vanderbilt University Press; Nashville; 1971), and J. F. Weston and S. K. Mansinghka, "Tests of the Efficiency Performance of Conglomerate Firms," Journal of Finance (vol. 26; no. 4; September 1971), pp. 919-946. Hence, in some cases, a firm's low profitability may increase its likelihood of acquiring some other firm, rather than its likelihood of being acquired.

22. R. L. Marris, "Galbraith, Solow and the Truth About Corporations," The Public Interest (Spring 1968), p. 44.

23. W. G. Lewellen, Executive Compensation in Large Corporations (Columbia University Press; New York; 1968), ch. 8.

24. W. G. Lewellen and B. Huntsman, "Managerial Pay and Corporate Performance," American Economic Review (vol. 60; no. 4; September 1970), pp. 710-720. In addition, stock options, which permit the holder to purchase a certain number of the company's shares at a specified price within some stated time period, are virtually worthless if the specified option price remains below the prevailing market price throughout the stated time period. Stock options become less attractive during recessions due to the generally downward trend in share prices. See R. Ricklefs, "Stock Options' Allure Fades, So Firms Seek Different Incentives," Wall Street Journal (May 27, 1975).

25. D. R. Roberts, Executive Compensation (Free Press; Glencoe, Ill.; 1959); J. W. McGuire, et al., "Executive Incomes, Sales, and Profits," American Economic Review (vol. 52; no. 4; September 1962), pp. 753-761. For a review of these and subsequent studies see G. K. Yarrow, "Executive compensation and the objectives of the firm," in K. Cowling, ed., Market Structure and Corporate Behavior: Theory and Empirical Analysis of the Firm (Gray-Mills; London; 1972), pp. 149-173, in which Yarrow also reports the results of his own study that show that executive compensation is more closely correlated with the size of the firm than with its profitability. For some more recent results see A. Cosh, "The Remuneration of Chief Executives in the United Kingdom," Economic Journal (vol. 85; no. 337; March 1975), pp. 75-94.

26. R. L. Marris, The Economic Theory of 'Managerial' Capitalism (Macmillan; London; 1964), pp. 46-109; J. Downie, The Competitive Process (Duckworth; London; 1958); and J. K. Galbraith, op. cit., pp. 171-173. Supportive empirical evidence comes from several sources. G. R. Roche recently reported the results of a study in which he found

that of the 1556 men and 10 women whose senior executive appointments were announced in the "Who's News" column of the Wall Street Journal during 1974, 87% were given new responsibilities within the same organization. See G. R. Roche, "Compensation and the mobile executive," Harvard Business Review (November-December 1975), p. 54. Such announcements are generally limited to the top executives (vice presidents and above) of the 500 largest industrial companies and of the 15 to 25 largest organizations in nonindustrial fields plus presidents and chairman of other listed or actively traded (but unlisted) companies. While the 87% figure can be quibbled over because Roche's sample did not include all large firms (some promotions may not have been announced and, due to the short period covered by the study, some of the relatively smaller firms may not have been in the sample because their presidents and chairmen remained the same throughout the period), in this writer's opinion the results do indicate a general preference for internal promotion. Roche finds that it is less expensive for an organization to promote internally than to search outside the firm for new talent. Ibid., p. 62. The long and careful (and costly) selection process involved in seeking higher level executives outside the firm is described in T. Levitt, "The managerial merry-go-round," Harvard Business Review (July-August 1974). Additional evidence of the preference for internal promotion is provided by Forbes magazine's 1975 roster of the 830 highest paid chief executives, which indicates that 707 of the 830 executives, or 85 percent, were promoted into the chief executive's position from within the company. See "Who Gets the Most Pay," Forbes (May 15, 1975).

27. An interesting summary of these alternative models is presented in J. B. Herendeen, The Economics of the Corporate Economy (Dunellen; New York; 1975), ch. 5. Some economists, notably Robert L. Heilbroner, have argued that profits *should not* be the single goal of modern corporations because of the price wars initiated by dominant firms, take-overs, and increases in market concentration that would be likely to result if large firms tried to maximize profits. See R. L. Heilbroner, et al., In the Name of Profit (Doubleday; Garden City, N.Y.; 1972). In contrast to the traditional theorists who argue that profit maximization is desirable, Heilbroner argues that profit maximizing behavior on the part of large firms may have ill side effects. The implication of his arguments is that, if it is found that companies do not maximize profits, it does not necessarily follow that society is worse off than it would be if profits were maximized.

28. W. J. Baumol, Business Behavior, Value and Growth, rev. ed. (Harcourt, Brace and World; New York; 1967), chs. 6-10. The first edition of the book appeared in 1959. Subsequently, Baumol published a second model in which managers seek to maximize the rate of growth of sales, rather than the level of sales. See W. J. Baumol, "On the Theory of Expansion of the Firm," American Economic Review (vol. 52; no. 5; December 1962), pp. 1078-1087, reprinted under the same title in G. C. Archibald, ed., The Theory of the Firm (Penguin; Middlesex, England; 1971), pp. 318-327. Though the sales maximization model is the better known of his two models, both are discussed in chapter two.

29. Baumol, *Business Behavior, Value and Growth*, *op. cit.*

30. Marris's model first appeared in R. L. Marris, "A Model of the 'Managerial' Enterprise," *Quarterly Journal of Economics* (vol. 77; no. 2; May 1963), pp. 185-209, and has been reprinted under the same title in Gilbert, *op. cit.*, pp. 211-237. A more expanded development of the model is provided in R. L. Marris, *The Economic Theory of 'Managerial' Capitalism*, *op. cit.* A revised version of the Marris model is given in R. L. Marris, "An Introduction to Theories of Corporate Growth," in R. L. Marris and A. Wood, eds., *The Corporate Economy: Growth, Competition and Innovative Potential* (Harvard University Press; Cambridge, Mass.; 1971), ch. 1. Both formulations of the Marris model and the connection between them are discussed in chapter two. The managerial utility function discussed in the text appears in the second formulation (though not in the first).

31. Williamson's model first appeared in O. E. Williamson, "Managerial Discretion and Business Behavior," *American Economic Review* (vol. 53; no. 5; December 1963), pp. 1032-1057, reprinted under the same title in Gilbert, *op. cit.*, pp. 238-266. A simplified version of the model was provided in O. E. Williamson, *The Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm* (Prentice-Hall; Englewood Cliffs, N.J.; 1964) as well as in O. E. Williamson, *Corporate Control and Business Behavior* (Prentice-Hall; Englewood Cliffs, N.J.; 1970), ch. 4. Both versions are discussed in chapter two. The objective function referred to in the text appears in the simplified version of the model.

32. The literature is extensive. See, for example, R. Monsen and A. Downs, "A Theory of Large Managerial Firms," *Journal of Political Economy* (vol. 73; no. 3; June 1965), pp. 221-236, reprinted under the same title in Gilbert, *op. cit.*, pp. 347-370, which focuses on goal conflicts among the different layers of management; R. M. Cyert and J. G. March, *A Behavioral Theory of the Firm* (Prentice-Hall; Englewood Cliffs, N.J.; 1963); K. J. Cohen and R. M. Cyert, *Theory of the Firm: Resource Allocation in a Market Economy* (Prentice-Hall; Englewood Cliffs, N.J.; 1965), ch. 16; and W. J. Baumol and M. Stewart, "On the Behavioral Theory of the Firm," in Marris and Wood, *op. cit.*, ch. 5. For a concise comparison of the various views of the firm and its objectives see F. Machlup, "Theories of the Firm: Marginalist, Behavioral, Managerial," *American Economic Review* (vol. 57; no. 1; March 1967), pp. 1-33.

33. Baumol and Stewart, *op. cit.*, pp. 118-120.

34. Marris and Wood, *op. cit.*, p. xvii.

35. R. E. Wong, "Profit Maximization and Alternative Theories: A Dynamic Reconciliation," *American Economic Review* (vol. 65; no. 4; September 1975), pp. 689-694. This argument assumes that the optimum size remains constant over time. If, as technological change occurs and

as the economy expands, the optimum size increases, firms could continue to grow at a positive rate forever. However, unless this optimum size could continue to increase at a sufficiently high rate (i.e. faster than the maximum rate at which firms in the industry could expand), there would still have to be a tendency for larger firms to grow more slowly than smaller ones.

36. The technology of production for each firm is embodied in a production function, which is a relationship between the inputs a firm employs and the outputs it produces. Returns to scale is an attribute of the production function. For the simple case of a firm producing a single commodity, when all inputs are increased by a proportion α, it is said that there are (i) decreasing returns to scale if output increases by a proportion smaller than α, (ii) constant returns to scale if output increases by the proportion α, and (iii) increasing returns to scale if output increases by a proportion greater than α. With decreasing returns to scale, at least beyond some rate of output, and with constant or increasing input prices, the long run average total cost curve will eventually turn upward. The point at which long run average total cost is minimized defines the optimum size of the firm.

37. Marris and Wood, op. cit., p. xvii.

38. O. E. Williamson, "Managerial Discretion, Organization Form, and the Multi-division Hypothesis," in Marris and Wood, op. cit., pp. 343-386.

39. Marris and Wood, op. cit., p. xvii, and J. L. Eatwell, "Growth, Profitability and Size: The Empirical Evidence," in Marris and Wood, op. cit., pp. 399-408.

40. Galbraith, op. cit., pp. 198-210.

41. Marris, Managerial Capitalism, op. cit., p. 177.

42. The distinction between 'internal' and 'external' growth is different from the distinction that is made below between 'internal' and 'external' financing (of growth). The former refers to whether or not the increased physical capital stock was obtained via takeover, while the latter refers to how the required funds (i.e. financial resources, or money capital) were raised. For example, external growth can be financed internally by using retained earnings to buy a controlling interest in another firm and internal growth can be financed externally by using the proceeds of a bank loan to fund advertising and research and development.

43. To obtain a controlling interest in another firm, a firm must purchase more than 50 percent (precisely, at least 50 percent plus one share) of the outstanding common stock of the firm that is being taken over. As each share entitles its owner to one vote at the annual meeting of stockholders, and as only common stock confers voting rights, ownership of more than 50 percent of the outstanding common stock enables the possessor (or as he is sometimes called, the 'takeover raider') to remove the former management team and substitute his

own. In reality, control of less than 50 percent of the common stock may still give one firm considerable power over another — possibly even effective control — but the term 'takeover' is used in this paper to mean ownership of at least 50 percent plus one share of the common stock. A related term is 'merger', and while some economists prefer to draw a distinction between a takeover and a merger, this paper uses the terms interchangeably.

44. Mergers are of three types: (i) vertical mergers, which take place between two firms that lie along the path a product takes as it moves from its original raw materials state through to its final state and distribution to consumers; (ii) horizontal mergers, which take place between two producers of the same product; and (iii) conglomerate mergers, which include all mergers not covered by (i) or (ii). Of the three, the Antitrust Division of the United States Justice Department has been tolerant only of the third form whenever large firms are involved, though, as evidenced by its challenge in ITT's takeover of Hartford Insurance and Avis Rent-A-Car, it can adopt a stricter stance with regard to conglomerate mergers as well.

45. L. G. Martin, "The 500: A Report on Two Decades," Fortune (May 1975), p. 239. The largest 500 industrial corporations are ranked annually on the basis of total sales revenue.

46. Ibid., p. 239.

47. Ibid., p. 241.

48. L. G. Martin, "How Beatrice Foods Sneaked Up on $5 Billion," Fortune (April 1976), p. 119.

49. Ibid., pp. 118-119.

50. Monsen and Downs, op. cit., pp. 221-236, and O. E. Williamson, Managerial Discretion, Organization Form, op. cit., pp. 343-386.

51. Ibid., p. 354.

52. See, for example, B. E. Davis, G. J. Caccappolo, and M. A. Chaudry, "An Econometric Planning Model for American Telephone and Telegraph Company," Bell Journal of Economics and Management Science (vol. 4; no. 1; Spring 1973), pp. 29-56.

53. O. E. Williamson, Managerial Discretion, Organization Form, op. cit., pp. 343-386.

54. Ibid., pp. 353-368.

55. The term 'uncertainty' and the closely related term 'risk' are defined precisely in section C of this chapter.

56. D. Vickers, The Theory of the Firm: Production, Capital, and Finance (McGraw-Hill; New York; 1968), and E.R. Arzac, "Structural Planning under Controllable Business Risk," Journal of Finance (vol. 30; no. 5; December 1975), pp. 1229-1237.

57. The basic features of the mean-variance approach are set out in J. Mossin, Theory of Financial Markets (Prentice-Hall; Englewood Cliffs, N.J.; 1973), and also in J. Hirschleifer, Investment, Interest, and Capital (Prentice-Hall; Englewood Cliffs, N.J.; 1970), pp. 277-310.

58. J. Lintner, "Optimum or Maximum Corporate Growth under Uncertainty," in Marris and Wood, op. cit., pp. 172-241.

59. The rudiments of time-state-preference theory are set out in K. J. Arrow, "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies (vol. 31; no. 86; April 1964), pp. 91-96, and also in Hirshleifer, op. cit., pp. 231-276.

60. The board of directors is, in theory at least, supposed to represent the shareholders' interests. Recently, the board system has come under heavy criticism. Typically, in large corporations the firm's professional managers dominate the firm's board of directors and the board meets only once a month or once a quarter. All too often the board of directors acts as a mere rubber stamp for management. See C. C. Brown, Putting the Corporate Board to Work (Macmillan; New York; 1976), for further discussion on this point.

61. The recent well-publicized firing of the chairman of Gulf Oil is a good example, though share ownership in Gulf is not widely dispersed, but rather, is heavily concentrated in the hands of the Mellon family. It was the Mellon interests who were primarily responsible for the ouster of Gulf's chairman. See also footnote 9, and "Why Insurgents Won the Day at Southdown," Business Week (July 19, 1976), which describes how a severe drop in the profits of Southdown Inc. has led to a fight for control of the company that so far has resulted in three dissident shareholders being elected to the company's board of directors.

62. See Galbraith, op. cit., pp. 77-80, on this point.

63. Ibid., pp. 81-82.

64. Singh, op. cit., pp. 148-149.

65. Diminishing returns means that the addition to profit (net of outlays for research and development) that results, ceteris paribus, from each additional dollar spent for research and development falls as research and development spending is increased.

66. The term 'competitors' is used here loosely to refer both to other producers of the same good and to producers of different products that are substitutes for the good in question.

67. Penrose, op. cit., ch. 4.

68. Marris, Managerial Capitalism, op. cit., ch. 3.

69. For example, under uncertainty the firm's total profit in any future time period becomes a random variable. Thus, the phrase 'maximizing total profit' becomes meaningless, and a suitable objective that takes into account the existence of uncertainty, such as maximizing expected profit or maximizing the expected utility of profit, must be adopted. This point is discussed further in chapter two.

70. This distinction between 'obtaining' and 'characterizing' a solution is discussed further later in this section.

71. For a brief survey of these techniques see M. D. Intriligator, Mathematical Optimization and Economic Theory (Prentice-Hall; Englewood Cliffs, N.J.; 1971), chs. 2-5.

72. The problems associated with trying to specify a collective utility function are addressed in sections G and K of chapter two.

73. See footnote 72.

74. For example, the problem

$$\text{maximize } f(x_1, x_2) \text{ subject to } g(x_1, x_2) = b$$

can be reformulated equivalently as the unconstrained problem

$$\text{maximize } f(x_1) \text{ , provided } \partial g/\partial x_2 \neq 0$$

(or as the unconstrained problem

$$\text{maximize } f(x_2) \text{ , provided } \partial g/\partial x_1 \neq 0 \text{) .}$$

75. To be able to convert (2) into (3) it is necessary and sufficient that

$$\text{rank} \begin{pmatrix} \partial g_1/\partial x_1 & \partial g_1/\partial x_2 & \cdots & \partial g_1/\partial x_n \\ \partial g_2/\partial x_1 & \partial g_2/\partial x_2 & \cdots & \partial g_2/\partial x_n \\ \vdots & \vdots & & \vdots \\ \partial g_m/\partial x_1 & \partial g_m/\partial x_2 & \cdots & \partial g_m/\partial x_n \end{pmatrix} = m \quad .$$

76. See subsection 2 in section G of chapter two.

77. These models are discussed in section L of chapter two.

78. Five such models are discussed in section L of chapter two.

79. See, for example, the Leland model in section L of chapter two.

80. This is the case in all the models, other than Leland's, that are discussed in section L of chapter two.

81. For a survey of these approaches and the conditions under which each is applicable see Intriligator, op. cit., chs. 11-14. A fuller discussion of the calculus of variations and its economic applications appears in A. Takayama, Mathematical Economics (Dryden; Hinsdale, Illinois; 1974), ch. 5, and a more detailed discussion of Pontryagin's maximum principle and its economic applications appears in ibid., ch. 8. It should be noted that all three approaches are closely related mathematically. Indeed, the necessary conditions of the calculus of variations can be derived from Pontryagin's maximum principle. Intriligator, op. cit., pp. 353-355. It should also be noted that, of the three approaches, the maximum principle is often the most useful. In particular, it is superior to dynamic programming in terms of characterizing the nature of the solution to (4).

82. Similarly, Pontryagin's maximum principle can be interpreted as an extension of the method of Lagrange multipliers to dynamic optimization problems. For more on this point see ibid., ch. 14.

83. Baumol and Stewart, op. cit., pp. 119-120.

84. Ibid., p. 142.

85. It should be noted that 'imperfect information' and 'uncertainty' are not synonymous. In essence, the difference is that, under imperfect information, all firms do not have free and equal access to all available information concerning the future, while under uncertainty, all firms may have free and equal access to whatever information is available, but that information is insufficient for them to determine what the future holds for them with certainty.

86. Such a comparison is undertaken in the discussion of the Lintner model in section J of chapter two.

87. From a more practical standpoint, it remains to be seen whether such a problem could be formulated for a large firm and be both meaningful in a business sense and tractable mathematically. This point will be discussed further in chapter six of the author's dissertation.

88. This definition reflects both the traditional emphasis placed on the productive characteristics of the firm and the modern emphasis placed on the organizational characteristics of the firm. For an example of the former, see Intriligator, op. cit., p. 178, and for an example of the latter, see Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 191.

89. Specifically, the corporation is able to raise larger sums of money, its existence does not cease when one of its owners dies, and its owners enjoy limited liability (to the extent of their shareholdings) for the debts of the corporation.

90. R. Sherman, The Economics of Industry (Little, Brown; Boston; 1974), p. 215.

91. For a discussion of these difficulties see ibid., pp. 215-216.

92. For a comprehensive treatment of the aerospace industry in general, and its composition under different definitions of the industry, see W. C. Ridder and M. K. Heinz, "Structure, Conduct, and Performance of the United States Aerospace Industry," unpublished M.S. thesis (Naval Postgraduate School; Monterey, Calif.; March 1976).

93. For example, in defining the military airframe industry, Stekler includes Martin-Marietta, which at the time of his study was a major producer of missiles, though it no longer produced airplanes. H. O. Stekler, The Structure and Performance of the Aerospace Industry (University of California Press; Berkeley; 1965), p. 47. In a later study, Carroll adopted a wider definition, including in addition to Martin-Marietta, other missile producers. S. L. Carroll, "The Airframe Industry," unpublished Ph.D. dissertation (Harvard University; Cambridge, Mass,; August 1970), ch. II.

94. Shephard has established the conditions that must be satisfied in order that the firm's production relations can be expressed in implicit form as in (5). See R. W. Shephard, Theory of Cost and Production Functions (Princeton University Press; Princeton; 1970) and R. W. Shephard and R. Färe, "The Law of Diminishing Returns," Zeitschrift für Nationalökonomie (vol. 34; nos. 1-2; 1974), pp. 69-90.

95. G. Bannock, R. E. Baxter, and R. Rees, The Penguin Dictionary of Economics (Penguin; Middlesex, England; 1972), p. 55. It should be noted that the measurement of capital involves a number of difficult issues, as for example, whether equipment should be valued at historic cost net of depreciation, at replacement cost, at current market (i.e. salvage) value, or on some other basis. For more on this see J. Robinson, "The Production Function and the Theory of Capital," Review of Economic Studies (vol. 21; 1953-4), pp. 81-106; D. G. Champernowne, "The Production Function and the Theory of Capital: A Comment," Review of Economic Studies

(vol. 21; 1953-4), pp. 112-135; and G. C. Harcourt, Some Cambridge Controversies in the Theory of Capital (Cambridge University Press; Cambridge; 1972). It should also be noted that a more general definition of the firm's capital stock has been suggested by Uzawa, who defines the capital stock to include "managerial and administrative abilities of the firm as well as the quantites of physical factors of production such as machinery and equipment." See H. Uzawa, "Time Preference and the Penrose Effect in a Two-Class Model of Economic Growth," Journal of Political Economy (vol. 77; no. 4, part 2; July-August 1969), pp. 628-652. Though quantification is difficult, Uzawa does suggest how an index for measuring the aggregate stock of corporate assets might be constructed. Ibid., pp. 637-639.

96. P. A. S. Taylor, A New Dictionary of Economics, 2nd ed., (Routledge & Kegan Paul; London; 1969), pp. 26-27, defines 37 different types of capital. For an in-depth discussion of several different types of capital see Hirshleifer, op. cit., ch. 6.

97. It is in the sense of money capital — the amount of money available for purchasing some item — that the term capital appears in common usage.

98. The role of the firm's financial policy is carefully examined in sections I and K of chapter two.

99. See, for example, D. Solomons, "Economic and Accounting Concepts of Income," in R. H. Parker and G. C. Harcourt, eds., Readings in the Concept & Measurement of Income (Cambridge University Press; Cambridge; 1969), pp. 106-119. For a fuller discussion of the difficulties encountered in defining and measuring profit see E. O. Edwards and P. W. Bell, The Theory and Measurement of Business Income (University of California Press; Berkeley; 1972).

100. For definitions of the various accounting notions of profit see G. A. Welsch and R. N. Anthony, Fundamentals of Financial Accounting (Irwin; Homewood, Illinois; 1974), ch. 3.

101. The residual left after opportunity costs have been charged against all inputs consists of normal profit plus excess profit. *Normal profit* is that amount of profit that is just sufficient to cause the firm to remain in its present industry. In a world of certainty and perfect competition, normal profit is zero for all firms in long run equilibrium. Under uncertainty, however, normal profit need not be zero. *Excess profit*, as the term implies, is profit over and above normal profit. It may arise as a result of monopoly power, a successful innovation, or a windfall gain.

102. The above definition of profit is appropriate for a single period model of the firm. In a multiperiod setting, however, specifying

exactly what constitutes profit becomes more difficult. In models of the firm profit is corporate income that is available for distribution to the shareholders as dividends. That portion not distributed as dividends is called retained earnings and is available for expenditure on plant and equipment. Since one pierod's expenditures and pricing policies can affect the firm's profitability in future periods, the definition of profit should reflect this intertemporal dependence. One might then want to define profit as the maximum amount the firm could distribute as dividends and still expect to be able to distribute the same amount in each ensuing period. This definition is based on Hick's Income No. 2. See J. R. Hicks, Value and Capital, 2nd ed. (Oxford University Press; Oxford; 1946), ch. 14. It is the definition used by Krouse, whose model is discussed in section L of chapter two. See C. G. Krouse, "On the Theory of Optimal Investment, Dividends, and Growth in the Firm," American Economic Review (vol. 63; no. 3; June 1973), p. 271. A similar definition that explicitly takes into account the existence of uncertainty has been proposed by John Lintner. Specifically, profit for any period is "the maximum cash dividend which (expectationally) *could* be paid in that period consistent with (*pro-forma*) no outside financing *and* with equally large *expected values* of the (level stream of) dividends which *could* be paid in future periods subject to the same *pro-forma* financing constraint." (Italics and parentheses are Lintner's). See J. Lintner, "Optimal Dividends and Corporate Growth Under Uncertainty," Quarterly Journal of Economics (vol. 78; no. 1; February 1964), p. 55. The Lintner model is discussed in section J of chapter two.

3. In the business literature reported profit is frequently referred to as 'earnings'. Henceforth in this paper the terms 'reported profit', 'net income", and 'earnings" are used synonomously.

4. As demonstrated in section G of chapter two, discretionary profit normally exceeds reported profit for the managerial firm since such a firm's managers will tend to spend more on advertising, more on staff, etc., than a profit maximizing firm.

5. This definition differs somewhat from Oliver Williamson's definition of discretionary profit. According to the Williamson definition, discretionary profit is reported profit (net of tax) less the minimum after-tax profit "necessary to assure the interference-free operation of the firm to the management." See O. E. Williamson, Managerial Discretion and Business Behavior, op. cit., p. 1035. The definition in the text is preferred since managers exercise some discretion in setting expenditure levels for research and development, advertising, etc., which are subtracted out when reported profit is calculated.

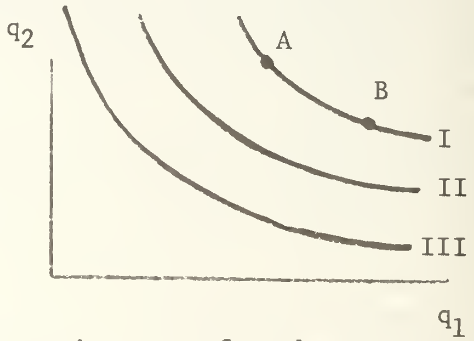
6. Ibid., p. 1036.

107. Marris, Theories of Corporate Growth, op. cit., p. 19.

108. See P. A. Samuelson, Foundations of Economic Analysis (Harvard University Press; Cambridge, Mass.; 1947), chs. 5,7.

109. Ibid., pp. 107-112; P. A. Samuelson, "Consumption Theory in Terms of Revealed Preference," Economica (new series, vol. 15; no. 60; November 1948), pp. 243-253; H. S. Houthakker, "Revealed Preference and the Utility Function," Economica (new series, vol. 17; no. 66; May 1950), pp. 159-174; and J. R. Hicks, A Revision of Demand Theory (Clarendon Press; Oxford; 1956).

110. In the simple case of a utility function having two arguments, the indifference map consists of a collection of curves, which are typically drawn convex to the origin as in the figure to the right. Along each curve, all combinations yield the same level of utility. For example, A and B are equally satisfying. If the axioms of revealed preference are satisfied — and, in essence, these require that an individual's preferences exhibit consistency and transitivity — the references given in footnote 109 show that the existence of such curves, as well as several important theorems that verify that the indifference curves obtained via revealed preference exhibit the basic properties that such curves have long been assumed to possess, can be proved.



111. How such a test procedure works in consumer theory is described in J. M. Henderson and R. E. Quandt, Microeconomic Theory: A Mathematical Approach, 2nd ed. (McGraw-Hill; New York; 1971), pp. 39-42.

112. Penrose, op. cit., p. 199. A discussion of various measures of the size of firms is contained in G. Schroeder, The Growth of Major Steel Companies, 1900-1950 (Johns Hopkins Press; Baltimore; 1952), pp. 24-35.

113. N. R. Collins and L. E. Preston, "The Size Structure of the Largest Industrial Firms, 1909-1958," American Economic Review (vol. 51; no. 5; December 1961), pp. 986-1011, use book value of total assets; J. M. Samuels, "Size and the Growth of Firms," Review of Economic Studies (vol. 32; no. 90; April 1965), pp. 105-112, and A. Singh and G. Whittington, Growth, Profitability and Valuation (Cambridge University Press; Cambridge; 1968) use book value of net assets; J. Steindl, Random Processes and the Growth of Firms (Hafner; New York; 1965) and W. J. Baumol, On the Theory of Expansion of the Firm, op. cit., use sales revenue; P. E. Hart, "The Size and Growth of Firms," Economica (new series,

vol. 29; no. 113; February 1962), pp. 29-39, uses the stock market value of the firm; and J. Steindl, Small and Big Business (Blackwell; Oxford; 1945) uses the total number of employees as an indicator of the size of the firm.

114. Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 192. In order that the different measures of size yield perfectly equivalent results, it is necessary that the firm be on a steady state growth path. However, this is at best an approximation to reality. Unless care is taken in selecting the period over which growth is to be measured, changes in factor proportions (as the firm adjusts to technological change or to changes in demand) or rapid bursts of takeover activity coupled with falling profitability can cause these different indicators of size to yield conflicting signals. Generally, the longer the time period involved, the greater is the likelihood that these short term distortions would be smoothed out, and the greater would be the likelihood that the different measures of size would be highly correlated. See the next footnote for a reference.

115. J. Bates, "Alternative Measures of the Size of Firms," in P. E. Hart, Studies in Profit, Business Saving and Investment in the United Kingdom, 1920-1962, vol. I (Allen & Unwin; London; 1965), ch. 8.

116. This is the measure of size used in Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 192.

117. What follows is a brief discussion of the distinguishing features of the four types of market structure. For a more precise economic definition of each of the market structures see S. C. Webb, Managerial Economics (Houghton Mifflin; Boston; 1976), ch. 20. An excellent description of the economic significance of each of these market structures may be found in either Sherman, op. cit., or F. M. Scherer, Industrial Market Structure and Economic Performance, (Rand McNally; Chicago; 1970). Monopoly, oligopoly, and monopolistic competition are often collectively referred to as *imperfect competition*.

118. The dividing line between oligopoly and monopolistic competition is not well-marked. For example, an industry that consists of five large producers and twenty small producers will behave more in accordance with the oligopolistic model the smaller is the twenty small producers' collective share of the market. For this reason Utton eschews the less precise notions of oligopoly and monopolistic competition in favor of structural characteristics specified in terms of the number of firms, the extent of industrial concentration (percentage of sales rung up by the top four firms), and the size ratio of firms (the average size of the largest three firms divided by the average size of all other firms in the industry), in his study of how market structure influences the behavior of firms. See M. A. Utton, Industrial Concentration (Penguin; Middlesex, England; 1970).

119. See the references listed in footnote 146.

120. This distinction was first emphasized in F. H. Knight, Risk, Uncertainty and Profit (Houghton Mifflin; New York; 1921).

121. Hirschleifer, op. cit., p. 215.

122. See Arrow, Optimal Allocation of Risk-Bearing, op. cit.; K. J. Arrow, Aspects of the Theory of Risk-Bearing (Yrjö Johnssonin Säätiö; Helsinki; 1965); and G. Debreu, The Theory of Value (Wiley; New York; 1959), ch. 7.

123. The opposite sort of behavior is called *risk seeking* behavior and the in-between case is *risk neutral* behavior.

124. This type of behavior is explained in terms of the *diminishing marginal utility of income*: the increase in utility that would result from winning the gamble is smaller than the decrease in utility that would result from a corresponding equiprobable loss. See, for example, Hirshleifer, op. cit., pp. 224-226.

125. In contrast, the indifference curves for a risk seeking individual would bend downward and to the right, as illustrated in Figure A, and the indifference curves for a risk neutral individual would be horizontal, as illustrated in Figure B.

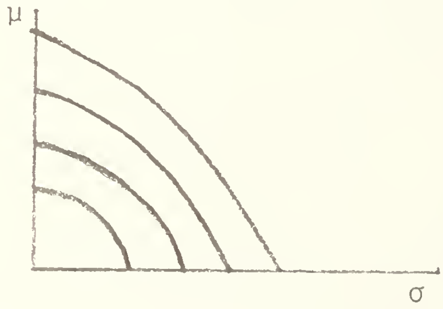


Figure A Indifference Curves for a Risk Seeker

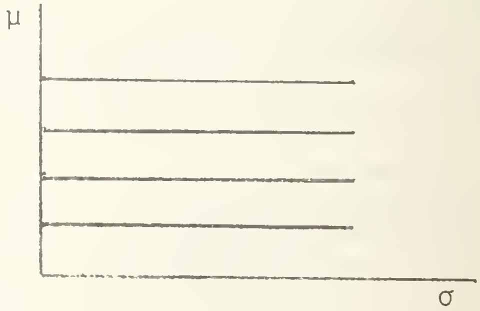


Figure B Indifference Curves Under Risk Neutrality

126. Ibid., ch. 8. This point is developed further in section K of chapter two, where several expected utility maximization models due to Leland are discussed.

127. Takeovers are of three types: *horizontal*, which involve producers of the same product, or similar products that are sold in the same market, *vertical*, which involve firms that operate at different stages of the same industrial process, and *conglomerate*, which include takeovers other than those of the horizontal and vertical varieties. For a discussion of the economic significance of these three types of takeovers, see Sherman, op. cit., ch. 10. The significance of this distinction is that the takeover wave in the United States in the late 1960's was concentrated in the third category of takeover.

128. Singh, op. cit., p. xi, and D. A. Kuehn, Takeovers and the Theory of the Firm: An Empirical Analysis for the United Kingdom, 1957-1969 (Macmillan; London; 1975), pp. 38-39. Kuehn describes a procedure for reclassifying as a takeover what is, strictly speaking, a merger. The procedure entails identifying the acquiring firm and the acquired firm on the basis of the contributions of each partner to the management of the merged firm. Ibid., pp. 39, 157-161.

129. Ibid., p. 39. For a discussion of the motivations underlying mergers and takeovers see P. O. Steiner, Mergers: Motives, Effects, Policies (University of Michigan Press; Ann Arbor; 1975). A convenient summary of the economic theory of takeovers and a survey of some recent empirical results are provided in B. Hindley, "Recent Theory and Evidence on Corporate Merger," in Cowling, op. cit., pp. 1-17.

130. Singh, op. cit., pp. 148-149.

131. Marris, Managerial Capitalism, op. cit., ch. 4, and Penrose, op. cit., ch. 8.

132. Marris, Managerial Capitalism, op. cit., p. 30.

133. The difference between a depression and a recession is largely a matter of degree. While both involve a reduction in the aggregate level of economic activity (as measured by gross national product (GNP), i.e. the nation's output of goods and services), a depression involves a deeper and more prolonged drop in GNP.

134. Marris, Theories of Corporate Growth, op. cit., p. 13.

135. R. Marris, "Why Economics Needs A Theory of the Firm," Economic Journal (vol. 82; no. 325s; March 1972 (Supplement)), p. 321.

136. Machlup, op. cit., p. 9.

137. Marris, Why Economics Needs A Theory of the Firm, op. cit., p. 321.

138. For a brief discussion of the issues involved see A. Wood, "Economic Analysis of the Corporate Economy: A Survey and Critique," in Marris and Wood, op. cit., pp. 64-65.

139. O. E. Williamson argues that the internal allocation (between the divisions of a conglomerate) of capital is more efficient than that achievable by the stock market (between single product firms) due to superior internal information flows and more effective internal control. See O. E. Williamson, Managerial Discretion, Organization Form, op. cit., pp. 368-377.

140. G. William Miller, Chairman of Textron, Inc., argues that only large diversified companies can bear the risks inherent in producing large expensive weapons systems for the government. See C. N. Stabler, "Impact of Big Combines on the Economy Defies Old-Fashioned Analysis," Wall Street Journal (August 12, 1968).

141. One question of current interest is the following. Should the major oil companies be broken up? One of the central issues concerns the extent to which the internalization of certain market transactions — through vertical integration — has resulted in excessively high prices, and as a result, abnormal profits. See J. Carberry, "Big Oil Besieged: Is Industry Too Strong? Its Opponents Say Yes And Call for a Breakup," Wall Street Journal (February 9, 1976), and "The flaky arguments over breaking up big oil," Business Week (August 16, 1976).

142. Economic consultant G. C. Means blamed roughly half the inflation during the current recession on industries in which there is little price competition. See "Industries With Little Competition Cited For About Half of Recession's Inflation," Wall Street Journal (April 15, 1975).

143. Numerous economists have argued that the large diversified corporation is more conducive to economic growth than single product firms. For example, see Wood, op. cit., p. 65, where it is argued that economies of scale in planning and forecasting and in developing new products, as well as more effective internal control give the conglomerate organization an advantage. See also A. Wood, A Theory of Profits (Cambridge University Press; Cambridge; 1975), for a discussion of related micro-macro issues.

144. A survey of these models is provided in L. Engwall, Models of Industrial Structure (D. C. Heath; Lexington, Mass.; 1973). While Engwall considers separately simulation models using discrete size and simulation models using continuous size, this paper treats them collectively. It should be noted that one other approach — a game theoretic one — has also been attempted. Unfortunately, the game theoretic approach has thus far proven to be less amenable than these others to empirical testing. R. Marris, "The Modern Corporation and Economic Theory," in Marris and Wood, op. cit., pp. 283-304, describes a game theoretic model that allows for the entry and exit of firms. Game theory, as applied to duopoly, is discussed in section C of chapter two of this paper.

145. In models using discrete size this means specifying the distribution of firms among the discrete size classes, and in models using continuous size this means identifying the continuous distribution that characterizes firm size. In both cases it is really how the distribution is changing over time, and the

asymptotic distribution toward which the sequence of actual distributions at successive points in time is converging (if such a distribution exists), that is of primary interest.

146. J. S. Bain, "Changes in Concentration in Manufacturing Industries in the United States, 1954-1966: Trends and Relationships to the Levels of 1954 Concentration," Review of Economics and Statistics (vol. 52; no. 4; November 1970), pp. 411-416. Concentration has not increased in all industries, and it has tended to increase least in those industries that grew most rapidly. See W. F. Mueller, "Industrial Structure and Competition Policy," in Studies by the Staff of the Cabinet Committee on Price Stability (Washington, D.C.; January 1969), p. 62, and J. F. Weston and S. I. Ornstein, "Trends and Causes of Concentration — A Survey," in J. F. Weston and S. I. Ornstein, eds., The Impact of Large Firms on the U.S. Economy (D. C. Heath; Lexington, Mass.; 1973), pp. 3-21. Aggregate concentration, as measured by the percentage of total manufacturing corporation assets controlled by the largest 100 firms, has also increased. See Scherer, op. cit., pp. 41-44. Both trends are observable in other countries. See K. D. George, "The Changing Structure of Competitive Industry," Economic Journal (vol. 82; no. 325s; March 1972 (Supplement)), pp. 353-368; K. D. George, "A Note on Changes in Industrial Concentration in the United Kingdom," Economic Journal (vol. 85; no. 337; March 1975), pp. 124-128; and Scherer, op. cit., pp. 44-45.

147. The prevailing view is that greater concentration — i.e. greater departures from the competitive ideal — result in higher prices, more excessive profits, and a less efficient allocation of the economy's scarce productive resources. See R. E. Caves, American Industry: Structure, Conduct, Performance, 3rd ed. (Prentice-Hall; Englewood Cliffs, N.J.; 1972) and J. S. Bain, "Relation of Profit Rates to Industry Concentration," Quarterly Journal of Economics (vol. 65; no. 3; August 1951), pp. 293-324. See also J. S. Bain, Industrial Organization, 2nd ed. (Wiley; New York; 1968); J. S. Bain, Barriers to New Competition (Harvard University Press; Cambridge, Mass.; 1956); J. M. Blair, Economic Concentration (Harcourt, Brace, and Jovanovich; New York; 1972); B. Bock and J. Farkas, "Concentration and Productivity," SBE No. 103 (The Conference Board; New York; 1969); Y. Brozen, "The Persistence of 'High Rates of Return' in High Stable Concentration Industries," Journal of Law and Economics (vol. 14; October 1971), pp. 501-512; N. R. Collins and L. E. Preston, Concentration and Price-Cost Margins in Manufacturing Industries (University of California Press; Berkeley; 1968); B. T. Allen, "Market Concentration and Wage Increases: U.S. Manufacturing, 1947-1964," Industrial and Labor Relations Review (vol. 21; no. 3; April 1968), pp. 353-366; P. Asch, "Industry Structure and Performance: Some Empirical Evidence," Review of Social Economy (vol. 25; March 1967), pp. 167-182; D. F. Greer, "Advertising and Market Concentration,"

Southern Economic Journal (vol. 38; no. 1; July 1971), pp. 19-32; C. J. Sutton, "Advertising, Concentration and Competition," Economic Journal (vol. 84; no. 333; March 1974), pp. 56-69; K. Cowling, "Optimality in firms' advertising policies: an empirical analysis," in K. Cowling, ed., Market Structure and Corporate Behavior, op. cit., pp. 85-103; J. Cable, "Market structure advertising policy and intermarket differences in advertising intensity," in K. Cowling, ed., Market Structure and Corporate Behavior, op. cit., pp. 105-124; and J. W. Markham, "Market Structure, Business Conduct, and Innovation," American Economic Review (vol. 55; no. 2; May 1965), pp. 323-332. For a critical review of earlier studies and a discussion of some important policy implications see H. Demsetz, "Industry Structure, Market Rivalry, and Public Policy," in Weston and Ornstein, op. cit., pp. 71-82; S. I. Ornstein, "Concentration and Profits," ibid., pp. 87-102; and W. S. Comanor and T. A. Wilson, "Advertising, Market Structure and Performance," Review of Economics and Statistics (vol. 49; no. 4; November 1967), pp. 423-440.

148. This leads Marris to suggest a game theoretic approach. See Marris, Modern Corporation and Economic Theory, op. cit., pp. 283-304. However, in the opinion of this writer, a great deal of work needs to be done before this approach begins to bear fruit. It is also this writer's opinion that the starting point for such research should be the individual firm.

149. See Engwall, op. cit., chs. 3-5. See also H. A. Simon and C. P. Bonini, "The Size Distribution of Business Firms," American Economic Review (vol. 48; no. 4; September 1958), pp. 607-617; I. G. Adelman, "A Stochastic Analysis of the Size Distribution of Firms," Journal of the American Statistical Association (vol. 53; no. 284; December 1958), pp. 893-904; and S. H. Archer and J. McGuire, "Firm Size and the Probabilities of Growth," Western Economic Journal (vol. 3; no. 3; Summer 1965), pp. 233-246. An interesting discussion of some of the practical limitations of these models is provided in Scherer, op. cit., pp. 125-130.

150. While it is normally assumed that the transition probabilities are constant, it is not necessary to do so. P. K. Newman and J. N. Wolfe, An Essay on the Theory of Value (Purdue University Press; West Lafayette, Ind.; 1960) have generalized on Adelman's results by permitting the transition probabilities to vary.

151. In empirical studies it is necessary to specify the size of the pool from which potential entrants may enter the industry. Only then can the transition probabilities in the first column be estimated. See Adelman, op. cit., p. 899.

152. The basic reference for the terminology and basic results that follow is W. Feller, An Introduction to Probability Theory and Its Applications, vol. I, 3rd ed. (Wiley; New York; 1968), pp. 387-394.

153. Ibid., pp. 393-394.

154. R. Gibrat, Les Inégalités Economiques (Sirey; Paris; 1931). In the economic literature the law of proportionate effect is also referred to as Gibrat's law.

155. Eatwell, op. cit., p. 403. See also Hart, op. cit., and E. Mansfield, "Entry, Gibrat's Law, Innovation, and the Growth of Firms," American Economic Review (vol. 52; no. 5; December 1962), pp. 1023-1051. Singh and Whittington report similar results for U.K. quoted companies. Singh and Whittington, op. cit. For a discussion of some contrary results see J. M. Samuels and A. D. Chesher, "Growth, survival and the size of companies 1960-9," in Cowling, op. cit., pp. 39-59.

156. Eatwell, op. cit., pp. 403-406. See also Mansfield, op. cit., pp. 1030-1035; J. M. Samuels, "Size and the Growth of Firms," Review of Economic Studies (vol. 32; no. 90; April 1965), pp. 105-112; S. Hymer and P. Pashigian, "Firm Size and Rate of Growth," Journal of Political Economy (vol. 70; no. 6; December 1962), pp. 556-569; and J. M. Samuels and D. J. Smyth, "Profits, Variability of Profits and Firm Size," Economica (vol. 35; no. 138; May 1968), pp. 127-139.

157. Steindl, Random Processes, op. cit., and Singh and Whittington, op. cit.

158. Y. Ijiri and H. A. Simon, "Business Firm Growth and Size," American Economic Review (vol. 54; no. 2; March 1964), pp. 77-89.

159. Simon and Bonini, op. cit.

160. H. A. Simon, "On a Class of Skew Distribution Functions," Biometrica (vol. 42; December 1955), pp. 425-440.

161. The characteristics of the Yule distribution are summarized in N. L. Johnson and S. Kotz, Distributions in Statistics, vol. 1 (Discrete Distributions) (Houghton Mifflin; New York; 1969), pp. 244-247, and the characteristics of the Pareto distribution are summarized in N. L. Johnson and S. Kotz, Distributions in Statistics, vol. 2 (Continuous Univariate Distributions — 1) (Houghton Mifflin; New York; 1970), ch. 19. In fact, there are several distributions, each generated by a different stochastic growth process, that fit the skewed firm size distribution typical of real world industries well enough that it is very difficult to distinguish one from another statistically. See Simon, Skew Distribution Functions, op. cit., and R. E. Quandt, "On the Size Distribution of Firms," American Economic Review (vol. 56; no. 3; June 1966), pp. 416-432.

162. Simon and Bonini, op. cit., p. 612.

163. Engwall, op. cit., p. 46.

164. Steindl, Random Processes, op. cit., p. 187.

165. Quandt, op. cit.

166. Steindl, op. cit., p. 194.

167. Adelman, op. cit., pp. 893-904.

168. Ibid., p. 900.

169. Engwall, op. cit., ch. 5, and Archer and McGuire, op. cit., pp. 233-246.

170. Adelman, op. cit., p. 901.

171. Ibid., p. 901.

172. S. J. Prais, "Measuring Social Mobility," Journal of the Royal Statistical Society (ser. A, vol. 118; 1955), pp. 56-66.

173. The index of mobility for time t is given by

$$M_t = \sum_{i=1}^{M} [\pi_i/(1-\pi_i)] \Big/ \sum_{i=1}^{M} [s_i^t/(1-p_{ii})] ,$$

where s_i^t is the proportion of firms in class i at time t. The quantity $1/(1-\pi_i)$ is the expected number of periods a firm would spend in class i if the industry were perfectly mobile, and the quantity $1/(1-p_{ii})$ is the number of periods a steel firm in class i in period t would expect to remain there. The sums are just weighted averages of these values taken over the M size classes.

174. Adelman, op. cit., p. 903.

175. See Engwall, op. cit., chs. 3 and 6. Studies of industrial structure that employed models using continuous size include Hart, op. cit.; P. E. Hart and S. J. Prais, "The Analysis of Business Concentration: A Statistical Approach," Journal of the Royal Statistical Society (ser. A, vol. 119, pt. 2;1956), pp. 150-181; Singh and Whittington, op. cit.; and Samuels, op. cit.

176. Gibrat, op. cit.

177. The lognormal distribution has been applied to a wide range of economic phenomena. The characteristics of the lognormal distribution are summarized in Johnson and Kotz, vol. 2, op. cit., ch. 14. For a survey of its uses in economics see J. Aitchison and J. A. C. Brown, The Lognormal Distribution, with Special Reference to its Uses in Economics (Cambridge University Press; Cambridge; 1957).

178. D. G. Champernowne, "Discussion on Paper by Mr. Hart and Dr. Prais," Journal of the Royal Statistical Society (ser. A, vol. 119, pt. 2; 1956), p. 182.

179. There is, however, the possibility that the two assumptions are invalid but that their effects are counterbalancing. If average cost curves are L-shaped, as Bain has suggested, then it is not unreasonable to expect that deviations from the law of proportionate effect would be more common among smaller than among larger firms. If growth rates are more variable among smaller firms, then chance will occasionally permit a small firm to grow very rapidly. On average small firms might even grow faster than larger firms. For a constant population of firms this would cause actual concentration to be higher than what the model would predict. (See Engwall, op. cit., pp. 67-68, or Scherer, op. cit., pp. 128-129, for more on this point). If, in addition, the number of firms is growing (as more firms enter than leave), then concentration would tend to decrease. Putting these two opposite effects together, the net effect may be very small.

180. Hart and Prais, op. cit., p. 170; Quandt, op. cit., pp. 425-427; Simon and Bonini, op. cit.; and Engwall, op. cit., ch. 6. In their study Simon and Bonini tested the Yule distribution — which is a generalization of the lognormal distribution in the sense that the law of proportionate effect still holds but births are permitted — against data for the United States steel industry and reported close fits. For some contrary results see I. H. Silberman, "On Lognormality as a Summary Measure of Concentration," American Economic Review (vol. 57; no. 4; September 1967), pp. 807-831.

181. See footnote 146 for references.

182. Hart and Prais, op. cit.

183. The underlying relationship is

$$\log [S_{t+1}] = \beta \cdot \log [S_t] + \varepsilon ,$$

which will generate a lognormal distribution provided $\beta = 1$. If the law of proportionate effect is valid, then S_t and ε are independent, and taking variances leads to equation (10).

184. Bain, Industrial Organization, op. cit.

185. Eatwell, op. cit., pp. 405-406. See also Samuels, op. cit., and Singh and Whittington, op. cit.

186. Eatwell, op. cit., p. 405. Two standard errors from the mean was used as the criterion of significance.

187. A description of simulation models is provided in Engwall, op. cit., chs. 7-9. Chapter 8 presents a simulation model using discrete size and chapter 9 describes a simulation model using continuous size. Two earlier simulation models are those of Ijiri and Simon, op. cit., and of F. E. Balderston and A. C. Hoggatt, Simulation of Market Processes (Institute of Business and Economic Research; Berkeley; 1962).

188. Ijiri and Simon devised the following weak form of the law of proportionate effect in order to build serial correlation into their model: "the expected percentage change in size of the totality of firms in each size stratum is independent of stratum." See Ijiri and Simon, op. cit., p. 79.

189. Two partial, though useful, surveys of the literature are presented in W. D. Nordhaus, "Recent Developments in Price Dynamics," in O. Eckstein, ed., The Econometrics of Price Determination Conference (Board of Governors of the Federal Reserve System and the S. S. R. C.; Washington, D. C.; 1972) and M. Rothschild, "Models of Market Organization with Imperfect Information: A Survey," Journal of Political Economy (vol. 81; no. 6; November-December 1973), pp. 1283-1308.

190. S. G. Winter, Jr., "An SSIR Model of Markup Pricing," unpublished paper (University of Michigan; Ann Arbor; 1971).

191. L. J. Maccini, "An Aggregate Dynamic Model of Short-Run Price and Output Behavior," Quarterly Journal of Economics (vol.90; no. 2; May 1976), pp. 177-196. Maccini's model generalizes on a previous model due to Phelps and Winter. Both models assume that firms are expected-present-value maximizers, but only the Maccini model deals with dynamic inventory adjustments. In the Phelps-Winter model the firm faces a dynamic demand function, which causes the firm's rate of change of sales to depend on the difference between its current price and the market average price. The time path of the firm's price gradually approaches the market average price, with the rate of price adjustment dependent on both the difference between the firm's price and the expected market average price and the difference between the firm's market share (of customers) and its expected equilibrium market share (of customers). The model predicts homogeneity of degree one of the rate of price adjustment with respect to the expected rate of inflation. See E. S. Phelps and S. G. Winter, Jr., "Optimal Price Policy under Atomistic Competition," in E. S. Phelps, et al., eds., Microeconomic Foundations of Employment and Inflation Theory (W. W. Norton; New York; 1970), pp. 309-337.

192. See P. Wiles, "Cost Inflation and the State of Economic Theory," Economic Journal (vol.83; no. 330; June 1973), pp. 377-398, and D. E. W. Laidler and J. M. Parkin, "Inflation — A Survey," Economic Journal (vol. 85; no. 340; December 1975), pp. 741-809.

193. See S. Lombardini, "Modern Monopolies in Economic Development," in Marris and Wood, op. cit., pp. 242-269.

194. Marris, Why Economics Needs a Theory of the Firm, op. cit., p. 327. The model is described in appendix 2 of that paper.

195. See section G of chapter two.

196. An excellent discussion of the purpose of general equilibrium analysis is provided along with a survey of the relevant literature in K. J. Arrow, "General Economic Equilibrium: Purpose, Analytic Techniques, Collective Choice," American Economic Review (vol. 64; no. 3; June 1974), pp. 253-272. Arrow delivered the paper in Stockholm, Sweden, in December 1972 when he received the Nobel Prize in Economic Science.

197. See, for example, K. J. Arrow and G. Debreu, "Existence of an Equilibrium for a Competitive Economy," Econometrica (vol. 22; no. 3; July 1954), pp. 265-290, and L. W. McKenzie, "On the Existence of General Equilibirum for a Competitive Market," Econometrica (vol. 27; no. 1; January 1959), pp. 54-71.

198. T. Negishi, "Monopolistic Competition and General Equilibrium," Review of Economic Studies (vol. 28; 1960-61), pp. 196-201, and K. J. Arrow, "The Firm in General Equilibrium Theory," in Marris and Wood, op. cit., pp. 68-110. A more detailed version of the Arrow model with proofs appears in K. J. Arrow and F. H. Hahn, General Competitive Analysis (Holden-Day; San Francisco; 1971).

199. For references see footnotes 140 and 141.

II. MODELS OF THE FIRM

A. INTRODUCTION

1. Overview of the Chapter

This chapter surveys the economic literature dealing with the theory of the firm. Discussed in this chapter are a collection of models of the firm that, in the opinion of this writer, form a cross section of the models that constitute the theory of the firm. The models selected include those that have been cited most frequently in the literature, and in addition, several recent models whose contributions to the theory of the firm were judged by this writer to be significant — either because a more general treatment of the firm was given or because a novel modeling approach was employed (or both).

A total of thirty models are discussed. These are listed in table II-34 on page 435, which provides a summary look at their economic content, and also in table II-35 on page 444, which provides a summary look at the mathematical techniques employed in their analysis. While the information these tables convey will be more meaningful to the reader after he has read sections B through L, the reader may still find it helpful to refer to these tables during the course of this introductory section. The next subsection, which discusses the components of the analytical framework that is used throughout the chapter, explains the meaning and significance of the categories of information summarized in the two tables. The reader may also find it helpful to refer to these tables as he reads through the chapter in order to appreciate better the distinguishing features of each model.

In selecting models for inclusion in this chapter, the author decided for analytical purposes to treat the principal variations of each of several basic models separately. That is, the list of thirty includes many models that are, along with one or two other models on the list, variations of the same basic model. The three models of the firm under perfect competition, the two Baumol models, the two Marris models, the two Vickers models, the three Lintner models, the Leland quantity-setting and price-setting models, and the two Meyer models could in each case be thought of as variants of a basic model. In the discussion below, each of these variants is treated separately and the characteristics of each are summarized separately, but care is also taken to indicate the relationships that exist among the variants of each basic model.

In this writer's opinion, treating these variants as separate models helps clarify the analysis. In some cases, for example the Baumol and Marris models, the two versions were in each case developed sequentially, and treating the versions separately has the advantage of making the development of the basic model stand out more clearly. In other cases, for example the Leland quantity-setting and price-setting models, the two variants explore and compare the economic implications of two different types of behavior on the part of the firm. In yet other cases, for example the Vickers, Lintner, and Meyer models, the variations of the basic model represent successive modifications that relax restrictive assumptions — and thereby make the basic model more realistic.

Treating the versions of each of these models separately (as the authors did originally) has the advantage of bringing out the effects of the restrictive assumption(s) more clearly.

The models discussed in this chapter are presented in more or less chronological order, beginning with the traditional models of the firm, and then following the evolution of the theory of the firm through the more sophisticated treatments given to the objectives of the firm, to the links between the firm's financial decisions and its operating decisions, to the behavior of the firm under uncertainty, to the behavior of the firm in disequilibrium, and to the behavior of the firm in a multiperiod context. Each of the models is carefully set out and analyzed, and the contributions of each to the theory of the firm are evaluated in terms of the aforementioned attributes.

The remainder of this section discusses first, the analytical framework that is used throughout the chapter to characterize and to assess the contributions of the thirty models, and second, the sources and uses of the accounting information that will be needed in later sections of the chapter. In connection with the latter, the accounting information is related to the concepts of 'capital' and 'profit' that were defined in chapter one, as well as to other concepts, such as 'debt' and 'equity', that are of critical importance in models that incorporate a role for the firm's financial policies.

2. Characterization of the Models of the Firm

Each of the models discussed in sections B through L is classified on the basis of the firm's supposed objective(s) as *traditional*, *managerial*, *behavioral*, or *modern traditional*. In the traditional models, as indicated in table II-34, the objective of the firm is to maximize total profit. In the managerial models its objective is to maximize total sales, the rate of growth, or some (utility) function of variables that contribute to the satisfaction of the firm's managers. In the behavioral models there is no single objective; the firm exhibits satisficing behavior and seeks to meet several goals simultaneously. In the modern traditional models the firm maximizes the stock market value of the firm, the share price, or some other quantity that reflects the utility of the firm's shareholders. As shown in section F, the modern traditional objective functions can be interpreted as attempts to reformulate the traditional objective of profit maximization, where the common theme is the firm's supposed desire to maximize the welfare of the firm's owners.

Within these classifications, the models discussed below are characterized according to their economic content, as shown in table II-34, and also according to their mathematical form and to the mathematical techniques used in their solution, as shown in table II-35. The latter characterization includes the following: the nature of the optimization problem (i.e. static or dynamic, as described in chapter one) in which form the model was expressed; the existence of nonlinearities; the nature of the constraints (or the lack of constraints other than nonnegativity constraints); and the nature of the solution technique(s) employed in analyzing the model. Since this mathematical characterization involves concepts that are likely to be familiar to the reader,[1] the remainder of this subsection discusses the economic characterization only.

The economic content of each of the models of the firm discussed below is distinguished according to the objective(s) of the firm assumed in the model; the model's treatment of the firm's financial decisions; whether or not uncertainty is permitted; whether or not the behavior of the firm in disequilibrium is considered; and the treatment given the behavior of the firm over time. The objectives of the firm have been discussed. The remainder of this subsection deals with the remaining four elements of the characterization.

As discussed below, the traditional models do not incorporate financial considerations explicitly, but rather, they leave the role of finance subsumed within the general equilibrium analysis of a market economy. Most of the managerial and modern traditional models treat finance explicitly, although only the Baumol growth model, the Herendeen model, and Leland's managerial model explicitly consider both debt financing and external equity financing.

As indicated in table II-34, the traditional models and the managerial models assume away the existence of uncertainty. The assumption of certainty simplifies these models to a great extent, though at the cost of abstracting from a factor that can exert a large influence on the behavior of the firm. Where uncertainty is permitted, it is handled within either of two frameworks: the mean-variance framework or the time-state-preference framework. The mean-variance framework is discussed in section J and the time-state-preference framework is described in section K.

The third factor that needs to be discussed is the question of equilibrium and disequilibrium. The economic concept of *equilibrium* indicates a state of affairs in which opposing economic forces are in balance and in which there is consequently no tendency for change.[2] By way of contrast, *disequilibrium* denotes the state of affairs in which equilibrium has not been attained. For example, a market is said to be in equilibrium when quantity demanded equals quantity supplied at the prevailing price, and a firm is said to be in equilibrium when there is no tendency for it to alter its policies. Only the Meyer and Wong models and Lelands' managerial model explicitly consider disequilibrium. In the Meyer models disequilibrium occurs in the product markets served by the models' monopolist when quantity supplied falls short of quantity demanded at the established market price. The Meyer models deal with disequilibrium within a single period context, whereas the Wong model and Leland's managerial model deal with disequilibrium within a multiperiod context, as discussed next.

The fourth factor that needs to be considered is the model's treatment of the behavior of the firm over time. The first consideration in this regard is the question of the *time span* covered by the model. The traditional models are all single period models,[3] whereas most of the others are multiperiod models.[4] For multiperiod models of the firm there are questions regarding the *existence* of a multiperiod equilibrium for the firm,[5] the *stability* of the multiperiod equilibrium (if one exists) for the firm,[6] and the *dynamics* of the firm, i.e. exactly what its true time path looks like. The Baumol and Marris growth models and the Herendeen model are concerned with steady state equilibrium growth paths. That is, in their models the firm selects an equilibrium growth rate and expands forever at that rate, unless changes in what Marris calls the super-environment[7] disturb the equilibrium. The other multiperiod models are also concerned with characterizing the multiperiod equilibrium of the firm, although these others do not assume steady state growth. Only the Wong model and Leland's managerial model explicitly consider the stability question. In the Wong model the equilibrium question involves the size of the firm's capital stock. Equilibrium is attained once the capital stock of optimum size has been accumulated, but until the optimum has been reached, the firm pursues the dividend policy that leads it as rapidly as possible toward its optimum size (e.g. if the current capital stock is below the optimum, no dividends are paid in order that total profit may be used to increase the firm's capital stock). In Leland's managerial model the equilibrium question involves profit maximization, and with his model Leland establishes conditions under which the firm's optimal current policies converge to profit maximization as the firm's planning horizon becomes infinite.

For the convenience of the reader, the characterization of each model is summarized in a separate table that gives the classification of the model; states the firm's objectives and constraints; identifies the variables in the model as exogenously determined or as endogenously determined and also identifies the firm's decision variables; states the parameters in the model; characterizes the model's treatments of finance, of certainty/uncertainty, of equilibrium/disequilibrium, and of the behavior of the firm over time; and gives the type of mathematical model (i.e. mathematical programming problem or optimal control problem) and the solution technique(s) employed in the analysis. Tables II-4 through II-33 are provided below for this purpose.

3. The Typical Firm's Financial Statements

This section describes the sources and uses of the accounting information called for in many of the models discussed in this chapter. The three main sources of accounting information are the balance sheet, the income statement, and the statement of retained earnings.[8] An example of the first source is given in table II-1 and examples of the second and third sources are shown in tables II-2 and II-3, respectively. Each of these sources of information is discussed below.[9]

The typical firm's *balance sheet* is illustrated in table II-1.[10] The balance sheet summarizes the financial position of the firm at a particular point in time.[11] The balance sheet shows the total assets of the firm, which represents the total investment that has been made in the firm. The assets portion of the balance sheet shows the structure of this investment, i.e. what portion is in the form of current assets, which include cash and items that are expected to be converted into cash or consumed within the normal operating cycle (normally one year), what portion is in the form of fixed assets, and what portion of total assets is in some other form, such as intangible assets, land, or long-term investments in the stock of other corporations. Fixed assets include plant and equipment, valued at historical cost less accumulated depreciation, and correspond to what was defined in section C of chapter one as *fixed capital*. Total fixed assets, then, is a dollar measure of the firm's (physical) fixed capital resources. It represents the difference between the value of these assets at the time of purchase and accumulated depreciation, which is the accountant's measure of the 'wearing out' of these physical productive resources.[12]

Table II-1 The Typical Firm's Balance Sheet

Assets			
Current Assets:			
Cash		xxx	
Accounts receivable (net) ...		xxx	
Inventories		xxx	
Total current assets			xxx
Fixed Assets:			
Plant	xxx		
Less accumulated depreciation	xxx	xxx	
Equipment	xxx		
Less accumulated depreciation	xxx	xxx	
Total fixed assets			xxx
Other Assets			xxx
Total Assets			xxx
Liabilities			
Current Liabilities:			
Accounts payable	xxx		
Short-term notes payable	xxx		
Total Current Liabilities		xxx	
Long-term Liabilities:			
Lont-term notes payable	xxx		
Bonds payable	xxx		
Total long-term liabilities		xxx	
Total Liabilities			xxx
Stockholders' Equity			
Contributed Capital:			
Preferred stock	xxx		
Common stock	xxx		
Total contributed capital		xxx	
Retained Earnings		xxx	
Total stockholders' equity			xxx
Total Liabilities and Stockholders' Equity ..			xxx

The balance sheet also shows the structure of the ownership claims on these assets. The liabilities-stockholders' equity portion of the balance sheet shows how these ownership claims are distributed between creditors and stockholders. It is a basic accounting identity that the sum of these ownership claims must equal total assets. Therefore, in table II-1, 'total liabilities and stockholders' equity' must equal 'total assets'. The former represents the total *money capital* employed in the firm, so that this fundamental accounting identity says simply that the total investment made in the firm must equal the total money capital employed. In other words, the two portions of the balance sheet represent different perspectives on the firm's financial structure.

The liabilities-stockholders' equity portion of the balance sheet shows the distribution of ownership claims between *debt* and *equity*. The distinction between debt and equity, as made by most economists, differs from the accounting distinction between liabilities and stockholders' equity. Debt includes preferred stock in addition to total liabilities. The reason for including preferred stock in debt, rather than in equity, is that preferred stock is, in some important respects, very much like a bond.[13] In particular, holders of preferred stock are paid a specified percentage return; all dividends currently owed holders of preferred stock must be paid before any dividends on common stock can be paid; holders of preferred stock have preference in the distribution of assets over holders of common stock in the event the firm goes bankrupt; and holders of preferred stock often are not given the right to vote at stockholder meetings. Adding preferred stock to total liabilities gives debt, which is that portion of the firm's money capital with which are associated (what may be treated as) fixed interest obligations. Stockholders' equity less preferred stock represents what the economist calls equity. Holders of equity are the residual claimants on the assets of the firm in the event the firm goes bankrupt. They are also the individuals most directly affected when the firm varies its dividend policies, and they are the persons to whom a takeover raider would have to pitch its offer.

The second source of accounting information, the *income statement*, is illustrated for a typical firm in table II-2. The income statement summarizes the profit performance of the firm for a specific period of time.[14] Two entries in table II-2 are particularly important. The first is *net operating income*, which is obtained by subtracting from net sales — i.e. gross sales revenue less an allowance for returned items — the costs of operating the business, including a dollar measure of depreciation. It should be emphasized that these dollar cost figures do not necessarily equal the opportunity costs of the respective inputs. Net operating income is the accountant's measure of the excess of revenue over costs generated from the firm's operations that can be used to pay debt holders and equity holders (after allowance for profit taxes, of course).

Table II-2 The Typical Firm's Income Statement

Total Sales (net)		xxx	
Less operating costs:			
Variable factor costs	xxx		
Selling expense	xxx		
Administrative salaries	xxx		
Research and development expense	xxx		
Depreciation	xxx		
Other operating expenses	xxx		
Total operating costs		xxx	
Net Operating Income			xxx
Less interest expense on notes payable and on bonds payable ..			xxx
Income before tax			xxx
Less income tax expense			xxx
Net Income			xxx

The second important entry in table II-2 is *net income*, which is obtained by subtracting interest expense[15] and income tax expense from net operating income. The entries 'income before tax' and 'net income' in table II-2 are generally referred to as 'pretax profit' and 'aftertax profit', respectively.[16] As discussed in chapter one, the accounting notions of profit differs from the economists'. In particular, the operating costs in table II-2 would have to be the true opportunity costs of the resources employed, depreciation would have to be the true physical wearing out of the firm's plant and equipment, and the interest expense would have to be the true opportunity cost of the firm's total assets in order for net income to be consistent with what the economist calls profit.

The third source of accounting information, the *statement of retained earnings*, is illustrated for a typical firm in table II-3. The statement of retained earnings explains the increase or decrease in retained earnings for the period covered by the firm's income statement. Net income can be used for either of two purposes. It may be distributed as dividends to the holders of the firm's preferred stock and common stock or it may be added to retained earnings and used to purchase additional assets. One of the firm's important policy decisions involves determining its payout ratio — i.e. what portion of net income will be paid out as dividends.

Table II-3 The Typical Firm's Statement of Retained Earnings

Beginning balance, retained earnings	xxx
Add net income for the year	xxx
Total	xxx
Less total dividends paid during the year	xxx
Ending balance, retained earnings	xxx

Throughout the chapter tables II-1, II-2, and II-3 are used to establish the accounting identities that are invoked, usually when an expression is needed for dividends paid in terms of the firm's policy variables. The distinction that has been made in this section, as well as in chapter one, between the economists' notions of capital, profit, etc., and the accountants' measurement of these stocks and flows does not hinder the analysis in later sections of this chapter. The same identities continue to hold — although the numbers are different and their interpretation is different — when the economists' notions of capital, profit, etc. are substituted for the accountants'.[17] In the remainder of this chapter the terms capital, profit, etc. are given their respective economic meanings, but the reader is asked to keep in mind the fact that, from a practical standpoint, measuring these stocks and flows involves certain difficulties and that these measurement problems cause some of the items reported in a firm's financial statements to differ in concept from the stocks and flows that appear in the various models of the firm discussed below.

4. Summary

This section has provided an overview of the chapter. It has also described the analytical framework that is used throughout the chapter to characterize and to assess the contributions of the models that are surveyed in the chapter, and it has also discussed the sources and uses of the accounting information that will be needed in the course of the survey.

The survey begins with a description of the traditional models, each of which places the individual firm within a different market structure: perfect competition and monopoly (the models in section B), duopoly (Cournot's model in section C), oligopoly (the kinked demand curve model in section D), and monopolistic competition (Chamberlin's model in section E). The remainder of the chapter discusses the modern revisions to the theory of the firm. Each of the modern models of the firm differs from the traditional models in at least one of the following four respects: (i) the nature of the firm's objective(s) (for example, do firms exhibit some sort of maximizing behavior, and if so, what is it they seek to maximize: shareholders' utility, managers' utility, or some other quantity?); (ii) the extent to which financial considerations are allowed for explicitly in the model (for example, of what significance is the manner in which firms finance their activities?); (iii) the treatment of uncertainty (for example, some models explicitly allow for uncertainty, while other models ignore it altogether); (iv) the treatment of disequilibrium questions (for example, some models consider the behavior of the firm in disequilibrium, while others are concerned with equilibrium exclusively); and (v) the treatment of time (for example, some models treat the firm within a single period, while others deal with the firm within a multiperiod steady state framework, while yet others permit non-steady state growth within the multiperiod context).

B. PERFECT COMPETITION AND MONOPOLY: OPPOSITE ENDS OF THE SPECTRUM

Traditional economic theory distinguishes four models of the firm, one for each of the following market structures: perfect competition, monopoly, oligopoly, and monopolistic competition. As described in section C of chapter one, perfect competition can be interpreted as a form of market structure within which firms are so numerous that no single firm can affect the market price by altering its own behavior, and in which all firms in the industry therefore act as price takers. In contrast, within the other three market structures firms are not passive acceptors of environmental information. Under these alternative market structures firms possess varying degrees of market power, i.e. each acts to some extent as a price maker. Under monopoly there is a single firm that produces and sells the entire output of some commodity;

the monopolist treats the market demand curve as the demand curve for his product and sets the price that maximizes total profit. Under oligopoly there are several producers who compete with one another; each must weigh the potential reactions of rivals before altering the price of its product. Under monopolistic competition there are numerous producers who, due to differences in the products they sell, enjoy some limited price-setting ability in their respective shares of the market. Thus, in terms of market power, as measured by the number of firms serving the market, perfect competition and monopoly are seen to be opposite ends of the spectrum with oligopoly and perfect competition falling somewhere in between.

The traditional models share several common features. Each postulates that the firm's objective is to maximize total profit. Each treats the firm within a single time period. Each assumes that market demand curves, production functions, and the supply curves for factors of production are known with certainty. Each is concerned with characterizing the nature of equilibrium for the firm. Within each the problem of financing the operations of the firm is not considered explicitly as part of the model of the individual firm, but rather, is subsumed within the general equilibrium analysis of a market economy.[18] Where the traditional models differ is in the market structure within which the firm is assumed to operate — and by implication, in the degree of market power the firm is assumed to possess.

The traditional models of the firm are discussed in sections B through E. It should be noted at the outset that

> The model of the firm in [traditional economic] theory is not, as so many writers believe, designed to serve to explain and predict the

> behavior of real firms; instead, it is designed to explain and predict changes in observed prices (quoted, paid, received) as effects of particular changes in conditions (wage rates, interest rates, import duties, excise taxes, technology, etc.).[19]

In the traditional theory the firm is a theoretical construct, the purpose of which is to explain and predict the behavior of prices.[20] According to Machlup, to criticize the traditional models for failing to explain the behavior of real-world firms would be unfair. Yet, because several economists have claimed to have provided empirical support for the neoclassical theory of the firm (and others have based further theoretical work on these empirical findings)[21] and because these models of the firm form the basis for the discussion of the modern theory of the firm in economics textbooks, it is essential that a careful review of the traditional models be undertaken before a presentation of the modern 'revisions' is begun.[22]

1. Perfect Competition

The bedrock of the traditional theory of the firm is the model of perfect competition. To the economist it has long represented the ultimate in competitiveness, and to the welfare economist the performance of the perfectly competitive firm has stood as the norm against which the performance of firms should be measured; but strangely, were a perfectly competitive industry to be found in the real world, it would undoubtedly appear to most observers devoid of any competitive activity. Perfect competition requires that the following five conditions be met:[23]

(i) Each firm takes the ruling market price as given. As indicated in section C of chapter one, firms might act in this passive manner because each produces an output so small relative to the total output of the industry that it is unable to influence the market price by expanding or contracting the quantity of goods it offers for sale.

(ii) The products of all firms in the industry are homogeneous; consumers view the products as perfect substitutes; and product differentiation is impossible. Thus, there is no need either for consumers to shop around or for producers to engage in advertising.

(iii) New firms are free to enter the industry and to produce identical products under the same conditions as existing producers; there are no barriers to entry, such as financial restrictions or coercion on the part of existing producers; and existing producers are free to leave the industry to avoid losses. Thus, no firm will be able to maintain 'abnormal' profits because high profits will quickly attract new entrants who will eventually drive profits down to the 'normal' level.

(iv) Firms act independently in deciding whether to expand or to contract output and whether to enter or to leave an industry. Thus, there is no collusion among firms.

(v) All buyers and sellers in the market have complete knowledge as to the bid and offer prices of the other market participants. Thus, insider information is prohibited and no participant is ever taken advantage of in a trade due to insufficient knowledge.

Within a perfectly competitive market there is neither price competition, since all firms are price takers, nor non-price competition (such as advertising, differences in product design, or special credit arrangements), since all products are homogeneous and each producer can sell as much of his output as he wishes at the ruling market price. Since all firms pay identical prices for their inputs and have the same production function, there can be no cost competition either. The requirements for perfect competition are sufficiently restrictive that very few real-world markets come close to the ideal.[24]

Since competition in any one of the many forms it takes in the real world is absent in the model of perfect competition, one may reasonably ask why economists continue to profess an interest. Baumol argues that it is due to the model's tractability and the fact that once a set of (admittedly questionable) assumptions is granted, perfect competition automatically leads to an allocation of society's scarce productive resources that is in the Pareto sense optimal.[25]

To describe the perfectly competitive model of the firm it is best to proceed in the following manner. First, it will be assumed that the firm employs only two inputs, which will be denoted by L and K and which may be thought of as labor and capital, and produces a single output, which will be denoted by Q. For simplicity the symbols L, K, and Q will be used to represent both the identity of the resource (good) and the amount used (produced). The two-input-single-output case permits the use of geometry to further the exposition. Having explored this case, the model will then be generalized to permit n outputs and m inputs.[26] However, before explicitly considering the model of the firm under perfect competition it might be instructive to review from a mathematical standpoint the concepts of marginal productivity (of an input), average productivity, and the production function, all of which are basic to the theory of the firm.

In the case of two inputs and a single output the firm's production function is given by

$$Q = f(L,K) \; . \tag{1}$$

In the long run both inputs are variable. The partial derivatives of (1), $\partial f/\partial L$ and $\partial f/\partial K$, are called the marginal productivity of labor and the marginal productivity of capital, and are denoted by MP_L and MP_K , respectively. In the short run, by definition, the firm's plant and equipment — its stock of capital — is fixed, and if this fixed amount of capital is denoted by $\bar{K}$, then output varies as a function of labor only. The quantity $AP_L = \frac{f(L,\bar{K})}{L}$ is called the average product of labor. It is almost universally assumed that production functions exhibit the law of diminishing marginal productivity, which states that when one input is varied and all others are held fixed, the marginal productivity of the variable input will eventually decline. Figure II-1 shows the relationship between AP_L and MP_L , where the latter obeys the law of diminishing marginal productivity.[27]

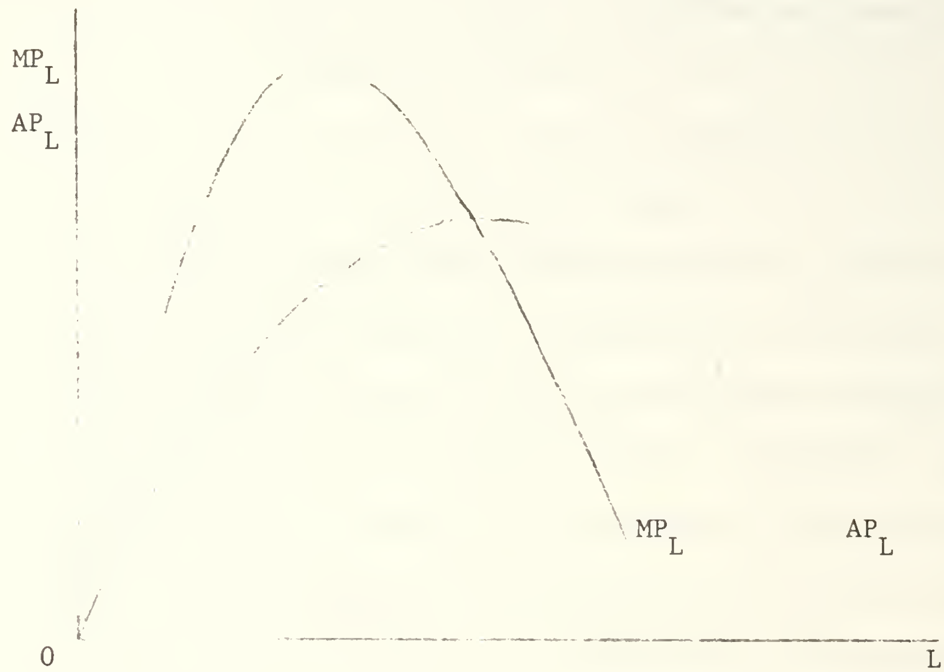


Figure II-1: Marginal Productivity and Average Productivity

The locus of combinations of L and K that yield a particular level of output, when combined with maximum technical efficiency, is called an isoquant. Denoting the particular level of output by Q_o , (1) becomes $Q_o = f(L, K)$. It is assumed that isoquants are continuous and convex to the origin as depicted by figure II-2.[28]

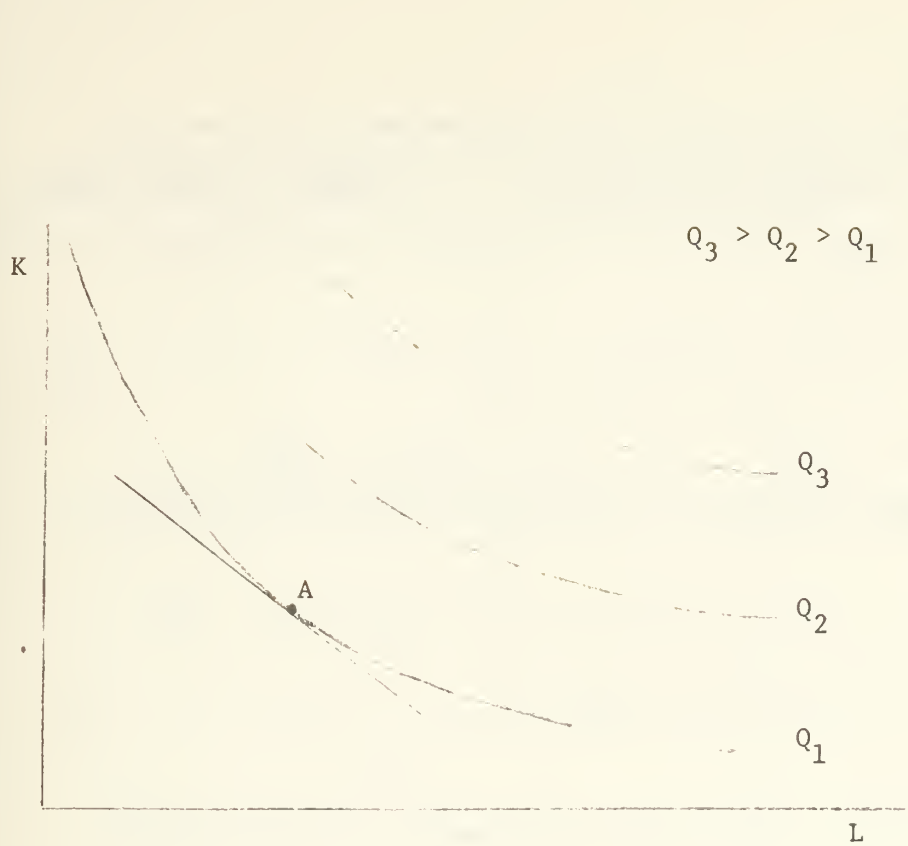


Figure II-2: Isoquants

The negative of the slope of an isoquant at a point, such as point A in figure II-2, is called the rate of technical substitution and is given by $RTS = - dK/dL$. The convexity of an isoquant means that the rate of technical substitution is diminishing as L is substituted for K. In words, as L is substituted for K, successively larger increments in L must be substituted for equal decrements in K in order to hold the level of output fixed.[29] Since within the feasible region of production an increase in the use of both inputs yields a greater output, isoquants further from the origin represent higher levels of output. Thus, in figure II-2 $Q_3 > Q_2 > Q_1$.[30]

When both product markets and factor markets are perfectly competitive, the firm takes factor and product prices as given. Let the unit prices of L, K, and Q be denoted by w, i, and p, respectively. The firm's total profit, π, is equal to total revenue, R, less total costs, C,

$$\pi = R - C \ , \tag{2}$$

where total revenue is equal to product price times the quantity of output,

$$R = p \cdot Q = p \cdot f(L, K) \ , \tag{3}$$

and where total cost is the sum of the payments to the two inputs,

$$C = wL + iK \ . \tag{4}$$

Substituting (3) and (4) into (2), the model of the perfectly competitive firm can be formulated as the following nonlinear programming problem:

$$\begin{aligned} &\underset{\{L,K\}}{\text{maximize:}} && \pi(L, K) = p \cdot f(L, K) - wL - iK \\ &\text{subject to:} && L \geq 0 \ , \quad K \geq 0 \ . \end{aligned} \tag{5}$$

In the short run K is fixed so that problem (5) simplifies to the following:

$$\begin{aligned} &\underset{\{L\}}{\text{maximize:}} && \pi(L) = p \cdot f(L, \overline{K}) - wL - i\overline{K} \\ &\text{subject to:} && L \geq 0 \ . \end{aligned} \tag{6}$$

First solving the short run optimization problem (6), the necessary conditions are the Kuhn-Tucker conditions:[31]

$$\left.\begin{aligned} \frac{d\pi}{dL} = p \cdot \frac{df}{dL} - w \leq 0 \\ \frac{d\pi}{dL} \cdot L = (p \cdot \frac{df}{dL} - w) \cdot L = 0 \\ L \geq 0 \end{aligned}\right\} \qquad (7)$$

Since $df/dL = MP_L$, conditions (7) require that

$$p \cdot MP_L = w \quad \text{if} \quad L > 0 \qquad (8)$$

$$L = 0 \quad \text{if} \quad p \cdot MP_L < w \qquad (9)$$

The product $p \cdot MP_L$ is interpreted as the value of the marginal productivity of L , measured as its contribution to total revenue. Since w is the cost of an additional unit of L , condition (8) states that, if labor is employed, the profit-maximizing firm will continue to hire additional labor up to the point at which the value of the marginal productivity of the last unit hired, $p \cdot MP_L$, just equals the ruling wage, w . Condition (9) states that no labor will be hired unless its contribution to revenue exceeds its cost.

If $L > 0$, then problem (6) is unconstrained and the second order optimization condition is

$$\frac{d^2\pi}{dL^2} = p \cdot \frac{d^2 f}{dL^2} < 0 \quad , \tag{10}$$

which requires that profit be declining with respect to further applications of L to production.[32] Since $d^2f/dL^2 = \frac{d}{dL}(df/dL)$, the rate of change of the marginal productivity of L, the law of diminishing marginal productivity guarantees that (10) holds over some range of L. What condition (10) means is that the profit maximizing producer would hire L_2 rather than L_1 units in figure II-3.

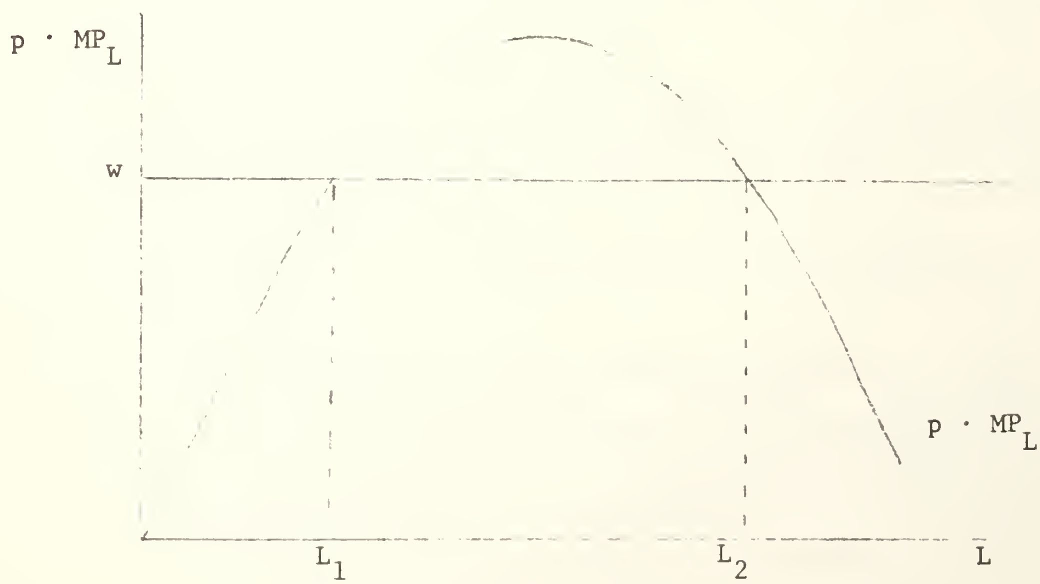


Figure II-3: Value of Marginal Productivity

Dividing both sides of $p \cdot MP_L = w$ by MP_L yields the condition[33]

$$p = \frac{w}{MP_L} \quad . \tag{11}$$

The ratio on the right-hand side of (11) is interpreted as the cost of an additional unit of L divided by the additional output due to the application in production, of an additional unit of L . This ratio is interpreted as marginal cost, or the cost of an additional unit of output. Denoting marginal cost by MC , equation (11) can be rewritten as the familiar

$$p = MC \tag{12}$$

rule for profit maximization. The profit maximizing producer should continue to expand output until the point at which price equals marginal cost. Since $MC = \frac{w}{MP_L}$ and since MP_L has the inverted U-shape depicted in figure II-1 while w is constant, it follows that MC will have the U-shape depicted in figure II-4. For the same reason that L_2 was preferred to L_1 in figure II-3, output level Q_2 is preferred to output level Q_1 in figure II-4, even though $p = MC$ for both.

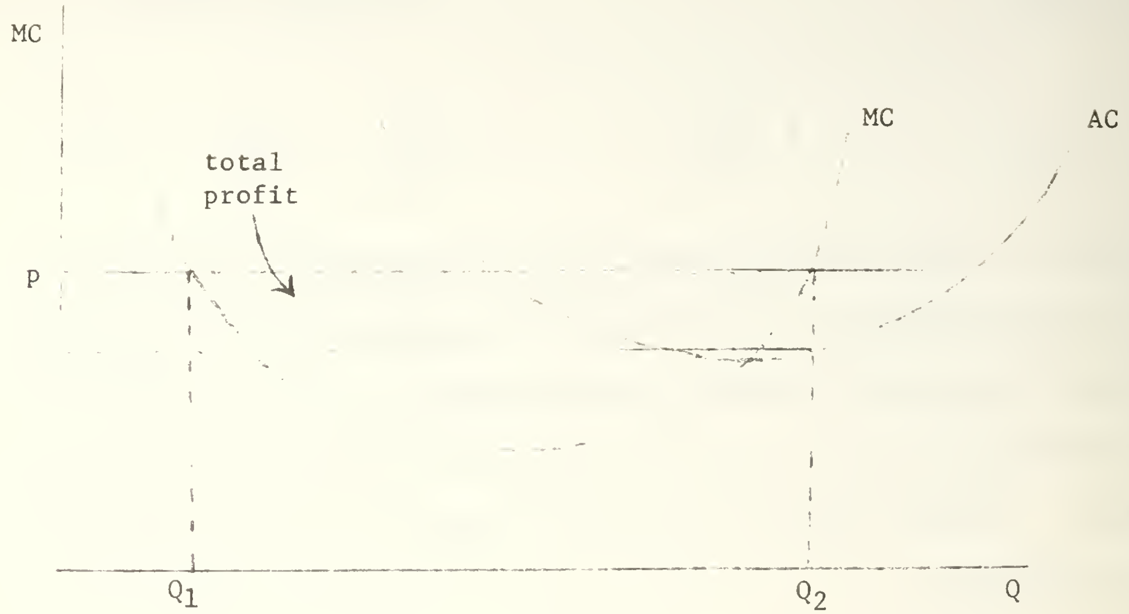


Figure II-4: Short Run Optimum for the Perfectly Competitive Firm

As long as inputs are used by the firm in the most technically efficient manner, the foregoing results demonstrate that maximizing profit in the factor market and maximizing profit in the product market are really just two ways of characterizing the same optimization process. This is the consequence of the fact that the solution to problem (6) yields both condition (8) for optimality in the factor market and condition (12) for optimality in the product market.

Turning to the long run, in which, by definition, the firm's capital stock is variable, the model of the perfectly competitive firm can be formulated as problem (5). The Kuhn-Tucker conditions for problem (5) are analogous to (7), with one set of conditions each for

L and K and with partial derivatives in place of simple derivatives:

$$\left.\begin{array}{ll} \dfrac{\partial \pi}{\partial L} = p \cdot \dfrac{\partial f}{\partial L} - w \leq 0 & \dfrac{\partial \pi}{\partial K} = p \cdot \dfrac{\partial f}{\partial K} - i \leq 0 \\ \dfrac{\partial \pi}{\partial L} \cdot L = (p \cdot \dfrac{\partial f}{\partial L} - w) \cdot L = 0 & \dfrac{\partial \pi}{\partial K} \cdot K = (p \cdot \dfrac{\partial f}{\partial K} - i) \cdot K = 0 \\ L \geq 0 & K \geq 0 \end{array}\right\} \quad (13)$$

With $L > 0$ and $K > 0$, conditions (13) lead to the optimality conditions

$$p \cdot MP_L = w \quad \text{and} \quad p \cdot MP_K = i \quad , \tag{14}$$

which are analogous to (8). Equations (14) can be interpreted as the requirement that each factor be hired up to the point at which the value of the marginal productivity of the last unit hired just equals its unit cost. Equations (14) can be rewritten to give

$$p = \frac{w}{MP_L} = \frac{i}{MP_K} \quad , \tag{15}$$

which once again can be interpreted as the requirement that price equal marginal cost. Condition (15) yields the well-known requirement that

$$\frac{MP_K}{MP_L} = \frac{i}{w} \quad . \tag{16}$$

Since $MP_K/MP_L = \frac{\partial f/\partial L}{\partial f/\partial K} = RTS$, condition (16) can be interpreted as the requirement that the profit maximizing firm combine inputs in such a way that the rate of technical substitution — the rate at which technology permits the factors to be substituted for one another in production — equals the ratio of the factor prices — the rate at which the market place permits the factors to be substituted for one another.

Condition (16) is better understood when it is recognized that the optimization problems (5) and (6) necessitate two simultaneous optimizations, one with respect to the product market and one with respect to the factor market(s). In order to earn maximum total profit, the firm must produce whatever level of output it selects at minimum cost. That is, the (simultaneous) determination of the optimal output level and the optimal input combination could be thought of as a two-step process. At step one, the firm determines the cost-minimizing combination of inputs and the associated total cost for each level of output. At step two, the firm selects the profit-maximizing level of output on the basis of both the demand conditions it faces and the cost relation determined at step one.

The first step in this process can be represented by a mathematical programming problem, and in the simple case of a single output and two inputs, it can also be represented geometrically. Consider the perfectly competitive firm in the long run. Both capital and labor are variable, and the firm's total cost is given by equation (4). For any particular level of output Q_o , the cost minimization problem can be expressed as the mathematical programming problem:

$$\begin{aligned} &\underset{\{L,K\}}{\text{minimize:}} && C = wL + iK \\ &\text{subject to:} && f(L, K) = Q_o \ , \end{aligned} \tag{17}$$

where for simplicity it is assumed that $L > 0$ and $K > 0$. This problem has been represented geometrically in figure II-5. The graph of $C = wL + iK$, where C is treated as a parameter, is linear and is called an isocost (equal cost) line. In order to produce Q_o units of output at minimum cost, the firm will find the input combination on isoquant Q_o that lies on the isocost line nearest the origin. In figure II-5 this occurs at the point A, where the isoquant and the isocost line are tangent, and hence, where the slopes of the curves are equal. Thus,

$$-\frac{w}{i} = \frac{dK}{dL} = -\frac{\partial f/\partial L}{\partial f/\partial K} = -\frac{MP_L}{MP_K} \ ,$$

which is the same as condition (16).[34]

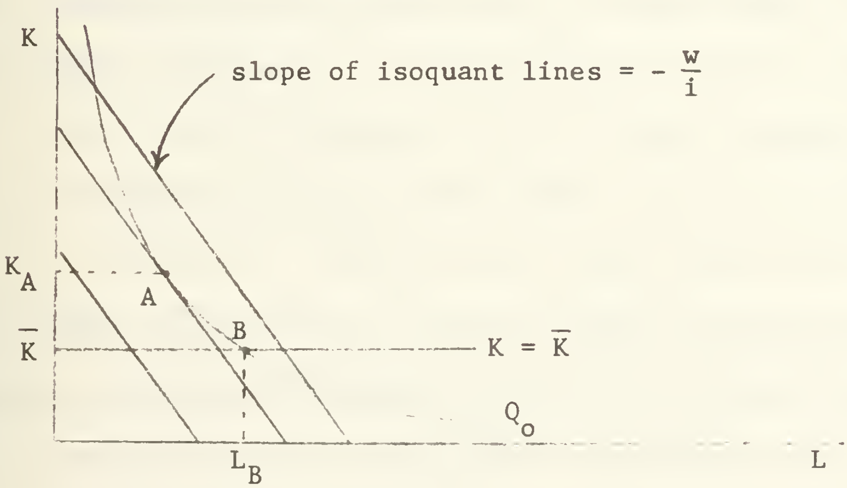


Figure II-5: Least Cost Combination of Inputs

If this procedure were followed for all feasible output levels, the set of ordered pairs $(Q, C(Q))$ would define a long run total cost function $C = C(Q)$, from which the long run marginal cost function $LRMC = C'(Q)$ could be computed by differentiation. A typical LRMC curve is shown in figure II-6 along with a typical LRAC curve, which is given by $LRAC = \frac{C(Q)}{Q}$. It should be emphasized that the LRAC curve, as just defined, gives the minimum average cost of producing each level of output when the plant is of optimum size for that particular output level. According to the traditional theory, the LRAC curve is U-shaped because of internal economies and diseconomies of scale which are caused either (i) by indivisibilities of the factors that give rise to a most efficient mix of men and machines, or (ii) by the increased difficulty of managing a larger organization that eventually outweighs economies resulting from the increased specialization that larger size makes possible (or (iii) by both of these factors reinforcing one another).

In the short run the firm's capital stock is fixed. The short run total cost function gives the total cost of producing each level of output, given this fixed stock of capital. The short run average cost function, SRAC, and the short run marginal cost function, SRMC, are constructed in a manner similar to that illustrated for the long run case. It should be noted that $LRAC < SRAC$ except for that level of output for which the current plant size is optimal. In terms of figure II-5, the firm's current plant size is $\overline{K}$, which is not optimal when $Q = Q_o$, so that $LRAC(Q_o) < SRAC(Q_o)$. Mathematically, the LRAC curve is the envelope of all the SRAC curves, of which there is one for each value of K . Each SRAC curve is tangent to the LRAC curve at the point at which the corresponding plant size is optimal, as in figure II-6. Note that for both the short run cost curves and the long run cost curves, marginal cost lies below (above) average cost when the latter is falling (rising).[35]

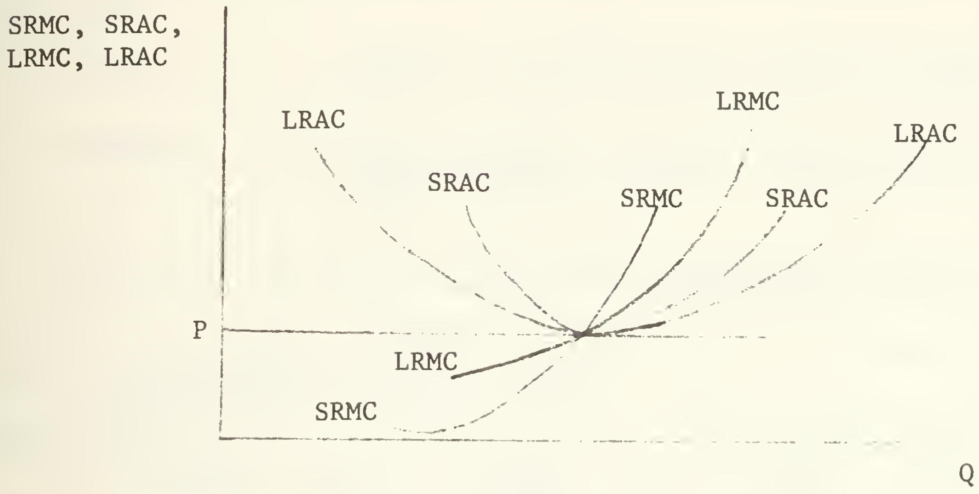


Figure II-6: Long Run Equilibrium for the Perfectly Competitive Firm

Given the short run and long run total cost functions, the profit maximizing firm's objective is to find the level of output Q that solves the problem

$$\text{maximize:} \quad \pi(Q) = p \cdot Q - C(Q) \ , \tag{18}$$

where $C(Q)$ represents the short run or long run total cost function depending on whether K is constant and where it is understood that in the short run version of problem (18) the plant size K is fixed, while in the long run version K is variable and $C(Q)$ gives the total cost of production when the optimum amount of capital K is employed. The first order condition for a solution to (18) is

$$p - C'(Q) = 0 \qquad \text{or} \qquad p = MC \ , \tag{19}$$

which is identical to conditions (12) and (15). The second order condition is

$$\pi''(Q) = - C''(Q) < 0 \qquad \text{or} \qquad C''(Q) = \frac{d}{dQ}(MC) > 0 \ . \tag{20}$$

Together (19) and (20) characterize the equilibrium position — as viewed from the product market — of the individual firm under perfect competition. According to (19) and (20), in order that the firm be in equilibrium, it is necessary that price equal marginal cost and that marginal cost be increasing. The short run case is depicted in figure II-4, where the firm earns positive profit equal to the area of the shaded rectangle.

By way of summarizing this section's discussion up to this point, the characteristics of the model of the firm under perfect competition, for the special case of one output and two inputs, are summarized in table II-4. The analytical framework on which the table is based was discussed above in section A.

Table II-4 Model Summary: Perfect Competition (1 output/2 inputs)

Class:	traditional (see (5), (6) in text)
Firm's Objective:	profit maximization
Constraints:	technological (embodied in the production function), nonnegativity constraints on the decision variables
Variables:	
Exogenous:	prices of inputs (w and i) and of output (p), capital stock ($\overline{K}$) in the short run model (6) only
Endogenous:	input level(s) (L in both models, K in the long run model (5) only), output level (Q), and total profit (π)
Decision:	labor input (L) in both models, capital input (K) in the long run model (6) only
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium
Time:	single period
Type of Model:	static optimization (nonlinear programming problem)
Solution Technique:	unconstrained optimization under assumption that solution is nontrivial; otherwise generalized Lagrange multipliers.

If the individual firm, which until now has been considered in isolation, is placed within a perfectly competitive market situation, it is possible to suggest the nature of market equilibrium. Over the long run positive profit cannot persist since, by assumption (iii) above, new firms are free to enter the industry. The effect of new entrants will be to reduce the market price p until it equals average cost. In long run market equilibrium, all firms earn zero economic profit, as depicted by figure II-6, and $p = SRMC = SRAC = LRMC = LRAC$.[36]

Having dealt with the two-input-single-output case the model of the profit-maximizing firm under perfect competition will now be generalized to permit n outputs and m inputs. The production function, which is most easily stated in implicit form, is

$$F(q_1 , \ldots, q_n , x_1 , \ldots, x_m) = 0 \quad , \tag{21}$$

where q_i denotes the output of the i-th product and x_j denotes the amount used of the j-th input, and where it is normally assumed that all first- and second-order partial derivatives are continuous and nonzero for all nontrivial solutions.[37] If all product and factor markets are perfectly competitive, then the firm takes all output prices and all input prices as given. Letting the price of each output be denoted by p_i and the price of each input be denoted by r_j, the firm's total profit, π, is given by

$$\pi = \sum_{i=1}^{n} p_i q_i - \sum_{j=1}^{m} r_j x_j \quad , \tag{22}$$

and the objective is to select q_i's and x_j's that maximize (22) subject to (21).[38] Stated more compactly in vector notation, the model of the firm is formulated as the following nonlinear programming problem:

$$\begin{array}{ll} \underset{\{\bar{q},\bar{x}\}}{\text{maximize:}} & \pi = \bar{q} \cdot \bar{p} - \bar{x} \cdot \bar{r} \\ \text{subject to:} & F(\bar{q} , \bar{x}) = 0 \quad , \end{array} \tag{23}$$

where $\overline{q}$ is the $1 \times n$ row vector of outputs, $\overline{p}$ is the $n \times 1$ vector of output prices, $\overline{x}$ is the $1 \times m$ row vector of inputs, and $\overline{r}$ is the $m \times 1$ vector of input prices.[39] Applying the method of Lagrange multipliers, the Lagrangian is

$$L_\lambda = \overline{q} \cdot \overline{p} - \overline{x} \cdot \overline{r} + \lambda \cdot F(\overline{q}\ , \overline{x})\ ,$$

where λ is the Lagrange multiplier, and the first-order conditions for an optimal solution are the following:

$$\begin{aligned} \frac{\partial L_\lambda}{\partial q_i} &= p_i + \lambda \frac{\partial F}{\partial q_i} = 0 \qquad i = 1, \ldots, n \\ \frac{\partial L_\lambda}{\partial x_j} &= - r_j + \lambda \frac{\partial F}{\partial x_j} = 0 \qquad j = 1, \ldots, m \\ \frac{\partial L_\lambda}{\partial \lambda} &= F(\overline{q}, \overline{x}) = 0 \end{aligned} \tag{24}$$

Taking any two of the first n equations, solving each for λ, equating the two expressions, and rewriting gives

$$\frac{p_k}{p_\ell} = \frac{\partial F/\partial q_k}{\partial F/\partial q_\ell} = - \frac{\partial q_\ell}{\partial q_k} \qquad \begin{matrix} 1 \leq k \leq n \\ 1 \leq \ell \leq n \end{matrix} \tag{25}$$

where the last equality follows from the implicit function rule. The quantity $- \partial q_\ell / \partial q_k$ is called the rate of product transformation, and it is interpreted as the rate at which one product can be traded off for another when the firm's production function, all inputs, and all other outputs are held fixed. Condition (25) is interpretable as requiring that, for every pair of outputs, the rate of product transformation equal the ratio of the prices of the two goods.[40] Next, taking any two from the second set of m equations and performing the same operation as that done to obtain (25) yields

$$\frac{r_k}{r_\ell} = \frac{\partial F/\partial x_k}{\partial F/\partial x_\ell} = -\frac{\partial x_\ell}{\partial x_k} \qquad \begin{matrix} 1 \leq k \leq m \\ \\ 1 \leq \ell \leq m \end{matrix} \tag{26}$$

Condition (26) is equivalent to condition (16) for the case of two inputs and is interpretable as requiring that, for every pair of inputs, the rate of technical substitution equal the ratio of the prices of the two inputs. Finally, taking any one of the first n equations and any one of the second m equations and performing the same manipulation gives

$$\frac{r_k}{p_\ell} = -\frac{\partial F/\partial x_k}{\partial F/\partial q_\ell} = \frac{\partial q_\ell}{\partial x_k} \quad \text{or} \quad r_k = p_1 \frac{\partial q_\ell}{\partial x_k} \qquad \begin{matrix} 1 \leq \ell \leq n \\ \\ 1 \leq k \leq m \end{matrix} \tag{27}$$

Condition (27) is equivalent to condition (14) for the case of two inputs and one output and is interpretable as requiring that the value of the marginal product of each input with respect to each output equal the input price. Collectively, conditions (25) - (27) are the necessary conditions[41] for profit maximization for the individual firm using m inputs to produce n outputs when all markets are perfectly competitive. Collectively, conditions (25) - (27) characterize the equilibrium position of the individual firm under perfect competition; if the firm is to be in equilibrium, it is necessary that its input and output levels satisfy these three sets of conditions.

In terms of the analytical framework set out in section A, the characteristics of the model are summarized in table II-5.

Table II-5 Model Summary: Perfect Competition (n outputs/m inputs)

Class:	traditional (see (23) in text)
Firm's Objective:	profit maximization
Constraints:	technological (embodied in the production function), nonnegativity constraints on the decision variables
Variables:	
Exogenous:	prices of inputs (r_j) and of outputs (p_i)
Endogenous:	input levels (x_j), output levels (q_i), and total profit (π)
Decision:	input levels (x_j) and output levels (q_j)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium
Time:	single period
Type of Model:	static optimization (nonlinear programming problem)
Solution Technique:	unconstrained optimization under assumption that solution is nontrivial; otherwise generalized Lagrange multipliers.

With the model of the individual firm under perfect competition having been described in mathematical terms, the comments made at the beginning of this section should be more meaningful. The model is concerned with characterizing the equilibrium position of the individual firm, and it is partly for this reason that the model appears devoid of any competition. But even in disequilibrium, the perfectly competitive firm would not, by assumption, engage in advertising or any other form of nonprice competition. Yet, there is considerable support for the view that, in the real world, the various forms of nonprice competition can play a significant economic role, as for example, advertising and sales promotional efforts that disseminate information and product differentiation that involves improvements in product quality and service.[42] The perfectly competitive firm is a price-taker, a passive participant in the economy. In contrast, Schumpeter has emphasized the importance of the active innovating role that firms play[43] — a role that is not in the repertoire of the perfectly competitive firm. In addition, the model of the firm under perfect competition does not treat the role of finance explicitly and it assumes away uncertainty, thereby abstracting from two significant real-world complications. For example, empirical evidence indicates that small firms, which,according to the standard economic realization of perfect competition, would be typical of a perfectly competitive industry, are constrained in their spending on research and development (and more so than larger firms) due to financial limitations.[44] So, whether or not the perfectly competitive model was ever intended to explain the behavior of actual firms, in the words of Baumol:

> It is likely to be ill-advised and inappropriate to hold perfect competition up as a model for the structure and conduct of industry in practice. Unfortunately, this precaution is not always carefully observed.[45]

But before a more acceptable standard can be devised, it is, in the opinion of this writer, necessary that economists develop a better understanding of the way in which actual business enterprises operate.

2. Linear Programming Formulation[46]

The model of the firm under perfect competition, or under any other market structure for that matter, could be formulated as a linear programming problem provided the objective function and the constraints could be made linear. Under perfect competition the objective function in problem (23) is linear in the decision variables because both output prices and input prices are taken as given, and hence, are treated as constants. It remains to be shown that the constraint set can be linearized. How this is accomplished is discussed below.

Before discussing the linear programming formulation, some conceptual differences between it and the nonlinear programming formulation (23) should be noted. Lagrange multipliers were used to characterize a solution to problem (23), that is, to develop rules (25) - (27) for determining the optimal usage of inputs and the optimal levels of output. In using linear programming the purpose is usually to obtain, rather than characterize, a solution. Linear programming is a technique for determining the optimal allocation of fixed amounts of inputs, and the solution to the linear programming problem gives the specific optimal allocation of these stocks of resources. Since there are fixed supplies of some inputs, the linear programming problems discussed in this section are best thought of as short run optimization problems.

In this subsection two types of linear programming problems are discussed. In the first there are n outputs and m inputs, but only one technique available for producing each output. In the second there is only one output, but there are n productive techniques available and again there are m inputs. These relatively simple problems bring out the distinguishing features of the linear programming formulation.[47]

In the first case the firm produces n outputs, the amount of each being denoted by q_j , $j = 1 , \ldots, n$. For each unit of the j-th output the firm sells it earns a profit[48] of c_j . The total profit it earns is the sum of the amounts earned on sales of the different products,

$$\pi = \sum_{j=1}^{n} c_j q_j \quad . \tag{28}$$

Assume there are m inputs, where $m < n$, and denote the amount available of each input by b_i , $i = 1 , \ldots, m$. Further assume that each unit of the j-th product requires a_{ij} units of the i-th input, where a_{ij} is a constant $1 \leq i \leq m$, $1 \leq j \leq n$. Then since the firm cannot use more than the available amounts of the m inputs, the optimal output levels must satisfy the m constraints

$$\left.\begin{array}{l} a_{11}\, q_1 + a_{12}\, q_2 + \ldots + a_{1n}\, q_n \leq b_1 \\ a_{21}\, q_1 + a_{22}\, q_2 + \ldots + a_{2n}\, q_n \leq b_2 \\ \quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \\ a_{m1}\, q_1 + a_{m2}\, q_2 + \ldots + a_{mn}\, q_n \leq b_m \quad . \end{array}\right\} \tag{29}$$

In addition, the optimal output levels must satisfy the nonnegativity constraints

$$q_j \geq 0 \ , \quad j = 1 , \ldots, n \quad . \tag{30}$$

The profit maximizing firm's objective is to maximize (28) subject to (29) and (30), which is easily carried out with the aid of the simplex algorithm. For convenience the optimization problem can be stated more compactly using vector-matrix notation as

$$\begin{aligned} \text{maximize:} \quad & \pi = \bar{c} \cdot \bar{q} \\ \text{subject to:} \quad & \bar{A} \cdot \bar{q} \leq \bar{b} \\ & \bar{q} \geq \bar{0} \quad , \end{aligned} \tag{31}$$

where $\bar{c}$ is the $1 \times n$ vector of contribution margins, $\bar{q}$ is the $n \times 1$ vector of outputs, $\bar{A}$ is the $m \times n$ matrix of fixed production coefficients, and $\bar{b}$ is the $m \times 1$ vector of resource availabilities. As a consequence of the extreme point theorem, the optimal solution to problem (31) can have no more than m decision variables at a positive level.[49] Thus, at least $n-m$ of the output levels are zero, i.e. it is not optimal for the firm to produce all n goods.[50] In terms of the analytical framework set out in section A, the characteristics of the model are summarized in table II-6.

Table II-6 Model Summary: Perfect Competition (linear programming formulation)

Class:	traditional (see (31) in text)
Firm's Objective:	profit maximization
Constraints:	technological (embodied in the fixed production coefficients), resource availabilities, and nonnegativity constraints on the decision variables
Variables:	
Exogenous:	contribution margins (vector $\bar{c}$), production coefficients (matrix $\bar{A}$), and stocks of resources (vector $\bar{b}$)
Endogenous:	input and output levels (vector $\bar{q}$), total profit (π)
Decision:	input and output levels (vector $\bar{q}$)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	determination of equilibrium input and output levels
Time:	single period
Type of Model:	static optimization (linear programming problem)
Solution Technique:	simplex method could be used.

As a consequence of the fixed production coefficients, the production function represented by the matrix A exhibits constant returns to scale. The optimal solution to (31) will be of the form

$$\bar{q}_b = \bar{B}^{-1} \cdot \bar{b} ,$$

where $\bar{B}$ is the matrix formed by the columns of $\bar{A}$ that remain in the basis at the final step of the simplex algorithm. If all inputs are increased by the same proportion k , while the coefficients c_j and a_{ij} are not altered, the basic variables do not change and the new optimal solution is

$$\bar{q}_b^* = \bar{B} \cdot (k \cdot \bar{b}) = k \cdot \bar{B} \cdot b = k \cdot \bar{q}_b \quad .$$

Thus, outputs increase in the same proportion k and returns to scale are constant. This result marks one of the differences between the linear and the nonlinear formulations: in the nonlinear case the production function could be made to exhibit either increasing or decreasing returns to scale, as well as constant returns to scale.

The dual linear programming problem to (31) is also of economic interest. Denoting by $\bar{w}$ the $1 \times m$ vector of dual variables, the dual of (31) is

$$\begin{aligned} &\text{minimize:} && \bar{w} \cdot \bar{b} \\ &\text{subject to:} && \bar{w} \cdot \bar{A} \geq \bar{c} \qquad (32) \\ & && \bar{w} \geq \bar{0} \quad . \end{aligned}$$

The dual variables serve the same mathematical role and have the same economic interpretation as the Lagrange multipliers in the nonlinear formulation. The variable w_j gives the rate of change of total profit, π , with respect to an increase in b_j . In symbols, $w_j = \partial\pi/\partial x_j$, and w_j is called the implicit price of an additional unit of input j . In the nonlinear formulation of the previous subsection for each input that is in fixed supply a constraint of the form

$g_i(x_1, \ldots, x_m) \leq b_i$ must be added and an additional Lagrange multiplier λ_i must be introduced when the problem is solved. This Lagrange multiplier is also interpreted as an implicit price $\lambda_i = \partial\pi/\partial b_i$.[51]

The second type of linear programming problem that is of interest is that in which the firm produces one output and has a choice of n productive techniques that it may use individually or in combination. In the linear programming formulation the notion of a production function is replaced by the somewhat simpler notion of a process, or activity.[52] The firm produces its output using one or more processes, for each of which the input coefficients are fixed for all levels of output. Thus, each process exhibits constant returns to scale. For example, suppose there are three production processes. Process 1 uses 3 units of K and 1 unit of L for each 5 units of output, while process 2 uses 2 units of K and 2 units of L and process 3 uses 1 unit of K and 4 units of L for each 5 units of output. Each of these processes is represented by a ray emanating from the origin in figure II-7, the slope of each being the constant input proportion that characterizes the process.

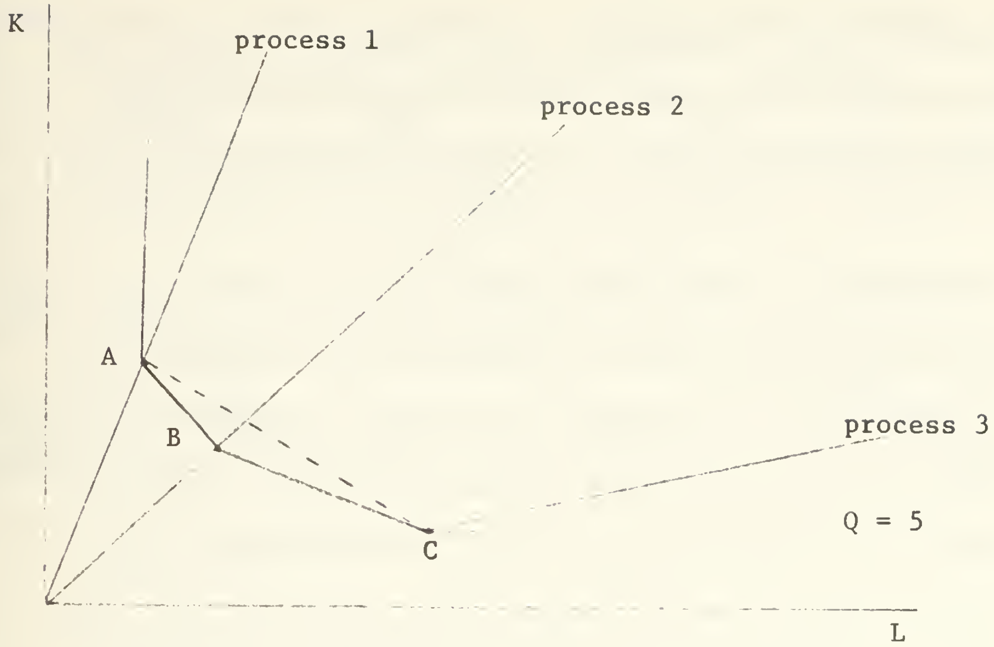


Figure II-7: Three Production Processes

To produce Q = 5 units of output the firm can use process 1 only, which is represented by point A in figure II-7; or it can use process 2 only (point B) or process 3 only (point C); or it can use the processes in combination. If it were to use processes 1 and 2 in combination, the firm could produce Q = 5 units of output using any of the input combinations lying on the line segment $\overline{AB}$.[53] Correspondingly, using processes 2 and 3 would require the input combinations lying on $\overline{BC}$, with the particular combination depending on the proportion of output produced using each process. Clearly the firm would not combine processes 1 and 3 since the required input combinations, which would lie on $\overline{AC}$, exceed the amounts that would be used to produce the same level of output if processes 1 and 2 or processes 2 and 3 were combined to yield the same input proportion. Thus, the line segment joining A and B and the one joining B and C define the

Q = 5 isoquant, which is analogous to the isoquants in figure II-2.[54] The difference is that in the linear case the isoquants are piecewise linear and are not differentiable at points where one process only is used.

If one were to increase the number of processes in figure II-7, the isoquants would begin to look more and more like the smooth isoquants in figure II-2. As the number of processes increases, the line segments shrink in length and the piecewise linear contour approximates a smooth contour more and more closely. The nonlinear case with its smooth isoquants can be viewed as the limiting case of the linear model in which there exist an infinite number of available processes.[55] Moreover, the conditions of optimality derived in the linear case carry over to the nonlinear case.[56]

The economic choice facing the firm is the selection of the levels at which the processes are to be operated. When operated at a higher level a process requires proportionately more of both inputs and yields proportionately more output. For example, the activity level of a process for producing cars could be the speed of the assembly line. Once the optimal operating levels of the various processes have been determined, the output level is uniquely established, and from the fixed input coefficients the input usage levels are also uniquely determined. Thus, unlike the nonlinear case in which the input levels were found directly, in the linear case the input usage levels are set indirectly.

To express the problem in linear programming format it is easier to express the objective function and constraints in terms of output levels rather than levels of operation of the various processes. The problem confronting the firm is then the same as (31), where q_j is the level of output using process j, c_j is the contribution margin earned on each unit of output produced using process j, and the j-th column of A contains the fixed input coefficients for the j-th process. Once again $\bar{b}$ is the vector of resource availabilities. For the above example, if 24 units of K and 16 units of L are available and the contribution margins are $c_1 = 5$, $c_2 = 7$, and $c_3 = 10$, then the problem is to

$$
\begin{aligned}
\text{maximize:} \quad & \pi = 5.0q_1 + 7.0q_2 + 10.0q_3 \\
\text{subject to:} \quad & .6q_1 + .4q_2 + .2q_3 \leq 24 \\
& .2q_1 + .4q_2 + .8q_3 \leq 16 \\
& q_1, q_2, q_3 \geq 0
\end{aligned}
$$

the solution to which is $q_1 = 20$, $q_2 = 30$, and $\pi = 310$.[57]

The two problems considered above are simple versions of the more general problem in which the firm must decide which products to produce and which processes to use. Since both problems are of the form of (31), it should be clear that the more generalized problem will be also. Indeed, by looking at each process available for producing each product as a process for producing revenue (rather than output), figure II-7 can be generalized to permit many products and many processes available for producing each. In this formulation isorevenue lines would replace isoquants, and each product produced by a different process would be treated as a separate q_j with its own contribution margin c_j, and the resulting linear programming problem would look like (31).

The problems discussed above were all market-oriented. One other important application of the linear programming technique is to internal corporate planning.[58] For example, the dual variables can serve as transfer prices when resources are transferred within the firm from one division to another or from one plant to another. Thus, as far as the internal workings of the firm are concerned, the dual problem is of somewhat greater interest than the primal problem.

3. Monopoly

Thus far it has been assumed that all markets are perfectly competitive. The individual firm has been unable to influence the prices it receives for the goods it sells or the prices it must pay for the inputs it purchases. In most cases, however, markets are imperfectly competitive. The actions of one or more buyers are perceived to have an influence on price. When the firm is able to influence the price of its output, it is said to possess some degree of monopoly power.[59] This section deals with the limiting case in which there is a single seller of a product for which there are no close substitutes — a market structure known as monopoly.[60] Sections C, D, and E discuss market structures in which there is more than one seller, with each having some degree of monopoly power.

A monopolist can influence the market price of its product by varying the amount of the good it offers for sale. Normally, consumers of a good would be willing to purchase more of a good only if the price were lowered, and the demand function

$$q = D(p) \tag{33}$$

expresses the quantity, q, consumers would be willing to purchase if the market price were p. The demand function (33) is single-valued with $dq/dp < 0$, so that it possesses an inverse function

$$p = p(q) \tag{34}$$

with $dp/dq < 0$, which shows how market price will vary in response to the changes in the amount of the good the monopolist offers for sale.

The demand curve facing the monopolist is downward-sloping like the DD curve in figure II-8.[61] Since the monopolist is the sole producer of the good, its demand curve is the market demand curve for the product. This is in contrast to perfect competition, where the market demand curve is also downward-sloping but the demand curve facing the individual producer is horizontal since, by assumption, each producer is able to sell as much as it likes at the prevailing market price.

The monopolist's total revenue, R, is equal to price times quantity, and since price is a function of quantity sold, so is total revenue:

$$R(q) = p \cdot q = p(q) \cdot q \quad . \tag{35}$$

Differentiating (35) with respect to q gives marginal revenue,

$$MR(q) = \frac{d}{dq} R(q) = p + \frac{dp}{dq} \cdot q \quad , \tag{36}$$

which is less than price since $dp/dq < 0$. Marginal revenue is interpretable as the net addition to total revenue resulting from the sale of an additional unit of output. Marginal revenue is less than price since an increase in quantity sold necessitates a decrease in the price of all units sold. The marginal revenue function (36) has been graphed as the MR curve in figure II-8.[62]

The monopolist may also have some influence over the prices it pays for inputs, with the price of each depending on how much it purchases.[63] The price it pays for the j-th input, r_j , is then a function of x_j , the amount purchased:

$$r_j = r_j(x_j) , \quad j = 1 , \ldots, m . \tag{37}$$

Since in general the firm can purchase more of an input only if it is willing to pay a higher price, $dr_j/dx_j > 0$, $j = 1 , \ldots, m$. The total outlay, C_j , for units of the j-th input is then

$$C_j = r_j \cdot x_j = r_j(x_j) \cdot x_j . \tag{38}$$

The derivative of (38) is interpreted as the marginal cost of the j-th input,

$$MC_j(x_j) = \frac{d}{dx_j} C_j = r_j + \frac{dr_j}{dx_j} x_j , \tag{39}$$

which is more than the price of the j-th input since $dr_j/dx_j > 0$.

The model of the monopolist can be formulated as the following mathematical programming problem:

$$\underset{\{q\ ,x_1,\ldots,x_m\}}{\text{maximize:}} \qquad \pi = p(q)\cdot q - \sum_{j=1}^{m} r_j(x_j)\cdot x_j \tag{40}$$

$$\text{subject to:} \qquad q = f(x_1\ ,\ \ldots,\ x_m)\ ,$$

where $q = f(x_1\ ,\ \ldots,\ x_m)$ is the firm's production function.[64] Once again focusing on nontrivial solutions and applying Lagrange multipliers, the Lagrangian is

$$L_\lambda = p(q)\cdot q - \sum_{j=1}^{m} r_j(x_j)\cdot x_j + \lambda(f(x_1\ ,\ \ldots,\ x_m) - q)$$

and the necessary conditions for an optimal solution to (40) are the following:

$$\left.\begin{aligned}
\frac{\partial L_\lambda}{\partial q} &= p(q) + \frac{dp}{dq}\cdot q - \lambda = 0 \\
\frac{\partial L_\lambda}{\partial x_j} &= -r_j(x_j) - \frac{dr_j}{dx_j}x_j + \lambda\frac{\partial f}{\partial x_j} = 0\ ,\quad j = 1,\ \ldots,\ m \\
\frac{\partial L_\lambda}{\partial \lambda} &= f(x_1\ ,\ \ldots,\ x_m) - q = 0\ .
\end{aligned}\right\} \tag{41}$$

The three conditions in (41) can be used to characterize the equilibrium position of the monopolist. The first condition in (41) requires that at optimality the Lagrange multiplier equal marginal revenue,

$$\lambda = p(q) + \frac{dp}{dq}\cdot q = MR \tag{42}$$

where the last equality follows from (36). The second set of m conditions requires that

$$\lambda \frac{\partial f}{\partial x_j} = r_j(x_j) + \frac{dr_j}{dx_j} x_j \;, \quad j = 1, \ldots, m \tag{43}$$

Using (39) and (42) and the fact that $\partial f/\partial x_j$ is interpreted as the marginal productivity of the j-th input, condition (43) can be reexpressed as

$$MR \cdot MP_j = MC_j \;. \tag{44}$$

The product $MR \cdot MP_j$ is interpreted as the marginal revenue product of input j and is the increase in revenue resulting from the use of an additional unit of input j. In words, condition (44) is interpretable as the requirement that usage of the j-th input be increased up to the point at which the net addition to revenue attributable to the last unit of the j-th input just equals the net addition to cost required to obtain it.[65]

The last condition in (41) is just the production function. It should be noted that the appearance of the production function as a necessary condition can be given the following economic interpretation. If problem (40) had been stated with the constraint expressed as the inequality

$$q \leq f(x_1, \ldots, x_m) \;,$$

and if the modified problem had been solved with the aid of generalized Lagrange multipliers, the set of necessary conditions would have included the following three necessary conditions:

$$q \leq f(x_1, \ldots, x_m) \qquad \lambda(f(x_1, \ldots, x_m) - q) = 0 \qquad \lambda \geq 0 \; .$$

If, at optimality, marginal revenue is positive (as it must be, except in the unusual case in which marginal revenue and marginal cost are both zero at optimality), it follows from (42) that $\lambda = MR > 0$. Since the middle of the three conditions requires that at least one of the terms in the product $\lambda(f(x_1, \ldots, x_m) - q)$ be zero, it follows that

$$q = f(x_1, \ldots, x_m) \; ,$$

which is interpretable as the requirement that, at optimality, the inputs be combined with maximum technical efficiency in order that maximum output be obtained from the amounts of inputs applied in production. Thus, the last condition in (41) can be interpreted to mean that profit maximization requires maximum technical efficiency.

Condition (43) can also be reexpressed so as to yield the product market solution. Dividing each side of (43) by $\partial f/\partial x_j$ and using (42) yields

$$\lambda = MR = \frac{r_j(x_j) + \dfrac{dr_j}{dx_j} x_j}{\partial f/\partial x_j} \; , \tag{45}$$

which is interpreted to mean that, at optimality, marginal revenue must equal the marginal cost of producing an additional unit of output in terms of each of the j inputs individually. But this common value of marginal cost in terms of each input is what was called previously the marginal cost of output, MC , so that (45) is equivalent to the familiar

$$MR = MC \tag{46}$$

rule for profit maximization under imperfect competition.[66] The optimum level of output is determined in figure II-8 by the intersection of MR and MC .[67] Total profit is equal to the area of the shaded rectangle.

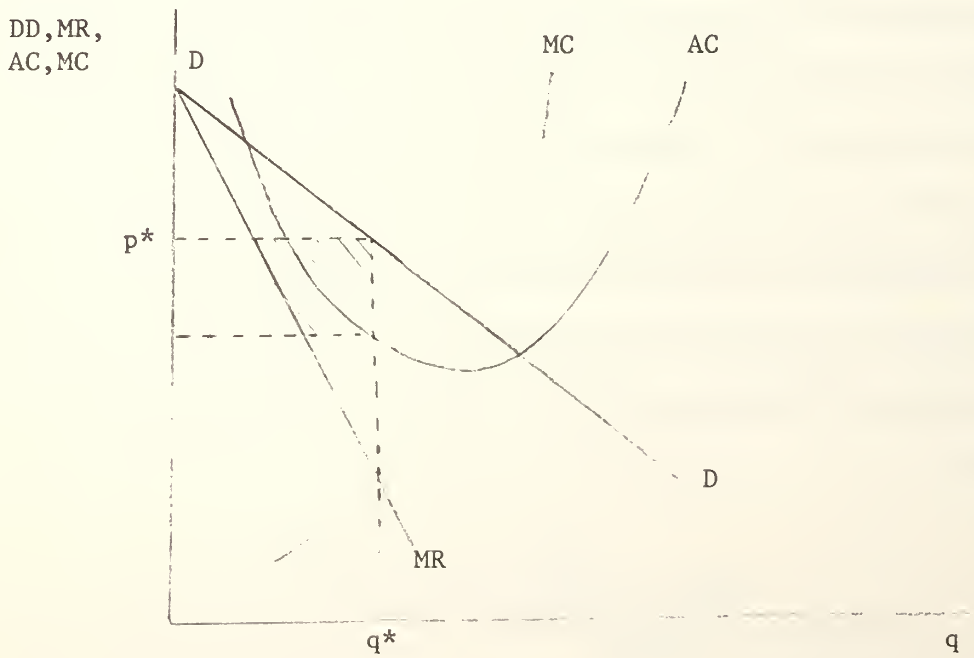


Figure II-8: Optimum Output for a Monopolist

The situation depicted by figure II-8, with the monopolist selling q* units of output at price p* and earning positive economic profit equal to the area of the shaded rectangle, could persist until some new entrant were able to break into the industry, in which case the industry would no longer be a monopoly. If entry were prohibited, monopoly profits could persist forever. This is in contrast to what happens in market equilibrium under perfect competition where free entry forces each firm to produce at the minimum point on its long run average cost curve and to earn zero economic profit in the long run. Under monopoly, the firm in long run equilibrium can earn positive economic profit. In addition, in long run equilibrium the monopolist will not minimize average total cost and its price will exceed marginal cost, as illustrated by figure II-8. In terms of the analytical framework discussed in section A, the characteristics of the model of the monopolistic firm are summarized in table II-7.

Monopoly is criticized because it tends to lead to output that is 'too low' and price that is 'too high', and by implication, any firm that possesses some degree of monopoly power is tainted. In the United States true monopoly is not a common occurrence, almost always being the result of either government policy, such as in the case of regulated utilities, or a temporary situation that results from a successful innovation and that lasts until a competitor can enter the market. In the first case, monopoly status may be conferred on a firm because of the existence of substantial economies of scale in production, and

Table II-7 Model Summary: Monopoly

Class:	traditional (see (40) in text)
Firm's Objective:	profit maximization
Constraints:	technological (embodied in the production function), nonnegativity constraints on the decision variables, market demand relation ($q = D(p)$), and input supply functions ($r_j = r_j(x_j)$)
Variables:	
Exogenous:	none (input prices if factor markets were perfectly competitive)
Endogenous:	price (p), output level (q), input levels (x_j), input prices (r_j), and total profit (π)
Decision:	output level (q) and input levels (x_j)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium
Time:	single period
Type of Model:	static optimization (nonlinear programming problem)
Solution Technique:	Lagrange multipliers (could have solved (40) as an unconstrained problem by substituting for q in the objective function, although the information conveyed by λ would have been lost) if nontrivial solution is assumed; otherwise generalized Lagrange multipliers.

in the second case, the temporary monopoly situation may be viewed as a reward for successful innovation.[68] With the exception of these special cases, it is generally believed that, on grounds of economic welfare, monopoly is undesirable, and more specifically, that whatever social benefits might be derived from taking greater advantage of economies of scale in production, distribution, research and development, etc., are outweighed by the potential loss of social welfare due to price exceeding marginal production cost, to long run average total cost not being minimized, and to the potential social costs associated with possible abuses of monopoly power.[69] Accordingly, government antitrust policy has sought to prevent monopolies from developing (e.g. as through mergers).[70] Unless this policy changes, monopoly in the American economy will continue to be confined to regulated industries and to temporary situations.

C. COURNOT'S MODEL OF DUOPOLY

A market structure in which there are a few sellers and in which each of these sellers perceives the other sellers serving the same market as rivals is called an oligopoly. Duopoly is a special case of oligopoly in which there are just two sellers. Because there are few sellers, under oligopoly competing firms can influence the market price, and the profits earned by any one firm therefore depend on the policies of all firms in the industry. Each firm chooses its policies in recognition not only of their direct effects, but also with a view toward their expected indirect effects — the likely reactions of their competitors. What makes duopoly so much simpler analytically than the more general case involving two or more sellers is that taking into account the reactions of one firm to the policies of its rival(s) is much simpler when there are only two firms.

The first part of this section describes the Cournot analysis of duopoly and mentions some extensions of the Cournot model.[71] The second part describes a game theoretic approach to the analysis of duopoly in which the behavior of the two firms is modeled as a two-person game.

1. Cournot's Model and Conjectural Variations

Assume there are two firms producing a homogeneous product. Total industry output of the product is the sum of the amounts produced by the two firms, $q = q_1 + q_2$, and market price is a function of this aggregate output $p = p(q_1 + q_2)$. Since each duopolist's output is sold at price p, the total revenue earned by each is dependent on its own output level as well as that of its rival:

$$\begin{aligned} R_1(q_1, q_2) &= q_1 \cdot p(q_1 + q_2) \\ R_2(q_1, q_2) &= q_2 \cdot p(q_1 + q_2) \end{aligned} \tag{47}$$

Similarly, each duopolist's total profit depends on its own and on its rival's output levels:

$$\begin{aligned} \pi_1(q_1, q_2) &= q_1 \cdot p(q_1 + q_2) - C_1(q_1) \\ \pi_2(q_1, q_2) &= q_2 \cdot p(q_1 + q_2) - C_2(q_2) , \end{aligned} \tag{48}$$

where it has been assumed that each firm's total cost depends on its own output only.[72] Each firm's objective is to select its own level of output so as to maximize its total profit. Differentiating each firm's profit function in (48) with respect to its own level of output and setting the result equal to zero gives:

$$\frac{d\pi_1}{dq_1} = p(q_1 + q_2) + q_1 \frac{dp}{dq} \cdot (1 + \frac{dq_2}{dq_1}) - \frac{dC_1}{dq_1} = 0 \qquad (49)$$

$$\frac{d\pi_2}{dq_2} = p(q_1 + q_2) + q_2 \frac{dp}{dq} \cdot (1 + \frac{dq_1}{dq_2}) - \frac{dC_2}{dq_2} = 0 \qquad (50)$$

The derivatives dq_2/dq_1 in (49) and dq_1/dq_2 in (50) are called conjectural variations.[73] The conjectural variation dq_2/dq_1 can be interpreted as the first duopolist's expectation as to how its rival's output, q_2, will respond to changes in its own output, q_1, and dq_1/dq_2 is interpreted similarly. In the Cournot analysis conjectural variations are always zero, i.e. $dq_2/dq_1 = dq_1/dq_2 \equiv 0$.

Equations (49) and (50) can be rewritten as

$$\begin{aligned} p + q_1 \frac{dp}{dq} (1 + \frac{dq_2}{dq_1}) &= \frac{dC_1}{dq_1} \\ p + q_2 \frac{dp}{dq} (1 + \frac{dq_1}{dq_2}) &= \frac{dC_2}{dq_2} , \end{aligned} \qquad (51)$$

where the expression on the left-hand side of each equation represents marginal revenue (the appropriate derivative of (47)) and the right-hand side represents marginal cost. Thus, equations (51) represent the familiar MR = MC necessary condition for profit maximization.[74] Equations (51) also imply that each firm's marginal revenue and equilibrium level of output are dependent on its conjectural variation.

Equations (49) and (50) cannot be solved for the equilibrium levels of output q_1 and q_2 unless the conjectural variations are specified. In the Cournot analysis it was assumed that each conjectural variation is zero, which implies that each duopolist assumes that variations in its output level will not induce changes in its rival's output level. Granted this assumption, equations (51) simplify to the following:

$$
\begin{aligned}
p + q_1 \frac{dp}{dq} &= \frac{dC_1}{dq_1} \\
p + q_2 \frac{dp}{dq} &= \frac{dC_2}{dq_2}
\end{aligned}
\qquad (52)
$$

which can be solved simultaneously for the equilibrium levels of output.

Another approach to finding the equilibrium output levels, one that describes the market process more fully, involves the use of reaction curves. This modern approach to the Cournot analysis of duopoly is described below. After the interpretation and use of reaction curves has been described, the reaction curve framework will be used to suggest how the original Cournot analysis might be extended to a multiperiod context and to a consideration of equilibrium/disequilibrium questions.[75]

A pair of reaction curves, one for each duopolist, can be constructed in the following manner. After the conjectural variations have been specified, equation (49) can be solved for q_1 as a function of q_2 and equation (50) can be solved for q_2 as a function of q_1 .

Each function is called a reaction function and the graph of each reaction function is called a reaction curve. Each producer's reaction curve shows its profit-maximizing output level contingent upon its rival's output level. By recognizing that equilibrium — both for the market and for each firm — can occur only when the levels of output q_1 and q_2 are such that first, each firm maximizes its profit, given its rival's output, and second, neither firm wants to alter its output level, the optimum output levels can be determined from the intersection of the reaction curves. In other words, the reaction curve is a convenient device — one that has also been used in international trade theory — for illustrating one producer's reactions to decisions made by its rival. Moreover, the use of the two producers' reaction curves permits a geometric representation of the notion of equilibrium for the two producers. As a simple example, the reaction curves for two firms having total cost functions

$$C_i = cq_i + d , \quad c > 0 , \quad d > 0 , \quad i = 1, 2 , \tag{53}$$

where the demand for their product satisfies

$$p = a - b(q_1 + q_2) , \quad a > 0 , \quad b > 0 , \tag{54}$$

are given in figure II-9, where it has been assumed that conjectural variations are zero. The equilibrium level of output for each producer is[76]

$$q_1 = q_2 = \frac{a - c}{3b} .$$

Provided $a > c$, the reaction curves intersect in the first quadrant, and a market equilibrium exists in which each firm produces a positive level of output.

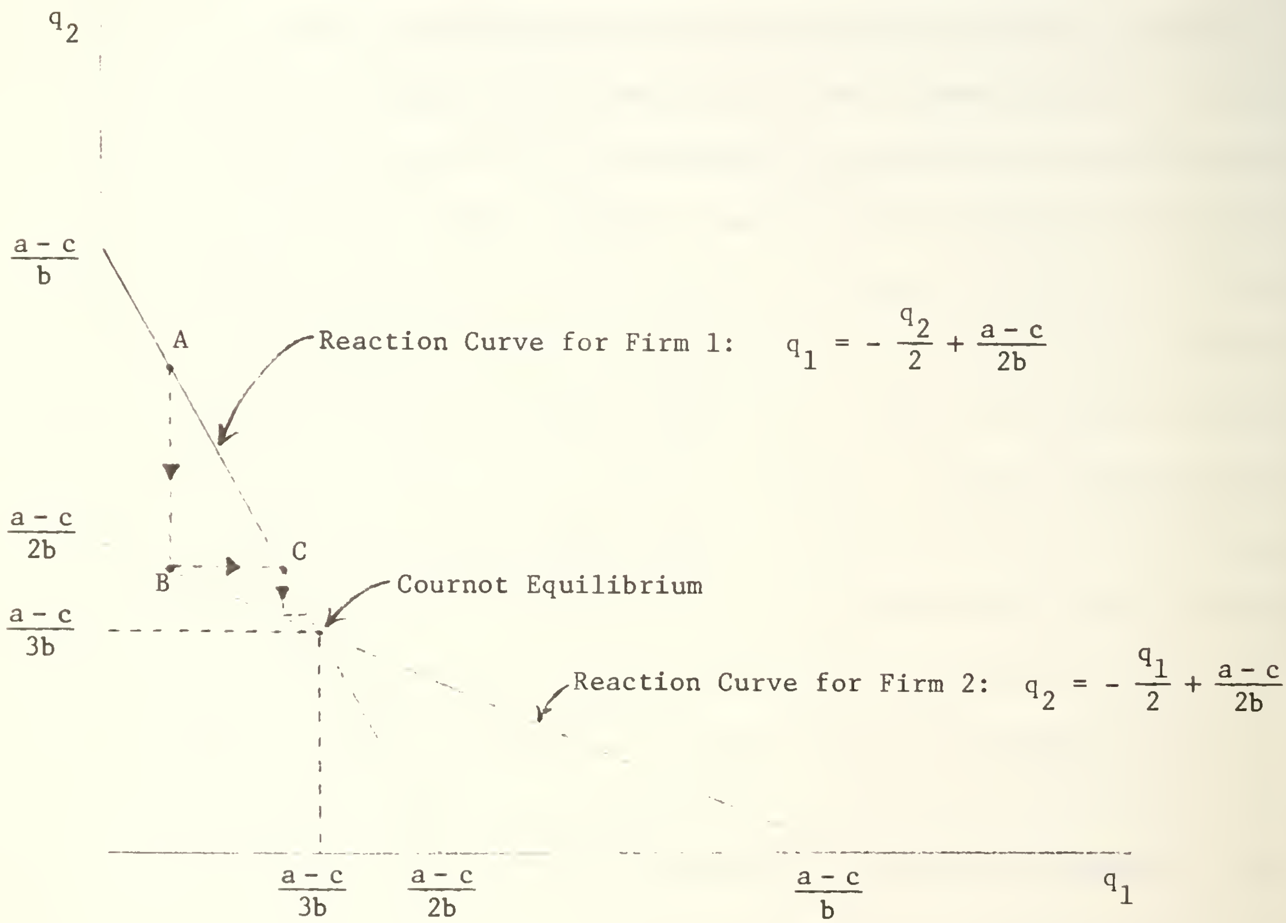


Figure II-9: Reaction Curves and Cournot Equilibrium

If each conjectural variation is zero, then figure II-9 can also be used to suggest the behavior of the two producers when the product market is in disequilibrium. In particular, the figure can be used to illustrate the point that the market is not in equilibrium unless each individual producer is in equilibrium. To see this, note first that if an output combination (q_1, q_2) lies on one of the producer's reaction curves, then the producer whose reactions are depicted by the curve is, by the way in which the reaction curve was constructed, in equilibrium. That is, according to the assumption of zero conjectural variations, the producer treats its rival's output level as a constant, and, by the construction of its reaction curve, the firm is producing the profit-maximizing level of output, given its rival's output level, and therefore has no incentive to alter its output. But suppose that only one producer is in equilibrium. For example, if the current output combination is represented by Point A in figure II-9, then firm 1 is in equilibrium, but firm 2 is not since the output combination does not lie on its reaction curve. Indeed, given the output level of firm 1, firm 2 would prefer to reduce output (and to shift the output combination to point B) and to earn higher profit. If firm 2 did reduce its output accordingly, it would be in equilibrium at B, given the output of firm 1. But firm 1 would not be in equilibrium at B. Given the output of firm 2, firm 1 would want to increase output (and to shift the output combination to point C). The adjustment process would cease once the Cournot equilibrium had been attained, since then each firm would lie on its reaction curve and neither would have any incentive to alter its output level. The above argument also suggests that, as long as each conjectural variation remains zero, the Cournot equilibrium is stable, i.e. whenever the market is out of equilibrium, forces are set in motion to return it to equilibrium.

The foregoing is also suggestive of how the duopolists might behave over time. That is, the successive movements toward equilibrium could be thought of as occurring in successive time periods as first one firm and then the other adjusted to the actions of its rival (all the while, however, continuing to assume that its rival would not retaliate by altering output). Attempting to extend the Cournot model in this manner serves to emphasize one of its major weaknesses. The assumption of zero conjectural variations implies that each firm is ignorant of the behavior of its rival.[77] In the multiperiod context, the firm would have to remain ignorant over time, i.e. there would be no learning process at work by which each firm would come to learn that its assumption that its rival would not retaliate is false.[78] The main reason for this limitation can be attributed to the model's primary concern with characterizing equilibrium in a single period context.[79]

Before considering a model of duopoly that permits nonzero conjectural variations, the characteristics of the Cournot model of duopoly are summarized below in table II-8.

The output levels given by (55) were the result of one of many possible modes of behavior on the part of the duopolists. Another interesting type of behavior is suggested by von Stackelberg.[80]
Each of the duopolists may act either as a follower or as a leader. A follower obeys its Cournot reaction function, adjusting its output so as to maximize its total profit, given the output level of its rival, whom it views as a leader. A leader does not obey its Cournot reaction

Table II-8 Model Summary: Duopoly

Class:	traditional (see (48) in text)
Firm's Objective:	profit maximization
Constraints:	market demand relation ($p = p(q_1 + q_2)$) and actions of the firm's rival, also, implicitly, the production function that underlies each firm's total cost function (which enters the model exogenously)
Variables:	
Exogenous:	implicitly, input prices (in order that each firm's total cost function depend on its own output level only)
Endogenous:	output levels (q_1 and q_2), market price (p), and total profit for each producer (π_1 and π_2); in addition, input usage could also be determined if the model were modified to allow the firm to select its optimal input combination
Decision:	each firm selects its output level (q_1 and q_2, respectively), and in the fuller analysis, input levels could also be determined
Finance:	subsumed
Certainty/ Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium, though the analysis can be extended with the aid of reaction curves to deal with disequilibrium issues
Time:	single period
Type of Model:	static optimization (model formulated as two unconstrained optimization problems — one for each firm)
Solution Technique:	unconstrained optimization problems solved simultaneously

function but rather, sets its output so as to maximize total profit, given the reaction function of its rival, whom it views as a follower. For example, in the linear case if firm 1 acts as a follower, then $dq_2/dq_1 = 0$, and if firm 2 acts as a leader, then $dq_1/dq_2 = -1/2$, the slope of firm 1's reaction curve. Any one of three possible situations may arise:

(i) Both firms behave as followers, which leads to the Cournot equilibrium. This equilibrium is unstable in this case because each firm could improve its profits by becoming a leader while the other remained a follower.

(ii) One firm behaves as a leader and the other behaves as a follower, with the leader earning more profit and the follower earning less than in the Cournot equilibrium.[81]

(iii) Both firms behave as leaders, which gives rise to the von Stackelberg disequilibrium. Each firm believes that the other will behave according to its reaction curve, but neither one does and both consequently earn smaller profit than they did under the Cournot equilibrium.[82] This situation is likely to lead to some sort of economic warfare until either one firm emerges as the leader or some form of collusive agreement is worked out. The eventual equilibrium is indeterminate.[83]

In the Cournot analysis conjectural variations are always zero. In the von Stackelberg analysis conjectural variations are zero or negative.[84] It is also possible for conjectural variations to be positive, with one or both producers anticipating that an increase in its output level will elicit an increase in its rival's level of output. There could be tacit collusion between the producers as they attempted to maximize joint profit ($\pi_1 + \pi_2$).[85] Generally, positive conjectural variations on the part of both duopolists would lead to lower output, higher price, and greater profits than in the Cournot equilibrium, whereas negative conjectural variations would have the opposite effect. But the exact solution depends on the exact behavior of the duopolists as reflected in specific values for the conjectural variations.

As in the Cournot analysis, the von Stackelberg analysis is mainly concerned with characterizing equilibrium within a single period context.[86] For this reason, it too does not lend itself well to extension to a multiperiod context.[87] In particular, the von Stackelberg model, to the extent that followers are present, suffers from the same limitations as the Cournot model.[88]

2. Duopoly As A Two-Person Game

Under duopoly the market price of the good and the profit earned by each producer depend on the output decision of both producers. The firms are players in a two-person game in which each player's strategy is represented by its output decision and the payoff to each player is the total profit it receives. By modeling duopoly as a two-person game the results of game theory can be employed to determine under what conditions a solution exists.

The theory of games originated from the work of von Neumann and Morgenstern.[89] Briefly, game theory is the study of situations involving two or more decision-makers, called players, each of whom makes policy decisions, called strategies, that collectively determine the rewards, called payoffs, each player receives on each play of the game. The collection of strategies each player may adopt together with the rules that determine the payoffs as a function of the strategies the players adopt is what constitutes the game. As such, game theory is concerned with situations like duopoly, or more generally oligopoly, in which conflict and cooperation play significant roles.

If a game involves two players and if one player's gain is the other player's loss, then the payoffs to the two players sum to zero and the game is called a two-person zero-sum game. Von Neumann and Morgenstern showed that if the number of pure strategies is finite and mixed strategies are allowed, a two-person zero-sum game always has a solution as long as the players are rational.[90] Each player will adopt a 'minimax' strategy (or set of strategies) — the one that minimizes its maximum expected loss (or equivalently, maximizes its minimum expected gain) for whatever strategies its rival might adopt. If duopoly were a zero-sum game, then presumably the duopoly problem could be solved by appealing to this result.

Unfortunately, as von Neumann and Morgenstern realized, the kinds of rivalry observed in the real world seldom are zero-sum situations.[91] Normally, there is the possibility of mutual gain or loss, as for example, when each firm can increase its profits by cooperating with its rival. As such, these situations correspond to nonzero-sum games. The payoff matrix might look like the one in figure II-10, where the strategies have been defined in terms of von Stackelberg's definition of follower and leader modes of behavior.[92] The payoffs are the profits earned by the two producers, the left-hand entry in each case referring to firm 1. The underlying demand curve is (54) with $a = 610$ and $b = 10$ and the underlying cost curve for each firm is

(53) with $c = 10$ and $d = 200$. The entries in the payoff matrix correspond to the von Stackelberg analysis of duopoly discussed in the previous subsection.[93]

		Firm 2's Strategy	
		follower	leader
Firm 1's Strategy	follower	(3800, 3800)	(2050, 4300)
	leader	(4300, 2050)	(2680, 2680)

Figure II-10: Payoff Matrix for Two Firms Illustrating the Prisoners' Dilemma

According to Figure II-10, either firm may adopt either strategy and both firms will earn positive economic profit. If both firms act as leaders, then the von Stackelberg disequilibrium results and each firm earns a profit of 2680. If both firms act as followers, then the Cournot equilibrium results and each firm earns a profit of 3800. Note, however, that this equilibrium is unstable because each firm has an incentive to alter its behavior. If one firm becomes a leader while the other remains a follower, then the leader increases its profit while the follower suffers a decrease in its profit. However, if both try to act as leaders, then the von Stackelberg disequilibrium results and both firms are worse off than they were in Cournot equilibrium. This illustrates the so-called Prisoners' Dilemma: the two players would be better off if they could agree to act as followers, yet under such an agreement, each is tempted to double cross the other, and if both cheat, then both are worse off than they were initially.[94] Of course, they would be best off if they could coordinate their activities perfectly and maximize joint profit.[95] In any case, the eventual solution is indeterminate.[96] Output and price will ultimately depend on whether a collusive agreement can be reached and then maintained, and accomplishing this may require a long and painful learning process before the two parties are able to work together.[97]

Though game theory has not furnished solutions — in the conventional sense at least — to the problem of determining price and output under duopoly, it has provided some interesting insights into the nature of conflict and the incentives for cooperation under duopoly, and more generally, under oligopoly. Unfortunately, as the number of player firms increases, the game becomes more complicated with opportunities for two or more players to form a coalition against the others. As a consequence of this, the game theoretic approach has thus far proven no more successful than other analytical approaches at providing general characterizations of the pricing and output decisions reached by firms under oligopoly.[98]

D. OLIGOPOLY AND THE KINKED DEMAND CURVE

In recent years many economists, including Galbraith, have argued that oligopoly has become the dominant form of industrial structure.[99] In many industries, including the automobile, aluminum, steel, and rubber industries, there are a small number of large firms that wield considerable economic power[100] that enables them to influence the markets for their products, but who must carefully weigh the likely reactions of their rivals when reaching their price and output decisions.[101] The classic approach to oligopoly is by extension (i.e. by increasing the number of participants) of the duopoly models of Cournot and von

Stackelberg that were discussed in section C.[102] Each of these approaches has been criticized and an almost bewildering array of alternative approaches has been proposed in their stead.[103] The purpose of this section is to describe just one of these models — the kinked demand curve model that seems to dominate textbook discussions of oligopoly.[104]

The kinked demand curve model is based on the assumption that an oligopolist's rivals will match a price decrease but will not follow a price increase. This is illustrated in figure II-11.

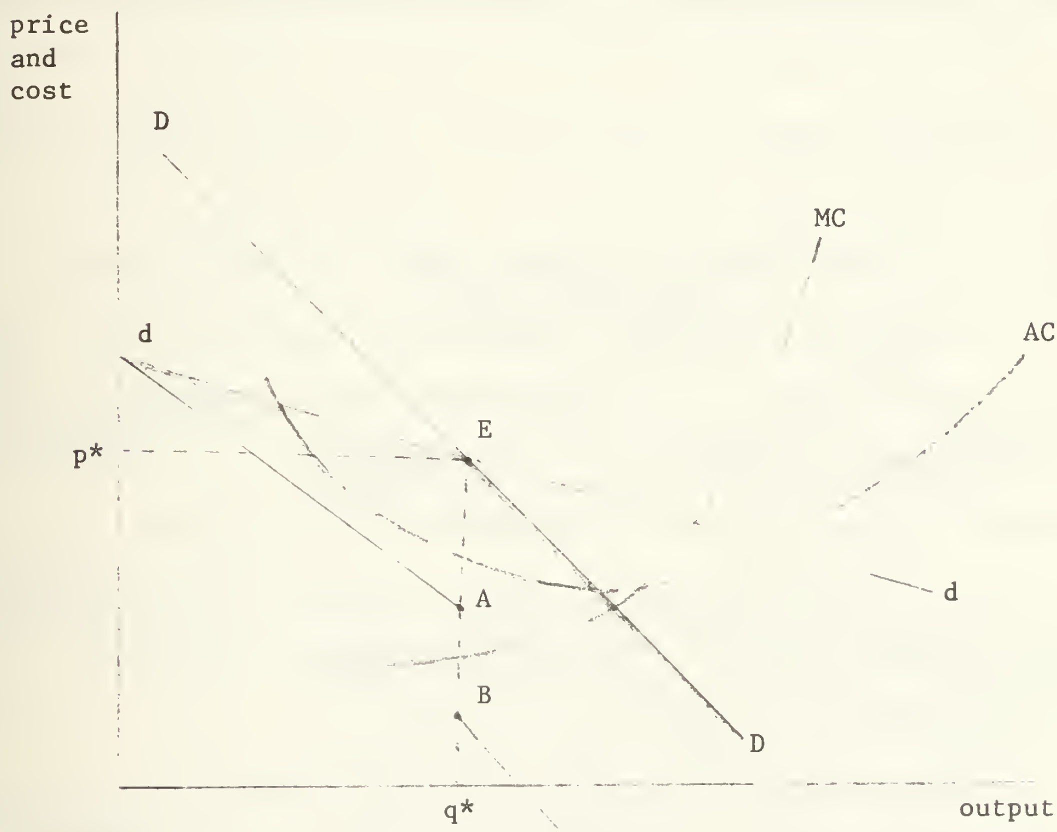


Figure II-11: The Kinked Demand Curve Model

The oligopolist's current price-output relationship is represented by the point E ; current price is p^* and current output is q^* . The demand curve dd has been drawn under the assumption that the oligopolist's price changes will not be matched by its rivals and demand curve DD has been drawn under the assumption that price changes will be matched. The curve dd is flatter, or more elastic, than DD because when price changes go unmatched, the oligopolist's unit sales, q , are more responsive to changes in price; by decreasing price it can increase sales at the expense of rivals who maintain their prices, but by increasing price it will lose part of its market share to its rivals.

The oligopolist's effective demand curve is comprised of that portion of dd that lies to the left of E and that portion of DD that lies to the right of E . This gives rise to a corner, or kink, at E . As a result of this kink the oligopolist's marginal revenue function is discontinuous at E , being comprised of the marginal revenue function corresponding to dd for output levels $q < q^*$ and of the marginal revenue function corresponding to DD for output levels $q > q^*$.[105] The oligopolist is unable to equate marginal revenue and marginal cost, but q^* is optimal since $MR > MC$ for $q < q^*$ while $MR < MC$ for $q > q^*$.

The jump discontinuity in MR at E implies that costs can increase or decrease without causing a change in either price or output as long as the MC curve continues to lie between A and B . However, a change in either DD or dd would lead to a change in price, unless the curves were to shift in such a way as to leave the kink at E . This implies that firms' price behavior will be relatively unresponsive to changes in cost, but will be responsive to changes in demand. This is a major weakness of the model, for most empirical studies have shown that in oligopolistic industries the reverse is true: price tends to be responsive to cost changes but unresponsive to changes in demand.[106]

The kinked demand curve model was originally proposed in order to explain the observed price rigidities in oligopolistic industries, yet a limitation of the model is its inability to explain how the kink got there in the first place. Furthermore, Stigler has offered empirical evidence against the existence of a kink in the demand curves of firms in several industries, including the steel, automobile, and potash industries.[107]

Table II-9 Model Summary: Oligopoly (kinked demand curve)

Class:	traditional (see figure II-11 in text)
Firm's Objective:	profit maximization
Constraints:	reactions of rivals; implicitly, the production function that underlies the cost curves in figure II-11
Variables:	
Exogenous:	current output and the current market price of output (which together locate the kink); implicitly, input prices (in order that the cost curves can be drawn in figure II-11, it is necessary that input prices remain fixed; when one or more input prices change, the cost curves shift)
Endogenous:	there are no endogenous variables, since the purpose of the model is to explain why prices tend to remain rigid, rather than to explain how they first got to where they are currently
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/Disequilibrium:	characterization of equilibrium
Time:	single period

In summary, the kinked demand curve model, the characteristics of which are summarized in table II-9, is not widely regarded as a satisfactory general model of oligopoly pricing.[108] A generally more valid model may be the mark-up pricing model, which has been described and subjected to considerable empirical testing by proponents of the behavioral school. This model is discussed in section H. One of the attractions of the mark-up pricing model is that it makes price more sensitive to changes in cost than to changes in demand, thereby correcting a major weakness of the kinked demand curve model.

E. CHAMBERLIN'S MODEL OF MONOPOLISTIC COMPETITION

In the Cournot analysis of oligopoly it is assumed that each producer takes the output level of each of its rivals to be fixed. While this appears to be a shaky assumption concerning the behavior of firms in markets where there are just a few producers, it is reasonable to hypothesize that, when the number of firms is large, so that each has only a small share of the overall market, each firm can treat the price and output of other producers as fixed when it sets its own price and output. Such a market situation corresponds to the model of monopolistic competition, according to which each firm's product is somehow differentiated through advertising, variations in style or quality, etc., from those of every other producer, and, as a consequence, each firm faces a downward-sloping demand curve for its product.[109] In such markets each firm possesses some degree of monopoly power — i.e. an ability to influence the price it receives for its product by adjusting quantity supplied and also an ability to influence through advertising expenditure — the quantity of output it sells — in its own corner of the market.

A practical realization of this model is a situation in which the number of firms is so large that a change in price or quantity by any one firm would not be perceived as having any impact on any other firm. As a result, in determining price and output in the short run, the firm assumes the corresponding choices of the other firms fixed and behaves like a monopolist in its corner of the market. Over the long run, the existence of a large number of close competitors produces a situation somewhat like the long run solution under perfect competition. In the long run all firms earn zero economic profit, although unlike perfect competition, all firms are operating slightly below optimal scale. The purpose of this section is to discuss the model of monopolistic competition due to Chamberlin and to illustrate these results.

For ease of exposition, let the demand functions facing the firms be linear and identical, satisfying the equation

$$p_i = a - b_1 q_i - b_2 \sum_{j \neq i} q_j \quad , \quad i = 1 , \ldots , n \quad , \qquad (56)$$

where there are n firms; p_i is the price charged and q_i is the output of the i-th firm; a , b_1 , and b_2 are all positive constants; and $\partial p_i / \partial q_j = -b_2$, $i \neq j$, is numerically small. Also let the firms have identical cost functions $C_i(q_i)$. Then, even though their products are differentiated (in the eyes of consumers, at least), the firms will have identical cost and revenue functions and will exhibit identical maximizing behavior. It should be noted that the demand relation (56) involves certain conceptual difficulties. The output levels of the rivals are summed as if these other goods were identical, even though the basic assumption underlying Chamberlin's model is that they are not identical. The conceptual problem associated with 'adding up' things that are, by assumption, 'different', is discussed further later in this section. Until then, suffice it to say that, while these assumptions are restrictive, they do permit the model to be developed in terms of the behavior of the 'representative' firm.

From (56) the profit earned by the representative firm is

$$\pi_i = p_i q_i - C_i(q_i) = (a - b_1 q_i - b_2 \sum_{j \neq i} q_j)\, q_i - C_i(q_i) \ . \quad (57)$$

Since b_2 is numerically small, any change in the i-th firm's output will have a negligible impact on its (n-1) rivals, and therefore, the i-th producer will act as if variations in q_i will have no effect on q_j , $j \neq i$, or in symbols, it will act as if $\partial q_j / \partial q_i = 0$, $j \neq i$. Differentiating (57) with respect to q_i , setting the result equal to zero, and rearranging terms yields

$$a - 2b_1 q_i - b_2 \sum_{j \neq i} q_j = C_i'(q_i) \ , \quad (58)$$

which is the familiar MR = MC rule for profit maximization.[110]

Since all firms will behave identically under the above assumptions, when one firm finds it profitable to adjust output, so will all the others. Thus, while the representative firm tries to adjust its price and quantity in accordance with the demand relation given by (56), the collective actions of producers will cause each individual producer's price and quantity to vary in accordance with the demand relation,

$$p_i = a - [b_1 + (n-1)b_2]q_i \ , \quad (59)$$

which has been obtained from (56) by setting $q_j = q_i$. The demand curve (56) for the individual firm is drawn as dd in figure II-12 under the assumption that the output levels of all other producers are fixed. In addition, the demand curve (59), which represents overall demand for the products of the n firms, is used to determine each firm's pro rata share of overall quantity demanded for each (common) price,[111] and this pro rata demand curve for the individual firm is drawn as DD in the same figure.[112] Note that even though b is small, $(n-1)$ is not. It is this latter term, which reflects the fact that all producers adjust output simultaneously, that accounts for the steeper slope of DD .

It should be noted that the demand relation (59) and the demand curve DD in figure II-12 involve the same conceptual difficulty noted in connection with (56). In (59) and in its geometric representation DD , the output levels of all n producers have been added together, even though the products themselves are 'different' in the sense of being differentiated either in terms of their actual characteristics or else in terms of their perceived characteristics. Chamberlin justifies this approach by introducing the concept of a 'group' of "producers whose goods are fairly close substitutes" for one another, as for example, a group of automobile manufacturers.[113] Accordingly, the demand curve DD , is meant to represent each automobile manufacturer's pro rata share of the overall market demand for automobiles when all automobile manufacturers charge identical prices. Yet, as Stigler argues, it is difficult to attach any economic meaning to the statement that physically different goods must have the same price.[114] As illustrated by the discussion of Chamberlin's model earlier in this section, Chamberlin further assumed that the "demand and cost curves for all the 'products' are uniform throughout the group."[115] As pointed out by Stigler, this uniformity assumption implicitly defines the products of the group as a set of physically homogeneous goods.[116] Chamberlin's model and the conclusions to be derived from it depend then, on a key component — the firm's pro rata share of overall demand for the products of the group (illustrated by DD in figure II-12) — the conceptual meaning of which is not unambiguous.

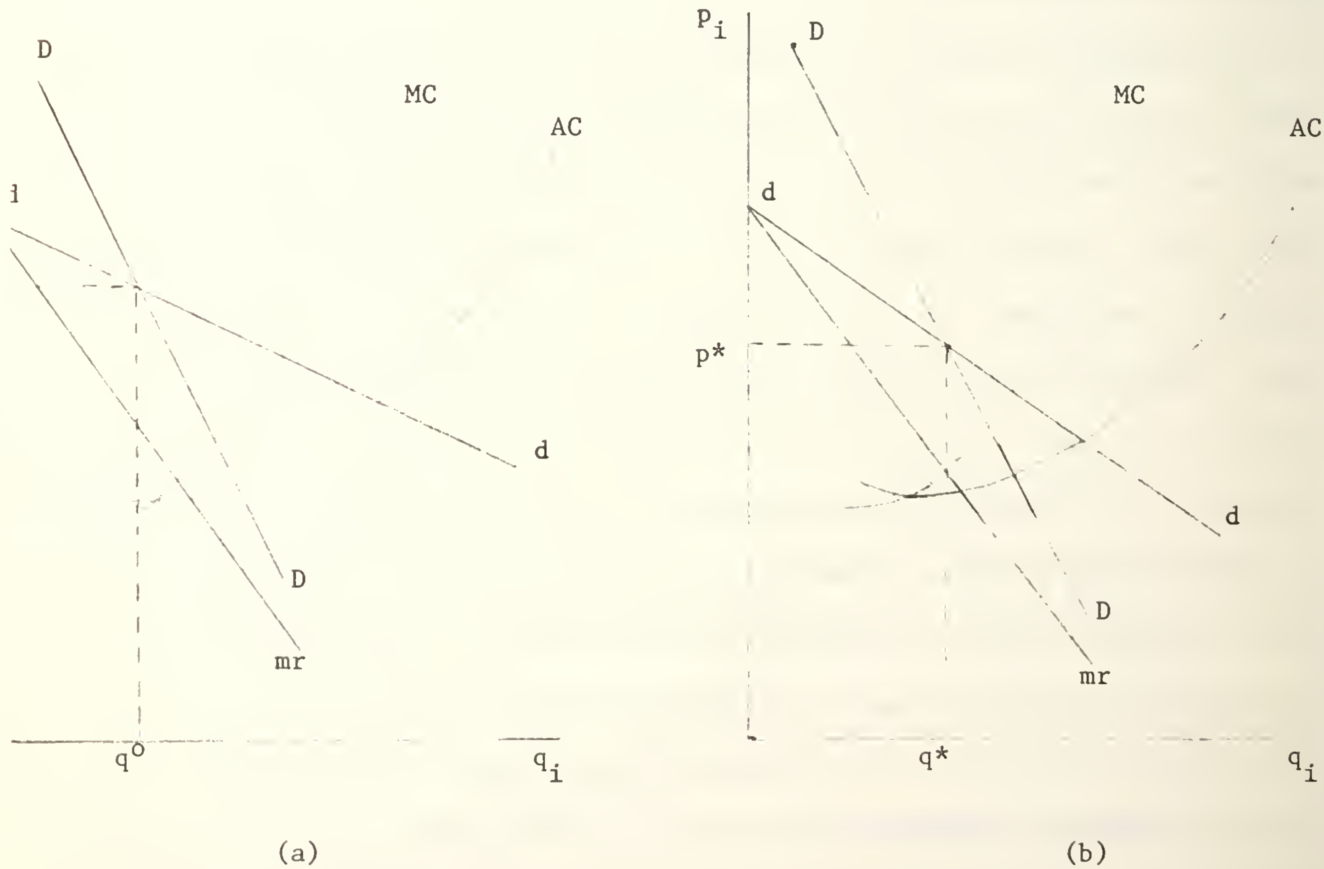


Figure II-12: Short Run Optimum for the Firm Under Monopolistic Competition

In setting price and output the representative firm treats dd as its relevant demand curve, but the collective action of all firms makes DD the representative firm's effective demand curve. As the representative firm's price and output are adjusted in accordance with DD , the demand curve dd 'slides along' DD , with the two demand curves always intersecting at the point representing current price and output. Thus, in figure II-12(a) the firm is selling q_o at price p_o , but it is not maximizing profit since mr (which corresponds to dd) exceeds MC at that point. The representative firm expands output, and the collective action of all firms causes price to fall and dd to shift downward. The process continues until condition (58) is satisfied, at which point mr intersects MC and total (short run) profit is maximized. This short run equilibrium situation is illustrated in figure II-12(b).

It should be emphasized that, even if the individual firm realizes that DD rather than dd is effectively its demand curve, it is unable to exploit this knowledge. For unlike oligopoly, under monopolistic competition the individual firm has no impact on its rivals' output levels. Each firm acts individually so as to maximize its own profit. Since the behavior of the individual firm under monopolistic competition has no perceptible impact on any of its rivals, and since adjusting price and output in accordance with dd holds out the prospect of greater profit (as long as these changes go unnoticed), the firm is likely to continue to treat dd as its relevant demand curve and to adjust price and output accordingly. Moreover, the firms are so numerous that collusion, tacit or otherwise, is virtually impossible, and should some sort of collusive agreement be reached, it would be likely that each individual firm's incentive to cheat would be strong enough — for the reasons just cited — to cause the arrangement to be shattered within a short time.

In the long run free entry and exit drive economic profit to zero. Unlike oligopoly, firms are so numerous that it is impossible for them to get together and erect barriers to entry. As new firms enter the industry, the representative firm finds that the amount of output it is able to sell at any given price decreases, and both dd and DD in figure II-12 shift to the left. Long run equilibrium is finally attained when the representative firm's perceived demand curve dd is tangent to the long run average cost curve. This long run equilibrium situation is illustrated in figure II-13. At the point of tangency, point A in figure II-13, dd and DD intersect each other and AC , so that price equals average cost and total profit is zero. The firm is maximizing long run profit, for at all output levels other than q^* , dd lies below AC . But since dd is downward sloping, the point of tangency must occur to the left of the minimum point on the AC curve. The firm produces less output at a greater average total cost than would obtain under perfect competition.

In terms of the analytical framework discussed in section A, the characteristics of Chamberlin's model of the firm under monopolistic competition are summarized in table II-10.

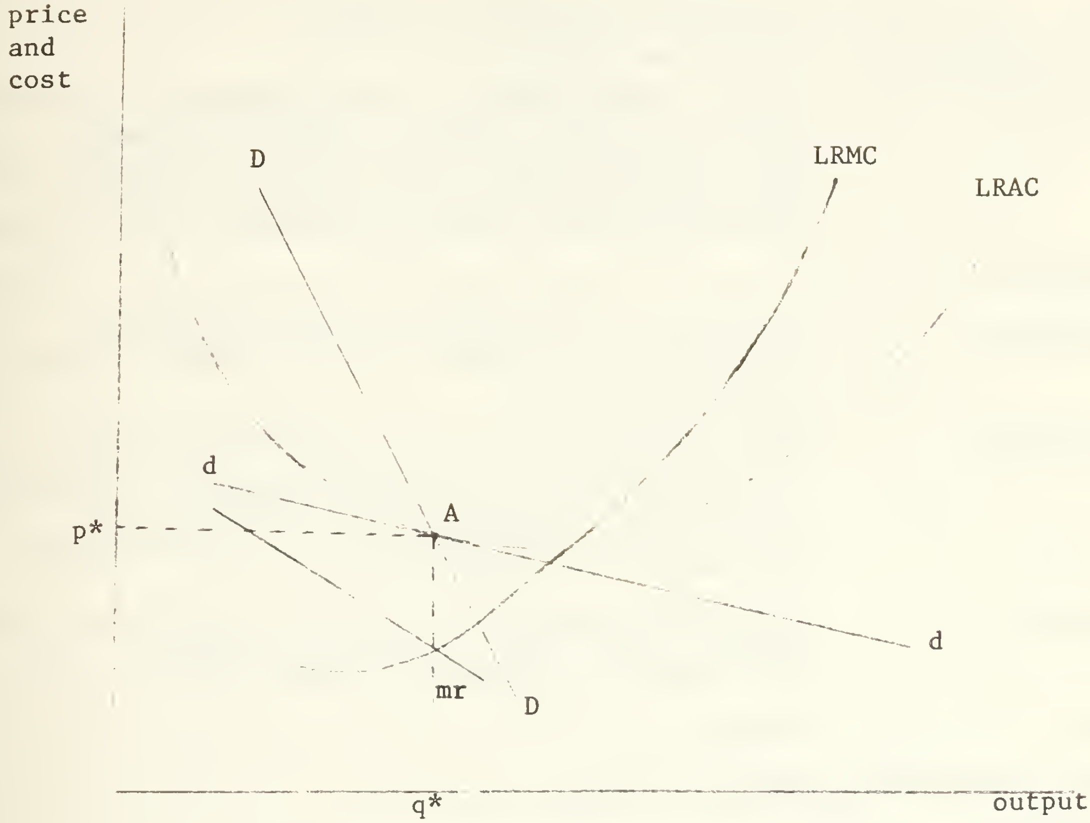


Figure II-13: Long Run Equilibrium for the Firm Under Monopolistic Competition

Table II-10 Model Summary: Monopolistic Competition

Class:	traditional (see (57) in text)
Firm's Objective:	profit maximization
Constraints:	overall market demand for the products of the group and market demand for individual firm's product; implicitly, the production function that underlies each firm's total cost function (which enters the model exogenously)
Variables:	
Exogenous:	implicitly, input prices (in order that each firm's total cost function depend on its own output level only)
Endogenous:	output levels (q_1, ..., q_n), market price (p), and total profit for each producer (π_1, ..., π_n); in addition, as in the duopoly analysis, input usage could also be determined if the model were reformulated to permit this
Decision:	each firm selects its output level (q_1, ..., q_n, respectively), and in the fuller analysis, input levels could also be determined
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium
Time:	single period
Type of Model:	static optimization (model formulated as an unconstrained optimization in terms of the 'representative firm')
Solution Technique:	unconstrained optimization for determining the short run equilibrium position of the individual firm in isolation; graphical analysis for determining the long run equilibrium of firms in the industry.

Chamberlin's model of monopolistic competition suggests what can happen when the assumption of product homogeneity made in section B is dropped. Even if firms are so numerous that the behavior of any one firm will not affect the behavior of its rivals, and even if there is free entry and exit, over the long run output will not be produced at minimum average total cost. However, as noted previously, Chamberlin's model is not without its shortcomings.

The assumption that one firm's behavior will not influence the behavior of the other firms in the industry — or equivalently, that every firm has a tiny share of the market — is atypical of the product markets in the American economy.[117] While there may be thousands of service stations, retail clothing stores, and supermarkets, the behavior of rivals generally cannot be ignored. Instead of behaving according to the model of monopolistic competition, industries such as the above three each behave more like a series of linked oligopolies.[118]

The model of monopolistic competition also shares the basic limitations of the models discussed in sections B through D. The models are concerned mainly with the determination of market price (or, in the case of the kinked demand curve model, with explaining the rigidity of current market price), and in achieving this determinancy, they have abstracted from several factors that are likely to have a significant impact on the behavior of actual firms. For example, it is assumed that each firm knows its cost and revenue functions with certainty, and while this might be acceptable under perfect competition where each firm takes prices as given and only needs to estimate its production function, in the real world uncertainty as to future costs and the future state of demand are a major source of concern to the corporate manager. In the remainder of this chapter models of recent vintage that attempt to explain the behavior of the modern corporation are examined.

F. REFORMULATING THE FIRM'S OBJECTIVE FUNCTION: MAXIMIZING THE STOCK MARKET VALUE OF THE FIRM

In the traditional models of the firm discussed in sections B through E it was assumed that the firm maximizes total profit. In those single period optimization models long run profit maximization is consistent with short run profit maximization, i.e. the firm does not suffer any harm to its long run profitability by pursuing the myopic policy of maximizing short run profit. Both in the short run and in the long run the firm selects the cost-minimizing combination of variable inputs with which to produce its chosen level of output, the only difference being that in the long run the set of variable inputs includes the firm's capital stock and in the short run it does not. Short run profit maximization is consistent with long run profit maximization because the former involves maximizing profit subject to the constraint that the firm's capital stock is fixed, while the latter involves the same optimization, but with capital variable.

In the real world, however, firms are typically faced with having to make intertemporal profit trade offs. Current decisions are likely to affect future profits, as for example, when a decision is made to reduce price this period (say, the price of automobiles), and as a result, an increase in the quantity of units sold occurs this period, but at the expense of units sold, and possibly at the expense of profits as well, next period. More recent attempts to model the behavior of the firm have taken into account the multiperiod context within which actual firms operate. In these modern models, intertemporal profit trade offs are explicitly taken into account. Such models conform with the widely held view that actual firms are not much interested in the quick kill that short run profit maximization implies and that, as a consequence, it is unlikely that an actual firm would as a general practice, intentionally pursue short run policies to the detriment of its long run objectives and continued survival.[119] Moreover, actual firms operate within a multiperiod context that is characterized by uncertainty, and in such an environment, the exact meaning of 'profit maximization' is unclear.[120]

In this section the traditional profit-maximizing model of the firm is reformulated. The single period objective of maximum total profit is replaced by the objective of maximizing the stock market value of the firm, one of several possible objective functions that explicitly take into account intertemporal profit trade offs. Then an example of such a model is presented. In the last part of the section, alternative objectives, such as sales maximization and growth maximization, and their relation to the objective of profit maximization are discussed.

1. Maximizing the Stock Market Value of the Firm

The most obvious way to adapt the objective of profit maximization to a multiperiod context is to have the firm maximize the present value of the future flow of profits. If the firm's profit in each period t is denoted by π_t, and if the firm's rate of discount, ρ, remains constant throughout future time periods, then the present value, V, of the future flow of profits is given by

$$V = \sum_{t=1}^{\infty} \frac{\pi_t}{(1+\rho)^t} , \qquad (60)$$

where it has been assumed that profit is realized at the end of each period.[121] If profit is initially π_o and is growing at an annually compounded rate of $g \cdot 100$ percent per annum, then (60) simplifies to

$$V = \sum_{t=1}^{\infty} \frac{\pi_o (1+g)^t}{(1+\rho)^t} = \pi_o \left(\frac{1+g}{\rho-g}\right) , \tag{61}$$

provided $g < \rho$. If profits remain constant forever (i.e. $g = 0$) , then (61) simplifies further to the present value formula[122]

$$V = \pi_o/\rho \ . \tag{62}$$

The use of present value calculations in deriving formulas (60), (61), and (62) takes into account intertemporal profit trade offs. If the firm attempts to maximize short run profit, it may wind up setting high prices that bring about a loss of consumer good-will and adversely affect future sales or it may cut costs by reducing product quality only to find that future sales suffer as a result. Given that most individuals — either in their role as shareholders or in their role as managers of firms — have a positive preference for income now rather than in the future,[123] the firm will be willing to some extent to sacrifice future profits for present profits.[124] But it is unlikely that the firm would, as a general operating policy, strive all out for maximum profit in the short run.[125]

Given that certain conditions are satisfied, V in formulas (60), (61), and (62) can be interpreted as the total market value of the firm. Modigliani and Miller have shown that, when capital markets are perfect,[126] the total market value of the firm — the market value of its equity plus the market value of its debt — is independent of the firm's capital structure — the distribution of ownership claims between equity and debt — when the capital markets are in equilibrium, and that, with additional assumptions, this result continues to hold even when there is uncertainty present.[127] Under Modigliani's and Miller's assumptions and with π_o interpreted as expected profit and with ρ interpreted as the appropriate discount rate,[128] formula (62) expresses the total market value of the firm under uncertainty when capital markets are in equilibrium.[129] Thus, the traditional model could be reformulated with either the present value of future profits or the total market value of the firm as the objective function, and the two reformulations would be equivalent when the capital markets are in equilibrium, provided capital markets are perfect and complete.[130]

Formulas (60), (61), and (62) would be appropriate as the objective function of the firm — interpreted as either the present value of the profit stream or the total market value of the firm — provided the firm were owner-controlled, because under owner control the firm's owners, rather than the firm's managers, decide how total profit is distributed between dividends and retained earnings. More importantly, the owners would determine the firm's investment policy and its financial policy, both of which will, in general, affect the firm's profitability and its total market value. As long as greater wealth is preferred to less, the owners could be expected to select those policies that, given the operating and financial decisions of all other firms,[131] maximize the total market value of the firm.

Given the separation of ownership from control, however, the shareholders of the firm have only an indirect say in the firm's operating policies and they do not control the distribution of profits. Their ownership is limited to the shares they hold and they have two sources of income: the dividends they receive periodically and the capital gains they realize upon sale of their shares.[132] For this reason, it is the stock market value of the firm's shares, rather than the total market value of the firm, that contributes directly to the utility of the owners. Since a higher market value of the firm's shares ceteris paribus increases shareholders' current wealth, which in turn implies greater shareholder utility, the criterion of maximizing the utility of the firm's shareholders is identified with the maximization of the current market value of the firm's shares,[133] although at the very least this requires that capital markets be perfect and, in an uncertain environment, it requires that additional conditions be satisfied.[134] Given that the appropriate assumptions are satisfied, the maximization of the stock market value of the firm's shares is the appropriate objective function for a model that is based on the view that the firm's managers act in a manner consistent with the best interests of the firm's shareholders.[135]

The reformulation of the traditional model's objective function in terms of the stock market value of equity has taken either of two forms: maximizing the share price[136] or maximizing the aggregate market value of the firm's equity.[137] In each case, however, a variety of specific formulations exist.[138,139] The two approaches are equivalent when the number of shares outstanding is held fixed.[140] It should be noted that the objectives of short run profit maximization, maximization of the total market value of the firm, maximization of the stock market value of the firm, and maximization of the share price cannot, in general, be used interchangeably. While an exhaustive examination of conditions under which different objectives are interchangeable lies beyond the scope of this paper, it should be mentioned that the four objectives can be used interchangeably (i.e. short run profit maximization is equivalent to the other three) when (i) the firm's policy choices one period do not affect its profitability in any other period, (ii) capital markets are perfect, and (iii) there is perfect certainty.[141] Since actual capital markets do not possess these characteristics, the four objectives cannot be regarded as interchangeable in models that purport to explain the behavior of actual firms.

In view of the apparent separation of ownership from control throughout much of the modern corporate sector, the next section describes a model in which the objective of the firm is to maximize the market value of the firm's equity. Similar, though more complex, models are discussed in sections I through L of this chapter.

2. A Value Maximization Model

This subsection presents a simple value maximization model of the firm. The model assumes that there is certainly with regard to future events and that capital markets are perfect, so that, in equilibrium at least, maximizing the firm's stock market value is equivalent to maximizing shareholder utility. A second value maximization model that assumes a certain environment is described below in section J. That model, the certainty version of the Lintner model, is more conveniently placed in section J along with Lintner's uncertainty models, which are shown to be straightforward generalizations of his basic certainty model. As the Lintner certainty model differs from the model discussed in this subsection — Lintner assumes steady-state growth whereas the model discussed below permits the firm's rate of growth to vary over time — the reader might find it interesting to read the first subsection of section J right after completing this subsection.[142]

Consider a firm that produces a single product that it sells in a less than perfectly competitive market. At each point in time it sells $Q(t)$ units of output at price $p(t)$.[143] Demand is a function of both price and the current level of advertising expenditure. Assuming that the demand function can be reexpressed with price $p(t)$ as the dependent variable, demand satisfies $p(t) = p[Q(t), A(t)]$, where $A(t)$ is the level of advertising expenditure at time t. Total revenue at time t, $R(t)$, is then also a function of quantity and the current level of advertising expenditure, $R(t) = R[Q(t), A(t)]$. The current rate of output, $Q(t)$, is a function of the amount of capital and the amounts of the variable inputs, such as labor, that are employed in production. Given the current capital stock, $K(t)$, and given perfect factor markets (i.e. constant market prices for all variable inputs), the firm's cost of production function $C[Q(t); K(t)]$, which is written in this manner to emphasize the function's dependence on the size of the firm's capital stock at time t, can be determined.[144]

Ignoring corporate income taxes, the firm can use the cash generated from its operations, $R[Q(t), A(t)] - C[Q(t); K(t)]$, for one of the following three purposes: paying dividends, advertising, and purchasing plant and equipment (i.e. gross investment). Letting $D(t)$ represent the amount of dividends paid at time t and letting

$I(t)$ represent the level of gross investment, the sources of cash must equal the uses of cash, so that the following accounting identity must always hold:

$$R[Q(t), A(t)] - C[Q(t); K(t)] = D(t) + A(t) + I(t) . \quad (63)$$

Solving (63) for $D(t)$ yields the following expression for the amount of dividends paid at time t :

$$D(t) = R[Q(t), A(t)] - C[Q(t); K(t)] - A(t) - I(t) , \quad (64)$$

where $D(t)$ is required to be nonnegative.

Under the assumptions of certainty and perfect markets and the additional assumption that the market rate of interest, r , remains constant over time, the current stock market value of the firm's equity, V , must, in equilibrium, equal the present value of the future flow of dividends,[145]

$$V = \int_0^{\infty} D(t)\, e^{-rt}\, dt . \quad (65)$$

Adopting the traditional criterion, it is assumed that the firm's objective is to maximize (65).

The optimization is not unconstrained, however. Allowance must be made for depreciation. Assume the capital stock wears out at a constant rate δ , so that $\delta \cdot K(t)$ represents the cost of capital depreciation.[146] Assuming that all investment is financed internally,[147] the growth rate of the firm's capital stock, $\dot{K}(t)$, where the dot denotes differentiation with respect to time, is identically equal to gross investment less depreciation,[148]

$$\dot{K}(t) = I(t) - \delta \cdot K(t) . \tag{66}$$

The objective of the firm is to maximize (65) subject to (66). Rewriting (66) as

$$I(t) = \dot{K}(t) + \delta \cdot K(t) \tag{67}$$

and performing two substitutions, the first involving (67) being substituted for $I(t)$ into (64) and the second involving the resulting expression for $D(t)$ being substituted into (65), the model of the value-maximizing firm can be formulated as the following unconstrained multiperiod optimization problem:[149]

$$\underset{\{Q(t),A(t),K(t)\}}{\text{maximize}} \quad V = \int_0^\infty \{R[Q(t),A(t)] - C[Q(t);K(t)] - A(t) - [\dot{K}(t) + \delta \cdot K(t)]\} \, e^{-rt} \, dt , \tag{68}$$

which can be solved by employing the classical calculus of variations.[150] The necessary conditions for an optimal solution to problem (68), which are used collectively to characterize the equilibrium position of the individual firm through time, are the following:

$$\left(\frac{\partial R}{\partial Q} - \frac{\partial C}{\partial Q}\right) e^{-rt} = 0 \tag{69}$$

$$\left(\frac{\partial R}{\partial A} - 1\right) e^{-rt} = 0 \tag{70}$$

$$\left(\frac{\partial R}{\partial K} - \frac{\partial C}{\partial K} - \delta\right) e^{-rt} - \frac{d}{dt}(-e^{-rt}) = 0 , \tag{71}$$

where equation (71) is the Euler equation.

Since $e^{-rt} > 0$, it follows from (69) that $\frac{\partial R}{\partial Q} = \frac{\partial C}{\partial Q}$ at each time t, which is just the familiar marginal revenue equals marginal cost rule for short run profit maximization. Thus, in setting its output level, the firm behaves as a short run profit maximizer. Similarly, (70) implies that $\frac{\partial R}{\partial A} = 1$ at each time t, which is just the same short run profit maximization rule applied to advertising instead of output. The firm should spend on advertising up to the point at which an additional dollar of advertising expenditure calls forth exactly an additional dollar of revenue.

Evaluating $\frac{d}{dt}(-e^{-rt})$, rearranging terms, and simplifying in equation (71), yields the following necessary condition for optimal investment in plant and equipment:

$$\frac{\partial R}{\partial K} - \frac{\partial C}{\partial K} = \delta + r . \tag{72}$$

According to (72), at each point in time the firm should invest in plant and equipment up to the point at which the addition to net revenue (i.e. total revenue less variable costs) resulting from the optimal use of the new facilities just equals the cost of capital, measured in (72) as the sum of the rate of depreciation and the rate of interest, i.e. the sum of depreciation costs and the opportunity cost of capital. More simply, the firm should continue to invest up to the point at which the marginal profitability of capital just equals the marginal cost of capital. This is just the neoclassical criterion for optimal capital investment (when capital goods prices are held fixed).[151] Collectively, conditions (69) - (71) have been shown to yield the equilibrium conditions for short run profit maximization, which bears out the statement made in the previous subsection that, in a world of certainty with perfect markets, short run profit maximization and maximizing the firm's share price are equivalent.

By way of summarizing this subsection, the characteristics of the value maximization model are summarized in table II-11.

Table II-11 Summary of Value Maximization Model

Class:	modern traditional (see (68) in text)
Firm's Objective:	stock market value maximization (where the stock market value of the firm is expressed as the present value of the flow of dividends)
Constraints:	implicitly, technological constraint (once again, embodied in the (implicit) production function) and implicit product demand (at each point in time) and factor supply (at each point in time) constraints; also implicitly, nonnegativity constraints on the decision variables and on dividend flows
Variables:	
Exogenous:	interest rate (r)
Endogenous:	for each time t , output (Q(t)), advertising expenditure (A(t)), capital stock (K(t)), investment (I(t)), and dividends (D(t))
Decision:	for each time t , output (Q(t)), advertising expenditure (A(t)), and capital stock (K(t))
Finance:	internal financing only
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium, and specifically, of the equilibrium position of the firm at each point in time and of the equilibrium time path of the firm's capital stock (K(t))
Time:	multiperiod
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	classical calculus of variations

3. Alternative Objectives

Thus far all the models that have been considered have assumed that the firm's managers try to maximize the welfare of the firm's owners, which amount to profit maximization in the single period world of sections B through E and maximizing the share value of the firm, which reduces to profit maximization when markets are perfect and the future is certain, in the multiperiod context of the previous subsection. Whether in the modern corporate environment of large, diversified firms whose managers enjoy considerable discretion it is still reasonable to assume either that economic forces are at work to ensure profit (or value) maximization or that managers for some reason feel obligated to do the best they possibly can for the firm's shareholders has been the subject of considerable debate, the fundamental issues of which were summarized in chapter one. A variety of alternative objectives have been proposed, and the purpose of this subsection is to indicate in broad terms (i) why these alternative objectives may not be very different from profit maximization and (ii) even if they are different, it may be very difficult, if not impossible in many cases, to determine which objective (or set of objectives) is 'best'.

The next section of this chapter describes models in which managers are assumed to act in their own best interest, maximizing sales or a utility function whose arguments are quantities that are of interest to managers. In each model profit appears as a constraint. If due to pressure from dissident shareholders or to adverse swings in business activity the profit constraint becomes binding, the firm will, in effect, be forced to maximize profit.[152] If the profit constraint is alternatively binding and nonbinding, then the firm's managers may maximize growth (or some other quantity) one period and profit the next. Trying to determine empirically whether the firm is a profit maximizer or a growth maximizer would, therefore, be very difficult unless this particular pattern of behavior were correctly identified.[153] So, while the firm generally cannot maximize two different quantities, say sales and profit, at the same time, it may pursue different objectives at different points in time.[154] This point is discussed further in the next section.

A second complication that makes it difficult to determine the true objectives of the firm is uncertainty. If the firm's managers are trying to maximize the market value of the firm, then modifying (62), they try to maximize $V = \frac{\pi}{\rho}$.[155] But where uncertainty exists, both π and ρ can vary, and if the firm can adjust its expenditures on advertising and promotion, research and development, and investment in plant and equipment in such a way as to reduce the fluctuations in profit over time, and thereby reduce the level of risk and the value of ρ, it may succeed in raising V, even though it is at the same time reducing π. During peak periods the firm may increase advertising or research and development expenditure (or both) beyond the point at which it is most profitable to do so in order to moderate a rapid increase in profits. While the firm is actually attempting to reduce the fluctuation in total profit, it may appear to an outside observer that it is attempting to maximize sales or growth (subject to maintaining some satisfactory level of profit, π).

A third complication is that suggested by the behavioralists. According to the behavioralists, the objectives of firms vary according to the goals of the social groupings that make up each firm, and the goals of any single firm can change over time as the compositions of the social groupings change and as power — the relative extent to which a single social grouping is able to influence the other social groupings to accommodate its goals within the set of goals established for the firm — shifts among the social groupings within the firm. There may at no time be any single overriding goal that the firm's managers seek to maximize. Rather, according to this view of the firm, the various social groupings have goals that conflict to some extent. The goals of the firm are established through a bargaining process that determines a set of compromise goals. For this reason, the firm is unlikely to optimize with respect to some single objective, but rather, it is likely to seek satisfactory performance with respect to each of the several goals that arise out of the bargaining process in order to try to satisfy each social grouping. The behavioral models, their strengths and their limitations, are discussed further in section H.

The next section describes a class of models that reflect the objectives of corporate managers. In such models profit and the stock market value of the firm play significant roles, though as constraints rather than as the single all-consuming goal of the firm.

G. MANAGERIAL MODELS: MAXIMIZING MANAGERIAL UTILITY

Managerial theories of the firm reflect the recognition by economists first, that corporate managers have objectives other than the maximization of total profit or the maximization of the stock market value of the firm, and second, that the separation of ownership from control and imperfections in the capital markets have given corporate managers some discretion to pursue their own objectives. According to Mueller:[156]

> ... managers maximize, or at least pursue as one of their goals, the growth in physical size of their corporation rather than its profits or stockholder welfare ... both the pecuniary and nonpecuniary rewards which managers receive are closely tied to the growth rate of their firm.

Moreover, "a growth-maximizing management can be expected to push its [growth] beyond the point that maximizes stockholder welfare."[157] Therefore, the managerial theories have replaced the traditional objective of profit (or stock market value) maximization with other goals, such as growth maximization, that reflect the objectives of corporate managers.

Managers are held to favor growth and large size for a variety of reasons. Managers are supposed to value salary, power, and status,[158] all of which are positively correlated with the size of the firm.[159] Larger size provides management with greater security against takeover[160] and with a heightened ability to raise finance internally that leads to improved corporate financial security;[161] enables firms to take advantage of economies of scale in production, research and development, and marketing;[162] leads to market power that facilitates effective corporate planning;[163] and makes it easier for firms to diversify into new markets.[164] Faster growth creates more opportunities for the internal promotion of lower and middle level managers;[165] lends the impression that the firm is 'progressive';[166] facilitates improvements in operating efficiency as the firm brings new people into the organization and adopts new and better productive techniques;[167] and, given the current size of the firm, enables the firm to achieve a larger size, with all the benefits that larger size entails, more quickly. Growth through diversification often carries the added advantages of a temporary monopoly position, when a new product is developed and other firms are slow to react,[168] and a reduction in the firm's overall level of risk.[169] According to Marris, professional managers are motivated to adopt policies that promote growth, not only because growth creates new openings in the corporate hierarchy, but also because the head of a growing division distinguishes himself as a productive member of the organization worthy of promotion.[170] Marris and others have formulated models of the firm in which measures of growth or size enter the objective function either directly or indirectly through the managerial utility function.[171]

Corporate managers are believed to have other objectives as well. Monsen and Downs argue that managers are interested in maximizing the present value of their lifetime incomes, which they argue, is not directly related to the current level of profits.[172] Oliver Williamson maintains that managers are utility maximizers and that managerial utility is a function of the size of the manager's staff, total emoluments, and the amount of discretionary profit at the manager's disposal.[173] Gordon argues that managers strive for security and may eschew even relatively safe ways of increasing total profit in order to avoid risk.[174] Though giving growth a primary role, Marris also incorporates a measure of security — the valuation ratio — in his version of the managerial utility function.[175]

This section describes the managerial models of Baumol, Marris, and Oliver Williamson, in that order.[176]

1. The Baumol Models

a. Baumol's Sales Maximization Model

Based in part on his experience as a business consultant, William Baumol has formulated a model of the large oligopolistic firm in which the firm's objective is to maximize total sales revenue.[177] Baumol argues that corporate managers are more interested in sales than profit, partly because sales are more amenable than profit to objective measurement and partly because the salaries of top managers are more closely correlated with the size of the firm than with its profitability.[178] Numerous statements by corporate executives and articles in the business literature would appear to bear this out. For example, the following somewhat remorseful statement by Brooks McCormick, President of International Harvester Co., "We became a sales-oriented organization, assuming the more volume we had, the more money we would make, We simply did not put sufficient emphasis on profitability."[179] As this statement attests, the firm cannot ignore its profitability. There is a profit constraint in the Baumol model, but, according to Baumol, once this minimum level of total profit has been attained, the firm's managers are willing to sacrifice any further increase in profit for an increase in sales revenue.[180]

The firms with which Baumol is concerned are oligopolistic. From the discussion in sections C and D it would appear that the oligopolistic interdependence of firms would have to be taken into account. Baumol circumvents this problem by arguing that the firm follows established practices in its day-to-day operations and ignores the behavior of its rivals in the short run.[181] This enables Baumol first, to focus on the oligopolistic firm in isolation and to draw the firm's downward-sloping demand curve without any kink, and second, to determine the firm's optimal production and advertising policies without having to take into account explicitly the reactions of the firm's rivals.

Since the demand curve is downward-sloping, maximizing sales revenue is not the same as maximizing the volume of physical output. In the absence of a profit constraint, maximizing sales revenue would require the firm to locate the level of output at which marginal revenue is zero, or equivalently, at which the price elasticity of demand is equal to one.[182] In figure II-14 the firm would produce Q_3 units of output, which it would sell at price P_3, and earn total revenue of $R(Q_3) = P_3 \cdot Q_3$. Since $MC > MR$ at Q_3, the firm would not earn maximum profit, and in fact, since $P_3 < AC$, it would experience a loss.

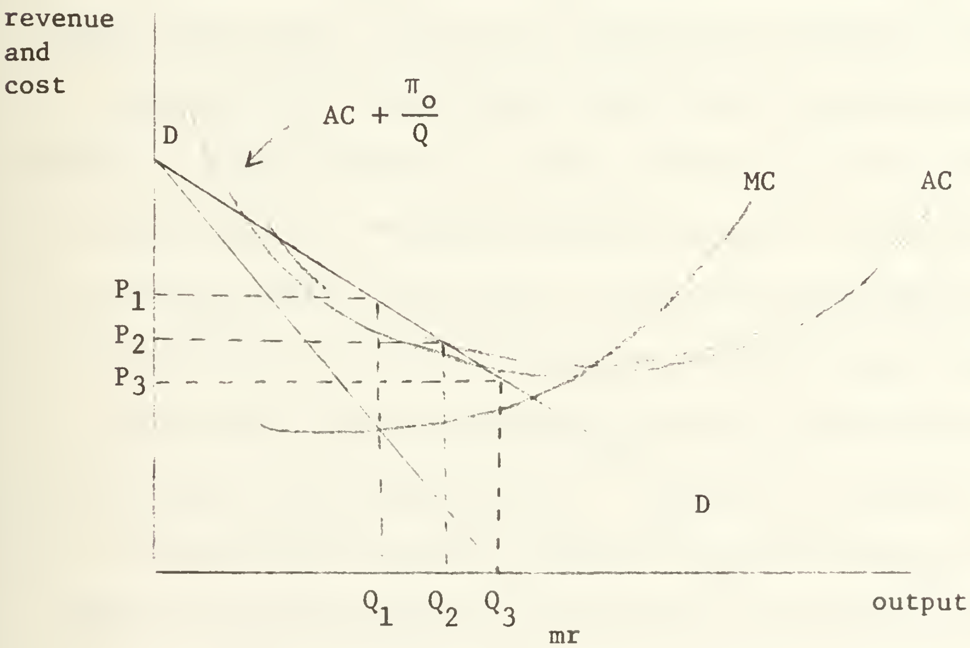


Figure II-14: Short Run Optima for a Revenue Maximizer and a Profit Maximizer

If there is a profit constraint, then the firm must take this into account when setting price and output. Because the firm in the Baumol model would be willing to sacrifice any additional profit beyond this minimum required amount if, by so doing, it could generate additional sales revenue, the firm treats the minimum required level of profit like an addition to its fixed cost. The selling price will be the sum of the average cost, AC , plus a mark-up just sufficient to meet the profit constraint. In figure II-14 the firm's profit must total at least π_o , and the curve $AC + \pi_o/Q$ has been drawn by adding π_o/Q , per unit required profit, to AC , average total cost, for each level of output Q . The constrained sales maximizer will produce Q_2 units of output, which it will sell at price $P_2 = AC(Q_2) + \pi_o/Q_2$, and earn total revenue of $Q_2 \cdot P_2 = Q_2 \cdot AC(Q_2) + \pi_o$ and total profit of π_o , as required.[183] Once again, total profit is not maximized because marginal cost exceeds marginal revenue at Q_2 .[184]

The foregoing has focused on conditions under which revenue maximizers and profit maximizers can be expected to act differently. Conditions under which they can be expected to exhibit identical behavior are also of interest. If the minimum required level of profit were equal to the maximum attainable profit, then a sales maximizer and a profit maximizer would behave in an identical fashion, employing the same combination of inputs, producing the same level of output, and charging the same price. Moreover, this would continue to be true if advertising were introduced into the model. In this more general case, the profit constraint would force the revenue maximizer to adopt advertising policies consistent with profit maximization.[185] As a second case of identical behavior, even if the maximum attainable profit exceeds the required minimum, for given levels of production cost and advertising expenditure, a sales maximizer and a profit maximizer would use the same combination of inputs to produce the same level of output. Since profit equals revenue minus cost, for given levels of total production cost and advertising expenditure, revenue maximization is tantamount to profit maximization. Indeed, as demonstrated below, even when production costs and advertising expenditure levels are permitted to vary, the sales maximizer, just like the profit maximizer, strives for maximum productive efficiency, for the lower are the revenue maximizer's production costs, the greater is its surplus of revenue over production costs, and the greater is the supply of funds that are available (beyond those needed to meet the profit constraint) to be spent on advertising.[186]

Baumol assumed that the marginal productivity of advertising expenditure is always positive, that is, that a one dollar increase in advertising expenditure will always call forth some additional sales at the current market price. For this reason, it is impossible for the firm to achieve maximum revenue before the profit constraint has been satisfied. In figure II-15 the firm has achieved maximum revenue by producing and selling Q_1 units of output, and profit exceeds the required minimum, given the firm's current level of advertising expenditure. The firm would react to this situation by spending the excess of profit above the required minimum on advertising, shifting the total revenue and total cost curves upward.[187] Beyond some point the marginal productivity of advertising expenditure will diminish steadily, and this will ensure that successive increases in advertising expenditure will produce successively smaller increments in total revenue, and eventually, successively greater decrements to total profit. At the optimum, Q_2 in figure II-15, the profit constraint is satisfied exactly.[188]

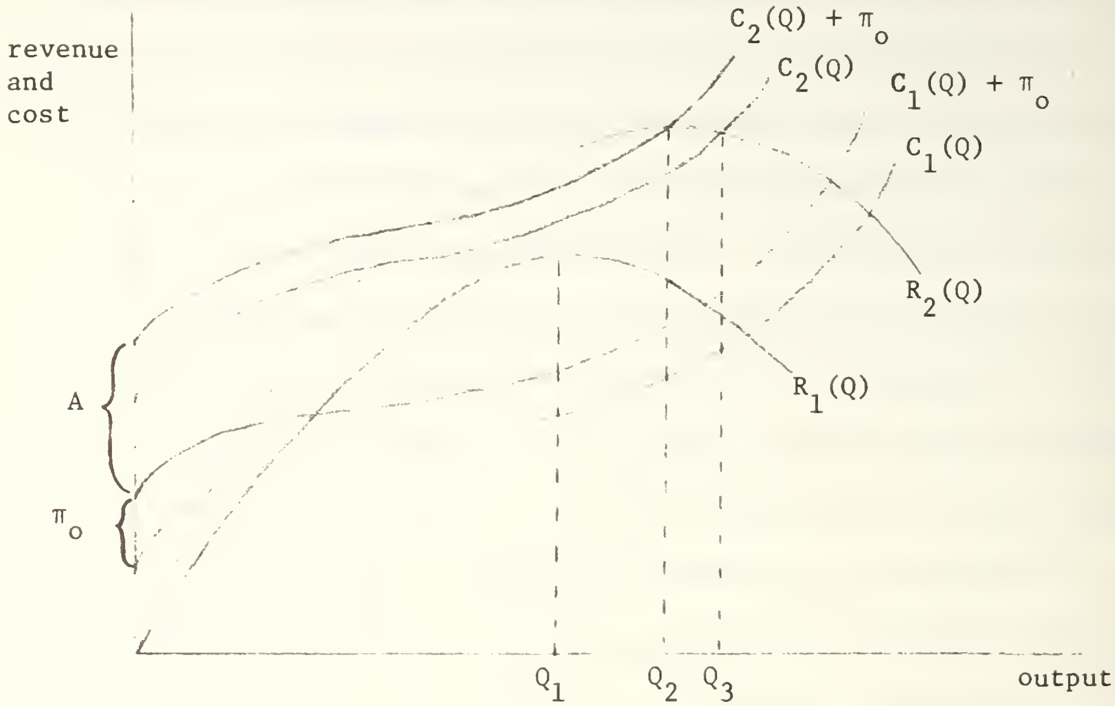


Figure II-15: Short Run Optimum for a Sales Maximizing Firm That Can Vary Its Advertising Expenditure

The foregoing discussion assumed the firm produced a single output. A more general model of the revenue maximizing firm, in which the firm uses m inputs to produce n outputs each with a downward-sloping demand curve DD, can be formulated as the following mathematical programming problem:[189]

$$\begin{aligned}
&\underset{\{q_i, x_j, A_i\}}{\text{maximize}} \quad \sum_{i=1}^{n} R_i(q_i, A_i) \\
&\text{subject to} \quad F(q_1, \ldots, q_n, x_1, \ldots, x_m) = 0 \\
&\qquad \sum_{i=1}^{n} R_i(q_i, A_i) - \sum_{j=1}^{m} r_j x_j - \sum_{i=1}^{n} A_i \geq \bar{\pi} \\
&\qquad q_i,\ x_j,\ A_i \geq 0\ ,\quad \forall_i, \forall_j
\end{aligned} \tag{73}$$

where q_i is the output of the i-th good; x_j is the amount used of the j-th input; A_i is the amount spent on advertising for the i-th good (which is assumed not to affect the demand for any of the firm's other products); $R_i(q_i, A_i)$ is the revenue earned from the sale of units of output i and is a function of the amount of output and the level of advertising expenditure;[190] r_j is the unit cost of the j-th input, which is assumed constant; and $\bar{\pi}$ is the minimum required level of profit. The objective function expresses total revenue earned as the sum of the amounts earned on the individual products; the first constraint is the familiar production function, and the second constraint is the profit constraint.

The solution to problem (73) can be characterized by applying the Kuhn-Tucker conditions for an optimum. The Lagrangian is[191]

$$\begin{aligned}
L_\lambda = &\sum_{i=1}^{n} R_i(q_i, A_i) + \lambda_1[F(q_1, \ldots, q_n, x_1, \ldots, x_m)] \\
&+ \lambda_2(\bar{\pi} - \sum_{i=1}^{n} R_i(q_i, A_i) + \sum_{j=1}^{m} r_j x_j + \sum_{i=1}^{n} A_i)
\end{aligned} \tag{74}$$

and the first order conditions are:

$$\frac{\partial L_\lambda}{\partial q_i} = \frac{\partial R_i}{\partial q_i} + \lambda_1 \frac{\partial F}{\partial q_i} - \lambda_2 \frac{\partial R_i}{\partial q_i} = 0 \ , \quad i = 1,\ldots, n \tag{75}$$

$$\frac{\partial L_\lambda}{\partial x_j} = \lambda_1 \frac{\partial F}{\partial x_j} + \lambda_2 r_j = 0 \ , \quad j = 1, \ldots, m \tag{76}$$

$$\frac{\partial L_\lambda}{\partial A_i} = \frac{\partial R_i}{\partial A_i} + \lambda_2 \left(- \frac{\partial R_i}{\partial A_i} + 1\right) = 0 \ , \quad i = 1, \ldots, n \tag{77}$$

$$F(q_1, \ldots, q_n, x_1, \ldots, x_m) = 0 \tag{78}$$

$$\left.\begin{array}{l} \sum_{i=1}^{n} R_i(q_i, A_i) - \sum_{j=1}^{m} r_j x_j - \sum_{i=1}^{n} A_i > \overline{\pi} \quad \text{and} \quad \lambda_2 = 0 \\ \text{or} \\ \sum_{i=1}^{n} R_i(q_i, A_i) - \sum_{j=1}^{m} r_j x_j - \sum_{i=1}^{n} A_i = \overline{\pi} \quad \text{and} \quad \lambda_2 < 0 \end{array}\right\} \tag{79}$$

Solving these necessary conditions leads to a characterization of the single period equilibrium position of the revenue maximizing firm. Solving each of the equations in (75) for $\frac{\lambda_2 - 1}{\lambda_1}$ gives $\frac{\partial F/\partial q_i}{\partial R_i/\partial q_i} = \frac{\lambda_2 - 1}{\lambda_1}$, and equating any two expressions for $\frac{\lambda_2 - 1}{\lambda_1}$ yields

$$\frac{\partial R_k/\partial q_k}{\partial R_\ell/\partial q_\ell} = \frac{\partial F/\partial q_k}{\partial F/\partial q_\ell} = - \frac{\partial q_\ell}{\partial q_k} \qquad \begin{array}{l} 1 \leq \ell \leq n \\ 1 \leq k \leq n \end{array} \tag{80}$$

where $-\partial q_\ell/\partial q_k$ is interpreted as the rate of product transformation. According to equation (80), in order for the firm to be in equilibrium, it is necessary that the rate of product transformation for each pair of goods equal the ratio of the marginal revenues of the two goods. It is noted that condition (80) is equivalent to (25), where $p_k = \partial R_k/\partial q_k$ and $p_\ell = \partial R_\ell/\partial q_\ell$. Next, solving each of the equations (76) for $-\lambda_2/\lambda_1$ gives $\dfrac{\partial F/\partial x_j}{r_j} = -\dfrac{\lambda_2}{\lambda_1}$, and equating any two expressions for $-\lambda_2/\lambda_1$ yields

$$\frac{r_k}{r_\ell} = \frac{\partial F/\partial x_k}{\partial F/\partial x_\ell} = -\frac{\partial x_\ell}{\partial x_k} \qquad \begin{array}{l} 1 \leq \ell \leq m \\ 1 \leq k \leq m \end{array} \tag{81}$$

where $-\partial x_\ell/\partial x_k$ is interpreted as the rate of technical substitution. According to equation (81), in order for the firm to be in equilibrium, it is also necessary that the rate of technical substitution between each pair of inputs equal the ratio of the costs of the two inputs. It is noted that condition (81) is identical to (26). Taken together, the equilibrium conditions (80) and (81) imply that the sales maximizer, like the profit maximizer whose optimal mixes of outputs and inputs satisfy (25) and (26), respectively, will avoid the production of relatively unprofitable outputs and the use of relatively unprofitable inputs.

Unlike the profit maximizer, the sales maximizer will produce 'too much' output and use 'too many' of its inputs, with 'too much' and 'too many' understood in the sense of 'more than a profit maximizing firm would'. To see why, solve any of the equations in

(75) and any of the equations in (76) for λ_1 and equate the two expressions to obtain

$$\frac{\lambda_2}{\lambda_2 - 1} \frac{r_k}{\partial R_\ell / \partial q_\ell} = - \frac{\partial F/\partial x_k}{\partial F/\partial q_\ell} = \frac{\partial q_\ell}{\partial x_k} \qquad \begin{array}{l} 1 \leq \ell \leq n \\ 1 \leq k \leq m \end{array} \tag{82}$$

Solving (82) for r_k gives

$$r_k = \frac{\lambda_2 - 1}{\lambda_2} \cdot \frac{\partial q_\ell}{\partial x_k} \cdot \frac{\partial R_\ell}{\partial q_\ell} \qquad \begin{array}{l} 1 \leq \ell \leq n \\ 1 \leq k \leq m \end{array} \tag{83}$$

From (79), $\lambda_2 \leq 0$. It follows from (77) and the assumption that $\frac{\partial R_i}{\partial A_i} > 0$, for all levels of advertising expenditure A_i , that $\lambda_2 < 0$. Otherwise, for $\lambda_2 = 0$, (77) gives $\frac{\partial R_i}{\partial A_i} = 0$, which is a contradiction. Note that $\lambda_2 < 0$ implies, by (79), that the profit constraint must be binding at optimality, as stated earlier in this subsection. Thus, from (79), $\lambda_2 < 0$, which implies that $\frac{\lambda_2 - 1}{\lambda_2} > 1$. Then from (83)

$$r_k = \frac{\lambda_2 - 1}{\lambda_2} \cdot \frac{\partial q_\ell}{\partial x_k} \cdot \frac{\partial R_\ell}{\partial q_\ell} > \frac{\partial q_\ell}{\partial x_k} \cdot \frac{\partial R_\ell}{\partial q_\ell} \qquad \begin{array}{l} 1 \leq \ell \leq n \\ 1 \leq k \leq m \end{array} \tag{84}$$

which implies that the sales maximizer, when in equilibrium would employ inputs beyond the point at which it is most profitable to do so. In equilibrium for the sales maximizer, the marginal revenue product of the k-th input, $\frac{\partial q_\ell}{\partial x_k} \cdot \frac{\partial R_\ell}{\partial q_\ell}$ is less than that input's marginal cost (in this case r_k), in contrast to (27), according to which the profit maximizer, when in equilibrium, would have hired inputs up to the point at which the marginal revenue product of each input just equals that input's marginal cost. Therefore, if each input's marginal revenue product is declining, $\frac{\partial q_\ell}{\partial x_k} \cdot \frac{\partial R_\ell}{\partial q_\ell} < 0$, as is typically assumed, condition (84) implies that the sales maximizer hires more of the k-th input than does the profit maximizer.

Since at optimality the marginal physical product of all inputs must be positive, i.e. $\frac{\partial q_\ell}{\partial x_k} > 0$, dividing each side of (84) by $\partial q_\ell / \partial x_k$ will not change the sense of the inequality. Thus,

$$\frac{r_k}{\partial q_\ell / \partial x_k} > \frac{\partial R_\ell}{\partial q_\ell} , \quad \text{or} \quad MC_\ell > MR_\ell , \quad 1 \le \ell \le n \tag{85}$$

Unlike the profit maximizer who equates marginal revenue and marginal cost, the sales maximizer expands production beyond this point so that, in equilibrium, marginal cost exceeds marginal revenue.[192] In the case of one output, this situation is depicted by figure II-14.

In view of the results just obtained, it would appear reasonable that in equilibrium the sales maximizer would balance the relative contributions to revenue and to profit of each output, each input, and the expenditure on advertising for each good. This is easily demonstrated. From (75),

$$\frac{\partial R_i}{\partial q_i} - \lambda_2 \left(\frac{\partial R_i}{\partial q_i} - \frac{\lambda_1}{\lambda_2} \cdot \frac{\partial F}{\partial q_i}\right) = 0 . \tag{86}$$

From (76), $\frac{\partial F/\partial x_j}{r_j} = - \frac{\lambda_2}{\lambda_1}$ so that $- \frac{\lambda_1}{\lambda_2} = \frac{r_j}{\partial F/\partial x_j}$ and substituting into (86) gives

$$\frac{\partial R_i}{\partial q_i} - \lambda_2 \left[\frac{\partial R_i}{\partial q_i} + \frac{r_j}{\left(\frac{\partial F/\partial x_j}{\partial F/\partial q_i}\right)}\right] = 0 . \tag{87}$$

But $\dfrac{\partial F/\partial x_j}{\partial F/\partial q_i} = -\dfrac{\partial q_i}{\partial x_j}$ and $\dfrac{r_j}{\partial q_i/\partial x_j}$ represents the marginal cost of the i-th output, so that the expression in brackets in (87) is interpreted as the marginal profit yield, $\partial\pi_i/\partial q_i$, of the i-th output. Substituting $\partial\pi_i/\partial q_i$ for the expression in brackets in (87) and solving for λ_2 gives

$$\lambda_2 = \frac{\partial R_i/\partial q_i}{\partial\pi_i/\partial q_i}, \quad 1 \leq i \leq n . \tag{88}$$

The Lagrange multiplier λ_2 is equal to the ratio of the marginal revenue of output i to the marginal profit yield of output i. A similar result holds for the inputs. Substituting $\lambda_1 = \dfrac{(\lambda_2 - 1)\,\partial R_i/\partial q_i}{\partial F/\partial q_i}$ from (75) into (76) and rearranging terms gives

$$\frac{\partial R_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_j} - \lambda_2 \left(\frac{\partial R_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_j} - r_j\right) = 0 , \tag{89}$$

where the expression in parentheses is interpreted as the marginal profit yield of input j. Solving (89) for λ_2 gives

$$\lambda_2 = \frac{\dfrac{\partial R_i}{\partial q_i} \cdot \dfrac{\partial q_i}{\partial x_j}}{\dfrac{\partial R_i}{\partial q_i} \cdot \dfrac{\partial q_i}{\partial x_j} - r_j}, \quad 1 \leq j \leq m , \tag{90}$$

which states that λ_2 is equal to the ratio of the marginal revenue product of input j to the marginal profit yield of input j.

Next considering advertising, from (77)

$$\lambda_2 = \frac{\frac{\partial R_i}{\partial A_i}}{\frac{\partial R_i}{\partial A_i} - 1} , \quad 1 \leq i \leq n , \tag{91}$$

which states that λ_2 equals the marginal revenue yield of an additional dollar of advertising divided by the marginal profit yield of an additional dollar of advertising. Combining conditions (88), (90), and (91) leads to the optimality conditions for sales maximization stated by Baumol: the sales maximizing firm should set its output, input, and advertising levels so that the ratio of the marginal revenue yield to the marginal profit yield is the same for all outputs, all inputs, and all categories of advertising expenditure.[193]

In Baumol's model the firm's minimum profit constraint, $\bar{\pi}$ in (73), is not determined, although Baumol does suggest its origin.[194] Needham and J.H. Williamson have shown how to extend the model so that $\bar{\pi}$ is determined.[195] The extension requires that the firm's objective be changed from single period sales maximization to the maximization of the present value of the stream of future sales. Then, as Needham demonstrates, given the minimum retention ratio permitted by the stock market, the interdependence of the different sales and profit levels over time guarantees that, once the firm has selected the current level of sales that maximizes the present value of the sales stream, its current minimum profit constraint $\bar{\pi}$ is determined.[196]

Baumol's sales maximization model implies that, ceteris paribus, the revenue maximizing firm will produce more output and register greater total revenue, but earn smaller profit, than a profit maximizing firm. When the sales maximizer is in equilibrium, marginal cost will exceed marginal revenue (and may even exceed price if average total cost rises sharply enough) and the marginal cost of each input will exceed its marginal revenue product. The distinguishing characteristics of Baumol's model of the revenue maximizing firm are summarized in table II-12.

Table II-12 Summary of Baumol's Sales Maximization Model

Class:	managerial (see (73) in text)
Firm's Objective:	maximize sales revenue
Constraints:	technological constraint (embodied in the production function), minimum profit constraints, and nonnegativity constraints on the decision variables; implicitly, product demand conditions (as embodied in $R_i(q_i,A_i)$)
Variables:	
Exogenous:	minimum required profit level ($\bar{\pi}$) and input prices (r_j); the profit level could be made endogenous, as suggested by Needham or by J.H. Williamson, although that would require that some stock market-related variable, such as a minimum retention ratio, be determined exogenously
Endogenous:	output levels (q_i), input levels (x_j), advertising levels (A_i), total revenue ($\sum_{i=1}^{n} R_i(q_i,A_i)$), and total profit
Decision:	output levels (q_i), input levels (x_j), and advertising levels (A_i)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of equilibrium position of the firm
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	generalized Lagrange multipliers

The Baumol sales maximization model has been criticized because there is no utility trade off to the firm's managers between revenue and profit. Up to the constraint, the firm maximizes profit, but once the constraint has been satisfied, the firm maximizes sales. A second criticism of Baumol's model is that it assumes certainty. Recently, Yarrow has reformulated Baumol's sales maximization model to incorporate uncertainty and has found that the introduction of uncertainty leads to predictions different from Baumol's regarding the behavior of the firm in response to various external stimuli, such as a change in the (lump sum) profits tax.[197] A third criticism of the model is that it ignores growth. Spending excess profit on advertising is not inconsistent with maximizing the firm's growth, and as Baumol later argued, there is much evidence that corporate managers are concerned with the growth of the firms they control.[198] The next subsection presents Baumol's growth maximization model.

b. Baumol's Growth Maximization Model

Subsequent to the development of his single period sales maximization model that was just discussed, Baumol developed a model in which the objective of the firm's managers is to maximize the rate of growth of sales, rather than the level of sales.[199] Unlike his single period model, in which the profit constraint appeared to be arbitrarily imposed from outside the firm, in the growth maximization model profit becomes a decision variable as management adjusts the level of profit according to the firm's need for internally generated funds with which to finance the firm's expansion.[200] Both of Baumol's models have a managerial character. In the single period model sales are maximized because that is the objective of the firm's managers.

Indeed, the objective function could just as easily have been formulated as $U(\sum_{i=1}^{n} R_i(q_i, A_i))$, where U stands for managerial utility.[201] Similarly, in presenting his growth maximization model, Baumol argued that top corporate managers maximize the rate of growth of sales, rather than the level of sales or some other quantity.[202] This subsection describes Baumol's growth maximization model.[203]

Define the following variables:

g = the firm's annual rate of growth of sales.[204]

I = the annual net (of dividends) flow of money capital into the firm, expressed as a percentage of the firm's current stock of fixed assets plus inventories, that is invested in fixed assets (i.e. plant and equipment) and in inventories (to support the growth in sales).[205]

π = total annual profit, expressed as a percentage of the firm's equity, which is the sum of total annual dividends paid, D , plus total annual retained earnings, E , each expressed as a percentage of the firm's equity. Thus, $\pi = D + E$, which is an accounting identity[206] that appears as a contraint in the model below.

The flow of money capital into the firm is reflected in the liabilities and stockholders' equity side of the firm's balance sheet, which is illustrated above in table II-1. If bonds are issued or if money is borrowed from banks (or from other sources) via the issuance of notes, then liabilities and total debt both increase. If shares of preferred stock are issued, then the contributed capital portion of stockholders' equity, and in addition debt, as defined in section A, both increase. If new shares of common stock are issued, then the contributed capital portion of stockholders' equity and equity, as defined in section A, both increase. The amounts of money capital raised from all these external sources are combined into the function $\phi(\pi, D)$, which expresses the amount of money capital that can be raised (annually) from external sources as a function of the firm's total annual profit, π , and total annual dividends paid, D , each expressed as the appropriate percentage indicated above. Implicitly, Baumol assumes this functional relationship to be time invariant. That is, both debt financing and external equity financing are permitted, although no distinction is made between them in the model. In addition, a third source — an internal source — of money capital is total retained earnings. Adding the amount of money capital generated internally, E , to the amounts raised externally (in each case, expressed as the appropriate percentage indicated above) gives an expression for the total amount of money capital raised annually, $I = \phi(\pi,D) + E$, which appears as a constraint in the model below.

It follows from the basic accounting identity between total assets and the sum of total liabilities and total stockholders' equity that any change in liabilities plus stockholders' equity must be reflected in an equal (in magnitude) change in total assets. According to the above definition of I , the increase in money capital is used by the firm to purchase additional plant and equipment and to build up inventories. This use of money capital is what is implied when I is included as an argument of the growth rate function $g(I,\pi)$ in the model below. Given the above definitions and relationships, Baumol's growth maximization model is formulated as the following mathematical programming problem.[207]

$$\begin{aligned} &\underset{\{\pi,D\}}{\text{maximize:}} && g = g(I,\pi) \\ &\text{subject to:} && I = \phi(\pi,D) + E \\ & && \pi = D + E \end{aligned} \qquad (92)$$

The objective function expresses the firm's growth rate as a function of its investment rate, I, and its profit rate, π. It is assumed that $\partial g/\partial I > 0$ and that beyond some point $\partial g/\partial \pi < 0$ (though, as in the Marris model discussed below, growth and profit may be directly related for low levels of profit and low growth rates).

The first constraint in (92) shows that a higher profit rate influences the amount of funds available for investment in two ways: directly through retained earnings and indirectly through the impact of the profit rate and the profits distributed as dividends on the firm's ability to raise funds externally. The second constraint in (92) is merely an accounting identity.

The relationship between π and g may be explained in the following manner.[208] The present value, R, of the stream of future sales is

$$R = \int_0^\infty S\, e^{gt}\, e^{-st}\, dt = \int_0^\infty S\, e^{-(s-g)t}\, dt = \frac{S}{s-g}, \qquad (93)$$

where S is the initial level of sales and s is management's subjective time rate of discount of sales, which reflects the value of a dollar in sales today relative to a dollar in sales a year from now.[209] From (93), $\frac{\partial R}{\partial g} = \frac{S}{(s-g)^2} > 0$, so that the present value of future sales is a strictly increasing function of g. On the cost side, let $C(g)$ represent the present value of all expected future costs as a function of the growth rate.[210]

As the firm grows, it must expand its management team. Increasing the rate at which new people are brought into the organization can lead to internal inefficiencies, and as a consequence, to higher costs, if the rate of expansion becomes so rapid that the new people cannot be properly indoctrinated as to the ways of the organization prior to assuming positions of responsibility.[211] If these costs of growth increase at an increasing rate as the firm's rate of growth increases, then $C'(g) > 0$ and $C''(g) > 0$, and eventually, the increasing cost of expansion will catch up with the increasing revenue derived from expansion.

The curves $R(g)$ and $C(g)$ are shown in figure II-16 where S and s have been held fixed so that R is a function of g only. The vertical distance between the curves defines the growth-profit function, $\pi(g) = R(g) - C(g)$. Up to g_m , $\pi(g)$ is an increasing function of g , and beyond g_m , $\pi(g)$ decreases as the growth rate increases. At higher growth rates profits compete with sales as the lower prices and higher advertising outlays needed to promote the rapid growth of sales cut into the firm's total profit. The profit maximizing firm would select growth rate g_m at which

$$\pi'(g_m) = R'(g_m) - C'(g_m) = 0 \quad \text{or} \quad R'(g_m) = C'(g_m) . \qquad (94)$$

Equation (94) is the familiar marginal revenue equals marginal cost condition for profit maximization, but with marginal revenue and marginal cost expressed as functions of the growth rate, rather than as functions of output as is usually the case.[212]

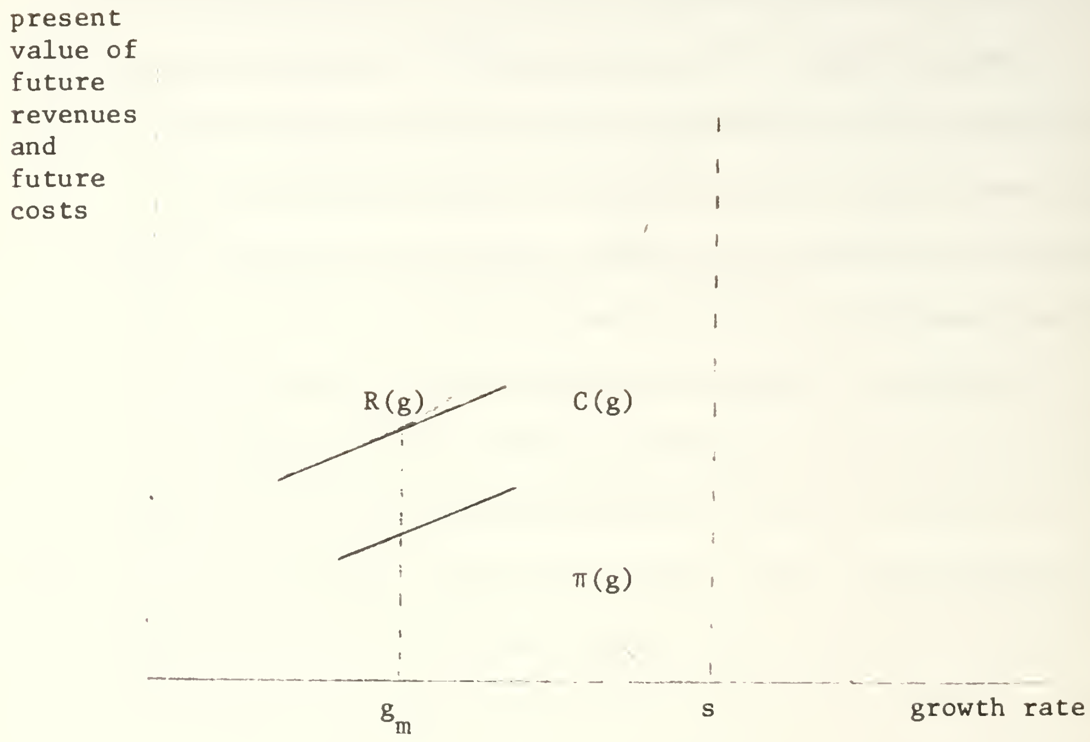


Figure II-16: Revenue from Expansion and the Cost of Expansion in the Baumol Growth Model

Baumol's growth maximization model (92) is a steady state growth model. In steady state in the Baumol growth maximization model, total profit, total sales, fixed assets, and all other quantities grow at a constant rate.[213] In addition, total annual profit, π , total annual dividends paid, D , and total annual retained earnings, E , each remains fixed as a percentage of the firm's equity, implying that total annual profit, total annual dividends paid, total annual retained earnings, and the firm's equity all grow at the same constant rate. While it is not required, in general, that these variables grow at the same rate as sales or as money capital, it can be shown that, if the firm produces a single good, if the firm's production function is linearly homogeneous, if input prices and the price of output are fixed (i.e. the product market and factor markets are all perfectly competitive), and if the amount of money capital raised externally grows at the same rate as the amount generated internally (i.e. retained earnings), then sales and total money capital will, when the firm is in equilibrium, grow at the same constant rate as total annual profit, total annual dividends paid, etc.[214]

Having discussed several features of Baumol's growth maximization model, the remainder of this subsection proceeds with an analysis of it. Baumol's model of the (sales) growth maximizer (92) can be simplified through the introduction of the retention ratio, r, which will, given the constancy of π, D, and E, remain constant in steady state.[215] Since $D = (1 - r)\pi$ and $E = r\pi$, it follows that the model can be reformulated as the unconstrained mathematical programming problem:

$$\underset{\{\pi, r\}}{\text{maximize:}} \quad g = g[\phi(\pi,(1 - r)\pi) + r\pi \ , \pi] \ . \tag{95}$$

Applying the chain rule and the fact that $\frac{\partial I}{\partial \phi} = 1$, the first order conditions for an optimal solution to (95) are found to be:

$$\frac{\partial g}{\partial \pi} = \frac{\partial g}{\partial I}\left[\frac{\partial \phi}{\partial \pi} + (1-r)\frac{\partial \phi}{\partial D} + r\right] + \frac{\partial g}{\partial \pi} = 0 \tag{96}$$

$$\frac{\partial g}{\partial r} = \frac{\partial g}{\partial I}\left[\frac{\partial \phi}{\partial D}(-\pi) + \pi\right] = 0 \tag{97}$$

By rearranging terms in (96) and by appealing to the implicit function theorem to write $-\frac{\partial g/\partial \pi}{\partial g/\partial I} = \frac{\partial I}{\partial \pi}$, provided $\partial g/\partial I \neq 0$, it follows that (96) is equivalent to

$$\frac{\partial I}{\partial \pi} = \frac{\partial \phi}{\partial \pi} + (1 - r)\frac{\partial \phi}{\partial D} + r , \tag{98}$$

which is the partial derivative with respect to π of the first constraint in (92), and hence, which is always satisfied as long as all the partial derivatives exist.

From (97) any one of three possibilities may hold: $\frac{\partial g}{\partial I} = 0$, $\frac{\partial \phi}{\partial D} = 1$, or $\pi = 0$. If $\partial g/\partial I = 0$, then from (96), $\partial g/\partial \pi = 0$ as well, and the firm has been able to generate enough finance so that the growth rate is an unconstrained maximum. If the firm is always able to use additional finance profitably, then $\partial g/\partial I > 0$, and this possibility can be ruled out.

If $\frac{\partial \phi}{\partial D} = 1$, then (98) simplifies to $\frac{\partial I}{\partial \pi} = \frac{\partial \phi}{\partial \pi} + 1$. The economic interpretation of the former is that at optimality the outflow of an additional dollar of dividends is just sufficient to enable the firm to raise an additional dollar of funds from external sources, so that increasing dividends will not alter the net inflow of funds. The firm is unable to increase its rate of growth of sales due to financial restrictions.

If $\pi = 0$, then there are no internally generated funds to spend for investment purposes or to distribute as dividends. Presumably, zero profit could not persist, unless there were investors who were willing to sacrifice all profit (and therefore all dividends) for growth. It might be conjectured that the nature of ϕ would cause

$\frac{\partial\phi}{\partial D} = 1$ to occur before $\pi = 0$. If so, then this third possibility can also be ruled out.

Thus, the firm finds its growth rate of sales constrained by financial restrictions. By pushing for growth, it reduces its profitability and its ability to pay dividends to the point where additional dividends will not call forth sufficient additional external finance to permit further expansion.

By way of summary, the distinguishing features of the Baumol growth maximization model are summarized in table II-13.

Table II-13 Summary of Baumol's Growth Maximization Model

Class:	managerial (see (95) in text)
Firm's Objective:	maximize the rate of growth of sales
Constraints:	financial constraint (embodied in the function ϕ); implicitly, product demand and factor supply conditions and, also implicitly, the technological constraint embodied in its production function (see footnote 214)
Variables:	
Endogenous:	profit (π), dividends paid (D), and retained earnings (E), each as a percentage of total equity; retention ratio (r); and growth rate of sales (g)
Decision:	profit as a percentage of total equity (π) and retention ratio (r)
Finance:	both debt financing and external equity financing permitted (in addition to internally generated funds, i.e. retained earnings)
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state growth path of the firm
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization

2. The Marris Model

Recognizing first, the considerable exercise of managerial discretion that is implied by the separation of ownership from control in the modern corporation, and second, the consequent implied pursuit of growth-promoting policies by corporate managers, Robin Marris has developed a managerial model of the firm[216] in which managers seek to maximize their own utility, which is a function of (i) the rate of growth of the firm's productive assets and (ii) security against takeover.[217] This subsection describes the two basic formulations of the Marris model and explores the important economic implications of each.

In Marris's view modern corporate managers are primarily interested in the growth of their firms because of the strong positive correlation between size and executive salaries and because growth adds to their prestige and creates opportunities for promotion. But if the firm strives all out for growth, ignoring its profitability and permitting the market valuation of its shares to fall, it runs the risk of a takeover bid.[218] There does exist some empirical evidence to support Marris's view that management can become so preoccupied with growth that profits suffer, the share value falls, and the disgruntled shareholders sell out their shares to a takeover raider, or worse yet, the firm goes bankrupt.[219] In Marris's view, corporate managers who value their own security will maximize growth, but subject to a valuation constraint.

In developing a growth maximization model of the Marris type there are at least two ways to proceed with the process of model formulation. One approach is to specify a managerial utility function

$$U = U(g,v) \ , \tag{99}$$

where g is the rate of growth of the firm's productive assets and v is the valuation ratio, which Marris defines as the ratio of the stock market value of the firm's equity capital to the book value of its net assets.[220] The valuation ratio serves as the measure of security; as it falls the probability of takeover increases. Having specified the utility function, the next step is to find what Marris calls the 'growth-valuation function'

$$v = v(g) \ , \tag{100}$$

which gives the valuation ratio corresponding to each (feasible) growth rate g. The problem facing the firm is to maximize (99) subject to (100). This represents Marris's latest specification of his model.[221]

The second approach — the first one adopted by Marris[222] — is to proceed via the important economic relationships that underlie (100) and to specify the security constraint in terms of market-imposed constraints on the firm's selection of values for key financial policy variables. Then the problem confronting the firm is to maximize g subject to these constraints. To distinguish the two formulations of the Marris model, the initial formulation will be referred to as the 'growth maximization' formulation and the recent formulation will be referred to as the 'managerial utility maximization' formulation.

Depending on the shape of U — whether there is a continual trade off between g and v along smooth managerial indifference curves, or else a clearly specified minimum permissible valuation ratio v^* and a lexicographic ordering of objectives such that first v^* is attained and thereafter g is maximized[223] — the two approaches can yield identical solutions. The discussion of the Marris model begins with the initial formulation and then proceeds to the second formulation and examines the relationship between the two.

a. Initial Formulation of the Marris Model: Growth Maximization[224]

The objective of the firm is to select the maximum rate of growth of productive assets consistent with the continued financial security of the firm. In the model the firm is required to grow in steady state;[225] that is, the model abstracts from the possibilities that the growth rate might vary with the size of the firm, as has been suggested by Penrose,[226] or that a particular policy might promise rapid growth after a few periods of relatively slow growth or vice versa.[227] In the model steady state growth requires that growth be balanced in the sense that the rate of growth of productive assets,[228] $\dot{C}$, must equal the growth rate of demand, $\dot{D}$, or in equation form,

$$\dot{C} = \dot{D} . \tag{101}$$

When growth is balanced and (101) is satisfied, it is possible to speak of 'the' growth rate of the firm, and this balanced growth rate will be denoted hereafter by g.

The growth of demand reflects not only increased sales in existing product lines, but also sales resulting from diversification, i.e. from the development of new products. Diversification plays a crucial role in the growth of demand because of the tendency for sales of a product to fall off as the market becomes saturated.[229] The rate of growth of demand, $\dot{D}$, depends largely, then, on the rate at which new products are added to the firm's catalogue. But the rate at which new products are added depends not only on the rate at which new products are tried, but also on the proportion of successes, for not all new products will prove to be profitable.[230] By increasing its spending on market research, product development, and advertising, the firm can increase the proportion of successes, though at the cost of a reduced profit margin. Denoting by d the diversification rate, i.e. the ratio of attempted new products to existing products, and by m the profit margin, i.e. the ratio of net operating income to total sales,[231] the rate of growth of demand can be expressed as a function of m and d, called the 'demand-growth' function,

$$\dot{D} = D(m,d) , \tag{102}$$

where $\partial\dot{D}/\partial m < 0$ and $\partial\dot{D}/\partial d > 0$. Several demand-growth curves are shown in figure II-17, where each demand-growth curve expresses the relationship between $\dot{D}$ and d for a different (given) profit margin m_i. A higher value for m_i leads to a lower demand-growth curve since $\partial\dot{D}/\partial m < 0$. For any given value of m_i the demand-growth curve is upward-sloping since $\partial\dot{D}/\partial d > 0$, but rises at a decreasing rate due to the assumption of diminishing returns.

Similarly, the growth rate of productive assets on the left-hand side of equation (101) is dependent on the firm's profit rate, which in turn is a function of the profit margin and the rate of diversification. The growth rate of productive assets is equal to the ratio of the supplies of new finance (both from retained earnings and from external sources such as the sale of bonds) available for that purpose each period to the stock of productive assets at the beginning of the period. Assuming no new stock issues, a constant leverage ratio (the ratio of debt to equity), and a constant retention ratio (the ratio of retained earnings to net income),[232] the growth rate of productive assets will be a constant proportion, a , of the profit rate (the ratio of net income to total assets), p :[233]

$$\dot{C} = a \cdot p \ . \tag{103}$$

Equation (103) defines the 'supply-of-finance' function.[234] Due to the desires of corporate managers for financial security, the retention ratio (and in an uncertain world where markets are incomplete, the leverage ratio) will be constrained, so that a cannot exceed some upper limit a^* imposed by the financial markets:

$$a \leq a^* \ . \tag{104}$$

The firm's rate of profit, p in equation (103), is a function of the profit margin and the rate of diversification, called the 'profit-rate' function,

$$p = p(m,d) \ , \tag{105}$$

where $\partial p/\partial m > 0$ and $\partial p/\partial d > 0$ over low rates of diversification and $\partial p/\partial d < 0$ over high rates of diversification. For a given diversification rate, a higher profit margin implies a higher profit rate. For a given profit margin, a higher rate of diversification at first leads to a higher profit rate as the firm's productive resources are shifted from less successful to more successful products and the firm's overall efficiency of capital utilization improves. Increasing the diversification rate, however, requires that the management team grow more rapidly, which will eventually lead to internal inefficiencies, a less efficient utilization of the firm's capital, and a falling profit rate.[235]

If the firm has selected the maximum permissible value for a, $a = a^*$ — its reason for making such a choice is discussed below — then the supply-of-finance function is of the form

$$\dot{C} = a^* \cdot p(m,d) \quad . \tag{106}$$

Several such supply-of-finance curves are drawn in figure II-17, where each supply-of-finance curve expresses the relationship between g and d for a different (given) profit margin m_i.

Since the supply-of-finance function is a constant multiple a^* of the profit-rate function, for each profit margin m_i the rate of growth of the supply of finance increases where $\partial p/\partial d > 0$ and decreases where $\partial p/\partial d < 0$. Since $\partial p/\partial m > 0$, a higher profit margin leads to a higher supply-of-finance curve.

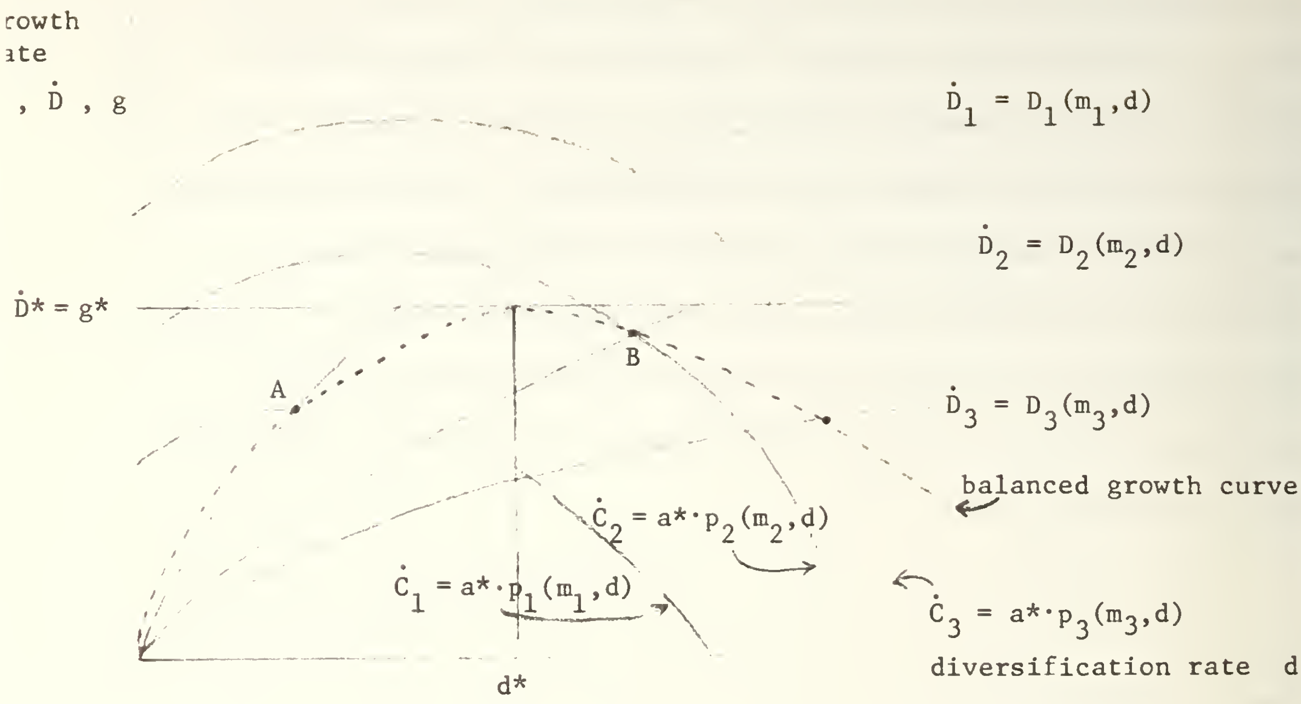


Figure II-17: Demand-Growth and the Supply-of-Finance in the Marris Model

Collecting equations (101), (102), (103), and (105), and the security constraint (104), the Marris model can be summarized as follows:

$\dot{C} = \dot{D}$ (balanced-growth condition) (101)

$\dot{D} = D(m,d)$ (demand-growth function) (102)

$\dot{C} = a \cdot p$ (supply-of-finance function) (103)

$a \leq a^*$ (security constraint) (104)

$p = p(m,d)$ (profit-rate function) (105)

where $\dot{C}$, $\dot{D}$, m , d , a , p , and a^* are as defined above. For each value of a that satisfies (104), the four equations (101), (102), (103), and (105) define a balanced-growth curve like the one shown as a dashed curve in figure II-17. The balanced growth curve shows the relationship between the balanced growth rate $g = \dot{C} = \dot{D}$ and the diversification rate d. It is found by varying the profit margin m, and for each value of m, locating the point of intersection of $\dot{C}$ and $\dot{D}$. In figure II-17 the points A, at which $\dot{C}_1$ and $\dot{D}_1$ intersect, B, at which $\dot{C}_2$ and $\dot{D}_2$ intersect, and C, at which $\dot{C}_3$ and $\dot{D}_3$ intersect, lie on the balanced-growth curve. Note that as a increases the family of supply-of-finance curves will rise, thereby increasing the ordinate of each point on the balanced-growth curve. Since the firm's objective is to maximize its growth rate, it will select $a = a^*$ in order to reach the highest permissible balanced-growth curve. Maximizing its growth rate along that curve leads the firm to select the equilibrium diversification rate d^* in figure II-17. In multiperiod equilibrium the firm grows in steady state at rate g^*, diversifies at the equilibrium rate d^*, and earns an equilibrium profit margin m^* which satisfies the equation $g^* = D(m^*,d^*) = a^* \cdot p(m^*,d^*)$.

The characteristics of the initial formulation of the Marris model — his growth maximization model — are summarized in table II-14.

Table II-14 Summary of Marris's Growth Maximization Model

Class:	managerial (see (101)-(105) in text)
Firm's Objective:	maximize rate of growth of productive assets (which is also the rate at which all other quantities in the model grow)
Constraints:	demand-growth function, supply-of-finance function, security constraint, and profit-rate function, which collectively subsume single period and multiperiod product demand, factor supply, technological, and financial constraints; also, the leverage ratio and the retention ratio are required to be constant (both are subsumed in a)
Variables:	
Exogenous:	maximum quasi-retention ratio (a*)
Endogenous:	balanced growth rate ($\dot{C} = \dot{D} = g$), profit rate (p), profit margin (m), diversification rate (d), and quasi-retention ratio (a)
Decision:	profit margin (m), diversification rate (d), and quasi-retention ratio (a)
Finance:	the only external source of finance permitted is debt issuance
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state growth path of the firm
Time:	multiperiod
Type of Model:	static optimization (set of simultaneous equations)
Solution Technique:	graphical (see figure II-17)

b. Recent Formulation of the Marris Model: Managerial Utility Maximization[236]

In the initial formulation of the Marris model the constant a* is exogenous. If there is no debt,[237] so that $a = r$, then in the initial formulation the stock market places a direct restriction on r. Greater insight is achieved if the restriction on r is linked to the stock market value of the firm's shares, for it is a low share value that presents a threat to management by increasing the likelihood of a takeover bid and prevents management from increasing r all the way up to one. In the more recent formulation of his model, Marris formulates the constraint in terms of the firm's valuation ratio.[238] In *The Economic Theory of 'Managerial' Capitalism* Marris suggests a way of proceeding from his initial formulation to his recent formulation.[239]

Given a retention ratio r, equations (101), (102), (103), and (105) can be solved to find a maximum balanced growth rate g_r, where the subscript r indicates the growth rate's dependence on r. For each new value of r the process can be repeated, yielding a value for g_r and associated values for p_r, m_r, and d_r.[240] With each value of r is associated the ordered pair (g_r, p_r). Since $dg_r/dr > 0$, it is possible to associate a single value of p_r with each value of g_r, and these ordered pairs define a function, which Marris calls the 'growth-profitability' function,

$$p = p(g) , \tag{107}$$

where $p'(g)$ may be positive for low growth rates but must eventually become negative.[241]

To obtain the 'growth-valuation' function, which expresses the valuation ratio as a function of the steady-state growth rate, an expression for the stock market value of the firm's shares is needed. Given the profit rate, p, the retention ratio, r, the current book value of total assets, K, a constant continuous rate of discount, i, and the steady-state growth rate, g, the market value of the firm's shares, V , is equal to[242]

$$V = \frac{p(1 - r)K}{i - g} \tag{108}$$

and the valuation ratio, v , is given by

$$v = \frac{V}{K} = \frac{p(1 - r)}{i - g} . \tag{109}$$

From (107) $p = p(g)$ and from (103) $g = rp$, so that (109) can be reexpressed in terms of g and i only:

$$v = \frac{p - rp}{i - g} = \frac{p(g) - g}{i - g} , \tag{110}$$

where $g \leq p(g)$ and $g \leq i$.[243] Equation (110) defines the 'growth-valuation' function. Note that if $v < 1$, then the market value of the firm's shares is less than the book value of the firm's assets. If the book value closely approximates the market value of the firm's assets, then $v < 1$ would be likely to encourage a takeover bid. For this reason, Marris argues that the firm will maintain $v \geq 1$, which implies from (110) that $p \geq i$.[244]

In the more recent formulation of the Marris model, the model of the firm is expressed as the following mathematical programming problem:

$$\begin{array}{ll} \underset{\{g\}}{\text{maximize:}} & U(g,v) \\ \text{subject to:} & v = v(g) = D(g) \cdot Y(g) , \end{array} \tag{111}$$

where $v(g)$ defined by (110) has been written as the product of two functions, $D(g) = p(g) - g$ and $Y(g) = \frac{1}{i - g}$.[245] The function $D(g)$, which Marris calls the 'general dividend function', expresses the dividend yield as a percentage of the firm's productive assets,[246] and if $p'(g) < 0$ for all g, then $D'(g) = p'(g) - 1 < 0$, so that the general dividend yield is a declining function of the firm's growth rate. The function $Y(g)$, which Marris calls the 'present-value' function, is the reciprocal of the dividend yield (on the share price).[247] For i constant, $Y'(g) > 0$.

The necessary conditions for an optimal solution to problem (111) can be used to characterize the equilibrium steady state growth path of the firm, and in particular, to determine the equilibrium growth rate g^* and the corresponding equilibrium valuation ratio v^*. For comparative purposes, note that, under the traditional criterion, the firm would select g so as to maximize the valuation ratio. Differentiating $v(g)$ with respect to g, setting the derivative equal to zero, and rearranging terms yields the necessary condition for optimum v:

$$-\frac{D'(g)}{D(g)} = \frac{Y'(g)}{Y(g)}, \qquad (112)$$

which requires that, for the value-maximizing firm to be in equilibrium, the firm must select g such that the percentage decrease in the general dividend yield is equal (numerically) at the margin to the percentage increase in the share price, with each percentage change (or 'semi-elasticity') determined with respect to a change in the firm's growth rate.[248]

A growth-valuation curve is shown in figure II-18, and the optimum valuation ratio is shown as v_{max}. Following Marris, the growth-valuation curve is drawn with the portion of the curve corresponding to low growth rates upward-sloping.[249] For low rates of growth an increase in the growth rate implies a higher valuation ratio. Note that in this case the value maximizer will grow, i.e. $g_{v_{max}} > 0$. This result depends, of course, on the shape of $v(g)$, for if $v'(g) < 0$ for all g, then v_{max} occurs where $g = 0$.[250]

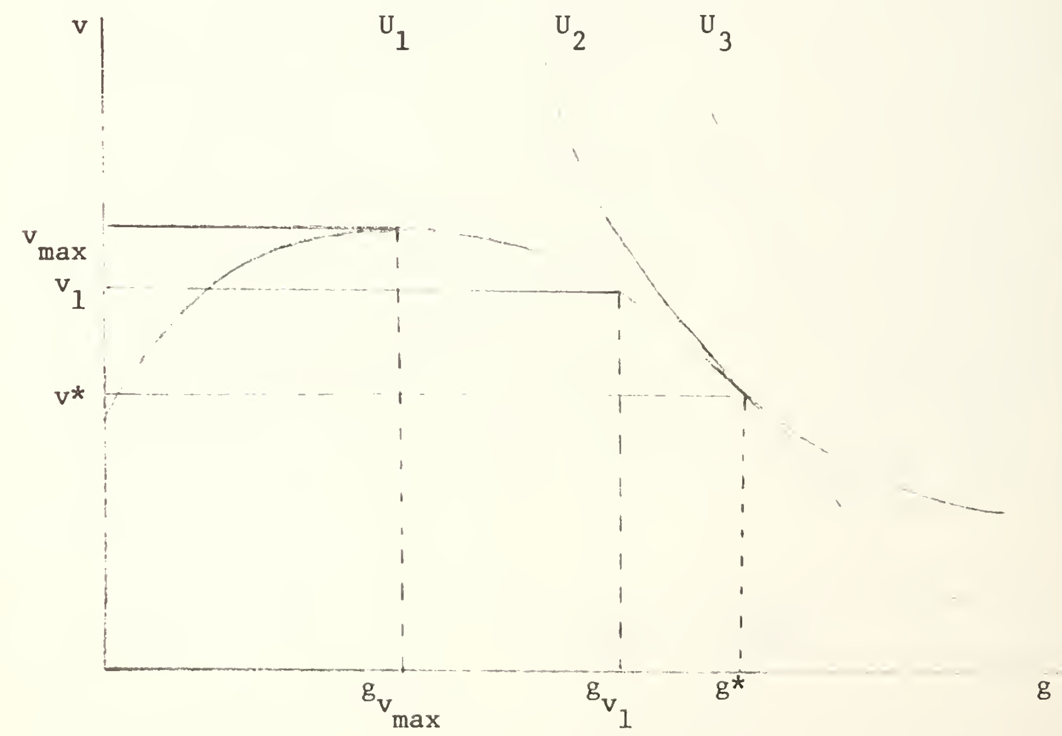


Figure II-18: Optimum for a Value Maximizer and for a Growth Maximizer

For managerial utility maximizers, the optimum value of g satisfies the necessary condition:[251]

$$\frac{dU}{dg} = \frac{\partial U}{\partial g} + \frac{\partial U}{\partial v} \quad [D'(g)Y(g) + D(g)Y'(g)] = 0 \ , \tag{113}$$

which may be rewritten as

$$- \frac{\partial U/\partial g}{\partial U/\partial v} = D'(g)Y(g) + D(g)Y'(g) \ . \tag{114}$$

The expression on the left is interpreted as the managers' marginal rate of substitution between growth and security, and the expression on the right is interpreted as the marginal cost of additional growth in terms of stock market valuation, and when the managerial utility maximizing firm is growing along its equilibrium steady state growth path, these two quantities must be equal. In terms of figure II-18, condition (114) requires that, at the point of optimality, the slope of the managerial indifference curve equal the slope of the growth-valuation curve, i.e. the familiar tangency condition necessary for utility maximization.[252]

From equation (113) it is easily seen that the value maximizer and the managerial utility maximizer will grow at the same rate and achieve the same valuation ratio only if $\partial U/\partial g = 0$, that is, only if the managerial indifference curves become horizontal so that the point of tangency occurs where $v = v_{max}$. This would imply that beyond some point faster growth brings no additional utility. If managers' desires for growth are never satiated, so that the indifference curves are always downward-sloping, then $v^* < v_{max}$. Ceteris paribus, a managerial utility maximizer of the Marris type (a 'growth maximizer') grows faster, but has a smaller valuation ratio, than a value maximizer. Since $p'(g) < 0$, the growth maximizer also has a lower profit rate, and since $r = g/p$, it also retains a larger proportion of total profit than a value maximizer.[253]

To summarize the discussion up to this point, the distinguishing features of Marris's model of the managerial utility maximizer are listed in table II-15.

Table II-15 Summary of Marris's Managerial Utility Maximization Model

Class:	managerial (see (111) in text)
Firm's Objective:	maximize managerial utility expressed as a function of the firm's growth rate and valuation ratio
Constraints:	growth-valuation function; implicitly, single period product demand and factor supply conditions and conditions restricting the potency of the firm's 'growth-promoting' expenditures, and implicitly, the technological constraint embodied in the firm's production function
Variables:	
Endogenous:	growth rate (g) and valuation ratio (v)
Decision:	growth rate (g)
Finance:	restriction to internal financing
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state growth path of the firm
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (this follows from converting (111) into an equivalent unconstrained optimization problem, as explained in footnote 251)

Having presented the managerial utility maximization formulation of the Marris model, the relationship between that formulation and the earlier growth maximization formulation is considered next. The equilibrium steady state growth path characterized by the optimal solution to the mathematical programming problem (111) leads to an equilibrium value for v and to an equilibrium value for r. If the firm were to take the equilibrium value v^* as the constrained valuation ratio, the model of the firm (111) could be reformulated as one involving the maximization of g subject to $v = v^*$. Alternatively, the corresponding equilibrium value for r could be used, as in the initial formulation of the Marris model, to obtain the same optimal g. Adopting the former approach, suppose that a minimum value for v is determined outside the model. Then the model of the firm can be formulated as the following mathematical programming problem:

$$\begin{aligned} &\text{maximize:} && g \\ &\text{subject to:} && v(g) \geq v_1 \ , \end{aligned} \tag{115}$$

where v_1 is the minimum valuation ratio necessary to prevent takeover. The model (115) incorporates what may be called the strong form of the valuation ratio constraint, while the model (111) incorporates the weak form of the valuation ratio constraint.[254] As long as $v_1 \leq v_{max}$, problem (115) has a feasible solution. Since $v'(g) < 0$ at the optimal solution, $v^* = v_1$ and $g = g_{v_1}$ (see figure II-18).

It is possible to generalize the Marris model to permit, for example, the discount rate i to depend on the firm's leverage ratio, i.e. the ratio of debt to equity. Rather than carry out the generalization in this section, it will prove more fruitful first to develop the Vickers and Lintner models in sections I and J and then to suggest how the presence of uncertainty and the need to consider the method of financing the firm's activities that are incorporated in these models affect the behavior of growth maximizers of the Marris type as well as that of value maximizers.

c. Evaluation of the Marris Model

Empirical support for the growth maximization hypothesis has come from three sources.[255] One source is the tests carried out by Douglas Kuehn[256] of the hypothesis that firms are growth maximizers against the alternative hypothesis that firms are profit maximizers. Kuehn finds that the valuation ratio is the most consistent indicator of whether a firm will be taken over and that, among takeover raiders at least, the growth maximization hypothesis is more consistent with the pattern of growth rates, profit rates, retention ratios, and valuation ratios than the profit maximization hypothesis. While Kuehn conducted his tests using British data, his findings are consistent with those of Friend and Puckett, Reid, and others, which are based on American data and which are discussed below.[257] The second source of support is the finding that "when stock prices are related to current dividends and retained earnings, higher dividend payout is usually associated with higher price-earnings ratios,"[258] for this suggests that firms tend to operate with a retention ratio in excess of that which would maximize the share value, which is consistent with the Marris model's implication that a growth maximizer will have a higher retention ratio and a lower valuation ratio than a value maximizer.[259] The third source of support for the growth maximization hypothesis are the empirical findings of Reid and others that generally support the view that companies merge not for the sake of profits, but for the sake of increasing size.[260]

On both theoretical grounds and empirical grounds, then, there is good reason to believe that firms are interested in growth and that they are willing to accept somewhat lower profitability and somewhat lower market valuation of their shares in order to increase the growth rate to the maximum consistent with continued financial security.

Nevertheless, the Marris model has certain limitations. The model assumes that the firm selects a steady state growth rate and that share prices are determined as the present value of the dividend stream growing at this constant rate, but in order for such a share valuation model to have empirical validity, it is necessary that growth and profit rates be stable over time and that the stock market be aware of this.[261] Even under the assumption of certainty, which Marris makes, such a valuation model requires that capital markets be perfect and in equilibrium. Yet, Nerlove shows that there is substantial disequilibrium in the capital market — "both suppliers of capital to the firm and investors in common stock share an imperfect and dim perception of the profitable opportunities for investment open to the firm."[262] Marris's model rests, then, on a theory of share valuation that requires that certain strict conditions, including the existence of perfect capital markets, be met. In fairness to Marris, this is not as much a criticism of his model as it is a recognition of the limitations of the particular share price formula he adopted.[263] The significance of the choice of valuation formula lies in the impact it may have on the shape and location of the growth-valuation curve, and hence, on the equilibrium steady state growth path chosen by the firm.

A second limitation of the Marris model is that it assumes certainty. Yarrow has modified Marris's model to incorporate uncertainty,[264] although he leaves the stock market valuation of shares subsumed within what he calls the managerial security function[265] (which is a generalization of Marris's growth-valuation function). Another approach — one that takes security valuation into account explicitly — might be to reformulate Marris's model first, with expected utility maximization in place of utility maximization and second, with a growth-valuation function based on a security valuation formula, such as the one suggested by Sharpe, Lintner and Mossin, that takes uncertainty into account explicitly.[266]

Two other limitations of the model are its treatment of managerial utility and its assumption of steady state growth. Is managerial utility a function of the growth rate and the valuation ratio only, or are there other arguments, such as staff and managerial emoluments, that have been left out?[267] As the model purports to explain the mode of behavior of actual large firms, if there are other sources of managerial utility that have been left out of the model, then, depending on their importance relative to growth and security, the equilibrium steady state growth path chosen by the firm might be affected. A possibly more serious question concerns how the collective managerial utility function is formed.[268] Do executives at all levels have the same marginal rate of substitution between growth and security, or do the top executives, who are more likely to lose their jobs following a takeover, place a relatively greater value on additional security than lower level executives, who are more interested in opportunities for promotion — or does the managerial utility function reflect the trade offs of top management only? Might there be a sociological process at work within the firm by which the firm's objectives are determined?[269]

Second, it is unlikely that, in planning for the future, firms ignore the business cycle. In an expanding economy, with markets growing and profits rising, managers not only desire growth, they are forced by competition to expand in order to protect their market shares. It is also advantageous to bring out new products — i.e. to diversify — in an expanding economy because the firm is better able to obtain the financial resources with which to finance test marketing and other selling expenses and consumer attitudes are generally more favorable than in the downswing. In the downswing, however, the emphasis is likely to shift from growth to profits.[270] While this shift can be accommodated within the Marris model, it requires that managerial utility be maximized with respect to security and growth lexicographically — i.e. it requires that the firm act so as to maximize v whenever $v < v_1$.[271]

The above criticisms were not meant to detract from the significance of Marris's contribution. The explicit recognition of the separation of ownership from control, and with it the appearance of growth as one of the modern corporation's primary objectives, in a model of the firm is highly significant. In this writer's opinion, any model that hopes to explain the behavior of the modern corporate enterprise should include growth among the firm's objectives.

3. The Oliver Williamson Model

In the Marris model managerial utility is a function of the firm's growth rate and its valuation ratio. One consequence of faster growth is that any particular size — and the salary, power, prestige, etc. that are positively correlated with size and that contribute to managerial utility — is attained more quickly. Oliver Williamson has developed a managerial model in which size, and in particular, the size of managers' staffs, plays a more direct role.[272] In his model managers exhibit an expense preference, enlarging their staffs, increasing managerial emoluments, and spending funds available for discretionary investment in order to increase their own utility. In the Williamson model managerial utility is expressed as a function of the total expenditure on staff, total managerial emoluments, and discretionary profit.[273] Staff expenditure includes the salaries of managerial personnel and expenditures on advertising and research and development. Larger staffs create advancement opportunities and contribute to the security, status, and prestige of managers.[274] Emoluments represent that portion of managerial salaries and perquisites that is discretionary in the sense that their removal would not cause managers to leave the firm to seek other employment.[275] Discretionary profit is the amount by which reported profit exceeds the required (by the need for financial security) minimum and is the amount that managers have available for spending on plant and equipment. Investment expenditures, then, are determined by managerial, as well as by economic, considerations. In contrast to the Marris model, where the separation of ownership from control enables managers to pursue growth at least partly at the expense of the firm's share value, the separation of ownership from control in the Williamson model manifests itself in the ability of managers to exercise some degree of discretion over the allocation of corporate income.

Williamson's model can be formulated as the following mathematical programming problem:

$$\begin{aligned} &\text{maximize:} && U = U(S, M, \pi_d) \\ &\text{subject to:} && \pi_r \geq \pi_o + T \end{aligned} \tag{116}$$

where U represents managerial utility, S is staff expenditures, M is managerial emoluments, π_r is reported profit, π_o is the exogenously-determined minimum acceptable level of after-tax profit, and T is the amount of corporate profit taxes. Reported profit is equal identically to total actual profit, π, less managerial emoluments,

$$\pi_r = \pi - M . \tag{117}$$

Letting t denote the exogenously-determined tax rate (assumed constant) on reported profit, discretionary profit, π_d, satisfies the identity,

$$\pi_d = \pi_r - \pi_o - T , \tag{118}$$

and substituting $T = t \cdot \pi_r$ and then (117) for π_r, (118) becomes[276]

$$\pi_d = (1 - t)[\pi - M] - \pi_o \quad . \tag{119}$$

Total profit, π , is equal identically to total revenue minus production cost minus staff expenditure, or in equation form,

$$\pi = \pi(Y,S) = R(Y,S) - C(Y) - S \ , \tag{120}$$

where Y is output, R is total revenue expressed as a function of output and staff expenditure, and C is production cost, which depends on the level of output.[277] Substituting (120) for π in (119) gives

$$\pi_d(Y,S,M) = (1 - t)[R(Y,S) - C(Y) - S - M] - \pi_o \quad . \tag{121}$$

Substituting (121) for π_d the objective function in the model (116) becomes:

$$\underset{\{Y,S,M\}}{\text{maximize:}} \quad U = U(S,M,(1-t)[R(Y,S) - C(Y) - S - M] - \pi_o) \quad , \tag{122}$$

where all quantities in (122) are measured in dollar units. Note that the constraint in problem (116) may be rewritten as $\pi_d \geq 0$, that is, discretionary profit must be nonnegative. But π_d is one of the arguments of the managerial utility function. Since it is reasonable to expect that, because discretionary profit contributes to managerial utility, $\pi_d > 0$ at optimality, and hence, that the constraint in problem (116) is satisfied, the model (116) can be reformulated equivalently as the unconstrained optimization problem (122).[278] The necessary conditions for an optimal solution to problem (122) are the following:

$$\frac{\partial U}{\partial Y} = \frac{\partial U}{\partial \pi_d} (1 - t)\left(\frac{\partial R}{\partial Y} - \frac{dC}{dY}\right) = 0 \tag{123}$$

$$\frac{\partial U}{\partial S} + \frac{\partial U}{\partial \pi_d} (1 - t)\left(\frac{\partial R}{\partial S} - 1\right) = 0 \tag{124}$$

$$\frac{\partial U}{\partial M} + \frac{\partial U}{\partial \pi_d} (1 - t) (-1) = 0 \tag{125}$$

The necessary conditions (123) - (125) for an optimal solution to (122) can be used to characterize the single period equilibrium position of the Williamson-type utility maximizing firm. Equation (123) yields the familiar condition,

$$\frac{\partial R}{\partial Y} = \frac{dC}{dY} . \tag{126}$$

In equilibrium, the Williamson-type firm equates marginal revenue (with advertising held fixed) and marginal production cost. Given the optimal level of advertising (i.e. staff) expenditure, equation (126) implies that the firm will produce the profit-maximizing level of output. From equation (124),

$$\frac{\partial R}{\partial S} = 1 - \frac{1}{(1-t)} \frac{\partial U/\partial S}{\partial U/\partial \pi_d} , \tag{127}$$

where $\frac{\partial U/\partial S}{\partial U/\partial \pi_d}$ is the marginal rate of substitution between discretionary profit and staff. In the traditional profit-maximizing firm, the staff has no value in excess of that which is associated with its productivity, so that this marginal rate of substitution is zero for the short run profit maximizer and $\partial R/\partial S = 1$. But in the managerial firm, $\partial U/\partial S > 0$ and $\partial U/\partial \pi_d > 0$, so that $\partial R/\partial S < 1$ in equation (127). Since the marginal cost of staff expenditure is just one (since, by assumption, S is measured in dollars) equation (127) implies that the managerial firm pushes staff expenditures beyond the profit-maximizing level. How much beyond depends, of course, on the tax rate t and on the marginal rate of substitution between staff and discretionary profit. Note the similarity to the sales maximization case, where in order to maximize sales (and managerial utility) the firm carried advertising expenditure beyond the profit-maximizing level. Here the motivation is different, but the effect is much the same. Last, from equation (125)

$$\frac{\partial U}{\partial M} = (1 - t)\frac{\partial U}{\partial \pi_d}. \tag{128}$$

With the constraint satisfied as an inequality (or at least with $\partial U/\partial \pi_d > 0$) equation (128) implies that the firm will absorb some portion of actual profit as emoluments (i.e. $M > 0$). The optimal levels of M and π_d must satisfy the condition $\frac{\partial U/\partial M}{\partial U/\partial \pi_d} = (1 - t)$. The marginal rate of substitution between emoluments and discretionary profit must equal one minus the tax rate.[279]

By defining staff expenditure to include managerial emoluments (i.e. by subsuming M in S), the Williamson model (122) can be simplified to permit a geometric representation. With this new definition of S, reported profit becomes identical to actual profit,

$$\pi = \pi_r = R(Y,S) - C(Y) - S \tag{129}$$

and discretionary profit becomes

$$\pi_d = (1-t)\,[R(Y,S) - C(Y) - S] - \pi_o \quad . \tag{130}$$

Note that with t and π_o fixed, maximizing (130) is equivalent to maximizing (129). For any level of staff expenditure, S, maximizing (130) with respect to Y leads to a point on the curve $\pi_d = \pi_d(S)$ in figure II-19. The utility function to be maximized is $U = U(S,\pi_d)$. In figure II-19 managerial utility is maximized at the point (S^*,π_d^*) at which the indifference curve and the curve $\pi_d = \pi_d(S)$ are tangent, and where the marginal rate of substitution between staff and discretionary profit just equals the marginal cost of additional staff in terms of discretionary profit. In contrast, the profit-maximizing firm would spend $\bar{S}$ on staff and earn $\bar{\pi}_d$ discretionary profit in figure II-19, i.e. spending less on staff and earning greater profit than the managerial utility maximizer.

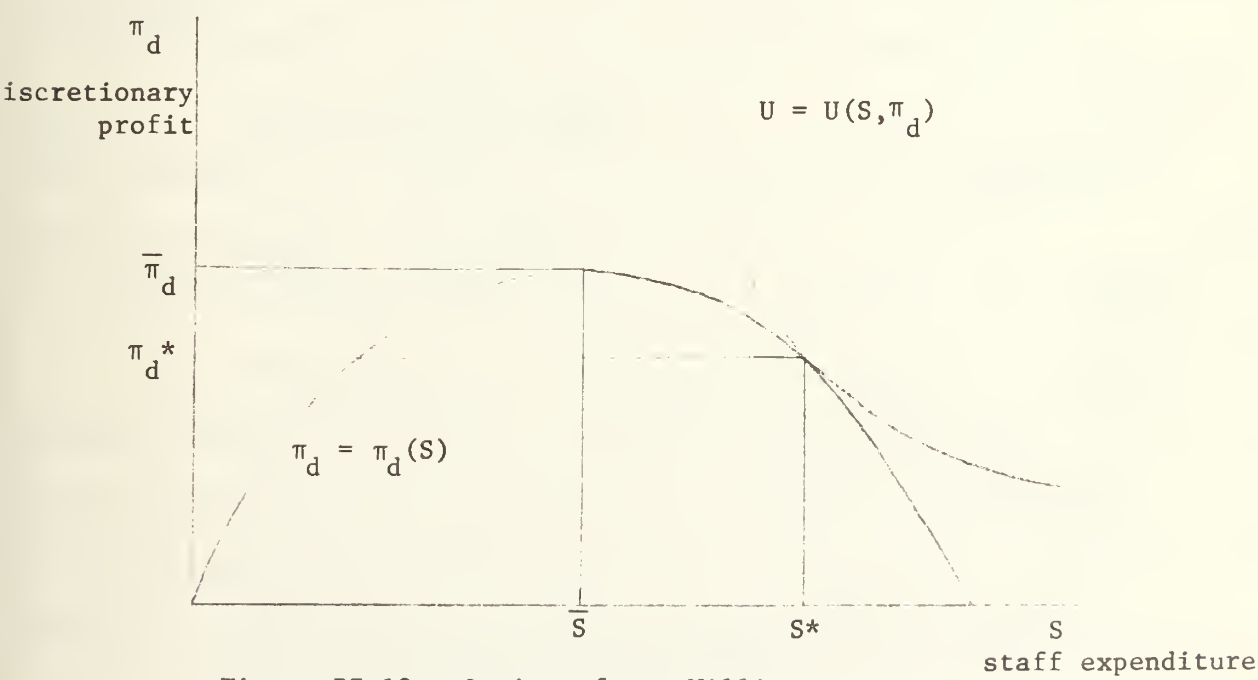


Figure II-19: Optimum for a Williamson-type Utility Maximizer and for a Profit Maximizer

Figure II-19 implies that the Williamson-type firm will reach an output decision different from that of the short run profit maximizer. The Williamson-type firm pushes staff expenditure (which includes advertising) beyond the profit maximizing level, and if marginal returns to advertising are always positive, then the Williamson-type firm can be expected to produce more output than a profit maximizer. Therefore, the Williamson-type firm is not merely a profit maximizer in which managers exercise control over a share of the (maximum) profit, but rather, is one in which managers increase staff and their own emoluments to the detriment of profit.

The product market solution for the Williamson-type firm is exhibited by figure II-20. The firm produces output Q_1 at which marginal revenue equals marginal production cost.[280] The firm charges price P_1 and reports (pre-tax) profit amounting to $(P_1 - C_2) \cdot Q_1$, of which $T = (C_3 - C_2) \cdot Q_1$ is paid as corporate profit tax, $\pi_o = (C_4 - C_3) \cdot Q_1$ represents the minimum acceptable level of after-tax profit, and $\pi_d = (P_1 - C_4) \cdot Q_1$ is discretionary profit.

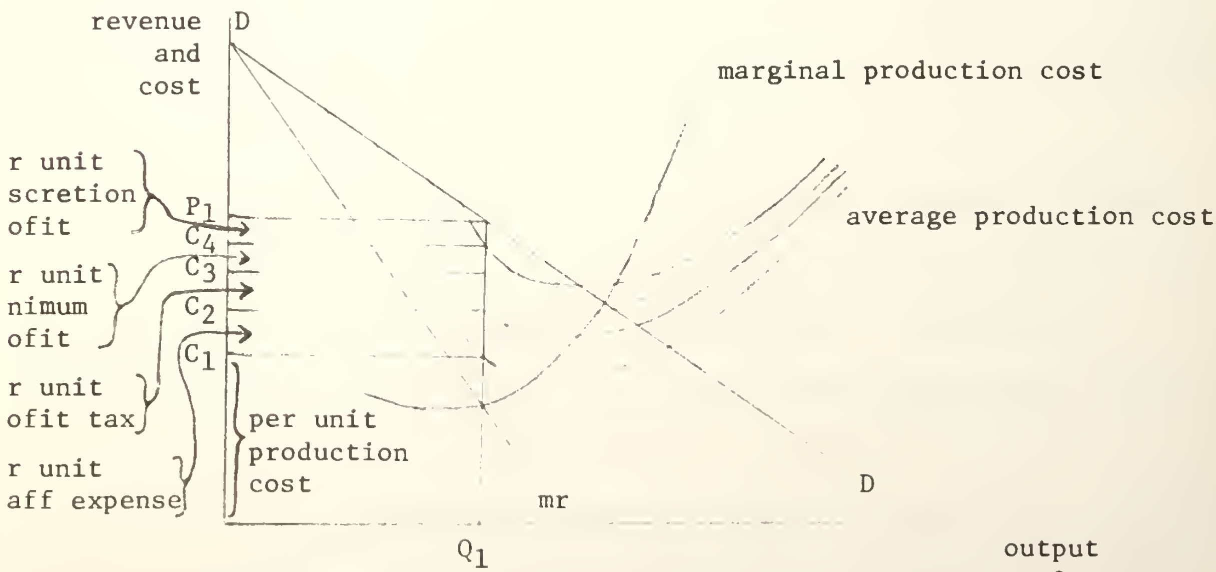


Figure II-20: Product Market Solution for the Williamson-type Firm

By way of summarizing the discussion thus far, the distinguishing features of the Williamson-type managerial firm are set out in table II-16.

Table II-16 Summary of O.E. Williamson Model

Class:	managerial (see (122) in text)
Firm's Objective:	maximize managerial utility expressed as a function of staff, emoluments, and discretionary profit (and in the simpler version discussed in the text, with staff and emoluments combined into a single variable)
Constraints:	minimum acceptable level of after-tax profit; and implicitly, product demand and factor supply conditions, and also implicitly, the technological constraint embodied in the firm's production function
Variables:	
Exogenous:	minimum acceptable level of after-tax profit (π_o) and the tax rate (t)
Endogenous:	output (Y), staff (S), emoluments (M), and discretionary profit (π_d)
Decision:	output (Y), staff (S), and emoluments (M)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of single period equilibrium position of the firm
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization

The Williamson model has two important implications. First, managers exercise considerable discretion as to how the surplus of revenue over production cost will be spent and this has implications for the efficiency with which resources are allocated within the firm. Managers exercise their discretion by enlarging staffs, paying themselves emoluments, and possibly even carrying out investment (paid for out of discretionary profit) beyond the point at which each type of expenditure is most profitable in order to increase thereby their own utility.

The second implication is that, if the firm's profitability were to fall, in order to continue to satisfy the minimum profit constraint various types of expenditure would have to be curtailed, and possibly even discontinued altogether. Williamson provides the following example of what can happen when profits fall sharply; in response to a sharp fall in profit one firm reduced salaried employment company-wide by 32 percent; cut headquarters staff by 41 percent, research and development staff from 165 to 52, and personnel and public relations staff from 57 to seven; and reduced emoluments of all kinds.[281] Moreover, all of this occurred with production unchanged.[282] This sort of abrupt cost-cutting behavior is easily explained by Williamson's model, but it is not well-explained by the profit-maximizing model.

While its implications are consistent with observations of the behavior of actual firms, the Williamson model does have three limitations. First, it subsumes the financial decisions of the firm. Though the Williamson-type firm is held to value discretionary profit, which is available for investment purposes, and though discretionary profit could be linked to retained earnings and the minimum required level of after-tax profit could be linked somehow to stock market-imposed restrictions on the firm's choice of dividend policy, the question regarding how the utility maximizing firm (of the Williamson-type) should set its financial policies is not dealt with explicitly in the model. Second, the minimum acceptable level of after-tax profit is exogenous; the model fails to explain how this minimum acceptable level is determined. Third, the model does not relate discretionary expenditures directly to the important managerial goals of growth and security. These three criticisms are related. Presumably, π_o is determined, at least partly, by the demands of shareholders, which in turn places restrictions on the firm's retention policies, and hence, on the amount that can be spent on staff and emoluments and the amount left over as discretionary profit to be spent on plant and equipment.[283] Thus, while the Williamson model's treatment of discretionary expenditures on staff, emoluments, and plant and equipment makes the model a significant contribution to the literature on the theory of the firm, the model's failure to incorporate financial considerations explicitly as a management objective is, in the opinion of this writer, a serious limitation.

4. Summary of the Managerial Models

The managerial models of the firm reflect the effect the separation of ownership from control is supposed to have had on the goals of the modern corporate enterprise. According to the managerialists, the firm no longer strives for maximum profit or for the maximum value of the firm's shares, but rather, sacrifices some of its maximum attainable profit for the sake of promoting sales, raising the rate of growth, enlarging the staff, or increasing managerial emoluments, each for the ultimate purpose of maximizing managerial utility.

Though maximum profit is no longer the single most important goal of the firm, the firm's profitability continues to play a critical role in the managerial models as a constraint. Given the managerial firm's objectives, it will attempt to find the most profitable output mix and the most profitable input mix that will enable it to attain its objectives. Given the level of advertising for the Baumol-type sales maximizer or the level of staff expenditure by the Williamson-type utility maximizer, it was found that the firm would select the level of output for which marginal revenue equals marginal (production) cost. In addition, John Williamson has asserted that a value maximizer and a growth maximizer would produce the same short term output, and hence, charge the same price (although a sales maximizer would produce more).[284] In the Marris model the behavior of the firm is not inconsistent with maximizing residual (i.e. after-tax and after-dividend) net revenue (i.e. total revenue less total production cost), for such behavior generates the maximum supply of funds available for growth-promoting expenditure. For this reason, Peterson has suggested that there is little practical difference between maximizing profits and maximizing growth.[285] In the managerial models, then, though the firm is not maximizing total profit, it is, in effect, trying to maximize the amount of funds available to be spent at the discretion of management on those things that most enhance managerial utility.

Given that firms behave as the managerial models describe, an important question arises. What are the consequences of this sort of behavior, i.e. can one expect to be able to distinguish empirically between profit-maximizing behavior and managerial utility-maximizing behavior? The Kuehn study cited earlier has lent support to the growth maximization hypothesis.[286] Kuehn's conclusions, however, apply to a restricted sample. More generally, Solow demonstrates that, at the theoretical level at least, the effect of alternative motivations on the firm's response to various external stimuli is to produce responses that differ in degree rather than in direction or kind:[287]

> ... growth-oriented and profit-oriented firms would respond in qualitatively similar ways to such stimuli as changes in factor prices, discount rate, and excise and profit taxes. On the evidence only of its behavior in that kind of situation, an observer would find it hard to distinguish one kind of firm from the other.

Even if non-profit maximizing behavior can be distinguished from profit maximizing behavior, there still remains the question as to how far the non-profit maximizer is deviating from profit maximizing behavior, and because this would require either the determination of what profit would have been had the firm maximized profit or else a comparison of the relative profitability of the two types of firms, it might prove to be an impossible question to answer.[288] Indeed, there may be entrepreneurs who permit nonprofit considerations to enter their decision-making and professional managers may be sufficiently skilled and dedicated to their firms that they run their organizations with greater efficiency than an entrepreneur would, with the consequence that a cross-section analysis that distinguished owner-controlled firms from manager-controlled firms might discern little or no difference in the profitability of the two types of firms.[289] The empirical evidence accumulated thus far bears this out.[290] Except in some special cases, such as the takeover raiders in Kuehn's study, testing the managerial models against the traditional models would probably be very difficult.

This section has described the managerial models in which the utility of the firm's managers is maximized. These models were contrasted with the traditional models in which the utility of the firm's owners — measured as total profit for the entrepreneurial firm and as the share value for the corporation — is maximized. The next section describes the behavioral theory of the firm in which there are various social groups, each having its own goals, within the firm, but in which no single group's goals are ascendant.

H. BEHAVIORAL MODELS

Proponents of the behavioral theory of the firm[291] argue that the maximization models discussed earlier in this chapter offer little help in explaining the behavior of real firms. The profit maximization and value maximization models are criticized because, it is argued, in a world of uncertainty maximands such as total profit or the present value of the stream of future profits are very difficult to define in an empirically meaningful way.[292] Models that assume a single all-embracing goal — such as sales or growth maximization or the maximization of managerial utility — are criticized by the behavioralists on the grounds that the modern corporation is comprised of many social groupings, such as top managers, middle managers, lower level managers, professional staff, shareholders, etc., whose goals are likely to conflict. Organizational goals are established through a continual bargaining process that goes on within the firm among the various social groupings, and as a result, are more likely to reflect a series of compromises than the particular goal(s) of any single grouping and are likely to change through time as the experience of the firm conditions the bargaining process.[293] According to the behavioralists the compromises and trade offs that are made during the bargaining process, together with the uncertainties inherent in the real world and the often high cost of searching for information with which to reduce uncertainty, cause the firm to exhibit 'satisficing' rather than 'maximizing' behavior.[294] That is, the firm does not seek maximum performance with respect to any single objective, e.g. by trying to maximize total profit, but rather, seeks only satisfactory performance with respect to each of the several goals established through the internal bargaining process. As would be expected, this has led to a debate over whether managers try to maximize or are content to satisfice. But rather than arguing over the nature and sources of managerial motivation, the behavioralists suggest that economists develop a better understanding of the managerial decision-making process within actual firms.[295]

The behavioralists, and notably the works of Simon, Cyert and March, and Cohen and Cyert, have drawn attention to the actual decision-making processes within firms.[296] Relying on direct observations of actual decision-making processes, the behavioralists have tried to develop a theory general enough to transcend the specific firms studied.[297] While their work has resulted in several interesting empirical studies that have shown that at least in some firms some rather simple minded rules of thumb, such as setting prices by applying a fixed mark-up to the average variable cost of the item, are used with remarkable consistency,[298] and while their models have been very successful in predicting simple business decisions, such as future prices, on the basis of these rules of thumb,[299] the behavioral analysis has provided little in the way of interesting analytical implications.[300] It has failed to explain either how the rules of thumb are determined or how they will vary in response to changes in the values of economic variables exogenous to the firm.[301]

1. The Cyert and March View of the Firm

The behavioral theory of the firm is viewed by its proponents as a supplement to, rather than as a substitute for, the conventional theory of the firm.[302] Whereas the traditional theory of the firm is mainly concerned with the way in which the price system brings about the allocation of resources among markets, the behavioral theory is more concerned with the way in which resources are allocated within the firm. Since the behavioral theory is designed to answer questions of resource allocation different from those with which the traditional theory is concerned, the assumption of profit maximization, which may serve the traditional theory adequately, is neither necessary nor sufficient for answering the questions about internal decision-making and internal resource allocation with which the behavioral theory is concerned.[303] Nevertheless, if the behavioral theory is intended as a supplement to, rather than as a substitute for, the traditional theory, there remains the problem of resolving the traditional and behavioral theories into one broadly explanatory model with which the interface between internal markets (e.g. for the allocation of money capital within the firm) and external markets (e.g. product markets and factor markets) can be studied.

The behavioral theory of the firm, according to Cyert and March, requires the development of the following four major subtheories (i) a theory of organizational goals that describes the process of goal formation; (ii) a theory of organizational expectations that explains the information-gathering behavior of organizations; (iii) a theory of organizational choice that describes the process of selecting alternatives, comparing them, and deciding which one is best; and (iv) a theory of organizational control that provides needed insight into the difference between decision-making and implementation.[304]

Cyert and March conceive of the firm as a coalition, possibly consisting of smaller subcoalitions whose members share common goals. Members of the coalition, who may include workers, stockholders, and customers as well as managers, are responsible for decisions regarding the goals of the organization. Strictly speaking, the organization as such cannot have goals. Only the individuals within the organization can have goals.[305] The term 'organizational goals' refers, then, to agreement among the members of the coalition as to what objectives the organization should pursue. Cyert and March identify five organizational goals[306] — a production goal, an inventory goal, a sales goal, a market-share goal, and a profit goal — which are inconsistent in the sense that optimizing with respect to one, say profit, precludes optimizing with respect to some other, for example, sales. Members of the coalition have different interests, and consequently, rank the five goals differently. The 'final' set of goals of the organization is the result of a bargaining process among members of the coalition.[307] All five goals are set at attainable levels (or at least it must be expected that they are attainable in order that agreement among coalition members be reached) and conflicts among members of the coalition are resolved by various forms of 'side-payments', which may take the form of either monetary payments or policy commitments, such as committing more resources to new product development in order to appease the vice-president of marketing who had set a higher sales goal than the coalition was willing to grant. Members of the coalition are continually making demands on the coalition in accordance with their individual interests, and this requires that the goals of the organization be adapted continually. The demands are not mutually consistent, but by handling the demands sequentially, the organization can remain viable. This process of mutual accommodation has two important effects: (i) the goals of the organization are set at satisficing rather than maximizing levels in order that all five goals be attainable, and (ii) the goals change over time in response to the demands of different members of the coalition.[308]

Provided sufficient resources are available with which to make side-payments, the organization remains viable. Problems arise, however, when the organization is unable to accommodate the demands of its members. Since demands will adjust to actual payments, as well as to alternatives external to the firm, it can be expected that coalition demands, which are analogous to factor prices in the traditional theory, will equilibrate with payments in the long run. But in the short run a disparity may exist between total resources and total payments. Cyert and March term the difference between total resources and total necessary payments 'organizational slack'. In the traditional theory (at least in equilibrium) organizational slack is zero. Slack refers to payments to members of the coalition in excess of the minimum needed to keep them in the organization, as for example, dividend payments in excess of what is needed to keep shareholders loyal to the firm or executive compensation in excess of what is needed to keep them in the organization.[309] When market conditions are favorable slack becomes large, and when market conditions worsen, slack tightens as the firm reduces payments to some members of the coalition and seeks ways to cut costs.[310] Cyert and March do not argue that firms create organizational slack intentionally. Rather, they argue that it occurs naturally, and that it acts as a stabilizing factor for the organization over time.[311] Of course, the existence of slack also implies a less-than-optimal allocation of the firm's resources and a less-than-optimal long run rate of growth for the firm.[312]

Cyert and March also describe how the firm's decisions evolve over time and produce a simplified model with which they study the key decision processes at work in a firm that has only three goals (profit, production, and sales) and has to make decisions on price, output, and sales effort in each time period. They simulated the pricing decision in a department store and were able to predict to the penny the prices of 384 items out of the 414 in their sample.[313] In a later simulation model Cohen and Cyert produced equally impressive results utilizing a duopoly model to explain output decisions for two producers of metal cans.[314]

The time path of the firm's decisions can be explained on the basis of two types of models, the learning process model and the rule-of-thumb model. The learning process model described by Cyert and March is roughly the following. Suppose some change is under consideration. Management will apply two tests of its worthiness: (i) will it meet the firm's satisficing requirements (i.e. is it feasible)? and (ii) is it likely to improve the overall position of the firm (i.e. is it desirable)? If it passes both tests it will be implemented, and if the firm later finds that its expectations were met, or possibly even exceeded, then further changes of this type will be considered. According to this iterative process, prices, output levels, advertising outlays, etc., are gradually adapted over time. Adjustments take place through a trial and error process, rather than as the result of some grand optimization scheme. However, as the goals of the organization and conditions external to the organization are continually changing, the adaptations proceed toward a moving target so that the progressive adjustments do not necessarily converge to zero.

An example of such a model is the following:

$$\begin{aligned} \pi_t &= f(A_t) \\ A_{t+1} - A_t &= \alpha(\pi_t - \pi_{t-1})(A_t - A_{t-1})/|A_t - A_{t-1}| , \quad \alpha > 0 , \end{aligned} \tag{131}$$

where A_t is total advertising expenditure and π_t is total profit in period t. According to the first equation in (131), total profit is a function of the current level of advertising expenditure, and according to the second equation, the current change in advertising outlays will have the same sign as the preceding change provided total profit rose as a result, but will have the opposite sign if total profit fell. Quantitatively, the absolute change in the level of advertising expenditure each period is equal to α times the previous period's absolute change in the level of total profit.

The second type of model used to describe the time path of the firm's decisions, the rule-of-thumb model, is the subject of the next subsection.

2. A Mark-Up Pricing Model

The modern corporate enterprise typically produces a great variety of products that it sells in many different markets.[315]

Corporate managers have neither the time nor the money necessary to make detailed pricing decisions that take into account all relevant supply and demand conditions in each market. While the firm may have only a few competitors in any one market, overall it is likely to have a large number of competitors, and at least in theory, the potential reactions of each should be considered when policy decisions are made. Faced with this situation, the firm may find it convenient to rely on conventional rules of behavior.

The behavioralists have shown that in many markets conventions, or simple rules of thumb, that are known and accepted by all major participants have developed.[316] According to Baumol:[317]

> These rules of thumb do not work out too badly. They translate hopelessly involved problems into simple, orderly routines. They save executive time and permit a degree of centralized control over the firm's far flung operations. By and large, they probably contribute considerably to over-all operating efficiency. Most executives appear to recognize these rules for what they are — imperfect expedients designed to cope, in a rough and ready manner, with a difficult control and decision problem.

The use of rules of thumb is also intended to reduce the adverse effects of uncertainty, particularly the uncertainty surrounding the reactions of the firm's rivals, and in so doing, to bring order into the market.[318] To be workable these rules of thumb should satisfy three requirements:[319] (i) they must be easy to apply and based on criteria known to all participants; (ii) they must be accepted by all participants; and (iii) they must have some degree of flexibility so that they can change when a fundamental change in the participants' environment takes place.

One of the most common rules of thumb is the standard mark-up policy, such as that used by retail shops. Cyert and March tested a mark-up rule for department store pricing of standard items and found that the simple rule 'divide each cost by 0.6 ... and move the result to the nearest .95 ' predicted 188 out of 197 prices correct to the penny.[320] Baumol and Stewart retested the Cyert and March model for a similar, though unrelated, department store in a different city and found that the mark-up model applied, but that the mark-up applied to most items had increased (from 40 percent to 45 percent) over time in response to increasing costs.[321] Though the Cyert and March model exhibited remarkable accuracy in predicting prices, it neither explains why the mark-up was originally set at 40 percent nor gives any clues as to why or how the mark-up would change over time.

One way of determinng the mark-up, and at the same time showing that mark-up pricing may be consistent with profit maximization, is the following.[322] Suppose that the average variable cost of production, AVC, is constant and that the firm sets price, P, by applying a standard proportional mark-up m to average variable cost. Then

$$P = m \cdot AVC . \tag{132}$$

Since average variable cost is constant, marginal cost, MC, and average variable cost must be equal. To maximize total profit the firm must equate marginal revenue and marginal cost. Letting Q represent the level of output, this requires that

$$MR = \frac{d}{dQ}(P \cdot Q) = P + Q\frac{dP}{dQ} = P(1 + \frac{Q}{P}\frac{dP}{dQ}) = P(1 - \frac{1}{\eta}) \tag{133}$$

where η is the price elasticity of demand. But since $MR = MC = AVC$, substituting AVC for MR in (133) and solving for P yields

$$P = \left(\frac{\eta}{\eta - 1}\right) \cdot AVC \quad , \qquad (134)$$

which implies that the optimal mark-up in (132) is only a function of the price elasticity of demand.[323] From (134) it can be seen that the mark-up will increase as demand becomes less price elastic (i.e. as η falls) and that price will rise either as demand becomes less elastic or as average variable cost increases.[324] Such a model explains both the size of the mark-up and how it would be expected to change over time.[325]

3. Evaluation of the Behavioral Models

The behavioral approach has been characterized as one that strives for 'realism in process', in contrast to the managerial approach that aims toward 'realism in motivation'.[326] This represents a strength and at the same time a weakness. The behavioral simulation models emphasize the process of decision-making within actual firms, where goals continually change and the firm undergoes a learning process through which it continually adapts its goals and its behavior in light of its experience. But in order that the theory be given sufficient precision and sufficient generality that it can be useful for prediction on a wide scale, it must be given a good deal more content.[327] In order to use the behavioral models to study any single firm it is necessary to know a great deal about that organization's responses to different stimuli. The behavioral theories achieve realism but also give rise to certain ambiguities not present in the traditional and managerial theories. Depending on the goals of the coalition members and the nature of each organization's internal bargaining process, two organizations may respond differently to the same stimuli under identical circumstances. To the behavioralists this variability in response would undoubtedly be viewed as one of the strengths of their models, but to an economist primarily interested in the predictive usefulness of models of the firm, it would just as likely be viewed as a weakness.[328] However, as long as the behavioral contributions are seen in the proper light — as attempts to describe the decision-making processes within firms and to deal with problems of resource allocation within firms, rather than as an effort to supplant the traditional theories — their significance is more easily appreciated.

Possibly an even more important outcome of their work has been the recognition that a wide range of business decisions, which are not critical to the organization's continued existence and for which sufficient information with which to determine the profit maximizing course of action may be lacking, are made on the basis of rules of thumb. For example, for a company that sells thousands of items, the additional profit that may result from more efficient pricing of any one item may not justify the cost of gathering the needed information and carrying out the required calculations. That is, there exist transactions costs, as well as costs associated with the search process that precedes transactions, that are a real part of the profit maximizing calculations, but that are, in general, left out of the models of the firm. The existence of such costs implies that the adoption of reasonable rules of thumb might be a rational course of action,[329] and also, that the firm could be expected to continue operating in this manner until better information gathering or better analytical techniques become available.[330]

In order to summarize the discussion of the behavioral models themselves, the distinguishing features of these models are set out in table II-17.

Table II-17 Summary of Behavioral Models

Class:	behavioral
Discussion:	In general, the behavioral models are intended as a supplement to, rather than as a substitute for, the conventional theory of the firm. The behavioral models strive for 'realism in process', rather than 'realism in motivation'. They attempt to describe the decision-making processes within actual firms. In general, the behavioral models are non-optimization simulation models that are designed to explain the internal workings of the firm, rather than to characterize some optimal set of operating policies. The components of these models are, in general, of the following two types: - learning process models (see (131) in text) that seek to explain the time path of a firm's decisions in terms of an iterative process, and - rule-of-thumb models (see (134) in text) that seek to explain the time path of the firm's decisions in terms of decision rules that may be, in actual practice, optimally imperfect.

I. THE VICKERS MODEL AND THE ROLE OF FINANCE

In the models discussed thus far the problem of financing the activities of the firm was given only a very superficial treatment. In the traditional models discussed in sections B through E, the financial decisions the firm must make were subsumed within the general equilibrium analysis of a market economy, and hence, were not treated explicitly within any of the models. These traditional models were concerned with production decisions almost exclusively. In the models discussed in sections F and G, the role of finance was limited to the long term financing of corporate growth, and, with the exception of the Baumol and Marris growth maximization models, each model assumed that all investment funds were raised internally.[331] While this treatment was adequate given the assumptions of certainty and perfect capital markets, the actual business environment does not conform with these assumptions. For this reason, it is necessary to give the role of finance more explicit consideration.

Before the role of finance can be discussed, it is necessary that the distinction between financial capital and real capital be clearly made.[332] Real capital, which consists of plant and equipment, land, and inventories, is a factor of production. Financial capital, which represents the generalized purchasing power with which real capital, as well as labor services and raw materials, are purchased, is not. Ideally, real capital should be measured in physical units. However, since real capital is not homogeneous, attempts to aggregate different quantities of capital into a single measure have led to conceptual as well as practical difficulties. Traditionally, both real capital and financial capital have been measured in dollar terms, though in the case of the former, the dollar measure is, at best, a surrogate for the pure physical measure that is lacking.[333] Unfortunately, because both types of capital are measured in dollar terms, it is possible to confuse them. It deserves to be emphasized that the two are different, and that one denotes a physical quantity of productive resources, while the other denotes a money measure of the firm's financial resources. In the course of production the firm combines real capital with amounts of the other productive resources to produce output. But in order to command these factor services, the firm must first obtain the necessary purchasing power, i.e. the necessary financing, through either retained earnings or the issuance of financial securities (including notes issued to banks).[334] One of the important questions with which this section deals is the relationship between the firm's production decisions (including how much real capital the firm should invest in) and its financial decisions (including how it should finance the desired level of investment).

Until recently the theories of production and of finance had "developed along remarkably independent paths."[335] While the role of finance was not ignored completely,[336] in the words of Vickers:[337]

> The microeconomics literature, in addressing itself to the theory of the firm, has been preoccupied with a solution to the firm's production, factor use, price, and output problems, to the virtual exclusion of the questions of capital investment and financing.

Neoclassical production models have not needed securities markets because the assumption of profit maximization leads to a complete characterization of the single period equilibrium position of the firm when the firm's environment is certain and when capital markets are perfect, both of which the traditional theory assumes.[338] The reason for this is that, under the assumptions of certainty and perfect capital markets, the production decisions and the financial decisions of the firm are separable (i.e. they can be made independently of one another), at least in general equilibrium.[339] Moreover, as discussed below, this separability remains even under uncertainty, provided capital markets are complete.[340] But as the next subsection describes when markets are imperfect, say due to the existence of transactions costs or taxes,[341] or when uncertainty is present (and markets are incomplete),[342] the firm's production decisions and its financial decisions are no longer separable, and it becomes necessary to consider the role of finance explicitly in modeling the firm's production decisions.

Just as the traditional models of the firm did not treat the role of finance explicitly, so the pathbreaking models of financial equilibrium due to Sharpe, Lintner, and Mossin,[343] which contributed significantly to a better understanding of the workings of capital asset markets and investors' selections of optimal portfolios of risky assets, took the firms' production decisions as given and therefore had nothing to say concerning the relationship between the firms' production decisions and stock market equilibrium. It was not until some years later that Diamond and others began to examine the firm's production decisions in the context of stock market equilibrium.[344]

There were some earlier attempts, however, to build partial equilibrium models that linked the investment and financing decisions of the firm. Some of the earliest were the dividend capitalization models due to Gordon and Shapiro and Walter.[345] The various decision rules that emerged from these early models tended to be contradictory in that the different models would often imply different decisions under identical sets of conditions,[346] which Senchack attributes to the varying mixtures of stringent assumptions concerning modes of financing, the time structure of market discount rates, and so on.[347]

More recent efforts have been directed toward relaxing some of these restrictive assumptions. Lerner and Carleton modified the Gordon growth model and developed a sophisticated model that integrates the firm's investment and financing decisions.[348] Their equity valuation model takes into consideration product-, factor-, and financial-market constraints as well as investor expectations regarding the firm's rate of return and retention ratio. Davis extended the Lerner-Carleton comparative statics (i.e. single period) framework to a multiperiod framework by formulating his model as an optimal control problem in which the objective function expressed the share value as a function of the discounted flow of dividends (as in the Gordon model) and in which there were two constraints (in the form of differential equations), one on market price valuation and the other on capital expansion.[349] Subsequently Krouse adopted a control-theoretic approach to develop a model permitting market imperfections (in particular, transactions costs on new equity issues) and a time-varying market rate of discount.[350] Krouse's model excluded debt financing, and Senchack later generalized on Krouse's results by permitting debt financing.[351]

In view of the extensive efforts made toward incorporating financial considerations into models of the firm, it should be apparent that the firm's production and financial decisions are related. Yet, in view of the debate that went on for many years (and that does not appear to this writer to have been resolved completely) it is worth pausing to explore those conditions under which the firm's financial policy may be considered independent of its operating decisions. Following that discussion two models of the firm that give finance an important role to play will be presented.

1. Does the Firm's Financial Policy Really Matter?

According to the traditional view of capital structure and market valuation in the finance literature, there exists for each firm an optimal capital structure — mix of debt and equity — that simultaneously minimizes the firm's cost of capital[352] and maximizes the market value of the firm.[353] At the opposite extreme lies the Modigliani and Miller view, according to which the firm's cost of capital and total market value are independent of the firm's capital structure.[354] According to the traditional view expressed in the finance literature, the market value of the firm is a function of both its investment policy and its financial policy, whereas the Modigliani and Miller view maintains that the market value of the firm is based on the firm's investment policy only.[355]

The traditional financial argument regarding the firm's capital structure is essentially the following.[356] As debt is first introduced into the firm's capital structure, the overall cost of capital decreases and the market value of the firm increases.[357] This is due to the fact that bond interest payments are tax deductible whereas dividend payments are not, which has the effect of reducing the firm's cost of raising capital as the proportion of debt increases. As the proportion of debt is steadily increased, however, the firm's fixed interest obligations, which must be met out of profits, increase. Since the firm must satisfy these obligations or else go bankrupt, the increasing proportion of debt causes the firm's risk of bankruptcy to increase. Eventually the effect of the increased risk of bankruptcy will outweigh the effect of the tax advantages associated with debt issuance and will cause the firm's market value to fall and its cost of capital to rise. In between there lies a range of values for the proportion of debt in the firm's capital structure within which the cost of capital is minimized and the market value of the firm is maximized.[358] The traditional financial view of the importance of the firm's capital structure is illustrated in figure II-21.[359]

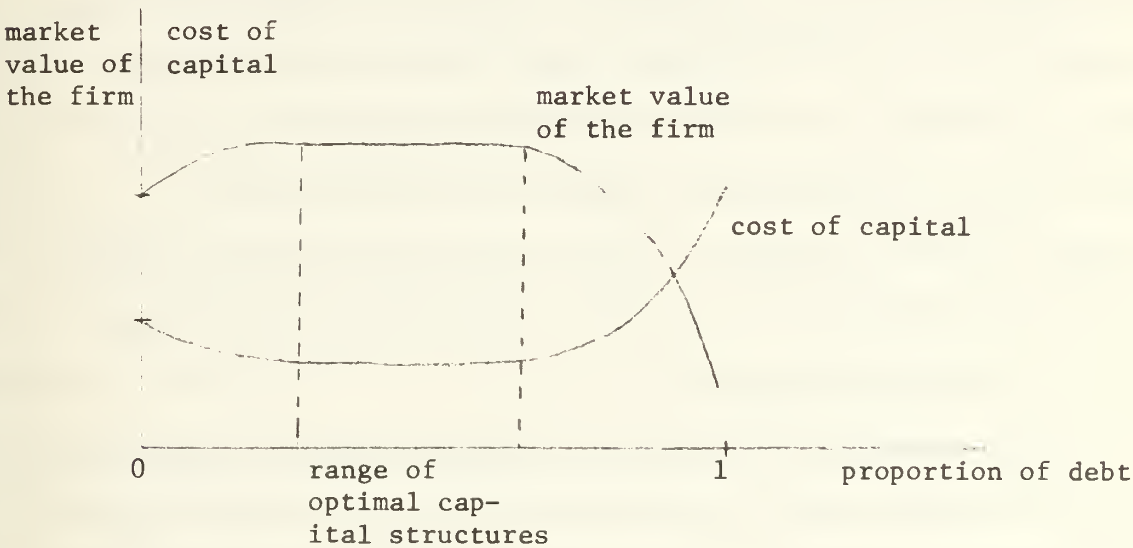


Figure II-21: The Firm's Optimum Capital Structure(s)

Modigliani and Miller argue that both curves in figure II-21 are horizontal,[360] given the firm's investment policy and provided that capital markets are perfect.[361] They argue that "the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worthwhile."[362] That is, they argue that the financing decision and the investment decision are separable.[363]

Modigliani's and Miller's conclusion that the firm's debt-equity mix is indeterminate even under uncertainty requires that assumptions in addition to the perfect markets assumption be made.[364] In their original analysis Modigliani and Miller assumed a zero default risk, so that all bonds were riskless assets and perfect substitutes for one another.[365] Mossin and Baron have shown that the market value of the firm remains independent of the firm's capital structure even when default risk is present (i.e. when the probability of the firm's going bankrupt is positive),[366] but Baron's paper and the earlier work of Hirshleifer indicate that this requires the existence of a set of complete markets for contingent claims and the absence of any bankruptcy penalties, such as the legal costs of liquidating the firm and the opportunity cost of investors' capital tied up in the firm during the liquidation process.[367]

The significance of perfect and complete markets is that each individual can create return opportunities equivalent to those that can be created by a firm as it alters its capital structure.[368] By selecting its investment policy so as to maximize the equilibrium market value of its outstanding securities, the firm is maximizing each individual investor's wealth. Given a set of securities and a set of (equilibrium) prices for those securities, the existence of complete capital markets ensures that the firm will, by maximizing its (equilibrium) market value, also enable each individual investor to achieve the widest possible range of investment opportunities. Since the existence of perfect capital markets permits each individual, by definition, to borrow or lend at the prevailing interest rate (and the same interest rate at which firms can borrow or lend), the existence of perfect and complete capital markets permits each individual to achieve his most desired pattern of returns independent of the firm's choice of capital structure. In such a world debt and equity become merely alternative ways of channeling ownership claims to income streams for investors, and the market value of the firm depends only on the total income stream and not on how it is channeled.[369]

Having recognized that the indeterminancy of the firm's debt-equity ratio depends on the existence of perfect and complete markets, it becomes easier to reconcile Modigliani's and Miller's claim that, given the firm's investment policy, its capital structure is irrelevant with the recently expressed fears that in American industry "debt has grown excessively in relation to equity ... and as capital, it is absolutely inferior;"[370] that only recently have equities markets appeared receptive to new issues;[371] and that "there is a tiny group of companies that can raise new equity capital, a larger group that can raise debt capital, and a very large group that cannot raise any capital at all."[372] In the real world markets are imperfect and incomplete.[373] Thus, the firm's capital structure is not irrelevant and in general "the cost to a firm of obtaining capital is a function of its capital structure."[374]

A second aspect of the firm's financial policy is its dividend policy. Modigliani and Miller showed that in a world of perfect capital markets the market value of the firm's equity is independent of the firm's dividend policy.[375] As in their earlier result concerning the indeterminancy of the debt-equity mix, the firm's operating policies are given and the result holds even under uncertainty.[376]

The irrelevance of the firm's dividend policy is easily demonstrated.[377] The remainder of this subsection develops a discrete time valuation model and uses the model to demonstrate the irrelevance of the firm's dividend policy when the future is known with certainty and when capital markets are perfect.[378] The model employs the following variables:

$d(t)$:	dividends paid per share in period t
$p(t)$:	price of a share (ex any dividend from the previous period) at the start of period t
$\rho(t)$:	market rate of interest during period t
$n(t)$:	number of shares outstanding at the beginning of period t
$m(t+1)$:	number of new shares issued during the period (sold without dividend at the closing price $p(t+1)$)
$V(t)$:	market value of the firm
$D(t)$:	total dividends paid to shareholders of record during period t.

Note that from the above definitions the following identities hold:

$$V(t) = n(t) \cdot p(t) \qquad D(t) = n(t) \cdot d(t)$$
$$n(t+1) = n(t) + m(t+1) \tag{135}$$

The assumptions of certainty and perfect capital markets imply that the following equation must hold when the capital markets are in equilibrium:[379]

$$\frac{d(t) + p(t+1) - p(t)}{p(t)} = \rho(t) \ , \tag{136}$$

or equivalently, that

$$p(t) = \frac{1}{1 + \rho(t)} [d(t) + p(t+1)] \quad . \tag{137}$$

Equation (136) has the following interpretation. In equilibrium in a world characterized by certainty and perfect capital markets, the yields on all securities must be identical and equal to the riskless rate of interest, $\rho(t)$. The left-hand side of (136) is interpreted as the yield on each equity share expressed as the sum of the dividend yield, $\frac{d(t)}{p(t)}$, and the percentage increase in share value (i.e. percentage capital gain), $\frac{p(t+1) - p(t)}{p(t)}$. Equation (137) expresses the same equilibrium condition as (136). The interpretation of (137) is that the share price at the beginning of each period must, in equilibrium, equal the discounted value of the sum of dividends paid during the period and the share price at the end of the period.

Multiplying each side of (137) by $n(t)$ and simplifying by using (135) yields

$$\begin{aligned} V(t) &= \frac{1}{1 + \rho(t)} [D(t) + n(t)\ p(t+1)] \\ &= \frac{1}{1 + \rho(t)} [D(t) + V(t+1) - m(t+1)\ p(t+1)] \quad . \end{aligned} \tag{138}$$

Equation (137) is one way of characterizing the equilibrium price of each share. Equation (138) is the analogous characterization for the equilibrium stock market value of the firm, i.e. for the aggregate stock market value of all shares outstanding. The interpretation of (138) is that the stock market value of the firm at the beginning of each period must, in equilibrium, equal the discounted value of the sum of net dividends (i.e. total dividends paid, $D(t)$, less receipts from new share issues, $m(t+1) \cdot p(t+1)$) and the stock market value of the firm at the end of the period.

Equation (138) implies that the firm's dividend decision affects the current market value of its equity, $V(t)$ (and its current share price, $p(t)$) in two conflicting ways: directly via $D(t)$ and indirectly via $-m(t+1)\, p(t+1)$.[380] In a certain world in which capital markets are perfect these effects cancel each other out. To see this, note that if the firm's operating policies are given, so that the level of investment, $I(t)$, and the current period's net income, $X(t)$, are to remain constant, then a higher dividend payout is possible only if the amount of new capital raised from external sources is increased by just enough to facilitate the increase in $D(t)$ while $I(t)$ and $X(t)$ are held constant, i.e. the dividend payment is financed from outside the firm (by new share sale or new borrowing).[381] The amount of outside capital needed by the firm is

$$m(t+1)\, p(t+1) = I(t) - [X(t) - D(t)] \quad , \tag{139}$$

and substituting (139) into (138) and canceling $D(t)$ gives

$$V(t) = \frac{1}{1 + \rho(t)} [X(t) - I(t) + V(t+1)] \quad . \tag{140}$$

Equation (140) reexpresses the characterization (138) of the equilibrium stock market value of the firm. The interpretation of (140) is that the stock market value of the firm at the beginning of each period must, in equilibrium, equal the discounted sum of net dividends (in this case expressed as net income less investment) and the stock market value of the firm at the end of the period.[382] Since $D(t)$ does not appear in (140) and since $X(t)$, $I(t)$, and $V(t+1)$ are all independent of $D(t)$, it follows that the current market value of the firm is independent of the current dividend decision. By proceeding in a similar fashion and showing that $V(t+1)$ is independent of $D(t+1)$, $V(t+2)$ is independent of $D(t+2)$, and so on, it can be shown that $V(t)$ is unaffected by the firm's (current and future) dividend policy.

This proposition on the irrelevance of dividend policy seems counterintuitive. But the apparent paradox arises from the tendency to overlook the important qualification 'given a firm's operating policies'. While it might seem reasonable that an additional cash receipt would always be welcomed, if the firm's operating policies are truly fixed, then this increased payout requires that additional funds be raised externally — by selling new shares and thereby reducing the share of future dividends that will be paid to current shareholders. In terms of equations (138) and (139), the fundamental principle of valuation assures that the effect on $V(t)$ of an increased dividend payout is exactly counterbalanced by the effect of a decrease in capital gains. What the irrelevance proposition is really saying, then, is that when capital markets are perfect[383] dividend decisions are independent of investment decisions,[384] so that the firm's investment decisions are not contingent upon its dividend decisions.[385] There are no financial illusions; values are determined by the firm's operating policies and not by how the firm chooses to 'package' its returns for distribution to its shareholders.[386]

Given the strong assumptions on which the irrelevance proposition rests, it should not be surprising that a number of arguments have been put forward and some empirical evidence has been gathered that contradicts the proposition. In an uncertain world it also seems reasonable that shareholders will generally prefer current dividends to future, less certain, capital gains.[387] There is also some empirical evidence to support the argument that, due to market imperfections, retained earnings are a cheaper source of investment funds than new issues, and that as a result, dividend policy is affected by investment policy (which determines the residual funds available for distribution as dividends).[388] There is also evidence that, at least over the short run, firms work to target dividend payout ratios.[389] Not only would this give dividend payments a high 'informational content' for both current and potential shareholders, but, to the extent that dividends and investments compete for the relatively limited supply of internal funds, it would also cause investment decisions to be affected by dividend decisions (which determine the proportion of internal funds available for investment).[390] Hence, there is good reason to believe that the firm's dividend policy is not irrelevant.[391] Yet Fama has recently offered empirical evidence that is strongly supportive of Miller and Modigliani, implying that whatever market imperfections exist in the real world are insufficient to invalidate the proposition that dividend policy is irrelevant.[392]

By way of summary, the conditions required for the firm's financial policies — its mix of debt and equity and its dividend decisions — to be irrelevant are not strictly satisfied in the real world corporate environment. While there does exist some empirical justification for assuming that the firm's production and investment policies are separable from its financial policies, this evidence is not conclusive. It is the opinion of this writer that a meaningful model of the firm should allow for the interaction of the firm's operating and financial policies.

2. The Vickers Model

Douglas Vickers has developed a single period model of the firm that integrates the firm's production, investment, and financing decisions.[393] Vickers deals with uncertainty by incorporating risk-adjusted interest rates and risk-adjusted capitalization rates. Risk is of two types: business risk and financial risk.[394] To simplify the analysis Vickers holds business risk fixed,[395] but as Turnovsky and Arzac have since shown, allowing business risk to vary does not materially affect Vickers's conclusions.[396,397]

Unlike the models discussed earlier in this chapter, the Vickers model takes into account the existence of risk, or synonomously, uncertainty.[398] There are two major frameworks available for incorporating uncertainty in a model of the firm: the mean-variance framework and the time-state-preference framework.[399] Vickers employs the mean-variance framework, as does Lintner, whose models are discussed in the next section of this chapter. Briefly, within the mean-variance framework, 'the' decision-maker (e.g. the firm's managers or its shareholders) has two objects of choice: expected returns, as measured by the mathematical expectation or some other statistical measure of the central tendency of returns, and risk, as measured by the variance, the coefficient of variation, or some other statistical measure of the dispersion of returns, where, by assumption, the returns (e.g. net income) are not known with certainty but their probability distribution is (known with certainty). In using the coefficient of variation of net income as a surrogate for financial risk,[400] Vickers is implicitly adopting the mean-variance framework. The second framework for incorporating uncertainty is described and several models of the firm that employ that framework are discussed below in section K. As pointed out in that section, the mean-variance approach is really just a special case of the time-state-preference approach to modeling the firm under uncertainty.

Vickers developed his model of the firm in two stages: first specifying a profit maximization model and then generalizing to a value maximization model.[401] The profit maximization model is the following:[402]

$$
\begin{aligned}
&\underset{\{X_1,\ X_2,\ D\}}{\text{maximize:}} && \pi = p(Q)\cdot f(X_1,X_2) - w_1X_1 - w_2X_2 - r(D)\cdot D \\
&\text{subject to:} && g(Q) + \alpha X_1 + \beta X_2 \leq E + D
\end{aligned}
\qquad (141)
$$

where π is net income, which is to be maximized, $p(Q)$ expresses price as a function of quantity demanded (i.e. the inverse of the demand function), $Q = f(X_1,X_2)$ is output, X_1 is a current factor of production (e.g. labor) with constant unit cost w_1, X_2 is a noncurrent factor of production (e.g. real capital) with constant (direct) unit cost w_2 that reflects depreciation, obsolescence, and scarcity rent but does not include interest charges,[403] r is the average rate of interest on debt capital,[404] which is an increasing function of the debt level, D is the debt level, E is the amount of equity which is assumed constant, $g(Q)$ expresses net working capital as a function of the output level, and α and β are the money capital requirement coefficients of the current and noncurrent factors of production, respectively.[405] The constraint in problem (141) expresses the money capital constraint. The money capital requirement is expressed by $g(Q) + \alpha X_1 + \beta X_2$ and money capital availability is the sum of equity capital and debt capital, $E + D$.[406] The constraint is an inequality, implying that requirements cannot exceed availability. Assuming prudent cash management on the part of the firm, requirements just balance money capital availability, the constraint becomes an equality, and the method of Lagrange multipliers can be used to obtain a characterization of the single period equilibrium position of the individual firm.

The necessary conditions for an optimal solution to (141) are the following:[407]

$$\frac{\partial L_\lambda}{\partial X_1} = MR(Q) \cdot \frac{\partial f}{\partial X_1} - w_1 - \lambda g'(Q) \cdot \frac{\partial f}{\partial X_1} - \lambda\alpha = 0 \qquad (142)$$

$$\frac{\partial L_\lambda}{\partial X_2} = MR(Q) \cdot \frac{\partial f}{\partial X_2} - w_2 - \lambda g'(Q) \cdot \frac{\partial f}{\partial X_2} - \lambda\beta = 0 \qquad (143)$$

$$\frac{\partial L_\lambda}{\partial D} = -[\frac{dr}{dD} D + r(D)] + \lambda = 0 \qquad (144)$$

where $MR(Q) = p'(Q) \cdot f(X_1, X_2) + p(Q)$ is marginal revenue. From equation (144)

$$\lambda = \frac{dr}{dD} D + r(D) \quad . \tag{145}$$

At the optimum the introduction of debt capital into the firm's capital structure will have been carried to the point at which the marginal cost of debt capital, $\frac{dr}{dD} D + r(D)$, just equals the marginal contribution of debt capital to net operating income, or equivalently, to the point at which the marginal contribution of debt capital to net income is zero.[408]

The Lagrange multiplier λ gives the implicit price of money capital (in terms of net operating income).[409] In view of this, equations (142) and (143) can be rewritten as

$$MR(Q) \cdot \frac{\partial f}{\partial X_1} = w_1 + \lambda \left[g'(Q) \frac{\partial f}{\partial X_1} + \alpha\right] = 0 \tag{146}$$

$$MR(Q) \cdot \frac{\partial f}{\partial X_2} = w_2 + \lambda \left[g'(Q) \frac{\partial f}{\partial X_2} + \beta\right] = 0 \tag{147}$$

where the left-hand side of each equation can be interpreted as the factor of production's marginal revenue product and the right-hand side can be interpreted as the factor's adjusted marginal cost, which is equal to the price of the factor plus the additional money capital required times its implicit price. Thus, it follows from (146) and (147) that, when the firm is in equilibrium, the marginal revenue product of each factor will exceed the factor's marginal cost by an amount just equal to the imputed marginal cost of the additional money capital required to employ that factor.[410] Solving equation (142) for $\partial f/\partial X_1$ and equation (143) for $\partial f/\partial X_2$ and then dividing yields the following expression for the marginal rate of technical substitution between the inputs when the firm is in equilibrium:

$$\text{MRTS} = -\frac{dX_2}{dX_1} = \frac{\partial f/\partial X_1}{\partial f/\partial X_2} = \frac{w_1 + \lambda\alpha}{w_2 + \lambda\beta}, \qquad (148)$$

which is similar to the expression for the marginal rate of technical substitution in the (conventional) neoclassical model,[411] except that in (148) the marginal cost of each factor is adjusted upward by the imputed marginal cost of the direct money capital requirement for that factor. Equation (148) can be interpreted as the requirement that, when the firm is in equilibrium, the marginal rate of technical substitution between the factors must equal the ratio of the adjusted marginal costs of the factors. It follows from (148) that when $\frac{\alpha}{\beta} \neq \frac{w_1}{w_2}$ and $\lambda \neq 0$, the money capital constraint causes the ratio of adjusted factor prices to differ from the ratio of unadjusted factor prices, thereby altering the firm's expansion path.[412]

Next, the product market equilibrium condition can be compared with the familiar (conventional) MR = MC rule for profit maximization. Solving equations (142) and (143) for marginal revenue gives

$$MR(Q) = \left(\frac{w_1}{\partial f/\partial X_1}\right) + \lambda\left[g'(Q) + \frac{\alpha}{\partial f/\partial X_1}\right] \qquad (149)$$

$$MR(Q) = \left(\frac{w_2}{\partial f/\partial X_2}\right) + \lambda\left[g'(Q) + \frac{\beta}{\partial f/\partial X_2}\right] \qquad (150)$$

where the term in parentheses on the right-hand side of each equation expresses marginal cost in the conventional sense. The expression in brackets in each equation expresses the additional amount of money capital needed to produce an additional unit of output. Hence, equations (149) and (150) state that, when the firm is in equilibrium, marginal revenue will exceed marginal production cost (i.e. the marginal cost of the factors of production used in producing the last unit of output) by an amount just equal to the imputed cost of the money capital required.

The product market solution for the Vickers model is represented in figure II-22. The traditional solution is price p_o and output Q_o where marginal revenue equals marginal production cost (i.e. marginal cost in the conventional sense).

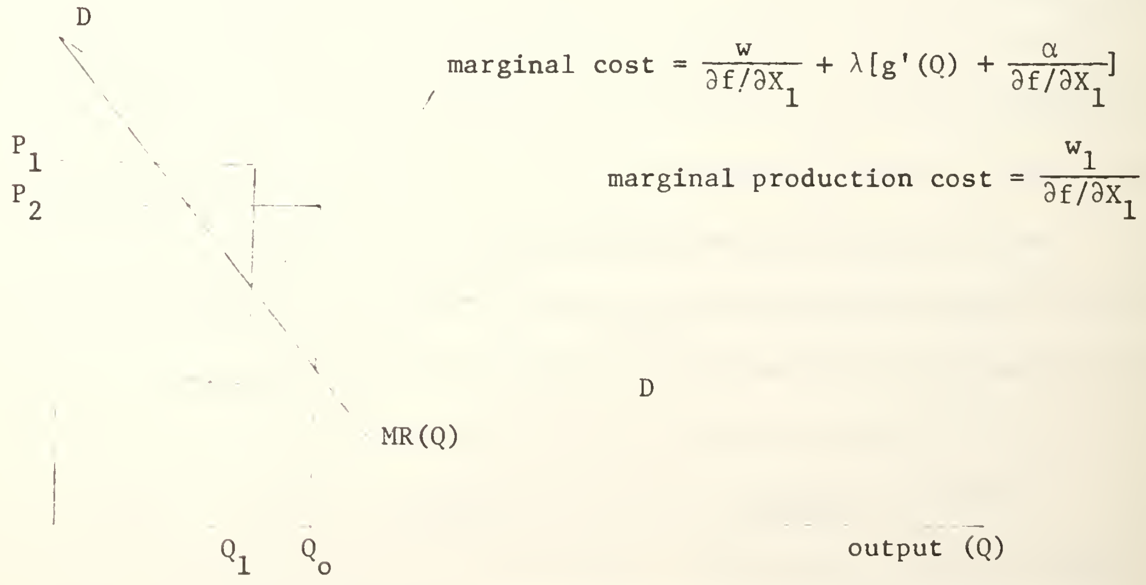


Figure II-22: Product Market Solution for the Vickers Model

The constrained solution is price p_1 and output Q_1 where marginal revenue equals marginal cost (inclusive of the imputed marginal cost of money capital). Note that the constraint on money capital (and in particular, on equity capital) has the effect of reducing the scale of output from Q_o to Q_1 (and correspondingly of raising the price of the good from p_o to p_1).[413] Putting this result together with equation (148) above, it follows that the money capital constraint affects both the scale of output and the expansion path.

To summarize the discussion of Vickers's profit maximization model, the main features of the model are presented in table II-18.

Table II-18 Summary of Vickers's Profit Maximization Model

Class:	modern traditional (see (141) in text)
Firm's Objective:	maximize total profit
Constraints:	technological (embodied in the production function) and financial (the money capital constraint); implicitly, product demand conditions, the average rate of interest on debt capital expressed as a function of the debt level, and the net working capital requirement
Variables:	
Exogenous:	direct factor costs (w_1 and w_2), money capital requirement coefficients (α and β), and the amount of equity (E)
Endogenous:	total profit (π), output (Q), price (p), input levels (X_1 and X_2), debt level (D), average interest rate on debt (r), and total money capital requirement ($g(Q) + \alpha X_1 + \beta X_2$)
Decision:	input levels (X_1 and X_2) and debt level (D)
Finance:	external finance (debt only) permitted (amount of equity held fixed)
Certainty/Uncertainty:	uncertainty allowed for (mean-variance framework)
Equilibrium/ Disequilibrium:	characterization of the equilibrium position of the firm
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	classical Lagrange multipliers

To generalize the profit maximization model to a value maximization model — in which the capitalized value of the net income stream being generated for the owners is the quantity to be maximized — the expression for net income, π in (141), is divided by the owners' capitalization rate, $\rho(D)$, which, for a given amount of equity, will be an increasing function of the amount of debt.[414] Letting V represent the market value of the firm's shares (expressed as the capitalized value of the net income stream) the Vickers model becomes the following:

$$\underset{\{X_1, X_2, D\}}{\text{maximize:}} \quad V = \frac{1}{\rho(D)} [p(Q) \cdot f(X_1, X_2) - w_1 X_1 - w_2 X_2 - r(D) \cdot D] \tag{151}$$

$$\text{subject to:} \quad g(Q) + \alpha X_1 + \beta X_2 \leq E + D$$

where the constraint is the same as before. Note that, since only one period's net income appears in the objective function in (151), the model is still a single period model. Though value maximization is not, in general, equivalent to profit maximization in (151), the trade off that exists between profits and market valuation is atemporal, rather than intertemporal, in nature. The difference between the models (141) and (151) is that, in general, the debt level affects both numerator and denominator in (151), so that the firm may stop short of profit maximization if further increases in the debt level raise the owners' capitalization rate, $\rho(D)$, proportionately more than net income, π , and thereby cause the market value of the firm's shares, V , to fall. Thus, in the Vickers model profit maximization and value maximization are not equivalent (unless ρ is a constant).

The necessary conditions for an optimal solution to problem (151) are the following:[415]

$$\frac{\partial L_{\lambda'}}{\partial X_1} = \frac{1}{\rho(D)} [MR(Q) \cdot \frac{\partial f}{\partial X_1} - w_1] - \lambda' g'(Q) \frac{\partial f}{\partial X_1} - \lambda'\alpha = 0 \qquad (152)$$

$$\frac{\partial L_{\lambda'}}{\partial X_2} = \frac{1}{\rho(D)} [MR(Q) \cdot \frac{\partial f}{\partial X_2} - w_2] - \lambda' g'(Q) \frac{\partial f}{\partial X_2} - \lambda'\beta = 0 \qquad (153)$$

$$\frac{\partial L_{\lambda'}}{\partial D} = - \frac{\pi}{[\rho(D)]^2} \frac{d\rho}{dD} - \frac{1}{\rho(D)} [\frac{dr}{dD} D + r(D)] + \lambda'$$

$$= - \frac{1}{\rho(D)} [V \frac{d\rho}{dD} + \frac{dr}{dD} D + r(D)] + \lambda' = 0 \qquad (154)$$

From equation (154)

$$\lambda' = \frac{1}{\rho(D)} [V \frac{d\rho}{dD} + \frac{dr}{dD} D + r(D)] \quad , \qquad (155)$$

which is analogous to (145). The Lagrange multiplier λ' measures the marginal value of money capital.[416] Equation (155) requires that, when the firm is in equilibrium, the marginal contribution of debt capital to capitalized net operating income must equal the capitalized value of its marginal cost, which in turn equals the capitalized value of the marginal interest cost, $\frac{1}{\rho(D)} [\frac{dr}{dD} D + r(D)]$, plus an amount $\frac{1}{\rho(D)} V \frac{d\rho}{dD}$ just sufficient to take account of the induced increase in the debt capitalization rate, $\rho(D)$, and the consequent fall in the market value of the firm's shares.

Next consider equations (152) and (153), which can be rewritten as

$$\frac{1}{\rho(D)} \left[MR(Q) \frac{\partial f}{\partial X_1}\right] = \frac{w_1}{\rho(D)} + \lambda' \left[g'(Q) \frac{\partial f}{\partial X_1} + \alpha\right] \tag{156}$$

$$\frac{1}{\rho(D)} \left[MR(Q) \frac{\partial f}{\partial X_2}\right] = \frac{w_2}{\rho(D)} + \lambda' \left[g'(Q) \frac{\partial f}{\partial X_2} + \beta\right] . \tag{157}$$

Equations (156) and (157) are interpreted as requiring that, when the firm is in equilibrium, the capitalized value of the marginal revenue product of each factor (the expressions on the left-hand side of (156) and (157)) must equal the capitalized value of that factor's marginal cost, $w_i/\rho(D)$, plus the imputed marginal cost of the money capital required, $\lambda' \left[g'(Q) \frac{\partial f}{\partial X_i} + \alpha\right]$.

Equations (152) and (153) together yield an expression for the marginal rate of technical substitution when the firm is in equilibrium. Solving equation (152) for $\partial f/\partial X_1$ and equation (153) for $\partial f/\partial X_2$ and dividing the former by the latter gives

$$\text{MRTS} = -\frac{dX_2}{dX_1} = \frac{\partial f/\partial X_1}{\partial f/\partial X_2} = \frac{\frac{w_1}{\rho} + \lambda'\alpha}{\frac{w_2}{\rho} + \lambda'\beta} , \tag{158}$$

which is analogous to (148). Equation (158) is interpreted as requiring that, when the firm is in equilibrium, the marginal rate of technical substitution must equal the ratio of the capitalized value of the direct factor cost w_1/ρ of the first factor adjusted upward by the imputed marginal cost of its direct money capital requirement, $\lambda'\alpha$, to the capitalized value of the direct factor cost plus the imputed marginal cost of money capital for the second factor.[417]

In order to summarize the discussion of Vickers's value maximization model of the firm, the distinguishing features of the model are presented in table II-19.

Table II-19 Summary of Vickers's Value Maximization Model

Class:	modern traditional (see (151) in text)
Firm's Objective:	maximize the stock market value of the firm (expressed in terms of one period's net income)
Constraints:	technological (embodied in the production function) and financial (the money capital constraint); implicitly, product demand conditions, the average rate of interest on debt capital and the owners' capitalization rate each expressed as a function of the debt level, and the net working capital requirement
Variables:	
Exogenous:	direct factor costs (w_1 and w_2), money capital requirement coefficients (α and β), and the amounts of equity (E)
Endogenous:	stock market value (V), output (Q), price (p), input levels (X_1 and X_2), debt level (D), average interest rate on debt (r), owners' capitalization rate (ρ), and total money capital requirement ($g(Q) + \alpha X_1 + \beta X_2$)
Decision:	input levels (X_1 and X_2) and debt level (D)
Finance:	external finance (debt only) permitted (amount of equity held fixed)
Certainty/Uncertainty:	uncertainty allowed for (mean-variance framework)
Equilibrium/ Disequilibrium:	characterization of the equilibrium position of the firm
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	classical Lagrange multipliers

The Vickers model is noteworthy for its integration of financial considerations into a model of the firm.

In both of the models of the firm, (141) and (151), finance, and in particular the money capital constraint, played an important role. In each case the optimal solution to the mathematical programming problem represented a simultaneous solution of the firm's production and financing problems and led to a characterization of the single period equilibrium position of the firm in which factor costs and the marginal cost of output were adjusted for the implicit cost of money capital.

One limitation of the Vickers model is its single period nature. Though the discounting by $\rho(D)$ in (151) might lead one to believe that Vickers's value maximization model is a multiperiod model, the complete absence of any intemporal profit trade offs and the appearance of only a single period's net income in the numerator of the valuation formula imply that the firm's behavior each period is being modeled as being independent of its behavior in every other period. Moreover, if one views Vickers's valuation formula as the formula for the present value of a perpetual annuity paying π per period when the discount rate ρ remains constant forever, then, in order for this valuation formula to hold, every period must be indistinguishable from every other period when judged from the standpoint of the firm and its behavior. Hence, both the profit maximization and the value maximization versions of the model are of the single period variety. The next subsection describes a model that not only incorporates finance, but that also allows for the growth of the firm.

3. The Herendeen Model

This subsection describes a model devised by James B. Herendeen.[418] In the Herendeen model the firm maximizes its rate of growth of total assets subject to a valuation constraint on permissible rates of growth. In this regard it is in the spirit of the Marris growth model discussed above in section G. But unlike the Marris model, in which all investment was financed internally and in which the role of the firm's financial policy was consequently suppressed, the Herendeen model gives external finance a prominent role to play. One of the firm's decision variables is the leverage ratio, which is the ratio of debt to equity, and the leverage ratio influences both the average interest rate on debt and the owners' capitalization rate. In this regard the Herendeen model is similar to the Vickers model. But unlike the Vickers model, in which the market value of the firm's shares was maximized, the Herendeen model assumes growth maximization and incorporates the market value of the firm's shares in the valuation ratio that appears in the model's constraint equation.

The development of the Herendeen model begins with a statement of some basic accounting identities. To appreciate better what follows the reader might find it helpful to refer to the typical firm's balance sheet illustrated in table II-1 and the typical firm's income statement illustrated in table II-2. Define the following variables: E is the book value of equity, D is the book value of total debt, A is the book value of total assets,[419] π is net income,[420] $L = D/E$ is the leverage ratio,[421] $i = i(L)$ is the average rate of interest on borrowed funds,[422] $\rho = \rho(L)$ is the owners' capitalization rate,[423] $e = \pi/E$ is the rate of return on equity, $p = \frac{\pi + iD}{A}$ is the rate of return on total assets (before deducting interest on indebtedness), and g is the rate of growth of total assets.

Herendeen assumes that $\frac{di}{dL} > 0$ and $\frac{d^2i}{dL^2} > 0$ for all L and that $\frac{d\rho}{dL} > 0$ and $\frac{d^2\rho}{dL^2} > 0$ at least in some interval containing the equilibrium leverage ratio.[424] One possible interpretation that can be given this set of assumptions concerning the signs of the first and second derivatives of $i(L)$ and $\rho(L)$ is that the level of risk borne by the firm increases as its leverage ratio, L, increases, thereby causing i and ρ to increase (at an increasing rate). Herendeen confines his consideration of risk to a single footnote,[425] though in light of the footnote and his assumptions concerning the derivatives of $i(L)$ and $\rho(L)$ it would seem that he is subsuming the effects of uncertainty within the functional forms of $i(L)$ and $\rho(L)$. Since he implicitly assumes that $i(L)$, $\rho(L)$, and the first and second derivatives of each are known with certainty, his model is, in effect, a model of the firm under certainty.

Continuing the development of the Herendeen model, net income is equal to gross profit minus interest payments, or in equation form, $Ee = pA - iD$. Dividing each side of this equation by E gives

$$e = (pA - iD)/E \quad . \tag{159}$$

From the basic accounting identity that states that the book value of assets equals the book value of total debt plus the book value of equity, $A = D + E$, it follows that $\frac{A}{E} = \frac{D + E}{E} = L + 1$, and upon substitution into equation (159), this leads to the identity

$$e = (L + 1)p - i(L)\cdot L = L(p) - i(L)) + p \;, \tag{160}$$

which states that the rate of return on equity is equal to the rate of return on assets plus the leverage ratio multiplied by the difference between the rate of return on assets and the interest rate on debt.[426]

If all net income is reinvested, if the firm maintains a constant leverage ratio, and if the rate of return on assets remains constant, then the rate of growth of total assets will equal the rate of return on equity, or $g = e$.[427] However, this abstracts from the firm's dividend policy (assuming the firm pays no dividends as was done above is tantamount to assuming the absence of any dividend policy whatsoever), the possibility of new equity issues, and corporate taxes. Next, these three factors are taken into account. Define the following variables: π_r is net retained current earnings, K is net dividend payments,[428] $k = K/E$ is the ratio of net dividends to the book value of equity, t is the corporate tax rate (assumed constant), and $e_r = \pi_r/E$ is the ratio of net retained current earnings to equity. Net retained current earnings is equal to net income less net dividend payments and corporate taxes paid, or in equation form, $\pi_r = eE - kE - teE$, and dividing through by E gives $e_r = (1 - t)e - k$. If all retained earnings are reinvested and if L and p remain constant, then $g = e_r$,[429] so that g may now be expressed in terms of the following identity:

$$g = (1 - t)e - k \quad , \tag{161}$$

which states that the rate of growth of total assets is equal to the rate of return on equity multiplied by one minus the tax rate less the ratio of net dividends to the book value of equity.

One further modification of the expression for the growth rate of total assets is needed. Equation (161) abstracts from the costs of maintaining the constant rate of return on assets, p, as the firm grows. In order to maintain any particular growth rate, g, the firm incurs real costs of growth in the form of (i) research and development costs intended to improve existing products and to develop new products, (ii) advertising expenditures and price reductions intended to increase sales of existing products, (iii) the cost of locating and developing new markets, and (iv) the cost of bringing new people into the organization.[430] Herendeen argues that, in general, these real costs of growth would tend to depress p, and the faster the firm tried to grow, the faster these costs would tend to increase, and the greater would be the downward pressure on p.[431] Following Herendeen, it is assumed that the real costs of growth are proportional to net income after taxes and dividends.[432] Letting B denote the total real costs of growth and $b = B/[(1-t)\pi - K]$ denote the ratio of the real costs of growth to retained earnings (i.e. net income after taxes less dividends),[433] it follows from $\pi = eE$ and $K = kE$ that $b[(1-t)e-k] = B/E$, the ratio of real growth costs to equity. Since the real costs of growth must be subtracted from net retained current earnings in order to determine the change in the firm's equity when the firm's profitability p is to be maintained, it follows that $b[(1-t)e-k]$ must be subtracted from equation (161). The growth rate of total assets is given by the following identity:[434]

$$g = (1-t)e-k-b[(1-t)e-k] = (1-b)[(1-t)e-k] . \qquad (162)$$

Substituting expression (160) for e into equation (162) gives

$$g = (1-b)\,[\,(1-t)\{(L+1)p - i(L)\cdot L\} - k]\ . \qquad (163)$$

Equation (163) states that the growth rate of total assets is equal to the rate of return on equity adjusted for taxes, dividend payments, and the real costs of growth.

All that remains to be done before the growth rate of the firm is fully specified is to express the rate of return on assets, p, as a function of the firm's total output and its selling outlays. Once this has been accomplished, substitution for p in equation (163) will yield the firm's objective function. Letting R be total revenue, C be total production costs,[435] and S be total selling outlays,[436] then by the definition of p given above,

$$p = (R-C-S)/A\ . \qquad (164)$$

Further define the rate of revenue[437] $r = R/A$; the rate of cost, $c = C/A$; the rate of selling outlays, $s = S/A$;[438] and the rate of output, $y = Y/A$, where Y is total output.[439] Assume the following two functional relationships: $r = r(y,s)$ and $c = c(y)$, where $\partial r/\partial y > 0$ but $\partial^2 r/\partial y^2 < 0$ since the demand curve is downward-sloping; where $\partial r/\partial s > 0$ but $\partial^2 r/\partial s^2 < 0$ due to diminishing returns to additional selling outlays; and where $dc/dy > 0$ and $d^2c/dy^2 > 0$, i.e. where the marginal rate of cost increases at an increasing rate as a function of the rate of output.[440] Then equation (164) may be reexpressed in terms of r , c , and s as follows:

$$p = r(y,s) - c(y) - s \ . \tag{165}$$

Substituting (165) for p into equation (163) yields the Herendeen objective function:

$$g = (1-b)\,[(1-t)\{(L+1)\,[r(y,s) - c(y) - s] - i(L)\cdot L\} - k] \ . \tag{166}$$

Next, the valuation constraint is formulated. Let V denote the market value of the firm's equity, and $v = V/E$ denote the firm's valuation ratio. Following Gordon, Marris, and Herendeen[441] define the market value of the firm's shares as the discounted flow of future dividends,

$$V = K/(\rho(L) - g) \ . \tag{167}$$

Dividing each side of equation (167) by E , substituting v for V/E and k for K/E , and rewriting, yields the equation:[442]

$$g - \rho(L) + k/v = 0 \ . \tag{168}$$

Substituting v_o , the exogenously determined minimum valuation ratio considered safe by the firm's managers, for v in equation (168) yields the valuation constraint.[443] In equation (168) k/v is the dividend yield, or the ratio of dividends per share to the market value of the share. Equation (168) can be rewritten as $\rho(L) = k/v + g$, which states that the owners' capitalization rate (i.e. the yield on equity) is equal to the dividend yield plus the growth rate (i.e. the rate at which the share price appreciates, or equivalently, at which capital gains accrue).[444]

According to the Herendeen model, the objective of the firm is to maximize (166) subject to (168).[445] In this model of the firm there are only three decision variables: the rate of output y, the rate of selling outlays s, and the leverage ratio L. The dividend rate k is assumed given. In reality, the firm's selection of a net dividend rate, k, is likely to be a matter of concern to shareholders, in which case ρ would vary as a function of both L and k, $\rho=\rho(L,k)$, with $\partial\rho/\partial k < 0$.[446] Permitting the net dividend rate, k, to become a policy variable and setting $v = v_o$ in (168), the Herendeen model of the firm, as modified by this author, can be expressed as the following mathematical programming problem:[447]

$$\begin{aligned}&\underset{\{y,s,L,k\}}{\text{maximize:}} \quad g(y,s,L,k) = (1-b)[(1-t)\{(L+1)[r(y,s)-c(y)-s]-i(L)\cdot L\}-k]\\ &\text{subject to:} \quad g(y,s,L,k) - \rho(L,k) + k/v_o = 0 .\end{aligned} \tag{169}$$

The necessary conditions for an optimal solution to problem(169), which can be used collectively to characterize the equilibrium steady state growth path of the firm, are, in addition to the constraint equation, the following:[448]

$$\frac{\partial L_\lambda}{\partial y} = (1-b)(1-t)(L+1)\left(\frac{\partial r}{\partial y} - \frac{dc}{dy}\right)(1+\lambda) = 0 \tag{170}$$

$$\frac{\partial L_\lambda}{\partial s} = (1-b)(1-t)(L+1)\left(\frac{\partial r}{\partial s} - 1\right)(1+\lambda) = 0 \tag{171}$$

$$\frac{\partial L_\lambda}{\partial L} = (1-b)(1-t)\left[p - \frac{di}{dL} L - i(L)\right](1+\lambda) - \lambda \frac{\partial \rho}{\partial L} = 0 \tag{172}$$

$$\frac{\partial L_\lambda}{\partial k} = (1-b)(-1)(1+\lambda) + \lambda\left(-\frac{\partial \rho}{\partial k} + \frac{1}{v_o}\right) = 0 \tag{173}$$

From equation (170) it follows that in steady state equilibrium

$$\frac{\partial r}{\partial y} = \frac{dc}{dy} \ . \tag{174}$$

Equation (174) is interpreted to require that, in order for the firm to be in steady state equilibrium, the marginal rate of revenue with respect to the rate of output must equal the marginal rate of (production) cost with respect to the rate of output. This equilibrium condition is analogous to the MR = MC profit maximization rule in the traditional model of the firm. Equation (171) implies that

$$\frac{\partial r}{\partial s} = 1 \ . \tag{175}$$

Equation (175) is interpreted to require that, in order for the firm to be in steady state equilibrium, the marginal rate of revenue with respect to the rate of selling outlays must equal one, which is the marginal rate of cost of selling outlays. Like (174), condition (175) is analogous to the traditional model's rule that advertising outlays should be expanded to the point where the addition to revenue (net of production costs) resulting from the last dollar of advertising expenditure just equals the cost of advertising (i.e. one dollar). Equations (174) and (175) also imply that

$$\frac{\partial e}{\partial y} = \frac{\partial e}{\partial s} = 0 \quad , \tag{176}$$

which is interpreted to require that, in order for the firm to be in steady state equilibrium, the rate of change of the rate of return on equity with respect to both the rate of output and the rate of selling outlays must be zero. Conditions (174), (175), and (176) imply that, given total assets A, the growth maximizer will select the same output level and the same level of selling outlays (exclusive, of course, of those selling outlays specifically intended to promote growth) as the profit maximizer.[449] The reason for this is that the firm's managers wish to use existing assets as efficiently as possible in order to generate the maximum possible net income, out of which dividends and taxes must be paid and further growth must be financed.

From equation (172) it follows that

$$(1-b)(1-t)p = (1-b)(1-t)(\frac{di}{dL} L + i(L)) + \frac{\lambda}{1+\lambda} \frac{\partial \rho}{\partial L} , \tag{177}$$

which is interpreted to requite that, in order for the firm to be in steady state equilibrium, the net rate of return on assets (i.e. net of taxes and net of the real costs of growth) must equal the net marginal rate of debt cost, $(\frac{di}{dL} L + i(L))(1-t)(1-b)$, plus the implicit marginal rate of debt cost, $\frac{\lambda}{1+\lambda}\frac{\partial\rho}{\partial L}$, as reflected in the change in the discount rate, ρ , induced by a change in the leverage ratio, L .[450] Equation (177) may be rewritten as[451]

$$(1-b)(1-t)\frac{\partial e}{\partial L} = \frac{\lambda}{1+\lambda}\frac{\partial\rho}{\partial L} , \qquad (178)$$

which is interpreted to require that, in order for the firm to be in steady state equilibrium, the after-tax rate of return on equity (which is figured net of the real costs of growth) must equal the implicit marginal rate of debt cost. Note that when the discount rate is constant, i.e. $\frac{\partial\rho}{\partial L} = 0$ for all L , equation (178) implies that $\frac{\partial e}{\partial L} = 0$. Thus with ρ constant the firm selects the leverage ratio L that makes the rate of change of the rate of return on equity with respect to the leverage ratio equal to zero. The significance of this result is that, with k fixed, maximizing the growth rate of total assets with respect to y , s , and L necessitates finding y , s , and L such that[452]

$$\frac{\partial e}{\partial y} = \frac{\partial e}{\partial s} = \frac{\partial e}{\partial L} = 0 . \qquad (179)$$

Thus, given the firm's dividend policy as reflected in k and given a constant rate of discount, ρ , growth maximization becomes equivalent to maximizing the stock market value of the firm and, by (179), is achieved by maximizing the firm's rate of return on equity.

The last of the necessary conditions is (173), which can be be rewritten as:

$$-(1-b) = \frac{\lambda}{1+\lambda}\left(\frac{\partial\rho}{\partial k} - \frac{1}{v_o}\right) . \qquad (180)$$

To interpret (180), note that the left-hand side is the partial derivative of the objective function in (169) with respect to the net dividend rate, k, and that the right-hand side is the product of $-\frac{\lambda}{1+\lambda}$ and the partial derivative of g with respect to k in the constraint equation of (169). Thus (180) is interpreted to require that, with y, s, and L at their respective equilibrium values, the marginal improvement in the growth rate resulting from a decrease in the net dividend rate, k, must equal the implicit marginal cost of a reduction in the net dividend rate (measured in terms of the resulting change in g necessary to offset the effect of the reduction in k and thereby ensure that $v = v_o$ remains satisfied) in order for the firm to be in steady state equilibrium.[453]

According to the Herendeen model (as modified by the author to include k as a decision variable), the firm maximizes its steady state rate of growth of total assets subject to a valuation constraint and accomplishes this by selecting its rate of output, rate of selling outlays, leverage ratio, and net dividend rate to satisfy conditions (174), (175), (177) (or equivalently, (178)), and (180). The distinguishing charcteristics of the model are summarized in table II-20. An important implication of the model is that the firm's growth rate depends not only on how profitably it utilizes its existing assets (conditions (174) and (175)), but also on how it obtains additional funds through borrowings, retained earnings, and new equity issues, and how its financial decisions affect risk, and indirectly, the

Table II-20 Summary of Herendeen Model

Class:	managerial (see (169) in text)
Firm's Objective:	maximize the growth rate of total assets
Constraints:	valuation constraint; implicitly, product demand conditions (as embodied in $r(y,s)$), factor supply conditions (as embodied in $c(y)$), technology (as embodied in $(r(y,s) - c(y))$), the average interest rate on debt and the owners' capitalization rate (each expressed as a function of the firm's leverage ratio, with the owners' capitalization rate also a function of the firm's net dividend rate), and the constraints on the firm's ability to grow (as embodied in $g(y,s,L,k)$)
Variables:	
Exogenous:	minimum valuation ratio considered safe by the firm's managers (v_o), ratio of the real costs of growth to retained earnings (b), and the tax rate (t)
Endogenous:	growth rate of total assets (g), rate of output (y), rate of selling outlays (s), leverage ratio (L), net dividend rate (k), rate of revenue (r), rate of cost (c), average interest rate on debt (i), and owners' capitalization rate (ρ)
Decision:	rate of output (y), rate of selling outlays (s), leverage ratio (L), and net dividend rate (k)
Finance:	external finance permitted
Certainty/Uncertainty:	effects of uncertainty subsumed within the functional forms $i(L)$ and $\rho(L,k)$, so that the model is, in effect, a model of the firm under certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state growth path of the firm
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	classical Lagrange multipliers

interest rate on debt and the owners' capitalization rate (conditions (177) and (180)). Maximizing its growth rate requires that the firm make not only the correct operating (i.e. production and selling) decisions but also the correct financial decisions.

The Herendeen model shares much in common with the Marris growth model (115) discussed above in section G of this chapter. It could be transformed into a managerial model like Marris's managerial utility maximization model (111) by letting v vary and by introducing a managerial utility function incorporating v and g as defined by equation (166) as arguments of the new objective function. Such a model, by permitting finance a more expanded role, would be more general in its treatment of the firm than the Marris managerial utility maximization model.[454]

While the Herendeen model is like the Vickers model discussed earlier in this section in that both models incorporate risk and make the firm's financial policy an integral part of the model, the Herendeen model differs from the Vickers model and most other models of the firm that appear in the financial literature[455] in that the firm is not assumed to maximize the market value of the enterprise (or the stock market value of the firm's shares or the share value). This difference arises from the Herendeen model's assumption that "it is the enterprise, rather than the shareholder, who is residual claimant against corporate income."[456]

One of the uses to which the Herendeen model might be put is to test the behavior of the firm under the alternative hypotheses of growth maximization and value maximization. Herendeen does this, but the analysis is similar and the results obtained are in agreement with those discussed in connection with the Marris model — namely, that growth maximizers tend to grow faster, retain a larger portion of their earnings, and have a lower valuation ratio than value maximizers.[457] As the model discussed in the next section demonstrates, the implications of the Herendeen model concerning the comparative behavior of growth maximizers and value maximizers continue to hold when uncertainty is permitted.

J. THE LINTNER MODEL:[458] CORPORATE GROWTH UNDER UNCERTAINTY

The models of the firm discussed in previous sections have either allowed for risk and uncertainty indirectly, or worse, have assumed prescience and thereby abstracted from the effects uncertainty may have on the firm's choice of an optimal operating policy. The models presented above in sections B through H belong to the latter category,[459] while the Vickers model of section I, which associated the firm's level of financial risk with the coefficient of variation of net income and which allowed for risk by making the average interest rate on debt and the owners' capitalization rate each an increasing function of the firm's debt level, belongs to the former category. In addition, the Herendeen model of section I, in which the average interest rate on debt and the owners' capitalization rate are increasing functions of the firm's leverage ratio, subsumes the effects of uncertainty in the functions $i(L)$ and $\rho(L,k)$, so that, in effect, the model becomes a model of the firm under certainty.

What distinguishes Lintner's model from these earlier discussed models is Lintner's more sophisticated treatment of uncertainty. The managers of Lintner's firm select optimal values for the policy variables in order to place the firm on the long run expected growth path that maximizes the stock market value of the firm. But the firm selects an expected rate of growth — the actual rate of growth realized during any particular time period is a random variable. In selecting its optimal operating policy the firm is actually determining a probability distribution of growth rates. The variance of this distribution constitutes a risk effect that, in the context of a risk averse stock market, enters the share valuation formula directly in the form of a risk premium added onto the market rate of discount. In the inherently uncertain environment within which the firm operates, the market value of equity, which Lintner expresses as the present value of the certainty equivalents of the prospective dividend stream,[460] is affected by both uncertainty and the degree of risk aversion present in the market for the firm's shares. Hence, the firm's selection of an optimal operating policy must explicitly take into account uncertainty and investors' attitudes toward risk.

In the Lintner model the firm plans for growth, but the firm is not a growth maximizer. Lintner retains the neoclassical objective function in which the firm maximizes the current stock market value of its equity,[461] and as described below, Lintner demonstrates that even under uncertainty, maximizing the (expected) value of shareholders' equity does not imply maximizing the (expected) growth rate of the firm. The growth maximizer increases retentions until the marginal growth rate is zero, whereas the value maximizer stops short of this point, increasing retentions only to the point at which the marginal gain — in terms of the effect of increased growth on the share price — no longer exceeds the marginal (opportunity) cost — in terms of the effect of decreased dividend payout on the share price.[462]

Lintner's firm selects an expected rate of growth that is best thought of as a target growth path.[463] It accomplishes this by selecting values for the retention ratio and what he calls the 'basic' risk variable.[464] The firm might still take advantage of unexpected short run opportunities, but only to the extent that its long run growth strategy is not compromised.[465] Thus, the firm in the Lintner model eschews the myopic profit maximization implied in the traditional single period optimization models. The target growth rate selected by the firm is a steady state growth rate in the sense that the firm expects outlays for research and development, outlays for plant and equipment, operating costs, dollar sales, earnings, dividend payments, book values of assets, stock market value of equity, etc., all to grow at the same average (exponential) rate.[466] Because growth rates are subject to stochastic disturbances these variables are not assumed to grow at the same constant rate forever (although all quantities do grow at the same stochastic rate), and this more realistic treatment of the growth process avoids one of the limitations of the Marris-type steady state growth model. Moreover, interpreting the expected growth rate selected by the firm as a target rate of growth also appears consistent with the way in which corporate planners behave in the real world.[467]

This section describes the Lintner model of corporate growth under uncertainty. The discussion follows Lintner's development of the model by specifying the model first under certainty, then under 'stable uncertainty', and finally, under 'increasing uncertainty'. The distinction between 'stable uncertainty' and 'increasing uncertainty' will be made clear below.

1. The Model Under Certainty[468]

The objective of the firm is to maximize the current market value of equity. If capital markets are perfect, then in capital market equilibrium the current dividend yield plus the rate of change of the share price must equal the current rate of interest.[469] Assume for simplicity that the current rate of interest, which is denoted by i, remains constant over time and that all growth is financed internally.[470] In setting its operating policy the firm considers only those policies that lead to steady state growth. Denoting the steady state rate of growth by g, the retention ratio by r,[471] and the current level of earnings per share[472] by X_o, it follows from the above assumptions that the current share price, P_o, is:[473]

$$P_o = \int_0^{\infty} (1-r)X_o e^{-t(i-g)}\, dt = \frac{(1-r)X_o}{i-g} . \qquad (181)$$

In maximizing (181) the firm will only consider for adoption policy mixes that are *efficient* in the sense that for all policy mixes requiring the same retention ratio, only the one offering the maximum growth rate would be considered for adoption. It is assumed that the alternative efficient policy mixes are sufficiently numerous that they can be used to define alternative steady state growth rates as a continuously differentiable function of r, $g = f(r)$.[474] The set of efficient policy mixes, as defined above, has the property that, within this set, faster growth requires greater retentions. It follows that within the efficient set $dg/dr > 0$ for all r in the relevant range.

Taking X_o as given, the firm's objective is to select r that maximizes (181). Differentiating with respect to r gives

$$\frac{dP_o}{dr} = P_o \left[- \frac{1}{1-r} + \frac{dg/dr}{i-g} \right] . \tag{182}$$

From (182) it follows that

$$\frac{dP_o}{dr} \geq 0 \Longleftrightarrow (1-r) \frac{dg}{dr} + g \geq i \quad , \tag{183}$$

which implies that an increase in the retention ratio leads to an increase in the share price so long as the dividend payout of the marginal increase in the growth rate $(1-r) \frac{dg}{dr}$,[475] plus the growth rate itself exceeds the interest rate i.[476] The inequality (183) can be interpreted as a characterization of disequilibrium. If the firm's retention ratio is such that equality fails to hold in (183), then the firm cannot be in equilibrium. Indeed, an increase in the firm's retention ratio would lead to a new steady state growth path associated with which would be a higher share price P_o. Rewriting (183) gives

$$\frac{dP_o}{dr} \geq 0 \Longleftrightarrow \frac{dg}{dr} \geq \frac{i-g}{1-r} = \frac{X_o}{P_o} = y_e \quad , \tag{184}$$

where y_e is the current earnings yield on the stock. Since $y_e > 0$, (184) implies that the value maximizing firm will never maximize its rate of growth. That is, when the value maximizing firm is growing along its equilibrium steady state growth path, $\frac{dP_o}{dr} = 0$ and $\frac{dg}{dr} = y_e > 0$, so that increasing the retention ratio would lead to faster growth. But since greater retentions would reduce the share price, the value maximizing firm will stop increasing the retention ratio before the growth-maximizing ratio has been achieved. This also implies that a growth maximizer will tend to retain a greater proportion of earnings than a value maximizer. This implication is in agreement with the implications of the Marris and Herendeen models discussed earlier.

In the next two subsections the certainty model just discussed is generalized to permit first, what Lintner calls 'stable uncertainty', and then, what he calls 'increasing uncertainty'. By way of summarizing the discussion thus far, the distinguishing characteristics of Lintner's certainty model are presented in table II-21.

2. The Model Under 'Stable Uncertainty'

In this subsection the model just described is generalized to permit 'stable uncertainty'. The objective of the firm's managers remains unchanged, namely, to maximize the market value of equity. The firm still plans for growth, but it is no longer assumed that the future consequences of current policy decisions are known with certainty. Associated with any policy mix is some specified retention ratio, r, and an expected growth rate, $\bar{g}$. In the presence of uncertainty the firm can set its policies so as to select an expected growth rate, but the actual growth rate realized over any period will be a random variable. Consequently future dividends and stock prices are also uncertain.

Following Lintner, in this subsection it is assumed that the probability distribution of growth rates for time intervals of fixed duration in the future is stationary.[477] In particular, this implies that the variance of the distribution of growth rates is constant regardless of the futurity of the time interval to which the distribution applies.[478] It is assumed that the distribution

Table II-21 Summary of Lintner's Certainty Model

Class:	modern traditional (see (181) in text)
Firm's Objective:	maximize the equilibrium market price of a share of common stock
Constraints:	relationship between growth rate (g) and retention ratio (r) embodied in the set of efficient policy mixes ($g = f(r)$); implicitly, product demand, factor supply, and technological conditions, all of which underlie $g = f(r)$ and limit the extent to which additional retentions can be used to promote growth
Variables:	
Exogenous:	current level of earnings per share (X_o) and current rate of interest (i)
Endogenous:	steady state rate of growth (g) and retention ratio (r)
Decision:	retention ratio (r)
Finance:	internal financing only
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state growth path of the firm; in addition, disequilibrium (change in the retention ratio called for when the share price is not a maximum) also considered
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (by substituting $f(r)$ for g in (181))

of growth rates is normal with mean $\bar{g}$ and variance σ_g^2 ,[479] which implies that assets, total profit, dividends, the share price, etc., all of which are growing at the same expected rate $\bar{g}$, will be lognormally distributed.[480] In particular, if the current dividend per share is D_o , then the dividend per share t periods[481] hence, D_t , is lognormally distributed with $E[\ln D_t] = \ln D_o + \bar{g} \cdot t$, and, under the additional assumption that growth rates in different nonoverlapping time periods are independent random variables, var $[\ln D_t] = t \cdot \sigma_g^2$. Further assume that (i) each investor assesses a jointly lognormal distribution of the total returns (dividends plus capital gains) over all available securities with respect to a common date in the future and also assesses the distribution of returns from the whole portfolio to be lognormally distributed;[482] (ii) all investors have hyperbolic utility functions and choose their portfolios so as to maximize the expected utility of end-of-period wealth (i.e. as of the common date in the future with respect to which they make their assessments of security and portfolio returns);[483] and (iii) the stock market is perfect.[484] Then, under these and the previously stated assumptions, in stock market equilibrium the current share price is[485]

$$P_o = \int_0^\infty D_o e^{\hat{g}t} \cdot e^{-\omega t} dt = \int_0^\infty (1-r) X_o e^{-t(\omega - \hat{g})} dt = \frac{(1-r)X_o}{\omega - \hat{g}} , \quad (185)$$

where it is required that $\hat{g} < \omega$ in order that the indefinite integral exist and where

$$\hat{g} = \bar{g} - \alpha C \sigma_g^2 - \alpha f(\sigma_{ij}) \quad . \quad (186)$$

In (185) r and X_o are as previously defined and ω , which serves as the discount factor (and which Lintner assumes to remain constant over time), is the market weighted average of the shadow prices of all investors' wealth constraints and is interpreted as the shadow price of the market's total wealth (i.e. the combined wealth endowments of all investors).[486] In (186) $\bar{g}$ and σ_g^2 are as previously defined,

α is the market weighted average of the (constant) elasticities of all investors' utility functions and may be interpreted as the stock market's wealth elasticity and as a measure of the market's degree of risk aversion,[487] $C > 1$ reflects the fact that uncertainties regarding the future dividend (as embodied in σ_g^2) are magnified by the somewhat greater uncertainties regarding future share prices, which is due at least in part to the knowledge that in the future additional information that may affect the prospective resale value of shares held may beoome available,[488] and $f(\sigma_{ij})$ is a function that reflects the covariance of the firm's share price with other stock prices.[489] The term $\alpha C\sigma_g^2$ reflects that portion of risk attributable to uncertainty regarding the future growth of dividends, and hence, that portion of risk attributable to variation in the share price independent of the variation in other share prices, and the term $\alpha f(\sigma_{ij})$ reflects that portion of risk attributable to variation in the price of the company's stock relative to the variation in stock market prices generally.[490] Given these interpretations of $\alpha C\sigma_g^2$ and $\alpha f(\sigma_{ij})$ in (186) $\hat{g}$ can be interpreted as the certainty equivalent of the rate of growth of dividends. Equivalently, $D_o e^{\hat{g}t} = D_o e^{t(\bar{g}-\alpha C\sigma_g^2-\alpha f(\sigma_{ij}))}$ in (185) can be interpreted as the certainty equivalent of the dividend (per share) expected at time t. With this interpretation of $D_o e^{\hat{g}t}$, equation (185) expresses the share price as the present value of the certainty equivalents of the dividend stream.[491] That is, under the above assumptions, the equilibrium price of each share will be equal to the present value of the certainty equivalents of the stream of dividends that will accrue to the shareholder. Comparing equations (181) and (185) it can be seen that admitting (stable) uncertainty to the model has been accomplished by letting certainty equivalents take the place of certain amounts in (181) and by replacing the riskless interest rate i by the shadow price ω of the market's total investable wealth. As suggested by (186), however, there is a significant difference in content, for the certainty equivalent involves not merely the expectation $\bar{g}$ and the variance σ_g^2, but also the stock market's assessment of the riskiness of the stock as reflected in $\alpha C\sigma_g^2 + \alpha f(\sigma_{ij})$.

The objective of the firm is to maximize (185). In the certainty case the firm had a single decision variable r and the efficient set of growth rates was a function of r only, $g = f(r)$. The optimal policy mix was found by maximizing (181) subject to the constraint $g = f(r)$ that the policy mix be efficient. Thus far, only the objective function has been modified to reflect uncertainty. Next the constraint must be modified.

Under uncertainty the set of efficient policy mixes is of the form

$$\bar{g} = f(r, \sigma_g^2), \tag{187}$$

where r, $\bar{g}$, and σ_g^2 are as previously defined.[492] When growth rates are normally distributed the two parameters $\bar{g}$ and σ_g^2 are needed to specify the distribution that results from the selection of a retention ratio r (which by assumption will be held constant over time). Given the risk aversion of investors, one contour of the set of efficient policy mixes is depicted by the solid line above the σ_g^2 axis in figure II-23 and, for the same reason that higher retentions led to faster growth in the certainty case, $r_2 > r_1$ leads to a higher contour of the efficient set.[493]

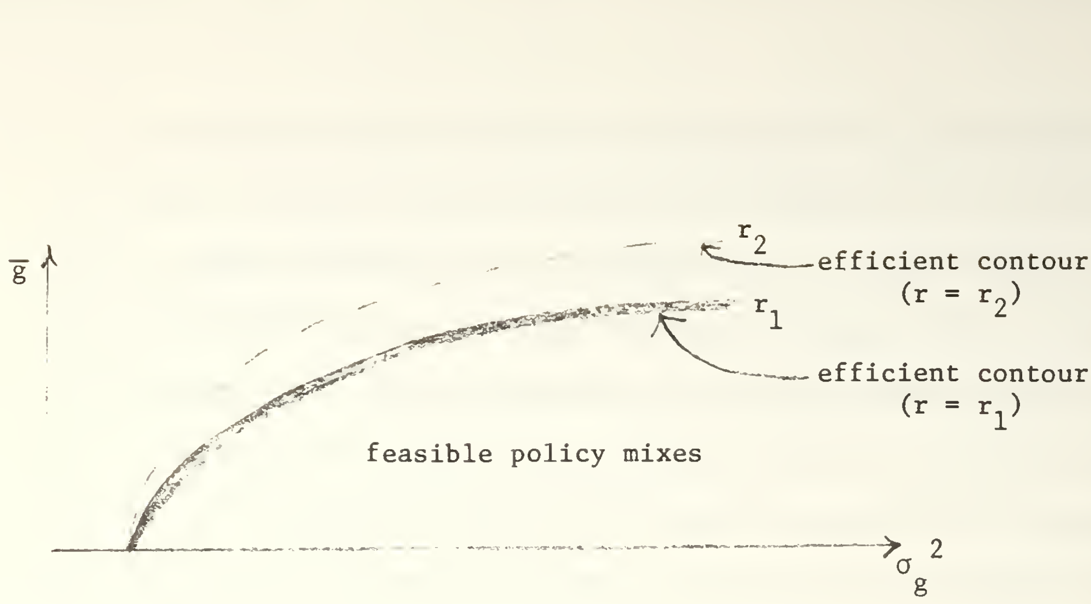


Figure II-23: Contours of the Efficient Set of Policy Mixes

Arguing that a larger retention ratio intended to raise the average growth rate also tends to increase the variability of the growth rate,[494] Lintner reexpresses σ_g^2 as a function of the retention ratio and what he calls the 'basic' risk variable, v_o :[495] $\sigma_g^2 = \sigma_g^2(r) = v_o + v(r)$. This enables (187) to be rewritten as

$$\bar{g} = f(r, v_o + v(r)) , \tag{188}$$

where $v(0) = 0$ and $v'(r) \geq 0$ and $v''(r) \leq 0$ for all r . The objective of the firm becomes the selection of the optimal pair of values (v_o, r) that maximizes (185) subject to (188).

The necessary conditions for maximizing (185) are the following:[496]

$$\frac{\partial P_o}{\partial v_o} = P_o \cdot \frac{\partial \hat{g}/\partial v_o}{\omega - \hat{g}} = 0 \Longrightarrow \frac{\partial \hat{g}}{\partial v_o} = 0 \tag{189}$$

$$\frac{\partial P_c}{\partial r} = P_o \cdot \left[\frac{-1}{1-r} + \frac{\partial \hat{g}/\partial r}{\omega - \hat{g}}\right] = 0 \Longrightarrow (1-r)\frac{\partial \hat{g}}{\partial r} + \hat{g} = \omega \tag{190}$$

According to (189) the firm should select the basic risk level v_o (conditional on r) that maximizes the certainty equivalent of the prospective dividend growth rate. According to (190) the firm should select the retention ratio r (conditional on v_o) that equates the sum of the dividend payout of the marginal certainty equivalent of the prospective growth rate, $(1-r)\ \partial\hat{g}/\partial r$, plus the certainty equivalent itself, g, to the shadow price of the market's total investable wealth (or what may be referred to as the 'riskless rate of interest') ω.

By expanding $\partial g/\partial v_o$ and $\partial g/\partial r$ equations (189) and (190) can each be given a richer interpretation. From (186) and the identity $\sigma_g^2 = v_o + v(r)$ it follows that (189) can be reexpressed as

$$\frac{\partial \hat{g}}{\partial v_o} = \frac{\partial \bar{g}}{\partial v_o} - \alpha C = 0 \Longleftrightarrow \frac{\partial \bar{g}}{\partial v_o} = \alpha C\ . \tag{191}$$

Thus, conditional on any particular value of r, the firm should vary its basic risk level along the corresponding efficient contour in figure II-23 until the point at which the slope of the contour, $\partial\bar{g}/\partial v_o$, equals αC, or equivalently, until the marginal improvement in the expected growth rate (i.e. the marginal value of the incremental basic risk) just equals the marginal cost of the incremental basic risk (as measured by the product of the market risk parameters α and C). Also note that as long as αC is finite (and this will usually be the case) equation (191) implies that the expected growth rate is positive and that the firm does not opt for the minimum level of risk attainable in the efficient set.[497] Next, expanding $\partial\hat{g}/\partial r$ gives

$$\frac{\partial \hat{g}}{\partial r} = \frac{\partial \bar{g}}{\partial r} + \frac{\partial \bar{g}}{\partial \sigma_g^{\,2}} v'(r) - \alpha C v'(r) = \frac{\partial \bar{g}}{\partial r} + \left(\frac{\partial \bar{g}}{\partial v_o} - \alpha C\right) v'(r) \ , \qquad (192)$$

from which it follows that $\frac{\partial \hat{g}}{\partial r} = \frac{\partial \bar{g}}{\partial r}$, or the marginal certainty equivalent of the growth rate equals the marginal expectation of the growth rate, whenever (i) the risk level is independent of the retention ratio (i.e. $v'(r) = 0$) or (ii) the firm is pursuing policies that are optimal with regard to the basic risk variable (i.e. $\frac{\partial \bar{g}}{\partial v_o} - \alpha C = 0$). In both cases (190) reduces to

$$(1 - r) \frac{\partial \bar{g}}{\partial r} + \hat{g} = \omega \ , \qquad (193)$$

and the optimal retention ratio is that which equates the sum of the dividend payout of the marginal expectation of the growth rate plus the certainty equivalent of the growth rate to the 'riskless interest rate'.

In comparison with (183), which gives the equilibrium condition for the certainty case (when the inequality is replaced by an equality), the marginal expectation of the growth rate $\frac{\partial \bar{g}}{\partial r}$ in (193) corresponds to the marginal growth rate $\frac{\partial g}{\partial r}$ and the 'riskless rate of interest' ω

in (193) corresponds to the riskless rate of interest i.[498] But the certainty equivalent growth rate $\hat{g}$ in (193) is always less than the expected growth rate $\bar{g}$ when uncertainty is present due to risk aversion.[499] Moreover, this difference increases as σ_g^2 increases. Consequently, under uncertainty the optimal retention ratio is generally lower (i.e. the optimal dividend payout is generally higher) than under certainty, and in general, the retention ratio varies inversely (and the dividend payout varies directly) with the level of risk borne by the firm.[500]

In the previous subsection (183) was used to show that a value maximizer will not maximize the rate of growth. Equation (193) can similarly be used to show that under uncertainty a value maximizer will not maximize the firm's expected rate of growth,[501] but rather, will tend to retain a smaller portion of earnings and grow more slowly than a growth maximizer. Solving equation (193) for $\frac{\partial \bar{g}}{\partial r}$ gives

$$\frac{\partial \bar{g}}{\partial r} = \frac{\omega - \hat{g}}{1 - r} = \frac{X_o}{P_o} = y_e > 0 \quad , \tag{194}$$

where y_e is once again the current earnings yield on the stock.[502] A growth maximizer would increase retentions to the point where $\frac{\partial \bar{g}}{\partial r} = 0$, but a value maximizer would stop short of this point. Thus, whether or not there is uncertainty, a value maximizer, as modeled by Lintner, retains a smaller portion of earnings and grows more slowly than a growth maximizer. But the existence of uncertainty forces both types of firms to retain less earnings, and therefore to grow at a slower average rate, than in the certainty case in order that the dividend payout be increased and shareholders be compensated for risk.

By way of summarizing the discussion of Lintner's model of the firm under 'stable uncertainty', the distinguishing features of the model are listed in table II-22.

Table II-22 Summary of Lintner's Stable Uncertainty Model

Class:	modern traditional (see (185) in text)
Firm's Objective:	maximize the equilibrium market price of a share of common stock
Constraints:	relationship between expected growth rate ($\bar{g}$) and retention ratio (r) embodied in the set of efficient policy mixes ($\bar{g} = f(r,v_o+v(r))$); implicitly, product demand, factor supply, and technological conditions, all of which underlie $\bar{g} = f(r,v_o+v(r))$ and limit the extent to which additional retentions can be used to promote (expected) growth; also implicitly, conditions in the stock market that determine the equilibrium share price (e.g. covariance function $f(\sigma_{ij})$)
Variables:	
Exogenous:	current level of earnings per share (X_o), the riskless rate of interest (ω), stock market's wealth elasticity (α), and magnification of uncertainty regarding future dividends (C)
Endogenous:	steady state expected growth rate ($\bar{g}$), retention ratio (r), variance of the distribution of growth rates (σ_g^2), and basic risk variable (v_o)
Decision:	retention ratio (r) and basic risk variable (v_o)
Finance:	internal financing only
Certainty/Uncertainty:	permits 'stable uncertainty' and utilizes the mean-variance framework
Equilibrium/ Disequilibrium:	characterization of the equilibrium steady state expected growth path of the firm; in addition, could consider disequilibrium as in the certainty version of the model (see table II-21)
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (by substituting $f(r,v_o+v(r))$ for $\bar{g}$ in (185))

3. The Model Under 'Increasing Uncertainty'

The model of the preceding subsection assumed that investors' assessments of future growth rates could be characterized by a time invariant probability distribution, so that, for example, there was no more uncertainty attached to assessments of growth rates ten years hence than there was to assessments of growth rates one year hence. This treatment of uncertainty — merely replacing the point estimate of g that was treated as a certainty in the initial formulation of the model with a stationary probability distribution for g — was overly simplified. Uncertainty tends to increase with futurity; uncertainties as to the outcome in any particular year, say the tenth, are the "compound resultant of the uncertainties in each of the intervening years";[503] and uncertainties as to outcomes in the tenth year will reflect uncertainties as to the outcome one year hence — compounded by uncertainties as to the outcome in each of the subsequent nine years. The model discussed in this subsection allows for uncertainties that increase with futurity.[504]

All but two of the assumptions stated in the preceding subsection still apply. In particular, it is again assumed that investors' utility functions are hyperbolic so that the current equilibrium price of each stock is still equal to the present value of the certainty equivalents of the elements of the future dividend stream discounted at the 'riskless interest rate' ω. The two new assumptions are the following.[505] (i) The changes in growth rates between all nonoverlapping pairs of short time intervals in the future are independent normally distributed random variables. More formally, it is assumed that $\tilde{g}(t+1) = g(t) + \tilde{u}(t)$, where the tilde denotes a random variable and where $E[\tilde{u}(t)] = 0$ and $\text{var}[\tilde{u}(t)] = 2t\sigma_u^2$ (for some positive constant σ_u^2) for all t and where $\text{cov}[u(t), u(t+t')] = 0$ for all pairs (t,t').[506] (ii) The variance of the currently assessed growth rate t periods hence exceeds the current variance of the growth rate, σ_g^2 (which is assumed time invariant), by the product $2t\sigma_u^2$, i.e. $\sigma_g^2(t) = \sigma_g^2 + 2t\sigma_u^2$. The variance of the cumulated growth beginning now and continuing over t periods is then $(\sigma_g^2 + t\sigma_u^2)\ t$. Under these assumptions $\text{var}\ [\ln D_t] = (\sigma_g^2 + t\sigma_u^2)\ t$, so that the logarithmic variance is a quadratic function of futurity, whereas in the preceding subsection it was only a linear function of futurity.

Under the above assumptions the certainty equivalent of the dividend (per share) expected at time t is[507]

$$D_o e^{\hat{g}t} = D_o e^{t(\bar{g} - \alpha C(\sigma_g^2 + t\sigma_u^2) - \alpha f(\sigma_{ij}))} \ , \qquad (195)$$

which differs from the certainty equivalent in the preceding subsection by the additional $-\alpha C t^2 \sigma_u^2$ term in the exponent. As illustrated by figure II-24, in the certainty case the logarithms of the certain future dividend receipts and in the stable uncertainty case the logarithms of the certainty equivalents of the future dividend receipts each increased at a constant linear rate over time, but the inclusion of the negative quadratic term in (195) causes the logarithms of the certainty equivalents in the increasing uncertainty case to reach a maximum and thereafter decline continuously (as long as $\sigma_u^2 > 0$ of course). The significance of this is that the indefinite integral expression (197) given below for the current share price must exist regardless of the size of the expected growth rate $\bar{g}$, so that the 'growth stock paradox' is no longer possible.[508]

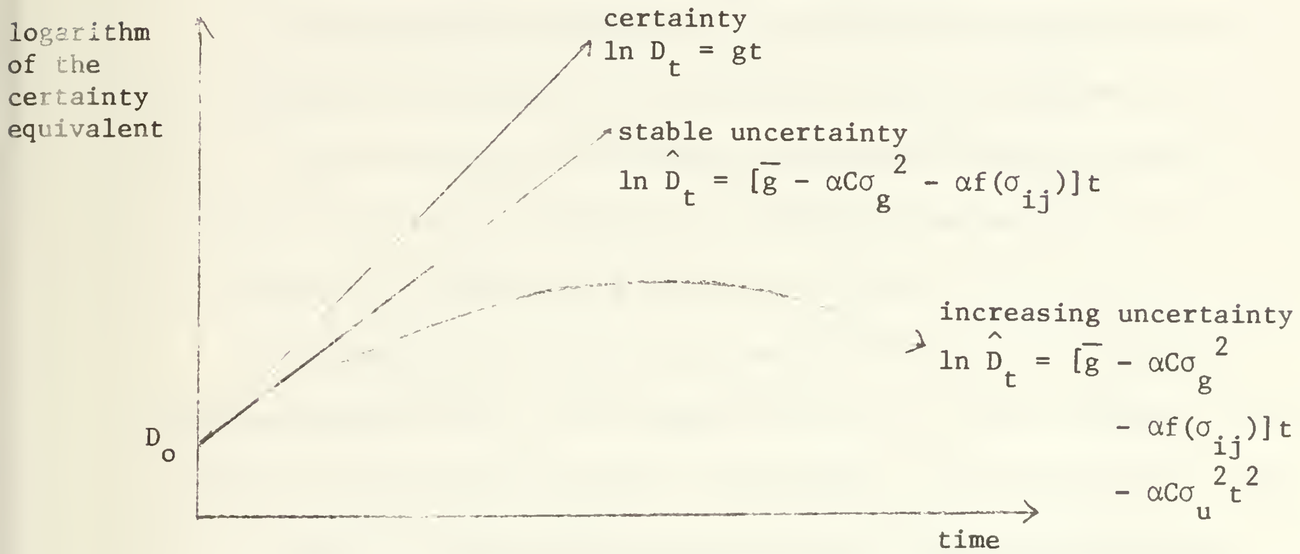


Figure II-24: Behavior of Certainty Equivalents in the Three Versions of the Lintner Model

From (195) the certainty equivalent of the growth rate is the expression in parentheses, which may be written as

$$\hat{g} = \bar{g} - \alpha C\sigma_g^2 - \alpha f(\sigma_{ij}) - \alpha C\sigma_u^2 t \quad , \tag{196}$$

which differs from (186) due to the $-\alpha C\sigma_u^2 t$ term. In contrast to (186), in which $\hat{g}$ was time invariant, $\hat{g}$ in (196) decreases with futurity (due to the effects of increasing uncertainty).

The current share price is

$$P_o = \int_0^\infty D_o e^{\hat{g}t} e^{-\omega t} dt = \int_0^\infty D_o e^{-t(\omega - \bar{g} + \alpha C \sigma_g^2 + \alpha f(\sigma_{ij}) + \alpha C \sigma_u^2 t)} dt \, , \quad (197)$$

which does not simplify as P_o did in (185) due to the presence of the quadratic term in the exponent. It is possible, however, to evaluate the integrals in (197) in terms of the probability density function and the inverse cumulative distribution function of the standard normal random variable.[509] This simplification does not require that any restrictions be placed on $\bar{g}$, and thus the 'growth stock paradox' cannot occur. The details of the calculations are omitted since the effect on P_o of changes in the various parameters in (197) can be seen directly from (197). Ceteris paribus the current share price varies directly with the expected growth rate $\bar{g}$ and inversely with each of the following factors: (i) the 'riskless interest rate' ω , (ii) the market's relative risk aversion α , (iii) the level of the growth covariance with other stocks $f(\sigma_{ij})$, (iv) the basic own-variance of the company's growth rate of dividends σ_g^2 , and (v) the time rate of increase in variances with futurity σ_u^2 .[510]

The objective of the firm is to maximize (197) subject to (188). Lintner distinguishes two cases, one in which σ_u^2 is independent of the firm's decisions and the other in which σ_u^2 varies with the basic risk level v_o . In each case there are the two decision variables v_o and r and the derivation and interpretation of the necessary conditions are similar to the treatment of the 'stable uncertainty' case in the preceding subsection.[511] These two cases are not considered further, although it is noted that the presence of σ_u^2 in both cases reduces the optimal retention ratio and the optimal expected growth rate (and they both vary inversely with σ_u^2),[512] and further, that making σ_u^2 dependent on v_o in the second case implies that the firm will no longer choose the basic risk level v_o that maximizes the certainty equivalents of its prospective growth rates.[513]

The distinguishing characteristics of the 'increasing uncertainty' version of the Lintner model of the firm are summarized in table II-23. By comparing the three versions, the characteristics of which are summarized in tables II-21, II-22, and II-23, the overall development of the Lintner model can be better appreciated.

The significance of the Lintner model lies in its sophisticated treatment of uncertainty. The objective of the firm is to maximize its current share price, which is determined by a risk-averse stock market that evaluates the effects the firm's choice of operating policies will have on the expected growth rate of dividends and the overall level of risk. Due to the existence of uncertainty, the firm will retain a smaller portion of earnings and therefore grow more slowly than it would under certainty. But even under certainty the value maximizer would still retain fewer earnings and grow more slowly than a Marris-type growth maximizer.

Two potential weaknesses of the Lintner model should be noted, however. The model restricts its attention to internal financing, though Lintner does note that the inclusion of debt financing would not alter any of the qualitative conclusions drawn from the model.[514] Second, the model assumes a stochastic steady state environment. Lintner justifies this by calling $\bar{g}$ a long run growth target.[515] Several models that permit non-steady state growth are discussed below in section L.

Table II-23 Summary of Lintner's Increasing Uncertainty Model

Class:	modern traditional (see (197) in text)
Firm's Objective:	maximize the equilibrium market price of a share of common stock
Constraints:	relationship between expected growth rate ($\bar{g}$) and retention ratio (r) embodied in the set of efficient policy mixes ($\bar{g} = f(r, v_o + v(r))$); implicitly, product demand, factor supply, and technological conditions, all of which underlie $\bar{g} = f(r, v_o + v(r))$ and limit the extent to which additional retentions can be used to promote growth; also implicitly, conditions in the stock market that determine the equilibrium share price (e.g. covariance function $f(\sigma_{ij})$)
Variables:	
Exogenous:	current level of earnings per share (X_o), the riskless rate of interest (ω), stock market's wealth elasticity (α), magnification of uncertainty regarding future dividends (C), and variance of growth disturbance term (σ_u^2)
Endogenous:	steady state expected growth rate ($\bar{g}$), retention ratio (r), variance of the current distribution of the growth rate (σ_g^2), and basic risk variable (v_o)
Decision:	retention ratio (r) and basic risk variable (v_o)
Finance:	internal financing only
Certainty/Uncertainty:	permits 'increasing uncertainty' and utilizes the mean-variance framework
Equilibrium Disequilibrium:	characterization of the equilibrium steady state expected growth path of the firm; in addition, could consider disequilibrium as in the certainty version of the model (see table II-21)
Time:	multiperiod
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (after substituting (188) into (197) and evaluating the integral as indicated in the text); actual solution not provided in the text; see Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 207-209, for details.

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A DYNAMIC STOCHASTIC MODEL OF CORPORATE BEHAVIOR OVER THE BUSINESS CYCLE WITH A SPECIAL APPLICATION TO THE MAJOR U.S. MILITARY AIRFRAME BUILDERS

John Dudley Finnerty

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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Corporate Behavior Over the Business Cycle
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This thesis contains a formulation of a dynamic stochastic model of corporate behavior over the business cycle and applies the basic model to firms in the U.S. airframe industry. The literature dealing with the theory of the firm is surveyed and a taxonomy is developed within which the major contributions to the literature are appraised.

The basic model is formulated as an optimal control problem.

block 19 continued

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The model is used to study the behavior of the firm over the business cycle and to suggest a possible reconciliation of the traditional and managerial theories of the firm. Financial considerations are incorporated into the model and the relationship between the firm's optimal operating decisions and its optimal financial decisions is examined. Organizational factors are introduced and some of the consequences of decentralized decision-making for the loss of control and X-efficiency are studied.

The basic model is extended to the major airframe builders by incorporating factors specific to that industry's institutional milieu. A model of a representative airframe builder is formulated as a stochastic optimal control problem and is used to study the impact of the government's progress payments policy and the likely impact of making interest expense an allowable cost under government contracts.

A Dynamic Stochastic Model of Corporate Behavior Over the Business Cycle With A Special Application To The Major U.S. Military Airframe Builders

by

John Dudley Finnerty
A.B., Williams College, 1971
B.A., Cambridge University, 1973
M.A., Cambridge University, 1977

Submitted in partial fulfillment of the requirements for the degree of

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K. CORPORATE BEHAVIOR UNDER UNCERTAINTY: THE TIME-STATE-PREFERENCE APPROACH

In the previous section uncertainty as to future outcomes (and in particular, uncertainty as to the actual rate of growth realized during any particular time interval) was handled within a mean-variance framework. The firm's selection of a retention ratio and a basic risk level determined both the expected (or mean) rate of growth and the overall risk level (i.e. the variance of the probability distribution of the growth rate). The mean-variance framework[516] is one of the two major lines of approach to the problem of risky choice over time. The other is the time-state-preference approach.[517]

This section describes the time-state-preference framework. It is shown that when a system of complete markets exists the problem of risky choice becomes formally equivalent to the problem of choice under certainty. In such a world, if markets are also perfect, the Modigliani-Miller proposition on the irrelevance of the firm's debt-equity ratio also holds,[518] and the firm's production decisions and its financial decisions are separable.

Later in the section two time-state-preference models, both due to Leland, and a related model due to Meyer, are presented. The first of Leland's models is primarily concerned with the firm's short run pricing and output decisions and the second is mainly concerned with the relationship between the firm's production decisions and the stock market value of the firm. Whereas Leland restricts his attention to the single product firm, Meyer allows the firm to produce several (i.e. $n > 1$) goods. Meyer also allows explicitly for the possibility that the amounts produced might prove insufficient to satisfy all demands, and incorporating this into his model leads to a probabilistic programming problem formulation of the model.

1. The Time-State-Preference Framework

One method of describing uncertainty about conditions in some future time period is to specify the set of possible states of nature that might prevail. For example, during some period there might just be two possible states of nature, say, war versus peace or prosperity versus depression.[519] There is one such set for each future time period (and these sets are not necessarily identical). Associated with each state of nature and each time period is a definite probability (possibly not fully known) of occurrence, with the probabilities of events for each time period summing to one. To continue the previous examples, war and peace might be equally likely or prosperity might be twice as likely as depression. For each time period and each state of nature both the configuration of the environment and the probability of its occurrence are assumed to lie beyond the control of the individual.[520] In addition, there exist securities, each of which is a complex entitlement to baskets of consumption claims[521] (loosely, dividends or interest payments) at various dates, where the amount received (e.g. the size of the dividend) at each date is dependent on the state of nature obtaining. For this reason these claims are called *contingent claims*. It is these contingent consumption incomes associated with alternative possible states of nature, rather than the securities themselves, that are assumed to be the individual's ultimate objects of preference.[522]

a. Pure Exchange

A description of a simple time-state-preference framework might better serve to convey the essential characteristics of this type of framework.[523] In the simplest case, there are only two time periods, the present (time "0") and the future (time "1"). The present state is known with certainty, and in the future one of two mutually exclusive states (state "a" or state "b") must obtain.[524] The individual does not know for certain what the future state of the world will be, although he can attach a (possibly subjective) probability, π_a and π_b, respectively (where $\pi_a + \pi_b = 1$), to the occurrence of each. To simplify the exposition by factoring out the decision as to how to allocate any one period's income among different commodities, it is assumed that there is a single generalized commodity, which will be referred to as "consumption claims" and which will be denoted by c. In the present period the individual consumes an amount c_o and in the future he will consume c_{1a} if state a obtains and c_{1b} if state b obtains.

The consumption claims c_o, c_{1a}, and c_{1b} are the individual's ultimate objects of choice. The individual's objective is to achieve the most desired time pattern of consumption in light of (i) his consumption preferences, (ii) his initial endowment, and (iii) market opportunities for exchanging dated contingent claims. The role of each of these three factors is described below. A discussion of the role of production is postponed until the next subsection.

The individual's preferences are represented by a utility function of the form $U = U(c_o, c_{1a}, c_{1b}; \pi_a, \pi_b)$. There are two dimensions of choice, one being the contemporaneous balance of risky claims between c_{1a} and c_{1b} and the other being the time-plus-risk choice between certain claims c_o and uncertain future claims c_{1a} and c_{1b}. The first of these choices is represented by figure II-25(a). The contours, each of which embodies the individual's subjective probability assessments π_a and π_b, are drawn convex to the origin,[525] just as in the two-good-single-period-certainty case (or more simply, the 'ordinary case') presented in introductory economics textbooks.[526] However, in contrast to the ordinary case in which a point on an indifference curve represents a combination of commodities, a point such as A in figure II-25(a) represents a pair of contingent claims. The amounts c_{1a}^A and c_{1b}^A cannot be consumed simultaneously; rather, c_{1a}^A will be consumed only if state a obtains and c_{1b}^A will be consumed only if state b obtains and the two states — and hence the amounts c_{1a}^A and c_{1b}^A — are mutually exclusive. Figure II-25(b) illustrates the choice between certain claims c_o and contingent claims c_{1a} (and a third figure with c_{1b} in place of c_{1a} could also be drawn).

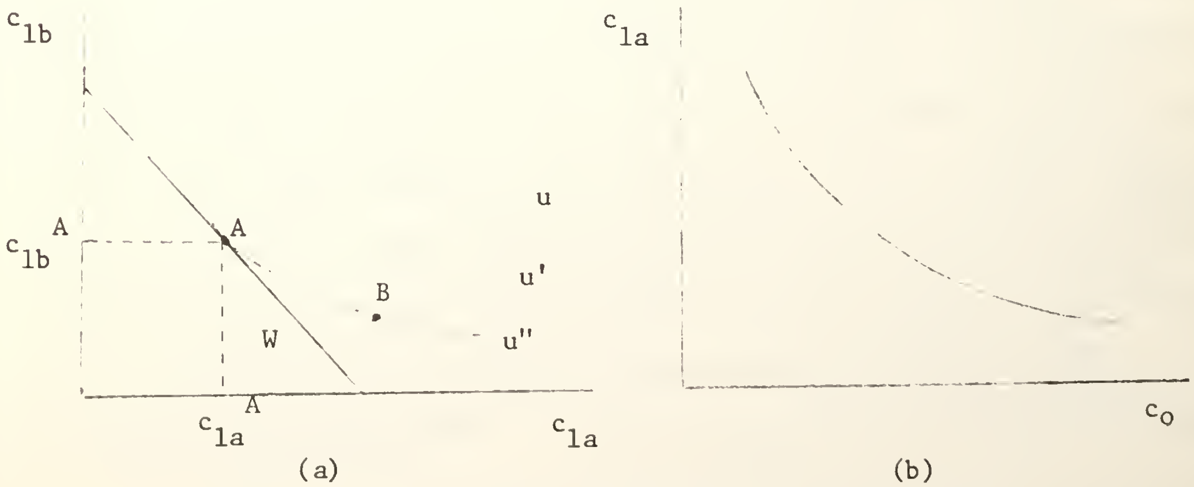


Figure II-25: Convex Indifference Curves Between State- and Time-Distributed Claims

The individual has an initial endowment of contingent claims (y_o, y_{1a}, y_{1b}). In the absence of exchange the consumption vector equals the initial endowment vector.[527] But where a market for contingent claims exists, the individual can exchange claims in order to reach his most preferred position (i.e. maximize his utility) Assume that the market for contingent claims is perfect, so that there are no taxes or transactions costs,[528] and *complete*, so that a sufficient number of securities exist to permit the individual to obtain whatever distribution of consumption claims (c_o, c_{1a}, c_{1b}) he most desires[529] — as restricted, of course, by his initial endowment and the market opportunities for exchange. For simplicity, assume there are just three securities, $C_o = (1,0,0)$, $C_{1a} = (0,1,0)$, and $C_{1b} = (0,0,1)$,[530] with prices ϕ_o, ϕ_{1a}, and ϕ_{1b}, respectively, so that, in effect, dated consumption claims are exchanged directly. To further simplify the analysis, certain consumption claims c_o may be chosen as the numeraire, so that $\phi_o = 1$, and the prices of C_{1a} and C_{1b} may be expressed in terms of the numeraire.[531] Denoting by W the individual's wealth, it follows that, in general equilibrium in the markets for contingent claims,

$$W = y_o + \phi_{1a}y_{1a} + \phi_{1b}y_{1b} = c_o + \phi_{1a}c_{1a} + \phi_{1b}c_{1b}, \quad (198)$$

where ϕ_{1a} and ϕ_{1b} are equilibrium prices for contingent claims and where the second equality follows from the fact that value is conserved in transactions (i.e. there are no "leakages" in the form of taxes or transactions costs) when markets are perfect.[532]

The problem confronting each individual is to exchange securities in such a way as to maximize his utility subject to his wealth constraint:[533]

$$\begin{aligned} &\underset{\{c_o, c_{1a}, c_{1b}\}}{\text{maximize:}} \quad U(c_o, c_{1a}, c_{1b}) \\ &\text{subject to:} \quad W = y_o + \phi_{1a} y_{1a} + \phi_{1b} y_{1b} = c_o + \phi_{1a} c_{1a} + \phi_{1b} c_{1b} \end{aligned} \tag{199}$$

and, for all individuals collectively, also subject to

$$\left.\begin{aligned} \Sigma c_o &= \Sigma y_o \\ \Sigma c_{1a} &= \Sigma y_{1a} \\ \Sigma c_{1b} &= \Sigma y_{1b} \end{aligned}\right\} \text{conservation relations} \tag{200}$$

where each individual's subjective probability assessments π_a and π_b are assumed fixed so that they can be suppressed when writing the expression for U; where each of the six sums is taken over all individuals; where the last three constraints require that contingent claims be neither created nor destroyed during the process of exchange; and where the nonnegativity constraints $c_o \geq 0$, $c_{1a} \geq 0$, and $c_{1b} \geq 0$ can be suppressed in light of the earlier convexity assumptions regarding the contours of U. Recalling that y_o, y_{1a}, and y_{1b} are given, the simultaneous solution of problem (199) yields the equilibrium conditions

$$\left.\begin{aligned} -\frac{\partial c_{1a}}{\partial c_o} &= \frac{1}{\phi_{1a}} \\ -\frac{\partial c_{1b}}{\partial c_o} &= \frac{1}{\phi_{1b}} \end{aligned}\right\} \qquad (201)$$

The left-hand side of each equation in (201) is interpreted as the marginal rate of substitution between a future contingent claim and a present certain claim and the right-hand side of each equation is interpreted as the price of a certain claim in terms of a future contingent claim.[534] Since, by assumption, ϕ_{1a} and ϕ_{1b} are the same for all individuals, equations (201) imply that, in general equilibrium in the markets for contingent claims, all individuals must have the same marginal rate of substitution between any particular pair of goods.

The problem just considered, in which a system of perfect and complete markets for contingent claims was assumed to exist, is a natural generalization of Fisher's theory of intertemporal choice[535] and is formally equivalent to the traditional analysis of consumer choice under certainty. In such a world there is uncertainty as to the future state of the world, but due to the existence of perfect and complete markets, individuals are free to exchange contingent claims so as to minimize the adverse impact of uncertainty, and as a result, there is not the vagueness one normally associates with uncertainty in the real world.

b. Production and Exchange

To generalize the time-state-preference framework of the preceding subsection to include production is easily carried out. To the economy of the preceding subsection are added one or more firms,[536] each having a production function of the form $Q(q_o, q_{1a}, q_{1b}) = 0$,[537] where the q's are negative if the input flow of the generalized commodity exceeds the output flow of the generalized commodity and are positive when the reverse is true. In general equilibrium in the markets for contingent claims, the market value of each firm, W_o^f , is equal to the net present certainty equivalent value of the firm's production set (q_o, q_{1a}, q_{1b}) :

$$W_o^f = q_o + \phi_{1a} q_{1a} + \phi_{1b} q_{1b} \tag{202}$$

The individual's utility function is as before, but his wealth constraint must be modified. Suppose there are F firms and that the individual owns a proportion α_f of the f-th firm. Then,in general equilibrium in the markets for contingent claims, his wealth is expressed by

$$W = y_o + \sum_{f=1}^{F} \alpha_f W_o^f + \phi_{1a} y_{1a} + \phi_{1b} y_{1b} = c_o + \phi_{1a} c_{1a} + \phi_{1b} c_{1b} \ . \tag{203}$$

Each individual tries to maximize his utility subject to (203), which leads to necessary conditions (201).

The conservation relations of the preceding subsection must be modified to take into account production. Since the amount of the generalized commodity available for consumption equals the initial endowment plus the amount produced less the amount used as inputs to production, the conservation relations (200) become

$$\left.\begin{aligned} \Sigma c_o &= \Sigma y_o + \Sigma q_o \\ \Sigma c_{1a} &= \Sigma y_{1a} + \Sigma q_{1a} \\ \Sigma c_{1b} &= \Sigma y_{1b} + \Sigma q_{1b} \end{aligned}\right\} \text{conservation relations} \qquad (204)$$

where, as before, the first two sums in each equation are taken over all individuals and where the third sum is taken over all firms. If, as is usually the case, q_o is negative and q_{1a} and q_{1b} are positive, it is easily seen from (202), (203), and (204) that, quite apart from individual preferences, each firm should attempt to maximize its market value, W_o^f. Regardless of the shapes of the firm's owners' utility functions, maximizing the market value of the firm is in the owners' best interests because that policy maximizes W and, under the assumption of complete markets, provides each owner with the most inclusive opportunity set (i.e. the set of attainable consumption triplets (c_o, c_{1a}, c_{1b})).[538] When markets are incomplete, however, the firm's owners' freedom to exchange contingent claims is restricted, so that the owners' interests might be better served by an investment policy that explicitly takes into account their consumption preferences.[539]

For the problem under consideration the firm seeks to maximize (202) subject to its production function as a constraint. This leads to the following additional equilibrium conditions (i.e.

to the additional necessary conditions for an optimum):

$$\left.\begin{aligned} -\frac{\partial q_{1a}}{\partial q_o} &= \frac{1}{\phi_{1a}} \\[2em] -\frac{\partial q_{1b}}{\partial q_o} &= \frac{1}{\phi_{1b}} \end{aligned}\right\} \qquad (205)$$

Equations (205) are interpreted to mean that, in general equilibrium in the markets for contingent claims, the marginal rate of technical substitution between a future contingent claim and a present certain claim must equal the price of the certain claim in terms of the future contingent claim.[540] Moreover, equations (205) imply that, for any particular pair of claims, the marginal rate of technical substitution must be the same for all firms.[541]

c. The Indeterminancy of the Firm's Optimal Capital Structure

One of the major sources of disagreement in the area of corporate finance is Modigliani's and Miller's Proposition I, the first part of which states that the total market value of the firm (i.e. the combined market value of its debt and equity) is independent of the firm's capital structure. This result, which was referred to earlier in sections F and I,[542] implies that the firm's investment (and production) decisions are separable from its financing decisions. One of the more interesting applications of the time-state-preference framework is the demonstration that Modigliani's and Miller's Proposition I holds under uncertainty provided markets are perfect and complete. The purpose of this subsection is to demonstrate this result.[543]

Let q_o denote the total of corporate funds to be committed to investment at time $t = 0$ and let d_o denote the portion coming from borrowings (i.e. debt) and e_o the portion coming from equity funds. Then[544]

$$q_o = d_o + e_o \quad . \tag{206}$$

Again assuming a two-period world (present and future) with two future states (a and b), all future returns from the firm's present investments will be distributed among bondholders and shareholders,[545] so that

$$\left.\begin{aligned} q_{1a} &= d_{1a} + e_{1a} \\ q_{1b} &= d_{1b} + e_{1b} \end{aligned}\right\} \tag{207}$$

where d_{1a} is the distribution to bondholders and e_{1a} is the distribution to shareholders at time $t = 1$ if state a obtains and d_{1b} and e_{1b} are interpreted similarly. Equations (207) imply that there are no leakages of investment returns into corporate or personal income taxes or into transaction costs.

If the market for contingent claims is complete, then, in equilibrium, the aggregate present certainty equivalent values of debt, D_o , and equity, E_o , can be expressed in terms of the streams of returns (d_o, d_{1a}, d_{1b}) and (e_o, e_{1a}, e_{1b}) and the equilibrium prices ϕ_{1a} and ϕ_{1b} as follows:[546]

$$\left.\begin{aligned} D_o &= d_o + \phi_{1a} d_{1a} + \phi_{1b} d_{1b} \\ E_o &= e_o + \phi_{1a} e_{1a} + \phi_{1b} e_{1b} \end{aligned}\right\} \tag{208}$$

From equation (202), the market value of the firm, W_o^f, is given by $W_o^f = q_o + \phi_{1a}q_{1a} + \phi_{1b}q_{1b}$. But when markets are perfect and complete, equations (206), (207), and (208) imply that

$$W_o^f = D_o + E_o , \tag{209}$$

which is Modigliani's and Miller's Proposition I. Given the firm's investment policy, its total market value is independent of how it finances that investment. The firm's investment and financing decisions are separable; the market's valuation of the firm is based on the firm's investment policy; and there is no single optimum capital structure (i.e. debt-equity mix). In a world of perfect and complete markets financing operations take place within a wealth constraint — they do not alter wealth.

The simplicity with which the indeterminancy result (209) was obtained should not obscure the significance of the assumption that a set of perfect and complete markets for contingent claims exists.[547] The absence of corporate and personal income taxes, transaction costs, and other external drains — which is assured when markets are perfect — ensures that individuals and firms collectively form a closed system so that (206) and (207) hold.[548] The completeness of markets, which underlies (208), permits each stream of contingent claims to be expressed as a single market-determined present certainty equivalent value. The existence of complete markets makes the market value of each firm insensitive to changes in the firm's capital structure. In the event a change in some firm's capital structure were to cause one or more of the markets for contingent claims to go into disequilibrium, the arbitrage mechanism would ensure that the disequilibrium was only temporary and would restore the equality between the sum of the market values of each firm's securities and the present certainty equivalent value of the firm's income stream.[549] When markets are incomplete (209) no longer holds necessarily because the equilibrating action of the arbitrage mechanism is impeded.

Having briefly set out the distinguishing features of the time-state-preference framework in this subsection, the next three subsections describe several models of the behavior of the firm under uncertainty. The first two focus on the individual firm in the short run in a partial equilibrium setting, while the third explores the link between the firm's production decisions and its stock market value.

2. The Behavior of the Firm Facing Uncertain Demand

In the traditional theory of the firm, which was discussed earlier in this chapter, it is assumed that the firm behaves as if its demand function, production function, and factor costs are known with certainty. Even if the true demand relation is not known with certainty, it is still assumed that the firm can formulate a best estimate of the demand relation, which it then treats as if it were nonstochastic. This treatment implies that once price or quantity is set by the firm, the value of the other variable is known with certainty. In the real world, of course, there may be a high degree of uncertainty as to the true relationship between price and quantity demanded, and it is reasonable to expect that the existence of such uncertainty will have an impact on the behavior of the firm.

In recent years there have been several attempts to incorporate uncertainty into the model of the firm by making demand random.[550] Typically these models have given random demand a specific functional form by introducing a random variable that is either additive or multiplicative into the demand relation.[551] One of the advantages of the model due to Leland discussed in this subsection is its more general formulation of random demand.

Introducing randomness into the demand relation necessitates a change in the firm's objective function. According to the traditional models of the firm, the objective of the firm under certainty is to maximize (single period) total profit. But under uncertainty, either price or quantity (or possibly both) is (are) stochastic, and so are total revenue and total profit. If risk is ignored, or if risk neutrality is assumed, then the objective function of the firm in the traditional models can be reformulated in terms of expected profit.[552] Since both managers and investors are generally regarded as risk averse,[553] it is more appropriate to caste the reformulation in terms of the expected utility of total profit,[554] since the expected utility calculation takes the attitude toward risk into account. The Leland model discussed below assumes the firm maximizes the expected utility of total profit.[555]

A third consideration is whether firms set market price or the quantity of output to be sold.[556] Uncertainty is usually introduced into the model of the firm under perfect competition by assuming that price is a random variable and that the firm can sell whatever quantity it wishes at the prevailing market price, and in the more general case of imperfect competition, by assuming that the firm makes its output decision prior to observing the market price.[557] The Leland paper discussed below considers the implications not only of this quantity-setting behavior but of price-setting behavior as well.[558]

a. Stochastic Demand[559]

Assume the firm produces a single output q, that it knows its total cost function with certainty, and that its objective is to maximize the expected utility of total profit. Demand is uncertain. Following Leland, uncertainty is introduced into the implicit demand relation $f(p,q) = 0$, where p is market price, by assuming that the implicit demand relation itself is random:

$$f(p,q,u) = 0 \ , \tag{210}$$

where u is not known at the time the firm sets quantity (or price), but has subjective probability density $dF(u)$. It is required that, for any value of u, p and q be inversely related in accordance with economic theory, and that larger values of u be associated with greater demand. The variable u represents the state of demand and larger values of u imply a more favorable state of market demand. With these restrictions (210) can be reexpressed in either of two forms:

$$p = p(q,u) \ , \quad \partial p/\partial q < 0 \ , \quad \partial p/\partial u > 0 \tag{211}$$

$$q = q(p,u) \ , \quad \partial q/\partial p < 0 \ , \quad \partial q/\partial u > 0 \ , \tag{212}$$

where in each case the distribution of the variable on the left-hand side of the equation can be obtained from the distribution of u, but is conditional on the value selected by the firm for the decision variable on the right.

b. The Quantity-Setting Firm

In this subsection the firm facing random demand must determine its quantity of output prior to observing the actual market price.[560] The probability distribution of price is conditional on the value of q selected, as implied by (211). The model of the firm is formulated as the following unconstrained optimization problem:

$$\underset{\{q\}}{\text{maximize}}\ E\,[U(\pi)]\ , \tag{213}$$

where $U(\pi)$ is the utility of total profit and where total profit π is given by

$$\pi = p(q,u)\cdot q - C(q) - F\ , \tag{214}$$

where $C(q)$ is total variable cost and F is fixed cost. Substituting (214) into (213) and differentiating $E[U(\pi)]$ with respect to q and setting this derivative equal to zero yields the necessary condition for an optimum:

$$E[(\partial\pi/\partial q)U'(\pi)] = E\{[MR(q,u) - MC(q)]U'(\pi)\} = 0\ , \tag{215}$$

where $MR(q,u) = p(q,u) + q[\partial p(q,u)/\partial q]$ and $MC(q) = C'(q)$. The sufficient condition for a solution to (213) is:

$$E[(\partial^2\pi/\partial q^2)\, U'(\pi) + U''(\pi) \cdot (\partial\pi/\partial q)^2] < 0 \ . \qquad (216)$$

It follows that when either (i) π is strictly concave in q for all u , so that $\partial^2\pi/\partial q^2 < 0$, and the firm is risk neutral or risk averse,[561] so that $U''(\pi) \leq 0$, or (ii) π is linear in q and the firm is risk averse $(U''(\pi) < 0)$,[562] (216) holds for all q so that a solution to (215) provides the global maximum of expected utility.

To explore the effect of uncertainty on the optimal output of the quantity-setting firm it is necessary to employ a comparative statics method of analysis. For this purpose, first construct a certainty demand curve equivalent to the random demand curve (210). Following Leland,[563] the certainty demand curve used below is the *expected price certainty demand curve* formed by associating with each output level, q , the expected price:[564]

$$p = E[p(q,u)] = f(q) \ . \qquad (217)$$

It follows from the linearity of the expectation and differentiation operators that for all q the expected marginal revenue, $E[MR(q,u)]$, is equal to the marginal revenue derived from the expected price certainty demand curve:

$$\begin{aligned} d[q \cdot f(q)]/dq &= f(q) + q[df(q)/dq] \\ &= E[p(q,u)] + q[\partial E[p(q,u)]/\partial q] \qquad (218) \end{aligned}$$

$$d[q \cdot f(q)]/dq = E[p(q,u)] + q[\partial p(q,u)/\partial q]$$

$$= E[MR(q,u)] \ .$$

It follows from what Leland calls the *principle of increasing uncertainty*[565] that a function $u^1(q)$ can be found such that

$$E[MR(q,u)] = MR[q,u^1(q)] \ . \qquad (219)$$

When profit is nonrandom, maximizing total profit is equivalent to maximizing the expected utility of total profit (provided, of course, nonsatiability is assumed). The profit maximizing firm selects output level q_c satisfying $\frac{d\pi}{dq} = \frac{d}{dq}[q \cdot f(q) - C(q) - F] = 0$, which by making use of (218) and (219) can be simplified to

$$MR(q_c, u^1_c) = MC(q_c) \ , \qquad (220)$$

where $u^1_c = u^1(q_c)$.

When demand is stochastic and u varies about $u^1(q)$ for all q , there are three possibilities:

(i) if q_c satisfies (215), then the introduction of uncertainty leaves the firm's optimal output unchanged;

(ii) if (215) is negative when $q = q_c$, but (216) is satisfied everywhere, then optimal output under uncertainty will be smaller than q_c , the optimal output under certainty; and

(iii) if (215) is positive when $q = q_c$ and (216) is satisfied everywhere, then output under uncertainty will exceed q_c .

Which of these possibilities will hold will depend on the attitude of the firm toward risk. Since firms would normally be expected to exhibit risk aversion,[566] the principle of increasing uncertainty would imply that the risk averse firm will produce less under uncertainty than it would under certainty (i.e. (ii) holds). This is demonstrated below.

Risk aversion implies that

$$\partial[U'(\pi)]/\partial u = U''(\pi) \cdot (\partial\pi/\partial u) < 0 ,$$

which implies that

$$U'(\pi) < U'(\pi^1) , \quad \text{for} \quad u > u^1_c , \tag{221}$$

where π^1 is total profit when $u = u^1_c$. From the principle of increasing uncertainty and (220),

$$MR(q_c,u) > MC(q_c) , \quad \text{for} \quad u > u^1_c . \tag{222}$$

Combining (221) and (222) yields

$$[MR(q_c,u) - MC(q_c)] \cdot U'(\pi) < [MR(q_c,u) - MC(q_c)] \cdot U'(\pi^1) , \tag{223}$$

for $u > u^1_c$. But when $u < u^1_c$ both inequalities (221) and (222) are reversed so that (223) continues to hold. Hence, (223) holds for all u and

$$E\{[MR(q_c,u) - MC(q_c)] \cdot U'(\pi)\} < U'(\pi^1) \cdot E[MR(q_c,u) - MC(q_c)] = 0 \tag{224}$$

where the equality on the right follows from (219) and (220). Thus, for q_a satisfying the necessary condition (215) for an optimum under uncertainty, it must be the case that $q_a < q_c$. Given the principle of increasing uncertainty, in the presence of uncertainty the risk averse firm will produce less than it would under certainty.[567] Under risk preference the opposite is true; inequalities (221) and (224) are reversed and the risk preferring firm will produce more under uncertainty than it would under certainty. In the intermediate case of risk neutrality, the existence of uncertainty will not affect the firm's output decision.

In the traditional theory of the firm, the firm's output decision in the short run is independent of the level of fixed costs. Under certainty a change in fixed costs would not alter optimal output (or price).[568] This is generally not true under uncertainty.[569] When the firm is risk averse and when absolute risk aversion[570] diminishes as total profit increases, increased fixed costs lead to smaller output.[571] Similarly, if absolute risk aversion increases with increasing profit, then greater fixed costs lead to greater output, and if absolute risk aversion is invariant with respect to total profit, then the level of fixed costs will not affect the firm's output decision. Intuitively, for any level of total revenue, higher fixed costs reduce total profit. Under diminishing absolute risk aversion (with increasing profit), the lower level of profit implies a relatively greater degree of absolute risk aversion. The firm compensates for this by setting lower output in order to experience a lower level of risk (i.e. dispersion of the probability distribution of total revenue).[572] Under increasing absolute risk aversion, the lower level of profit implies a relatively lower degree of absolute risk aversion. In effect, the firm is able to bear greater risk, and does so by raising its output level. In the intermediate case, the degree of absolute risk aversion does not change, and as a consequence, nor does the firm's output level.

The implication of the traditional theory with regard to the effect on output of an increase in demand must also be modified when uncertainty is introduced. This can be shown by utilizing comparative statics. Following Leland, an increase in demand will be defined as a parallel upward shift in the demand curve by a constant amount a for every state of nature.[573] Such a shift alters the expected value, but not the shape, of the conditional probability distribution of price.

The new demand curve is $p'(q,u) = p(q,u) + a$ and the first order condition (215) becomes

$$E\{[MR(q,u) + a - MC(q)] \cdot U'(\pi + qa)\} = 0 , \tag{225}$$

where π is defined by (214) and $MR(q,u) + a$ is the new expression for marginal revenue. To find the effect on q of a small change in a, take the total derivative of (225) and evaluate it at $q = q_a$ and $a = 0$ (i.e. optimal output for the initial demand curve). This gives

$$dq/da = - E[U'(\pi) + q(MR - MC)\ U''(\pi)]/D , \tag{226}$$

where $D = \partial[E(MR - MC)U'(\pi)]/\partial q < 0$ at $q = q_a$ and $\alpha = 0$ from (216). The overall effect of a shift in demand may be broken down into the sum of two effects. The first is $-E[U'(\pi)]/D$, which is always positive since $E[U'(\pi)] > 0$ and $D < 0$, and which may be called the "revenue-substitution effect."[574] As demand shifts upward, the increase in expected marginal revenue at $q = q_a$, with marginal cost unaffected, induces the firm to expand output in order to increase total profit. In the certainty case, the increase in marginal revenue, with marginal cost unchanged, has exactly this effect.

Under uncertainty there is a second effect, which is absent under certainty. The term $-q \cdot E[(MR - MC)U''(\pi)]/D = -q \cdot dq/dF$, is positive, zero, or negative, depending on whether absolute risk aversion is decreasing, constant, or increasing (given the principal of increasing uncertainty).[575] Leland calls this the "risk-income effect."[576] The importance of this effect is explained as follows. Under decreasing absolute risk aversion (i.e. the "normal" case) the risk-income effect would reinforce the revenue-substitution effect, thereby causing an even greater increase in output. Yet, it is possible that a firm could become increasingly risk averse as total profit rose to such an extent that the risk-income effect would outweigh the revenue-substitution effect, thereby causing output *to decrease* as demand increases.[577]

The next subsection discusses Leland's model of the price-setting firm under uncertainty. To facilitate a comparison of the model of the price-setting firm with the model of the quantity-setting firm, the main features of the latter model are summarized in table II-24.

Table II-24 Summary of Leland's Model of the Quantity-Setting Firm

Class:	modern traditional (see (213), (214) in text)
Firm's Objective:	maximize the expected utility of total profit
Constraints:	product demand relation ($p = p(q,u)$); implicitly factor supply and technological conditions embodied in $C(q)$
Variables:	
Exogenous:	level of fixed costs (F)
Endogenous:	quantity of output (q), price (p), and total profit (π)
Decision:	quantity of output (q)
Parameters:	level of fixed costs (F) and demand shift parameter (a)
Finance:	subsumed
Certainty/Uncertainty:	allows for uncertainty and utilizes the time-state-preference framework
Equilibrium/ Disequilibrium:	characterization of the equilibrium position of the firm and utilization of comparative statics to determine the effects of uncertainty on this equilibrium position
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (by substituting (214) into (213))

c. The Price-Setting Firm

For the firm that chooses to set price when demand is uncertain, two cases may be distinguished. First, the firm may set price and then adjust quantity to meet actual demand.[578] This case is discussed below. Second, the firm may fix both price and quantity, and if quantity demanded is greater (less) than quantity supplied, there is a shortage (surplus). This case, first discussed by Mills and further explored by Leland and Meyer,[579] is left to the next subsection. In the present section the first case is examined and it is shown that, while the choice of control variable (p or q) is immaterial under certainty, it is of considerable importance under uncertainty.

The firm sets price and then observes quantity demanded, the distribution of which is conditional upon the price p selected, as suggested by (212). Total profit is given by

$$\pi = p \cdot q(p,u) - C[q(p,u)] - F , \qquad (227)$$

which differs from the expression for profit (214) for the quantity-setting firm in that the randomness of q causes total variable cost to become random.[580] Maximizing the expected utility of profit, where profit π is given by (227), yields the following first- and second-order conditions:

$$E[(\partial\pi/\partial p)U'(\pi)] = E\{[MR(p,u) - C'[q(p,u) \frac{\partial q(p,u)}{\partial p}]U'(\pi)\} = 0 , \qquad (228)$$

where

$$MR(p,u) = q(p,u) + p[\partial q(p,u)/\partial p] ,$$

and

$$\frac{\partial}{\partial p} [E[(\partial\pi/\partial p)U'(\pi)]] < 0 . \qquad (229)$$

To explore the effect of uncertainty on optimal price it is necessary once again to utilize comparative statics, and as in the preceding subsection, to begin by defining an appropriate certainty demand curve. The obvious choice is the *expected quantity certainty demand curve*,

$$q = E[q(p,u)] = h(p) , \tag{230}$$

which is analogous to (217) in that with each price p is associated the expected quantity demanded. Under certainty the price-setting firm will select price $p = p_c$ satisfying

$$\left.\frac{\partial\pi}{\partial p}\right|_{p = p_c} = h(p_c) + p_c[dh(p_c)/dp] - C'[h(p_c)][dh(p_c)/dp] = 0 . \tag{231}$$

To evaluate the effect of uncertainty on optimal price it is necessary to evaluate the left-hand side of (228). If (229) is satisfied for all p, then the optimal price will be greater than, equal to, or less than p_c as (228) evaluated at $p = p_c$ is positive, zero, or negative. As in the case of the quantity-setting firm, the firm's attitude toward risk plays a crucial role in determining the effect of uncertainty on optimal price. But, in contrast with the quantity-setting firm, the shape of $C(q)$ will also have a bearing on how uncertainty affects the selection of optimal price.

Under risk neutrality with constant marginal cost, uncertainty has no effect on the price-setting firm's selection of p.[581] But when marginal cost is nonconstant, the effect of uncertainty is no longer zero — even if the firm is risk neutral.[582]

Under risk aversion the effect of uncertainty on optimal p depends critically on how the uncertainty of profit changes in response to a change in price, or in symbols, on how $\partial\pi(p,u)/\partial p$ in (228) behaves as a function of u. Results are readily obtainable if $\partial\pi(p,u)/\partial p$ is either invariant or changing monotonically (either increasing or decreasing) with respect to u.[583] For example, if it is monotonically increasing and if p_n is the optimal price charged by the risk neutral firm, then it can be shown that[584] $E\{U'(\pi)[\partial\pi(p_n,u)/\partial p]\} < 0$, which implies that the risk averse firm will charge a lower price than the risk neutral firm. Similarly, if $\partial\pi(p,u)/\partial p$ is independent of u, changing the degree of risk aversion will not cause the optimal price to change, while if $\partial\pi(p,u)/\partial p$ is monotonically decreasing with u, the risk averse firm charges a higher price than the risk neutral firm. Since there are no a priori grounds on which to base a conjecture on the sign of $\partial[\partial\pi(p_n,u)/\partial p]/\partial u$, the analysis is inconclusive.[585] Thus, since changes in cost uncertainty may or may not offset changes in total revenue uncertainty, it is impossible to say just how the uncertainty of profit changes in response to changes in price. As a consequence, the principle of increasing uncertainty, which was invoked in the previous subsection to draw definite conclusions regarding the effect of uncertainty on the optimal output of quantity-setting firms, is alone insufficient for deriving comparative conclusions for price-setting firms because of the random nature of these firms' costs.[586]

The distinguishing characteristics of Leland's model of the quantity-setting firm under uncertainty are summarized in table II-25.

Table II-25 Summary of Leland's Model of the Price-Setting Firm

Class:	modern traditional (see (213), (227) in text)
Firm's Objective:	maximize the expected utility of total profit
Constraints:	product demand relation ($q = q(p,u)$); implicitly factor supply and technological conditions embodied in $C(q)$
Variables:	
Exogenous:	level of fixed costs (F)
Endogenous:	quantity of output (q), price (p), and total profit (π)
Decision:	price (p)
Parameters:	level of fixed costs (F) could be varied as in the model of the quantity-setting firm
Finance:	subsumed
Certainty/Uncertainty:	allows for uncertainty and utilizes the time-state-preference framework
Equilibrium/ Disequilibrium:	characterization of the equilibrium position of the firm and utilization of comparative statics to determine the effects of uncertainty on this equilibrium position
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	unconstrained optimization (by substituting (227) into (213))

d. Summary

The Leland model demonstrates that, in contrast to the certainty model of traditional theory, the firm's selection of price and quantity under uncertainty is generally not insensitive to changes in fixed costs and the firm will, in general, behave somewhat differently according to whether it sets quantity or price (or both) prior to observing market demand.[587] In addition, due to risk aversion the quantity-setting firm will (if the principle of increasing uncertainty holds) generally produce less under uncertainty than it would under certainty, but whether the price-setting firm charges a higher or lower price will depend on how the uncertainty of profit changes as price changes.

The Leland models just discussed are single period models of the single product firm. Neither the restriction to a single period nor the restriction to a single product is a serious limitation, however. Zabel has developed a multiperiod model of the single product firm under uncertainty,[588] and several models of the multiproduct firm under uncertainty, including models due to Dhrymes and Meyer,[589] have appeared in the literature. The Meyer model is set out in the next subsection. By focusing their attention on the firm's pricing and output decisions, the Leland models discussed above subsume first, the firm's input decisions, and second, finance and the role of the firm's shareholders. Holthausen and others have studied the impact of uncertainty on the firm's input choices.[590] Holthausen found that, under risk aversion, the price-setting imperfect competitor employs a less-than-efficient expected capital-labor ratio and that increasing risk aversion tends to decrease the firm's capital stock.[591] The interaction between the stock market and the firm's production decisions has also been studied recently. The final model outlined in this section is a second model due to Leland that incorporates the stock market.

3. The Risk-Efficient Multiproduct Firm

In the previous subsection the firm produced a single output and set either price or quantity, but not both, prior to observing actual market demand. In this subsection the firm produces n goods and determines the price and quantity of each to be supplied to the market before actual demand is known. First, the basic model is developed for uncorrelated demands, and then the model is adapted to allow for correlations.

a. The Meyer Model[592]

A firm produces n goods, the amounts of which are denoted by $q_1, q_2, \ldots, q_n$.[593] In each of the product markets the firm enjoys a monopoly, so that the market demand curve for the i-th good, which will be denoted by $D_i = D_i(p_1, \ldots, p_n, u_i)$, $i = 1, \ldots, n$, is also the firm's demand curve for the i-th good, where D_i is the quantity demanded of the i-th good, p_i is the market price of the i-th good set by the firm, and u_i is the state of nature within the market for the i-th good.[594] Given the prices $p_1, \ldots, p_n$, quantity demanded has mean $E[D_i] = D_i^*$ and variance $E[(D_i - D_i^*)^2] = \sigma_i^2$, both of which are assumed finite. For comparison with riskless settings, the expected quantity certainty demand curve,

$$D_i^* = E[D_i(p_1, \ldots, p_n, u)] = h(p_1, \ldots, p_n), \quad (232)$$

which is analogous to (230) for the Leland model, is used. It is assumed that the firm's total cost function, which is denoted by $C(q_1, \ldots, q_n, Q)$, where Q represents capacity, is known with certainty.

The use of the capacity variable Q needs further explanation. In the peak-load pricing problem q_i units of output, say kilowatts of electricity, are sold each period. The firm (i.e. electric power utility) has a fixed capacity of Q kilowatts per period, so that there are n production constraints, $q_i \leq Q$, $i = 1 , \ldots, n$.[595] For the case of the discriminating monopolist, again say an electric utility, but this time selling to n different groups of customers in one period, there is the production constraint $\sum_{i=1}^{n} q_i \leq Q$.[596] In each case Q is a physical constraint on the number of units that may be produced. This treatment of capacity differs from the more conventional treatment of the firm in the short run in which the firm is always free to increase output by using additional amounts of the variable inputs and in which the law of diminishing returns ensures that with fixed capital stock the short run cost curves eventually become progressively steeper as output is increased.[597] This steepening and the conventional $MR = MC$ rule for profit maximization make it unnecessary to consider explicitly physical constraints on capacity. In many industries, such as electric power generation, marginal costs may rise very slowly, or possibly even remain constant or decline as capacity is approached,[598] so that the physical constraint on capacity must be stated explicitly. In keeping with Meyer's approach, the outputs $q_1 , \ldots , q_n$ must satisfy the capacity constraint[599]

$$g(q_1 , \ldots , q_n) \leq Q , \tag{233}$$

where g is a continuously differentiable function of the q_i's , which are, as usual, restricted to be nonnegative. When there is strict inequality in (233) there is said to be 'spare capacity'.

In addition to (233) and the nonnegativity constraints on the p_i's , q_i's , and Q , there are n chance constraints that relate the quantity supplied of each good to the random quantity demanded. In the certainty case the riskless constraint $D_i \leq q_i$ ensures that all demands are satisfied. For the case of the discriminating monopolist, at least, this also ensures that the market is in equilibrium (i.e. quantity demanded $\equiv$ quantity supplied at the prevailing price) when the monopolist is in equilibrium. But under uncertainty such a constraint is not meaningful since the actual quantity demanded, D_i , is a random variable. The most direct generalization of the riskless constraint is[600]

$$\text{Prob}\ \{D_i > q_i\} \leq \varepsilon_i \ , \quad i = 1 , \ldots , n \ , \tag{234}$$

which states that the probability that quantity demanded exceeds quantity supplied cannot exceed ε_i , where $0 < \varepsilon_i < 1$. For the case of the discriminating monopolist, the probabilistic constraint (234) implies that, when the monopolist is in equilibrium, the market may not be since quantity demanded can exceed quantity supplied (at the prevailing price). It will be simpler in what follows to be able to work with the riskless equivalent of (234):

$$q_i \geq D_i^* + N_i\sigma_i \ , \tag{235}$$

where N_i is the number of standard deviations above the mean demand just needed to satisfy (234).[601]

The objective of the firm's managers is to maximize the total market value of the firm,[602]

$$V = \frac{1}{\rho}\{E[\pi] - R\beta\sigma_\pi\} \quad , \tag{236}$$

where ρ is the riskless rate of interest, $E[\pi]$ is expected total profit, R is the market-determined price of risk, and $\beta\sigma_\pi$ gives risk as the product of the standard deviation of the firm's total profit, σ_π, times the coefficient of correlation, β, relating the firm's total profit to overall market returns (i.e. the returns on a portfolio made up of all firms' securities).[603] In (236) total profit, π, can be expressed as

$$\pi = \sum_{i=1}^{n} p_i D_i - C(q_1, \ldots, q_n, Q) \quad , \tag{237}$$

provided it is assumed that ε_i in (234) has been chosen small enough so that the number of lost sales is insignificant.[604] When ε_i is chosen in this manner, the probability that the market for the i-th good will be in disequilibrium when the firm is in equilibrium is made small. If it is assumed that the demands for the different goods are uncorrelated, then, from (237), the standard deviation of total profit is

$$\sigma_\pi = \left(\sum_{i=1}^{n} p_i^2 \sigma_i^2\right)^{1/2} \quad . \tag{238}$$

Substituting (237) and (238) into (236) and evaluating $E[\pi]$ in terms of $E[D_i] = D_i^*$ gives the new objective function

$$V = \frac{1}{\rho}\left[\sum_{i=1}^{n} p_i D_i^* - C(q_1, \ldots, q_n, Q) - R\beta\left\{\sum_{i=1}^{n} p_i^2 \sigma_i^2\right\}^{\frac{1}{2}}\right]. \tag{239}$$

To maximize (238) the firm must select optimal values for the p_i's , q_i's , and Q from among those values satisfying (232), (234), and the nonnegativity constraints. The model of the firm can be formulated as the following mathematical programming problem:

$$\underset{\{p_i, q_i, Q\}}{\text{maximize:}} \quad V = \frac{1}{\rho}\left[\sum_{i=1}^{n} p_i D_i^* - C(q_1, \ldots, q_n, Q) - R\beta\left\{\sum_{i=1}^{n} p_i^2 \sigma_i^2\right\}^{\frac{1}{2}}\right]$$

$$\text{subject to:} \quad q_i \geq D_i^* + N_i \sigma_i \ , \quad i = 1, \ldots, n \tag{240}$$

$$g(q_1, \ldots, q_n) \leq Q$$

$$p_i \geq 0 \ , \quad q_i \geq 0 \ , \quad i = 1, \ldots, n \ ; \quad Q \geq 0 \ .$$

The necessary conditions for an optimal solution to (240) are, in addition to (233) and (235), the following:[605]

$$\frac{\partial L_\lambda}{\partial p_i} = \frac{1}{\rho}\left[p_i \frac{dD_i^*}{dp_i} + D_i^* - R\beta\left\{\sum_{i=1}^{n} p_i^2 \sigma_i^2\right\}^{-\frac{1}{2}} \cdot p_i \sigma_i^2\right] - \mu_i \frac{dD_i^*}{dp_i} = 0$$

$$i = 1, \ldots, n \tag{241}$$

$$\frac{\partial L_\lambda}{\partial q_i} = -\frac{1}{\rho}\frac{\partial C}{\partial q_i} - \lambda \frac{\partial g}{\partial q_i} + \mu_i = 0 \ , \quad i = 1, \ldots, n \tag{242}$$

$$\frac{\partial L_\lambda}{\partial Q} = -\frac{1}{\rho}\frac{\partial C}{\partial Q} + \lambda = 0 \tag{243}$$

These necessary conditions can be used collectively to characterize the equilibrium position of the discriminating monopolist.

Optimal capacity investment, Q, is characterized by (243), which can be rewritten as follows:

$$\lambda = \frac{1}{\rho} \frac{\partial C}{\partial Q} . \tag{244}$$

The right-hand side of (244) is interpreted as the marginal cost of capacity, capitalized at the riskless rate of interest, and the left-hand side is interpreted as the 'risk-adjusted' quasi-rent on capacity,[606] measured as the instantaneous rate of change of the total market value of the firm with respect to an increase in capacity. Thus, condition (244) is just the familiar equilibrium condition equating marginal value to marginal cost, this time applied to investment in capacity under uncertainty.

The firm's optimal pricing policy is suggested by (241). By dividing each side of (241) by dD_i^*/dp_i and rearranging terms, (241) becomes

$$\mu_i = \frac{1}{\rho}\left[MR_i^* - \frac{dp_i}{dD_i^*} \frac{R\beta p_i \sigma_i^2}{\sigma_\pi}\right] , \quad i = 1 , \ldots, n , \tag{245}$$

where $MR_i^* = p_i + D_i^* (dp_i/dD_i^*)$ is the equilibrium value of riskless marginal revenue[607] and σ_π is given by (238). The expression in brackets is interpreted as risky marginal revenue, which is equal to riskless marginal revenue plus a marginal risk adjustment, $-\frac{dp_i}{dD_i^*} \frac{R\beta p_i \sigma_i^2}{\sigma_\pi}$. Equation (245) states that at optimality the Lagrange multiplier μ_i gives the capitalized value of risky marginal revenue for the i-th good.[608] Note that since R, p_i, σ_i^2, and σ_π are all nonnegative in (245), while dp_i/dD_i^* is negative and β may be either positive or negative (or zero), the sign of β determines the sign of the marginal risk adjustment. The significance of the sign of β is discussed below.

Optimal production policy is defined by (242). After using (244) to substitute for λ, equation (242) yields the equilibrium condition

$$\mu_i = \frac{1}{\rho}\frac{\partial C}{\partial q_i} + \frac{1}{\rho}\frac{\partial C}{\partial Q}\frac{\partial Q}{\partial q_i} \quad , \quad i = 1 , \ldots , n . \tag{246}$$

The right-hand side of (246) is interpreted as capitalized marginal production cost, $\frac{1}{\rho}\frac{\partial C}{\partial q_i}$, plus capitalized marginal capacity cost, $\frac{1}{\rho}\frac{\partial C}{\partial Q}\frac{\partial Q}{\partial q_i}$, and the left-hand side, as in (245), is interpreted as the capitalized value of risky marginal revenue. According to equation (246), given optimal prices and capacity, the firm should continue to expand the output of each good up to the point at which the capitalized value of risky marginal revenue just equals capitalized marginal production cost plus capitalized marginal capacity cost.[609]

How the existence of uncertainty affects the behavior of the firm is largely determined by β. When the profits of the firm are uncorrelated with overall market returns, so that $\beta = 0$, the marginal risk adjustment term in (245) disappears and (245) and (246) together yield the equilibrium condition under certainty,[610]

$$MR_i^* = \frac{\partial C}{\partial q_i} + \frac{\partial C}{\partial Q} \cdot \frac{\partial Q}{\partial q_i} \quad , \quad i = 1 , \ldots , n . \tag{247}$$

Provided the firm chooses ε_i small enough to ensure that $D_i{}^*$ is the average quantity sold, the behavior of the firm is not otherwise affected by the presence of uncertainty.[611] This is not true when $\beta \neq 0$. If the firm's profits are positively correlated with overall market returns (i.e. profits vary cyclically), then $\beta > 0$, risky marginal revenue exceeds riskless marginal revenue at each level of output, and it follows that, for given output levels q_i, the same effective demand for capacity would require lower prices under uncertainty than would be required under certainty. When profits are negatively correlated with overall market returns (i.e. profits vary countercyclically), then $\beta < 0$ and the opposite is true.

The next subsection generalizes the model just discussed to permit nonzero correlations between demands. By way of summarizing the discussion thus far, the distinguishing features of Meyer's model of the firm when demands are uncorrelated are presented in table II-26.

b. The Problem of Correlation Between Demands

Thus far in the discussion it has been assumed that demands are uncorrelated. This assumption permitted σ_π in (238) to take on a particularly simple form. To allow for nonzero correlations between demands requires two fundamental changes in the model (240). This subsection describes the needed changes.

First, the expression for σ_π, the standard deviation of the firm's total profit, in the valuation equation (239) must be modified. The basic valuation equation is still (236) and total profits, π, are

Table II-26 Summary of Meyer's Model (with Uncorrelated Demands)

Class:	modern traditional (see (240) in text)
Firm's Objective:	maximize the total market value of the firm
Constraints:	market demand conditions (as embodied in $D_i = D_i(p_1, \ldots, p_n, u_i)$), probabilistic constraint on proportion of unsatisfied demands (as embodied in $q_i \geq D_i^* + N_i\sigma_i$), and capacity constraint (as embodied in $g(q_1,\ldots,q_n) \leq Q$); and implicitly, technological and factor supply conditions (as embodied in $C(q_1,\ldots,q_n,Q)$)
Variables:	
Exogenous:	maximum proportion of demands for each good that go unsatisfied (ε_i) and hence the number of standard deviations (N_i) (of quantity supplied above expected quantity demanded), market price of risk (R), correlation coefficient (β), and the variance of quantity demanded for each good (σ_i^2)
Endogenous:	total market value of the firm (V), price of each good (p_i), quantity supplied of each good (q_i), capacity (Q), and expected quantity demanded of each good (D_i^*)
Decision:	price of each good (p_i), quantity supplied of each good (q_i), and capacity (Q)
Finance:	subsumed
Certainty/Uncertainty:	permits uncertainty and utilizes the mean-variance framework
Equilibrium/Disequilibrium:	characterization of the equilibrium position of the firm, while permitting one or more product markets served by the firm to be in disequilibrium
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	classical Lagrange multipliers (or generalized Lagrange multipliers if one or more inequality constraints are to be permitted to be non-binding at optimality — see footnotes 605 and 609).

still given by (237), but when demands are correlated the standard deviation of total profit is

$$\sigma_\pi = \left[\sum_{i=1}^{n} p_i^2 \sigma_i^2 + 2 \sum_{i>j} p_i p_j \sigma_{ij} \right]^{1/2} , \qquad (248)$$

where $\sigma_{ij} = E[(D_i - D_i^*)\ (D_j - D_j^*)]$ is the covariance between quantities demanded of the i-th and j-th goods.

Second, the character of random demand is more complex due to the correlations. The optimum output levels cannot be determined separately. The interactions among quantities demanded require that the n separate constraints given by (234) be replaced by the constraint

$$\text{Prob}\ \{D > q\} \leq \varepsilon , \qquad (249)$$

where $D = (D_1 , \ldots , D_n)$ is a random n-vector of quantities demanded, $q = (q_1 , \ldots , q_n)$ is a nonrandom n-vector of quantities produced, and $0 < \varepsilon < 1$. Letting $D^* = (D_1^* , \ldots , D_n^*)$ and assuming that $\Omega = E[(D - D^*)\ (D - D^*)^t]$ is nonsingular and positive definite,[612] the constraint (249) may be expressed as $\text{Prob}\ \{\Omega^{-\frac{1}{2}}(D - D^*) > \Omega^{-\frac{1}{2}}(q - D^*)\} \leq \varepsilon$, and if N is a n-vector of nonnegative constants such that the constraint is satisfied with $\Omega^{-\frac{1}{2}}(q - D^*) \geq N$, then the constraint (249) can be reexpressed in the equivalent riskless form[613]

$$q \geq D^* + \Omega^{\frac{1}{2}} N \ . \qquad (250)$$

Given these modifications, the model of the firm can be reformulated as the following mathematical programming problem:

$$\underset{\{p_i, q_i, Q\}}{\text{maximize:}} \quad V = \frac{1}{\rho}\Big[\sum_{i=1}^{n} p_i D_i^* - C(q_1, \ldots, q_n, Q) - R\beta\Big\{\sum_{i=1}^{n} p_i^2 \sigma_i^2 + 2\sum_{i>j} p_i p_j \sigma_{ij}\Big\}^{\frac{1}{2}}\Big]$$

$$\text{subject to:} \quad q_i \geq D_i^* + S_i \,, \quad i = 1, \ldots, n \tag{251}$$

$$g(q_1, \ldots, q_n) \leq Q$$

$$p_i \geq 0 \,, \quad q_i \geq 0 \,, \quad i = 1, \ldots, n \,; \quad Q \geq 0 \,,$$

where S_i is the i-th element of the n-vector $\Omega^{\frac{1}{2}}N$. Forming the Lagrangian[614] and differentiating would yield first order conditions very similar to (241) - (243). Indeed, (242) and (243) are unchanged and (241) is only slightly changed as follows:

$$\frac{\partial L_\lambda}{\partial p_i} = \frac{1}{\rho}\Big[p_i \frac{dD_i^*}{dp_i} + D_i^* - \frac{R\beta(p_i \sigma_i^2 + \sum_{i>j} p_j \sigma_{ij})}{\sigma_\pi}\Big] - \mu_i \frac{dD_i^*}{dp_i} = 0$$

$$i = 1, \ldots, n \tag{252}$$

The presence of correlated demands does not affect the production and investment decision rules, but it does affect the firm's pricing policy. This can be seen more easily by rewriting (252) as

$$\mu_i = \frac{1}{\rho}\Big[MR_i^* - \frac{dp_i}{dD_i^*} R\beta\Big\{\frac{p_i \sigma_i^2 + \sum_{i>j} p_j \sigma_{ij}}{\sigma_\pi}\Big\}\Big] \tag{253}$$

where MR_i^* is once again interpreted as the riskless marginal revenue and μ_i is once again interpreted as the capitalized value of risky marginal revenue.[615] But whereas MR_i^* is unchanged from (245), μ_i has been modified by the replacement of $p_i\sigma_i^2$ with $p_i\sigma_i^2 + \sum_{i>j} p_j\sigma_{ij}$ to reflect the fact that the standard deviation of the firm's profit, which is one component of risk, depends on the correlations between demands.

In a world of certainty the prices charged by a discriminating monopolist must always exceed marginal production cost whenever demand is less than perfectly elastic.[616] This is no longer true under uncertainty. The term in braces in (253) represents the marginal risk contributed by the i-th group of customers. Since prices are nonnegative, the marginal risk contribution can be negative when the correlations σ_{ij} are sufficiently negative. If, as is normally assumed, stock market investors are risk averse, then, for any given level of expected profit, the total market value of the firm will be maximized by selecting prices that minimize total risk.[617] It follows that those groups of customers for which the marginal risk contribution is negative will pay prices that are lower relative to marginal production cost than groups of customers for which the marginal risk contribution is nonnegative. Conceivably, the prices charged one or more groups of customers could lie below marginal production cost — something that could never happen in a riskless setting[618] — and possibly even lie in the range where riskless marginal revenue is negative.[619] The reason for this is that in a risky setting where investors are risk averse it will be in the firm's economic interest to sacrifice some expected profit whenever doing so will enable a reduction in overall risk sufficient to increase the total market value of the firm.

For the multiproduct firm, then, there are two reasons why a product might sell at a price below marginal production cost. The first, strong complementarities among the products, exists whether or not there is uncertainty. The second, a strong negative marginal risk contribution, is a consequence of market equilibrium in a risk averse stock market in an uncertain world.

To summarize the discussion of the Meyer model with nonzero correlations between demands, the distinguishing characteristics of the model are presented in table II-27.

The model discussed in the next subsection takes a much closer look at the link between the stock market and the firm's production decisions. In particular, it will be found that in a risky environment the stock market value of the firm has a crucial direct impact on the firm's output decision, and further, that the maximization of the stock market value of the firm, which was assumed in the Meyer model, does not in general lead to a Pareto optimal choice of outputs.

4. The Role of the Stock Market: Value Maximization Reconsidered

Earlier in this section it was found that when there is a complete set of markets for contingent claims it is in the best interests of every shareholder to have the firm's managers maximize the total market value of the firm. Each firm's time-state-contingent returns are evaluated at the established market prices of time-state-contingent claims, which are equal to the respective marginal rates of substitution for each shareholder, as indicated by the equilibrium conditions (201).

Table II-27 Summary of Meyer's Model (with Correlated Demands)

Class:	modern traditional (see (251) in text)
Firm's Objective:	maximize the total market value of the firm
Constraints:	market demand conditions (as embodied in $D_i = D_i(p_1,\ldots,p_n,u_i)$), probabilistic constraint on proportion of unsatisfied demands (as embodied in $q_i > D_i^* + S_i$), and capacity constraint (as embodied in $g(q_1,\ldots,q_n) \leq Q$); and implicitly, technological and factor supply conditions (as embodied in $C(q_1,\ldots,q_n,Q)$)
Variables:	
Exogenous:	maximum proportion of demands that go unsatisfied (ε) and hence the number of 'standard deviations' (N_i) (of quantity supplied above expected quantity demanded), market price of risk (R), correlation coefficient (β), and variance-covariance matrix (Ω)
Endogenous:	total market value of the firm (V), price of each good (p_i), quantity supplied of each good (q_i), capacity (Q), and expected quantity demanded of each good (D_i^*)
Decision:	price of each good (p_i), quantity supplied of each good (q_i), and capacity (Q)
Finance:	subsumed
Certainty/Uncertainty:	permits uncertainty and utilizes the mean-variance framework
Equilibrium/Disequilibrium:	characterization of the equilibrium position of the firm, while permitting one or more product markets served by the firm to be in disequilibrium
Time:	single period
Type of Model:	static optimization (mathematical programming problem)
Solution Technique:	classical Lagrange multipliers (or generalized Lagrange multipliers if one or more constraints are to be permitted to be nonbinding at optimality).

In such a world all shareholders would unanimously agree on the desirability of any proposed action by the firm that promised to increase the firm's total market value.

When markets are incomplete, shareholders are no longer able to insure against every contingency. Their marginal rates of substitution for time-state-contingent income claims may differ. Consequently, shareholders may disagree on the desirability of a proposed action by the firm because of the decision's effect on the time-state-distribution of returns. In particular, value maximization by the firm will not in general be Pareto optimal for the firm's shareholders,[620] and the firm cannot evaluate proposed investments merely on the basis of what effect they will have on the share price.[621]

In the Leland model discussed below it is shown that there can still be stockholder unanimity with respect to the production and investment decisions of the individual firm, independent of investors' subjective probability distributions and the shapes of their utility functions, provided that the alternative decisions open to the firm would not alter the set of state-distributions of returns available to individuals in the whole economy.[622] It is also shown that in general the output decisions favored by shareholders will not maximize the stock market value of the firm.[623]

Consider an economy in which there are $N + 1$ firms, indexed $j = 0, 1, \ldots, N$.[624] Each firm produces a single good and acts as a quantity-setter, as described by Baron and Sandmo.[625] The firm acts as a price-taker, where price is dependent on the state of nature θ. Cost functions are known with certainty, and the j-th firm's total profit, π^j, is given by

$$\pi^j = p^j(\theta) \cdot q^j - C^j(q^j), \quad j = 0, 1, \ldots, N, \tag{254}$$

where p^j and q^j are price and output, respectively, for the j-th firm. Assume the zeroth firm faces a certain price, so that its total profit is given by $\pi^o = \bar{p}^o \cdot q^o - C^o(q^o)$.[626] Each firm issues shares, which represent claims to profits such that ownership of a fraction s_i^j of the total shares of the firm outstanding entitles the i-th shareholder to total returns $s_i^j \pi^j$. Only equity financing is considered, so that the total market value of the j-th firm's equity, V^j , also represents the total market value of the firm.

Further assume there are M investors, indexed $i = 1 , \ldots , M$. Each investor can be described by the following three attributes:

(i) The i-th investor's current portfolio is represented by the vector $\bar{s}_i = (\bar{s}_i^o , \ldots , \bar{s}_i^N)$, where $\bar{s}_i^j$ is the fraction he owns of the i-th firm and $\sum_{i=1}^{M} \bar{s}_i^j = 1$, $j = 0 , 1 , \ldots , N$.

(ii) The i-th investor has utility function $U_i(R_i,\theta)$, which is unique up to a positive linear transformation and which gives utility as a function of return R_i and the state of nature. In addition, $U_i'(R_i,\theta) = \dfrac{\partial U_i(R_i,\theta)}{\partial R_i} > 0$ (implying nonsatiability) and $U_i''(R_i,\theta) = \dfrac{\partial^2 U_i(R_i,\theta)}{\partial R_i^2} < 0$.

(iii) The i-th investor's subjective probability measure over the states of nature is μ_i , and expectation with respect to μ_i is denoted by E_i .

Each investor is assumed to maximize expected utility, where utility is given by (ii). Given his current portfolio $\bar{s}_i$ and arbitrary market values $V = (V_o , V_1 , \ldots , V_N)$, the set of portfolios the i-th investor can afford, $B(V,\bar{s}_i)$ is given by

$$B(V,\bar{s}_i) = \{s_i \in R^{N+1} \,\Big|\, \sum_{j=0}^{N} V^j(s_i^j - \bar{s}_i^j) \leq 0\} \quad . \tag{255}$$

Nonsatiability implies that the only candidates for an optimal portfolio will be those that lead to equality in (255). Therefore, the equilibrium market values and portfolios must satisfy

$$\sum_{j=0}^{N} V^j(s_i^j - \bar{s}_i^j) = 0 \; , \quad \text{for all } i \; . \tag{256}$$

Define R_i , the total return to a portfolio s_i , as $R_i = \sum_{j=0}^{N} \pi^j s_i^j$, and using (256) to substitute for s_i^o , the total return to the i-th investor's portfolio becomes

$$R_i = r \sum_{j=0}^{N} V^j \bar{s}_i^j + \sum_{j=1}^{N} (\pi^j - rV^j) s_i^j \quad , \tag{257}$$

where $r \equiv \pi^o(q^o)/V^o$, the rate of return ('dividend yield') on the riskless security.

The collection of portfolios $\hat{s}_i = (\hat{s}_i^o , \ldots , \hat{s}_i^N)$, $i = 1 , \ldots , M$, together with the market values $\hat{V} = (\hat{V}^o , \ldots , \hat{V}^N)$ are said to constitute a *financial equilibrium* relative to the firms' production decisions $q = (q^o , \ldots , q^N)$ provided the following two conditions are satisfied:[627]

(i) Given q and $\hat{V}$, for each investor, i, $\hat{s}_i$ is optimal in $B(\hat{V},\overline{s}_i)$; that is, $\hat{s}_i$ is the solution to the problem:

$$\underset{s_i \,\varepsilon\, B(\hat{V},\overline{s}_i)}{\text{maximize:}} \qquad E_i[U_i(R_i,\theta)] \tag{258}$$

According to (258), the portfolio $\hat{s}_i$ is the one, selected from among the individual's set of affordable portfolios (i.e. B), that maximizes his expected utility of total portfolio return (i.e. R_i). Possessing this portfolio, the individual investor will be in equilibrium.

(ii) Given q, $\hat{V}$ equates the supply of and demand for each security; that is,

$$\sum_{i=1}^{M} \hat{s}_i^{\,j} = 1 \, , \quad j = 0 \, , \, \dots \, , N \, . \tag{259}$$

According to (259), the market values $\hat{V}$ must be equilibrium market values, i.e. for each j, $j = 0, \dots, N$, the market value V^j must be such that the associated price of each security (e.g. price of each share of common stock) equates the supply of to the demand for the outstanding shares of the j-th security.

The reason for fixing q in (i) and (ii) is that a change in q will alter the distributions of profits perceived by investors. Therefore, the financial equilibrium $\{\hat{s}_1, \dots, \hat{s}_M;\hat{V}\}$ is dependent on q.

Conditions (i) of financial equilibrium require that each investor solve problem (258). Using (257), which was obtained by using

(256) to eliminate s_i^o , to substitute for R_i yields an unconstrained maximization problem equivalent to (258). The necessary conditions for a maximum are

$$E_i[U_i'(R_i,\theta)\{\pi^j - r\hat{V}^j\}] = 0 \; ; \qquad j = 1 , \dots , N \; ; \quad i = 1 , \dots , M . \tag{260}$$

Substituting for π^j using (254) permits (260) to be rewritten as

$$E_i[U_i'(R_i,\theta)\{p^j(\theta)q^j - C^j(q^j) - r\hat{V}^j\}] = 0 \; ; \qquad j = 1 , \dots , N \; ; \quad i = 1 , \dots , M . \tag{261}$$

Combining equations (256), (259), and (260) (or (261)) produces a system of $(N + 1)(M + 1)$ equations in the $(N + 1)(M + 1)$ unknowns $\hat{s}_i^j$ and $\hat{V}^j$, $i = 1 , \dots , M$; $j = 0 , \dots , N$.[628] The solution to this system of equations provides the collection of individual portfolios and the set of security prices that constitute a financial equilibrium (which, it must be emphasized, is contingent upon the firms' output decisions).

A vector of outputs $\hat{q} = (\hat{q}^o , \dots , \hat{q}^N)$ is said to constitute a *production equilibrium* when outputs $\hat{q}$ are "in the stockholders' interests" — in the sense that there does not exist some other vector of outputs $\overline{q} = (\overline{q}^o , \dots , \overline{q}^N)$ at which all stockholders would be better off — when current portfolios and market values constitute a financial equilibrium relative to $\hat{q}$.[629] Thus, when production equilibrium and financial equilibrium hold simultaneously, there is no impetus for $\hat{q}$, $\hat{V}$, or any of the $\hat{s}_i$'s to change. This notion of a production equilibrium differs from the notion of the equilibrium level of output discussed previously in that the vector $\hat{q}$ results from general equilibrium, whereas in the models discussed previously each firm's optimal output level was determined in partial equilibrium (i.e. for a particular firm with the prices, output levels, etc., of all other firms held fixed).

One of the main results proved by Leland is the Unanimity Theorem,[630] which states that when firms function within a random environment in accordance with (254) and when current portfolios and market values are in financial equilibrium (given current production decisions $q = (q^o, \ldots, q^N)$), a firm's stockholders will vote unanimously for (or against) small changes in output. In symbols, for each firm k, the sign of $\frac{\partial E_i[U_i(R_i,\theta)]}{\partial q^k}$ is the same for all stockholders. One of the consequences of the Unanimity Theorem is a characterization of the production equilibrium. Since the details of the proof are provided in Leland's paper,[631] they will not be given here. Rather, only the main line of argument will be sketched.

For the riskless firm:

$$\frac{\partial E_i[U_i(R_i,\theta)]}{\partial q^o} = E_i[U_i'(R_i,\theta)\,\frac{\partial R_i}{\partial q^o}] \ ,$$

where R_i is given by (257).[632] As q^o changes, so also will $\hat{V}$, r, and $\hat{s}_i$. Using (260), $\bar{s}_i = \hat{s}_i$, and $\pi^o = \bar{p}^o q^o - C^o(q^o)$, the rate of change of the i-th investor's expected utility with respect to a change in q^o can be expressed as

$$\frac{\partial E_i[U_i(R_i,\theta)]}{\partial q^o} = E_i[U_i'(R_i,\theta)]\,\bar{s}_i^{\,o}\,[p^o - MC(q^o)] \ , \qquad (262)$$

where $MC^j(q^j) = \frac{dC^j(q^j)}{dq^j}$, $j = 0, \ldots, N$. But by the nonsatiability assumption, $E_i[U_i'(R_i,\theta)] \bar{s}_i^o$ is strictly positive for all investors with $\bar{s}_i^o > 0$ — i.e. for all stockholders of the riskless firm. Then (262) has the sign of $[p^o - MC(q^o)]$ for every one of the firm's shareholders. By a similar argument it is found for each risky firm that

$$\frac{\partial E_i[U_i(R_i,\theta)]}{\partial q^k} = E_i[U_i'(R_i,\theta)\{p^k(\theta) - MC^k(q^k)\}\bar{s}_i^k] . \qquad (263)$$

$$k = 1, \ldots, N$$

By using (261) to substitute for $E_i[U_i'(R_i,\theta) \cdot p^k(\theta)]$, (263) can be rewritten in an easier-to-interpret form:

$$\frac{\partial E_i[U_i(R_i,\theta)]}{\partial q^k} = E_i[U_i'(R_i,\theta)]\bar{s}_i^k[\frac{C^k(q^k) + r\hat{V}^k}{q^k} - MC^k(q^k)] . \qquad (264)$$

$$k = 1, \ldots, N$$

Nonsatiability again implies that for each firm, k , the sign of (264) will be the same as the sign of $\frac{C^k(q^k) + r\hat{V}^k}{q^k} - MC^k(q^k)$ for every one of the firm's shareholders.[633]

The Unanimity Theorem leads to the following characterization of production equilibrium, $\hat{q} = (\hat{q}^o, \ldots, \hat{q}^N)$. From (262) the optimal output level for the riskless firm is the output level $\hat{q}^o$ for which

$$p^o = MC^o(\hat{q}^o) , \qquad (265)$$

which is just the familiar price equals marginal cost rule for maximizing profit in perfectly competitive markets under certainty. From (264) the optimal output level, $\hat{q}^k$, for the k-th risky firm is that which satisfies

$$\frac{C^j(\hat{q}^j) + r\hat{V}^j(\hat{q})}{\hat{q}^j} = MC^j(\hat{q}^j) , \quad j = 1 , \ldots , N , \qquad (266)$$

where $\hat{V}^j(\hat{q})$ has been written to emphasize the dependence of $\hat{V}^j$ on $\hat{q}$. According to equation (266), in order for the j-th risky firm to be in equilibrium, the firm must choose $\hat{q}^j$ such that marginal production cost, $MC^j(\hat{q}^j)$, just equals the "total" average cost, which is the sum of average production costs, $\frac{C^j(\hat{q}^j)}{\hat{q}^j}$, and an imputed average capital cost, $\frac{r\hat{V}^j(\hat{q}^j)}{\hat{q}^j}$.[634] What (266) makes clear is that the risky firm's output decision is dependent on financial factors (i.e. its stock market value $\hat{V}^j(\hat{q}^j)$) as well as on real factors (i.e. production costs). In the certainty case, as indicated by (265), these financial factors are inconsequential. Under certainty, with perfect capital markets, the firm's production decisions are separable from its financial decisions, and Modigliani's and Miller's Proposition I holds.

If firms set their output levels according to (265) and (266), one may ask whether the equilibrium outputs $\hat{q} = (\hat{q}^o , \ldots , \hat{q}^N)$ will, in general, maximize each firm's stock market value. The answer is 'no',[635] and the method of proof is to show that, in general, $\left.\frac{\partial \hat{V}^j}{\partial q^j}\right|_{q^j = \hat{q}^j} \neq 0$, where $1 \leq j \leq N$. Differentiating (260) with respect to q^j, evaluating at $q^j = \hat{q}^j$, and simplifying, leads to the equation [636]

$$- \frac{1}{D_i} [E_i \{U_i'(R_i,\theta)\} (r \frac{\partial \hat{V}^j}{\partial q^j})] + \frac{\partial \hat{s}_i^{\,j}}{\partial q^j}$$

$$+ \frac{1}{D_i} [E_i\{U_i''(R_i,\theta)(\pi^j - r\hat{V}^j) (\hat{s}_i^{\,j} \frac{d\pi^j}{dq^j} + \sum_{k \neq j} (\pi^k - r\hat{V}^k) \frac{\partial \hat{s}_i^{\,k}}{\partial q^j})\}] = 0 , \tag{267}$$

where $D_i \equiv E_i\{U_i''(R_i,\theta)(\pi^j - r\hat{V}^j)^2\} < 0$. Summing (267) over all M individuals and noting that $\sum_{i=1}^{M} \frac{\partial \hat{s}_i^{\,j}}{\partial q^j} = 0$ gives

$$\left.\frac{\partial \hat{V}^j}{\partial q^j}\right|_{q^j=\hat{q}^j} = \frac{\sum_{i=1}^{M} \frac{1}{D_i} [E_i\{U_i''(R_i,\theta)(\pi^j - r\hat{V}^j)(\hat{s}_i^{\,j} \frac{d\pi^j}{dq^j} + \sum_{k \neq j} (\pi^k - r\hat{V}^k) \frac{\partial \hat{s}_i^{\,k}}{\partial q^j})\}]}{r \sum_{i=1}^{M} \frac{1}{D_i} [E_i\{U_i'(R_i,\theta)\}]} \tag{268}$$

in which the nonsatiability assumption and $D_i < 0$ ensure that the denominator is always negative, but in which the sign of the numerator is, in general, indeterminate.[637] Moreover, the numerator of (268) is, in general, nonzero, which implies that value maximization by firms does not lead to a Pareto optimal choice of outputs. If a firm were to choose its output level so as to maximize its market value, every shareholder could be made better off by an appropriate change in output away from the value-maximizing q^j.[638,639]

To summarize the discussion of Leland's model of the firm in the context of stock market equilibrium, the distinguishing characteristics of the model are presented in table II-28.

Table II-28 Summary of Leland's Stock Market Model

Class:	modern traditional (see (256), (259) and (261) in text)
Firm's Objective:	maximize each shareholder's expected utility
Constraints:	constraint on affordable portfolios (as embodied in $B(V,\bar{s}_i)$); implicitly, factor supply and technological conditions (as embodied in $C^j(q^j)$)
Variables:	
Exogenous:	probability distribution of price for each firm ($p^j(\theta)$) and investors' current portfolios ($\bar{s}_i$)
Endogenous:	equilibrium collection of portfolios ($\hat{s}_i$), equilibrium set of market values ($\hat{V}$), equilibrium output levels (q^j), and equilibrium expected utility levels ($E_i[U_i(R_i,\theta)]$)
Decision:	equilibrium portfolio for each investor ($\hat{s}_i$) and equilibrium output level for each firm (q^j)
Finance:	external financing (equity only) is permitted, although in the model discussed in the text the actual issuance of shares was subsumed in the collection of current portfolios ($\bar{s}_i$)
Certainty/Uncertainty:	permits uncertainty and utilizes the time-state-preference framework
Equilibrium/Disequilibrium:	characterization of the equilibrium output decision by each firm in the context of stock market equilibrium
Time:	single period
Type of Model:	system of $(N+1)(M+1)$ equations in $(N+1)(M+1)$ unknowns for the characterization of financial equilibrium; unconstrained optimization problem for each firm for the characterization of production equilibrium
Solution Technique:	unconstrained optimization (for each firm)

The significance of the Leland model lies in its integration of the firm's production decisions into a model of stock market equilibrium and its demonstration that, in general, maximizing the stock market (or total market) value of the firm does not necessarily lead to production decisions that are optimal from the standpoint of the firm's shareholders. In contrast to the traditional models, which subsumed the role of finance, Leland's model demonstrates the inextricability of production decisions and the stock market when capital markets are incomplete. Also, in contrast to the models discussed earlier in this paper that simply assumed value maximizing behavior to be in the shareholders' best interests, Leland's model shows that this simple criterion cannot be relied on to ensure a Pareto optimal choice of outputs, unless certain assumptions (e.g. perfect and complete capital markets) are satisfied.

L. CONTROL THEORY, DYNAMIC OPTIMIZATION, AND CORPORATE GROWTH

According to the traditional theory of the firm, the environment within which the typical firm operates is static and the decisions it must make apply to only a single time period. In the traditional models discussed above in sections B through E, the firm in the short run engages in myopic profit maximization by simultaneously selecting and instantaneously adopting the cost-minimizing mix of variable inputs and the profit-maximizing mix of outputs. Over the long run the firm is free to add to its capital stock and it does so to the extent that the added capital leads to increased productive efficiency and greater long run total profit. As in the short run, adjustments occur instantaneously. Moreover, short run profit maximization is perfectly consistent with long run profit maximization, so that intertemporal profit trade offs do not appear in the model. Even in the reformulated version of the neoclassical model discussed above in section F, in which the firm is assumed to maximize the stock market value of its outstanding shares in the absence of uncertainty, the firm's selection of its optimal operating policies is still in essence a single period optimization problem.

In many of the recently developed models, the firm is permitted to grow, and the decision-making problem it faces involves setting its operating policies so as to place it on the optimal growth path. But often, as in the Marris and Baumol growth models of section G, the Herendeed model of section I, and the Lintner model of section J,[640] the firm is assumed to grow in steady state, in which all quantities (e.g. the capital stock, total profit, total revenue, the stock market value of the firm, etc.) grow at the same rate and all ratios (e.g. profit rate, leverage ratio, retention ratio, etc.) remain constant.[641] By assuming steady state growth, economists have been able to express their models of the firm as static optimization problems, which can be solved either by applying the method of Lagrange multipliers or by appealing to the Kuhn-Tucker theorem, depending on the nature of the constraints.[642] In spite of the advantages due to this simplification and Marris's and Lintner's defense of this approach in terms of the firm's selection of optimal long run operating targets,[643] the steady state approach has been found wanting by many economists. To quote Krouse:[644]

> [Studies that assume steady-state growth] consider only the special equilibrium structure of corporate events where the economic state of the firm is identical in every period except by a constant scale factor, and all its decisions and exogenous variables (for example, discount rates) are required to be constant through time. Concomitantly, they limit the models to comparative dynamics, which is not useful for the analysis of the firm's decision rules or states as exogenous variables change over time, but only appropriate for the comparison of firms with different, time-constant values of these variables.

Steady state models are unable to explain the pattern of capital investment over time in the presence of either the business cycle or factors that might cause the growth rate of the firm to slow down as the firm matures. To deal with such problems effectively, a dynamic optimization technique is required.[645]

This section presents several multiperiod models of the firm. Each model is expressed in the form of an optimal control problem, in which form the multiperiod equilibrium position (i.e. the equilibrium growth path) of the firm can be characterized with the aid of either the classical calculus of variations or the maximum principle.[646] As pointed out in chapter one of this paper, the maximum principle can be considered the extension of the method of Lagrange multipliers to dynamic optimization. Indeed, the costate variables of the maximum principle and the Lagrange multipliers of static constrained optimization are interpreted similarly — as shadow prices — with the difference being that, in the dynamic optimization, the time path for each costate variable — rather than just a single value as in the case of a Lagrange multiplier — must be determined.[647,648]

The first model discussed below is due to Jorgenson. In his model Jorgenson reexpresses the neoclassical theory of the firm in terms of a multiperiod framework. The second model discussed below, which is due to Arrow, analyzes the firm's pattern of capital investment in the presence of the business cycle. The third subsection describes two approaches, one due to Wong, which utilizes the Jorgenson model, and the other due to Leland, to reconciling the controversy concerning whether firms maximize total profit, the rate of growth, or some other quantity. The final subsection explores a model developed by Krouse, which deals with the firm's financial decisions in the presence of capital market imperfections and which employs the discrete version of the maximum principle.

1. A Dynamic Version of the Neoclassical Theory: The Jorgenson Model

The Jorgenson model is directly concerned with determining the firm's optimal program of capital accumulation.[649] In the model it is assumed that the firm produces a single output, the amount of which is denoted by $Q(t)$, using two inputs, one a variable input (say, labor), the amount of which is denoted by $L(t)$, and the other a capital input, the amount of which is denoted by $K(t)$. In each case the t in parentheses denotes the variable's dependence on time.[650] The firm's gross investment is denoted by $I(t)$. The markets for the firm's output, for labor, and for durable investment goods are assumed to be perfectly competitive, so that the prices of output, labor, and investment goods, denoted by $p(t)$, $w(t)$, and $q(t)$, respectively, vary as functions of time only.[651] The net cash flow at time t, denoted by $R(t)$, is the difference between total revenue and total cash outlays for labor services and for new investment goods,

$$R(t) = p(t) \cdot Q(t) - w(t) \cdot L(t) - q(t) \cdot I(t) . \qquad (269)$$

The objective of the firm is to maximize its present value, which Jorgenson expresses as the present value of the future cash flow stream, which in turn is represented by the integral

$$V = \int_0^\infty R(t)e^{-rt}\, dt \quad , \qquad (270)$$

where r is the rate of discount, which, for simplicity, is assumed constant.

Present value is maximized subject to two constraints, one being the standard neoclassical production function.[652]

$$F(Q(t),\ L(t),\ K(t)) = 0 \quad , \qquad (271)$$

and the other being the expression for net investment, or the instantaneous rate of growth of the capital stock, as the difference between gross investment and the rate of depreciation:

$$\dot{K}(t) = I(t) - \delta \cdot K(t) \quad , \qquad (272)$$

where δ is some positive constant.[653] It is assumed that the function F in (271) has a full set of continuous second partial derivatives, and further, that the marginal rate of technical substitution between inputs and the marginal productivity of each input are all always positive. In (272) the rate of depreciation at time t , $\delta \cdot K(t)$, is proportional to the capital stock at time t ; that is, it is assumed that depreciation occurs at a constant proportional rate. Both (271) and (272) must hold at each time t .

Collecting (270)-(272), the Jorgenson model of the firm is expressed as the following optimal control problem:[654]

$$\begin{array}{ll} \underset{\{Q,L,I\}}{\text{maximize:}} & V = \int_0^\infty R(t)e^{-rt}\,dt \\ \text{subject to:} & \dot{K}(t) = I(t) - \delta \cdot K(t) \ , \ \forall t \\ & K(0) = K_o \\ & F(Q(t),\ L(t),\ K(t)) = 0 \ , \ \forall t \\ & Q(t),\ L(t),\ K(t) \geq 0 \ , \ \forall t \ , \end{array} \qquad (273)$$

where K_0 is the initial capital stock. Problem (273) can be solved with the aid of the calculus of variations. The Lagrangian is

$$L_\lambda = \int_0^\infty [Re^{-rt} + \lambda(t) \cdot F(Q,L,K) + \mu(t) \cdot (\dot{K} - I - \delta \cdot K)]\, dt \;, \qquad (274)$$

in which, for simplicity, the dependence of R, Q, L, K, $\dot{K}$, and I on t has been suppressed. For ease of exposition denote by $f_\lambda(t)$ the expression in brackets in (274). Then the Euler necessary conditions for an optimal solution to (273) are, in addition to the constraints in (273), the following:[655]

$$\frac{\partial f_\lambda}{\partial Q} = pe^{-rt} + \lambda(t) \cdot \frac{\partial F}{\partial Q} = 0 \qquad (275)$$

$$\frac{\partial f_\lambda}{\partial L} = -we^{-rt} + \lambda(t) \cdot \frac{\partial F}{\partial L} = 0 \qquad (276)$$

$$\frac{\partial f_\lambda}{\partial I} = -qe^{-rt} - \mu(t) = 0 \qquad (277)$$

$$\frac{\partial f_\lambda}{\partial K} - \frac{d}{dt}\frac{\partial f_\lambda}{\partial \dot{K}} = \lambda(t) \cdot \frac{\partial F}{\partial K} + \delta \cdot \mu(t) - \frac{d}{dt}\mu(t) = 0 \qquad (278)$$

Equations (275)-(278) can be used collectively to characterize the multiperiod equilibrium position of the Jorgenson-type firm.

Combining (275) and (276) yields the familiar factor market condition for profit maximization:

$$p \cdot \frac{\partial Q}{\partial L} = w \quad . \qquad (279)$$

According to (279), the firm should hire additional amounts of the variable input up to the point at which the marginal revenue product of the last unit hired just equals its price (i.e. the wage rate). The difference between (279) and the similar condition (8) derived in section B is that (279) must hold at every point in time over the indefinite future, whereas (8) holds only for a single instant. As in the single period case, (279) can be rewritten to yield the product market condition for profit maximization:

$$p = \frac{w}{\partial Q/\partial L} \qquad \text{or} \qquad p = MC \quad , \tag{280}$$

where (280) holds at every point in time, in contrast to (11) and (12) in section B, which hold only for a single instant.

A result perfectly analogous to (279) may be obtained for capital services if the quantity

$$i = q(r+\delta) - \dot{q} \quad , \tag{281}$$

where the dot denotes differentiation with respect to time, is interpreted as the unit price of capital services (i.e. the 'cost of capital').[656,657] From (277) $\mu(t) = -qe^{-rt}$. Differentiating with respect to t gives $\frac{d}{dt}\mu(t) = qre^{-rt} - \dot{q}e^{-rt}$. Substituting for $\mu(t)$ and $\frac{d}{dt}\mu(t)$ in (278) gives

$$\lambda(t)\frac{\partial F}{\partial K} - \delta qe^{-rt} - rqe^{-rt} + \dot{q}e^{-rt} = \lambda(t)\frac{\partial F}{\partial K} - i \cdot e^{-rt} = 0 \ . \tag{282}$$

Combining (275) and (282) gives

$$p \cdot \frac{\partial Q}{\partial K} = i \ , \tag{283}$$

which is analogous to (279), and which, in contrast to (14) in section B, holds at every point in time. Further, combining (279) and (283) gives the dynamic equivalent of (16) in section B,

$$\frac{\partial L}{\partial K} = \frac{\partial Q/\partial K}{\partial Q/\partial L} = i/w \ . \qquad (284)$$

It follows from (284) that, at each point in time, the marginal rate of technical substitution between the inputs must equal the ratio of the input prices.

Thus, in the Jorgenson model the firm maximizes its present value by maximizing total profit at each point in time.[658] This aspect of the model represents a strength and at the same time a weakness. On the plus side, the model demonstrates that the neoclassical theory of the firm does not have to be confined to static conditions and to single period decision-making. On the minus side, the model is barely dynamic.[659] That is, the firm never has to make any intertemporal profit trade offs; all it need do is to maximize total profit at each point in time. Under the assumptions of certainty and perfect markets, the myopic decision rules that lead to maximum single period total profit are also intertemporally optimal for the value maximizing firm. Moreover, in addition to the assumption of a perfect market for capital goods, it is assumed that the quality of the capital goods owned by the firm and the rate at which they depreciate are independent of the age of the goods (i.e. there is no technological obsolescence). Under these assumptions investment is reversible, and the firm at any point in time is able to sell any or all of its capital at the prevailing market price. In such a world, the rental price of capital services accurately reflects the interest rate, the rate of depreciation, and the rate of change of the price of capital goods, and consequently, the present value of the stream of future rentals just equals the current price of capital goods, and at the margin the firm is indifferent between renting and owning. This is a rather special case, as the discussion of the Arrow model in the next subsection will make clear.

To summarize this subsection, the main features of the Jorgenson model are presented in table II-29.

Table II-29 Summary of Jorgenson Model

Class:	modern traditional (see (273) in text)
Firm's Objective:	maximize the stock market value of the firm (expressed as the present value of the future cash flow stream)
Constraints:	technological constraint (embodied in the production function) and relationship between net investment, gross investment, and depreciation ($\dot{K} = I - \delta K$)
Variables:	
Exogenous:	prices of output, labor, and investment goods at each point in time (p(t), w(t), and q(t), respectively), initial capital stock (K_0), market rate of discount (r), and rate of depreciation (δ)
Endogenous:	rates of flow of output and labor services at each point in time (Q(t) and L(t) respectively), gross investment and capital stock at each point in time (I(t) and K(t), respectively), cash flow at each point in time (R(t)), and current stock market value of the firm (V)
Decision:	streams of output, labor services, and gross investment (Q(t), L(t), and I(t), respectively)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the multiperiod equilibrium position of the firm (and in particular, the time path of its capital stock)
Time:	multiperiod (continuous time)
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	classical calculus of variations

2. Optimal Capital Accumulation When Investment is Irreversible: The Arrow Model

In the Jorgenson model of the preceding subsection, investment was costlessly reversible, and as a consequence, the firm's decision as to how much capital to hold at any point in time was myopic — being independent of future product demand, future improvements in technology, etc. It followed from equations (281) and (283) that the firm should hold just that stock of capital for which the marginal revenue product of capital equals the cost of capital, where the latter is equal to the short term interest rate plus the rate of depreciation less the rate of appreciation of capital goods prices.[660] Thus, only forecasts of near term capital goods prices are needed. In this special case, investment is reversible so that the firm can buy a unit of capital, derive its marginal revenue product over some time span, and then sell the undepreciated balance at the prevailing market price.

In the Jorgenson model the myopic optimal accumulation policy implied by (283) depends crucially on the existence of a perfect capital goods market, which enables the firm at any point in time to buy or sell any quantity of capital goods at the established market price. In the Arrow model[661] discussed in this subsection, imperfections in the market for capital goods are introduced. Due to such factors as installation costs, which must be added to the purchase price, the purchase price may exceed the selling price (unless installation costs could somehow be recovered in the selling price[662]), and, due to such factors as specialization, the resale price may be substantially below book value. Thus, once an investment has been undertaken, the firm may not have the freedom to sell that is implied in the Jorgenson model — and in the extreme case developed by Arrow, investment is irreversible in that, once the capital good has been purchased, it cannot be resold.[663] The consequences of irreversibility are that, at times when the firm might wish to sell capital goods at the going price, it will not be able to do so, and that, at times when current calculations indicate that additional investment would be profitable, the firm may refrain from investing because it expects that changing business conditions in the near term would have led it to disinvest if it had been able to do so.[664] This latter effect of irreversibility, which might be interpreted as the firm's taking into account the business cycle (and in particular, an impending downswing in economic activity), introduces a dynamic element not present in the Jorgenson model.[665]

In the Arrow model there is one type of capital good, the stock of which at time t is denoted by $K(t)$. All other inputs are variable. It is assumed that, given $K(t)$, the variable inputs are combined to produce the outputs in such a way that total operating profit, defined as total revenue less the cost of variable inputs and denoted by $P(K(t),t)$, is maximized. The separate argument t in $P(K(t),t)$ reflects the fact that the functional relationship between P and K may shift over time due to improvements in technology, changes in supply or demand conditions, etc. That is, the separate argument t may be interpreted as a surrogate for technological change. As usual, the marginal productivity of capital is assumed strictly positive, so that $P_K = \frac{\partial P(K(t),t)}{\partial K} > 0$, and it is assumed that there are diminishing returns to capital, so that $P_{KK} = \frac{\partial^2 P(K(t),t)}{\partial K^2} < 0$. It is further assumed that the market for money capital is perfect, though interest rates may change over time (but in a known way). Let $\alpha(t)$ denote the factor to be applied to receipts at time t to discount them back to time zero. Then the short term rate of interest at time t, $\rho(t)$, is represented by

$$\rho(t) = -\dot{\alpha}(t)/\alpha(t) , \tag{285}$$

where, as before, the dot indicates differentiation with respect to time.[666] Selecting capital goods to be the numeraire, the price of capital goods is set equal to one, all other prices and interest rates are expressed in terms of capital goods, and the interest rate $\rho(t)$ is the short term money rate of interest less the rate of appreciation of capital goods prices.

Letting $I(t)$ denote the rate of gross investment at time t, Arrow's model of the firm is formulated as the following optimal control problem:

$$\begin{aligned} \underset{\{I(t)\}}{\text{maximize:}} \quad & V = \int_0^\infty \alpha(t)\,[P(K(t),t) - I(t)]\,dt \\ \text{subject to:} \quad & \dot{K}(t) = I(t) - \delta \cdot K(t) , \ \forall t \\ & K(0) = K_o \\ & I(t) \geq 0 , \ \forall t \end{aligned} \tag{286}$$

where δ and K_o are positive constants. The objective functional in problem (286) represents the present value of the future flow of cash, where the difference $P(K(t),t) - I(t)$ is the cash flow (in terms of capital goods) at time t. The first two constraints in (286) are the same as in the Jorgenson model, where δ is once again the constant percentage rate of depreciation. The third constraint in (286), which states that gross investment at each point in time must be non-negative, is the embodiment of the irreversibility of capital investment. According to (286), the objective of the firm is to select the time pattern of gross investment, $I(t)$, that maximizes the present value of the cash flow stream subject to the irreversibility of investment, $I(t) \geq 0$, and the restrictions that the time pattern of the capital stock, $K(t)$, is determined by its initial value, K_o, and by the firm's investment policy.

As demonstrated by Arrow, the model (286) can be reformulated as an equivalent model with $\delta = 0$.[667] This reformulation has the advantage of simplifying all subsequent calculations. The reformulated model is:

$$\begin{aligned} \underset{\{I(t)\}}{\text{maximize:}} \quad & V = \int_0^\infty \alpha(t)\,[\,P(K(t),t) - I(t)\,]\,dt \\ \text{subject to:} \quad & \dot{K}(t) = I(t), \ \forall t \\ & K(0) = K_o \\ & I(t) \geq 0, \ \forall t \end{aligned} \tag{287}$$

The solution to (287) can be characterized with the aid of Pontryagin's maximum principle. The necessary conditions so obtained can be used collectively to characterize the multiperiod equilibrium position of the value maximizing firm (and in particular, the time path of its capital stock) when investment is irreversible.

Introducing the costate variable $p(t)$ and forming the Hamiltonian function gives

$$H(K,I,p,t) = \alpha(t)\,[P(K(t),t) - I(t)] + p(t) \cdot I(t) . \tag{288}$$

The costate variable $p(t)$ is interpreted as the shadow price of capital at time t, and the Hamiltonian function H may be interpreted as the discounted value of investment at time t, which is expressed as the sum of the initial impact on current cash flow $\alpha(t)\,[P(K(t),t) - I(t)]$ plus the sum of discounted future benefits, $p(t) \cdot I(t)$ (i.e. the value of a gift of $I(t)$ at time t).[668] According to the maximum principle, $I(t)$ is chosen so as to maximize (288) subject to $I(t) \geq 0$, and $p(t)$ evolves through time according to the differential equation

$$\dot{p}(t) = -\frac{\partial H(K,I,p,t)}{\partial K} . \tag{289}$$

First, maximizing (288), define

$$q(t) = p(t) - \alpha(t) \tag{290}$$

and rewrite H as

$$H(K,I,p,t) = \alpha(t) \cdot P(K(t),t) + q(t) \cdot I(t) , \tag{291}$$

from which the maximization of H subject to $I \geq 0$ becomes trivial. If $q(t) < 0$, $\forall t$, then at optimality $I(t) \equiv 0$, $\forall t$. The reason for this is that $q(t)$, according to (290), expresses the difference between the shadow price, or value in terms of future cash flow, of capital goods discounted back to time zero and the market price of capital goods (which was taken to be one in current year dollars) discounted back to time zero. Thus, $q(t) < 0 \Longleftrightarrow p(t) < \alpha(t)$, which implies that the present value of cost exceeds the present value of future benefits, and no new investment is undertaken.

If $q(t) = 0$, then $I(t)$ in (291) can be any nonnegative quantity; from (290), $p(t) = \alpha(t)$, and at the margin the firm is indifferent between investing and not investing. However, $I(t)$ is not indeterminate, but rather, is determined by additional considerations discussed below. From (291), it would appear that $q(t)$ could also be positive. If $q(t)$ were positive, there would be no optimum for $I(t)$. From (290), $p(t) > \alpha(t)$, so that the shadow price of capital goods would exceed their cost, and it would pay the firm to invest an infinite amount at time t . But since the costate variable is assumed to be continuous, and since α is also, so is $q(t)$, which implies that $q(t)$ is positive over an interval. Therefore, infinite investment over an interval is called for, which is nonoptimal since any policy that led to this result could be improved upon by a policy that increased investment earlier.[669] Thus, the optimal investment policy must be such that

$$q(t) \leq 0 \ ; \quad \text{if } q(t) < 0 \ , \quad \text{then } I(t) = 0 \ . \tag{292}$$

Condition (292) might be thought of as the 'short run' optimum condition, for, according to (292), the appropriate course of action for the firm is dependent on the comparison between $p(t)$ and $\alpha(t)$ at time t .

Second, working with (289), taking the partial derivative of (291) with respect to K enables (289) to be simplified to

$$\dot{p}(t) = -\alpha(t) \cdot P_K , \tag{293}$$

which can be rewritten as $\alpha(t) \cdot P_K + \dot{p}(t) = 0$. Equation (293) states that the discounted current returns from a unit increase in the capital stock, $\alpha(t) \cdot P_K$, plus the rate of change of the discounted shadow price of capital should be zero.[670] Since (292) is specified in terms of $q(t)$, it will prove convenient to express (293) and its solution in terms of $q(t)$. From (290) and (293), $\dot{q}(t) = \dot{p}(t) - \dot{\alpha}(t) = \alpha(t)\,[-P_K - \dot{\alpha}(t)/\alpha(t)\,]$, which from the definition of ρ in (285) becomes

$$\dot{q}(t) = \alpha(t)\,[\,\rho(t) - P_K(K(t),t)]\ . \tag{294}$$

The first-order differential equation (294) is solved easily by integration. Between times $t = t_o$ and $t = t_1$,

$$q(t_1) - q(t_o) = \int_{t_o}^{t_1} \alpha(t)\,[\,\rho(t) - P_K(K(t),t)]\ dt$$

$$= \int_{t_o}^{t_1} \alpha(t) \cdot r(K,t)\ dt\ , \tag{295}$$

where $r(K,t) \equiv \rho(t) - P_K(K(t),t)$. The solution to problem (287) consists of the three functions of time $I(t)$, $K(t)$, and $q(t)$ jointly satisfying (292), (294), and the three constraints in (287).[671]

Necessary condition (292) implies that in the optimal solution to (287) are alternating periods of positive investment and zero investment. That is, the optimal time paths of $q(t)$ and $I(t)$ are such that *free* intervals,[672] which are characterized by $q(t) = 0$ and $I(t) > 0$ (i.e. by positive gross investment), alternate with *blocked* intervals, which are characterized by $q(t) < 0$ and $I(t) = 0$ (i.e. by zero gross investment).

In a free interval $q(t) = 0$, and hence $\dot{q}(t) = 0$, throughout the interval, so that by (294),

$$P_K(K(t),t) = \rho(t) \quad . \tag{296}$$

Note that equation (296) is really just the myopic investment decision rule represented by equation (284).[673] Following Arrow,[674] define the capital stock established by the myopic policy, $K^*(t)$, by the equation $P_K(K^*(t),t) = \rho(t)$. Under the assumption of diminishing returns, equation (296) has a unique solution, so that $K(t) = K^*(t)$ on the free interval. Thus, on a free interval the firm should expand capacity in accordance with the myopic policy embodied in (296). During periods of capacity expansion the Arrow-type firm follows the same investment decision rule as the Jorgenson-type firm.[675]

During a blocked interval the explanation of the firm's investment behavior is somewhat more complicated. Consider any blocked interval beginning at time $t = t_o > 0$ and ending at time $t = t_1 < \infty$.[676] Since t_o marks the end of one free interval and t_1 marks the beginning of another, and since, as Arrow shows,[677] neither $K(t)$ nor $q(t)$ can have any jump discontinuities (except possibly at $t = 0$), it follows that $K(t_o) = K^*(t_o)$ and $q(t_o) = 0$; $K(t_1) = K^*(t_1)$ and $q(t_1) = 0$; and $K^*(t_o) = K(t) = K^*(t_1)$ for all t in the blocked interval. Since, by definition, $q(t) < 0$ for all t on a blocked interval, it follows from this fact together with (295) and the definition of $r(K,t)$ that on any blocked interval (t_o, t_1) with $t_o > 0$ and $t_1 < \infty$:

$$\int_{t_o}^{t_1} \alpha(t) \cdot r[K^*(t_o),t]\, dt = 0 \tag{297}$$

$$\int_{t_o}^{t} \alpha(t) \cdot r[K^*(t_o),t]\, dt < 0 \quad \text{for} \quad t_o < t < t_1 \tag{298}$$

$$\int_{t}^{t_1} \alpha(t) \cdot r[K^*(t_o),t]\, dt > 0 \quad \text{for} \quad t_o < t < t_1 \tag{299}$$

Equation (297) and inequalities (298) and (299) characterize the firm's investment behavior (or more accurately, its lack of investment) during the blocked interval and have the following interpretation. If the firm were able to rent capital goods for some fixed time period at a (possibly time-varying) cost of $\rho(t)$, then $P_K - \rho = - r$ would represent the firm's instantaneous rate of profit resulting from the rental. Note that when markets are perfect and in equilibrium purchasing a capital good and selling it at the end of the period is perfectly equivalent to renting it for the entire period at the market rate of interest $\rho(t)$,[678] and further, that buying a capital good and holding it until a point in time at which the firm would wish to purchase a capital good is also equivalent to renting. Then, according to (297), at the margin the firm is indifferent between renting and not renting for the entire period (since the discounted value of the 'profit' stream is zero). According to (298), the firm would find it profitable to rent a capital good from time t_o until some time $t < t_1$; but, since the firm has to buy instead of rent, and since it can never sell the good (by the irreversibility assumption) nor does it wish to hold it at time t_1 , it eschews investment. According to (299), the firm would not wish to rent during any period beginning at time $t > t_o$ and ending at time t_1 , and therefore, neither does it wish to purchase any capital goods during the blocked interval.

Figure II-26 compares the investment behavior of an Arrow-type firm with that of a Jorgenson-type firm under the assumption that, except for the irreversibility of investment in the case of the Arrow-type firm, the two firms and the market environments within which each operates are identical. It is also assumed for simplicity that the initial capital stock for each is zero and that for the Arrow-type firm $t = 0$ marks the beginning of a free interval. In Figure II-26 $K(t)$ is the time path of the Arrow-type firm's capital stock and $K^*(t)$ is the time path of the Jorgenson-type firm's capital stock. Over the free intervals (a , c , and e) $K(t) = K^*(t)$ and the time paths are identical. Over the blocked intervals (b and d) $K(t)$

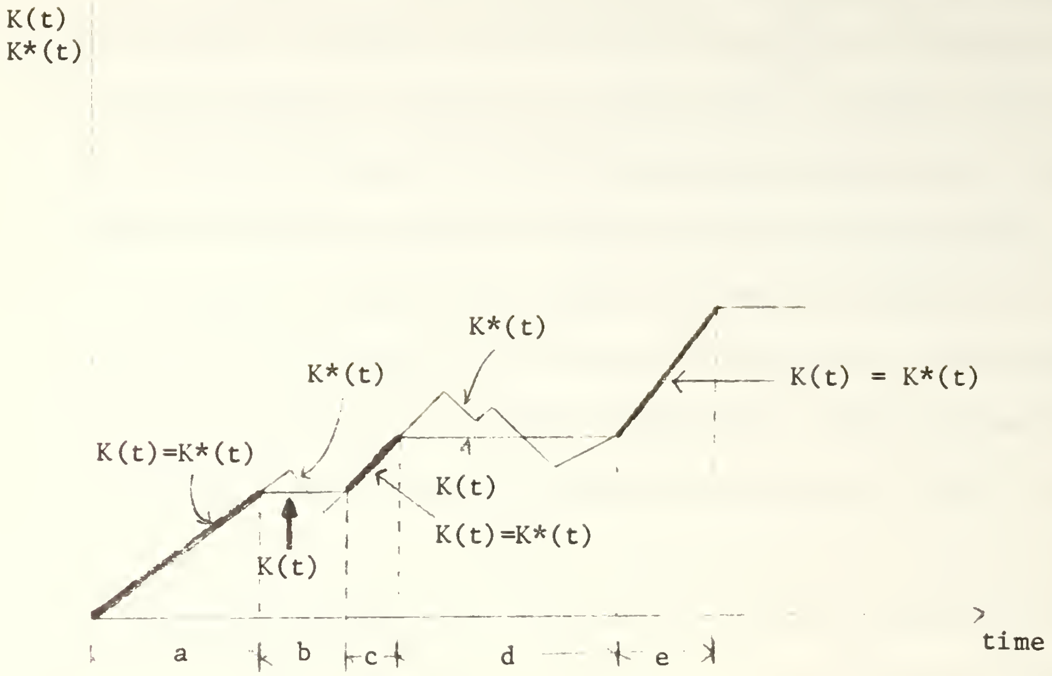


Figure II-26: The Time Pattern of the Capital Stock in the Jorgenson and Arrow Models

remains constant, whereas $K^*(t)$ fluctuates. Because the Jorgenson-type firm is able to disinvest by selling off capital it does not need, it is able to continue investing beyond the point in time at which the Arrow-type firm must cease investing. Due to the existence of perfect markets for capital goods in the Jorgenson model, the Jorgenson-type firm can sell off as much capital as it wants at the established market price of capital, so that during slack periods the Jorgenson-type firm can operate with $K^*(t) < K(t)$.

The irreversibility of investment, then, prevents the Arrow-type firm from investing at the beginning of a blocked interval and then selling capital goods sometime before the interval ends — an option that is available to the Jorgenson-type firm. In contrast to the myopic Jorgenson-type firm, which is able to invest right up to the end of a boom period and to disinvest quickly in the event of a downswing in economic activity, the Arrow-type firm must plan ahead, possibly reducing gross investment to zero before the end of a boom period if it believes the end of the boom period is imminent.[679]

To summarize the discussion of the Arrow model, the main features of the model are presented in table II-30.

Table II-30 Summary of Arrow Model

Class:	modern traditional (see (287) in text)
Firm's Objective:	maximize the stock market value of the firm (expressed as the present value of the future cash flow stream)
Constraints:	relationship between net investment and gross investment ($\dot{K} = I$, or in the original formulation (286), $\dot{K} = I - \delta \cdot K$) and nonnegativity of gross investment ($I \geq 0$); implicitly, product demand, factor supply, and technological conditions (as embodied in $P(K(t),t)$)
Variables:	
Exognenous:	present value factor at each time t ($\alpha(t)$) and initial capital stock (K_0) (and in the initial formulation (286) the rate of depreciation (δ))
Endogenous:	gross investment and capital stock at each time t ($I(t)$ and $K(t)$, respectively), total operating profit at each time t (P), and current stock market value of the firm (V)
Decision:	gross investment stream ($I(t)$)
Finance:	subsumed
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the multiperiod equilibrium position of the firm (and in particular, the time path of its capital stock) when investment is irreversible
Time:	multiperiod (continuous time)
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	maximum principle (continuous version)

3. Profit Maximization Versus Other Objectives: Toward a Dynamic Reconciliation

The previous two subsections were primarily concerned with capital theory, with the Jorgenson model describing the time pattern of the capital stock when investment is reversible and the Arrow model examining what happens when investment is irreversible. This subsection explores a second application of optimal control theory to the theory of the firm, namely, dynamic models that suggest that profit maximization may not be as unworkable an assumption as the managerialists seem to have implied.[680]

Two models are described in this subsection. The first, due to Wong, represents a life cycle view of the firm. During the firm's early years it maximizes its rate of growth until it reaches the most efficient size. From this point in time onward the mature firm maximizes total profit and invests just enough to keep its capital stock at its most efficient level. The second model, due to Leland, demonstrates that profit maximization may be perfectly consistent with a wide range of alternative goals because in a dynamic setting profits play a crucial role in furthering the growth of the firm, which in turn enhances the achievement of other long term goals. A third possible reconciliation, which has been suggested by O.E. Williamson,[681] is that the firm's behavior may alternate between profit maximization and utility maximization as changing conditions in the firm's environment alternately tighten and loosen, respectively, the profit constraint.

a. The Wong Model

The objective of the firm is to maximize the stock market value of the firm's equity, which is expressed as the present value of the future dividend stream.[682] In equation form the stock market value of the firm's equity, V, is given by

$$V = \int_0^\infty D(t)\, e^{-rt}\, dt \quad , \tag{300}$$

where $D(t)$ is the rate at which dividends are paid at time t and r is the discount rate, which for simplicity is assumed constant.

Dividends are paid to shareholders out of net receipts, which are defined to be total revenue less the cost of all variable inputs. For simplicity assume that product and factor markets are perfectly competitive and that the firm produces a single output and uses a single variable input. Denote the output price by $p(t)$ and the cost of the variable input (i.e. the 'wage rate') by $w(t)$. Net receipts not paid out as dividends are used for gross investment in capital, which is assumed to depreciate at a constant percentage rate δ. Total gross investment, $I(t)$, is given by

$$I(t) = p(t) \cdot f(K(t),L(t)) - w(t) \cdot L(t) - D(t) \quad , \tag{301}$$

where $f(K(t),L(t)) = Q(t)$ is the neoclassical production function and $K(t)$ and $L(t)$ are the amounts of capital and of the variable input, respectively, used in production. Net investment, $\frac{dK(t)}{dt}$, is equal to gross investment less depreciation,

$$\frac{dK(t)}{dt} = I(t) - \delta \cdot K(t) \quad . \tag{302}$$

Using (301) to substitute for I(t) in (302) and then combining (300) and (302) leads to the following formulation of the Wong model as an optimal control problem:

$$\begin{aligned}
&\underset{\{D,L\}}{\text{maximize:}} && V = \int_0^{\infty} D(t)\, e^{-rt}\, dt \\
&\text{subject to:} && \dot{K}(t) = p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - D(t) - \delta\cdot K(t), \ \forall\, t \\
& && K(0) = K_o \\
& && 0 \leq D(t) \leq p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) \ , \ \forall\, t \\
& && L(t) \geq 0 \ , \ K(t) \geq 0 \ , \ \forall\, t
\end{aligned} \tag{303}$$

where $\dot{K}$ once again denotes K differentiated with respect to time and where the last three constraints require that the time path of the capital stock begin at the initial level K_o, that the amount of dividends distributed and the amount of labor used at each point in time be nonnegative, and that the amount of dividends distributed never exceeds net receipts. Note that all investment is financed internally in problem (303).[683] Note also that problem (303) is a modified version of the Jorgenson model (273); the additional constraint (301) in problem (303) leads to a reduction from three to two in the number of the firm's decision variables and also causes investment to be irreversible (as in the Arrow model).[684]

The optimal solution to problem (303) can be characterized with the aid of the maximum principle. The Hamiltonian is

$$H(K(t),L(t),D(t),q(t),t) = D(t)\cdot e^{-rt} + q(t)\cdot e^{-rt}[\, p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - D(t) - \delta\cdot K(t)] \, , \tag{304}$$

where the costate variable has been written as $q(t)e^{-rt}$. The costate variable for problem (303) can be interpreted as the shadow price of capital, with $q(t)$ representing the shadow price of a unit of capital evaluated at time t in terms of future dividends and $q(t)e^{-rt}$ representing the shadow price of a unit of capital added at time t in terms of the value of future dividends discounted back to time zero. The following necessary conditions are used to characterize the firm's optimal dividend and hiring policies:

$$\left.\begin{array}{l} H(K^*(t),L^*(t),D^*(t),q(t),t) \geq H(K^*(t),L(t),D(t),q(t),t) \\ \quad \text{for all } L(t) \geq 0 \text{ and all } D(t) \text{ such that} \\ \quad 0 \leq D(t) \leq p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) \end{array}\right\} \tag{305}$$

$$\dot{K}(t) = p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - D(t) - \delta\cdot K(t) \tag{306}$$

$$\frac{d}{dt}[q(t)e^{-rt}] = -\frac{\partial H}{\partial K} = -q(t)e^{-rt}[p(t)\,\frac{\partial f}{\partial K} - \delta\,] \tag{307}$$

$$\lim_{t\to\infty} q(t)e^{-rt} = 0 \tag{308}$$

where $K^*(t)$, $L^*(t)$, and $D^*(t)$ in (305) are the optimal capital stock, the optimal hiring policy, and the optimal dividend policy, respectively. Equation (306) is just the first constraint in (303). The boundary condition for this differential equation is $K(0) = K_o$.

Condition (305) signifies the maximization of the Hamiltonian with respect to L and D . Setting $L(t) = L^*(t)$ in (305) and simplifying yields

$$D^*(t)\,[1 - q(t)] \geq D(t)\,[1 - q(t)] \quad . \tag{309}$$

From (309) it follows that[685]

$$\left.\begin{aligned} D^*(t) &= 0 \, , && \text{if } q > 1 \\ D^*(t) &= p(t)\cdot f(K^*(t),L^*(t)) - w(t)\cdot L^*(t) \, , && \text{if } q < 1 \\ D^*(t) &\in [0,p(t)\cdot f(K^*(t),L^*(t)) - w(t)\cdot L^*(t)] \, , && \text{if } q = 1 \end{aligned}\right\} \tag{310}$$

Setting $D(t) = D^*(t)$ in (305) and simplifying yields[686]

$$p \cdot \frac{\partial f(K,L)}{\partial L} = w \, , \tag{311}$$

which is the classical criterion for the optimal hiring of labor, namely, that labor should be hired up to the point at which the marginal revenue product of the last unit hired just equals the wage rate.

Using (310) to substitute for $D(t)$, (306) becomes

$$\dot{K}(t) = \begin{cases} p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - \delta\cdot K(t) , & \text{if } q > 1 \\ - \delta\cdot K(t) , & \text{if } q < 1 \\ p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - D(t) - \delta\cdot K(t), & \text{if } q = 1 \end{cases} \quad (312)$$

Note that since $q(t)$ measures the value of an additional unit of capital in terms of future dividends, $q > 1$ implies that the benefits to be derived from an additional unit of investment exceed the cash outlay. Therefore, the entire amount of net receipts should be invested, and no dividends should be paid in the current period. The opposite happens when $q < 1$. In that case the cash outlay for investment exceeds the benefits in terms of future dividends that are derivable, and so, gross investment should be zero and the firm should pay out the entire amount of net receipts as dividends. The third possibility, $q = 1$, is of considerable interest and is discussed further below.

Turning next to equation (307), evaluating $\frac{d}{dt}[q(t)e^{-rt}]$ and solving for $\dot{q}(t)$ yields the following expression:

$$\dot{q}(t) = - q(t) [p\cdot \frac{\partial f(K,L)}{\partial K} - (\delta + r)] . \quad (313)$$

Conditions (311), (312), and (313), together with (308), are the necessary conditions for the time paths $K^*(t)$, $L^*(t)$, and $D^*(t)$ to be optimal.[687] Moreover, these conditions are also sufficient if the production function $f(K,L)$ is concave.[688]

To aid in the determination of the optimal time paths of capital, labor, and dividends, a phase diagram showing the optimal dynamic paths of q and K has been constructed from (312) and (313) in figure II-27.[689]

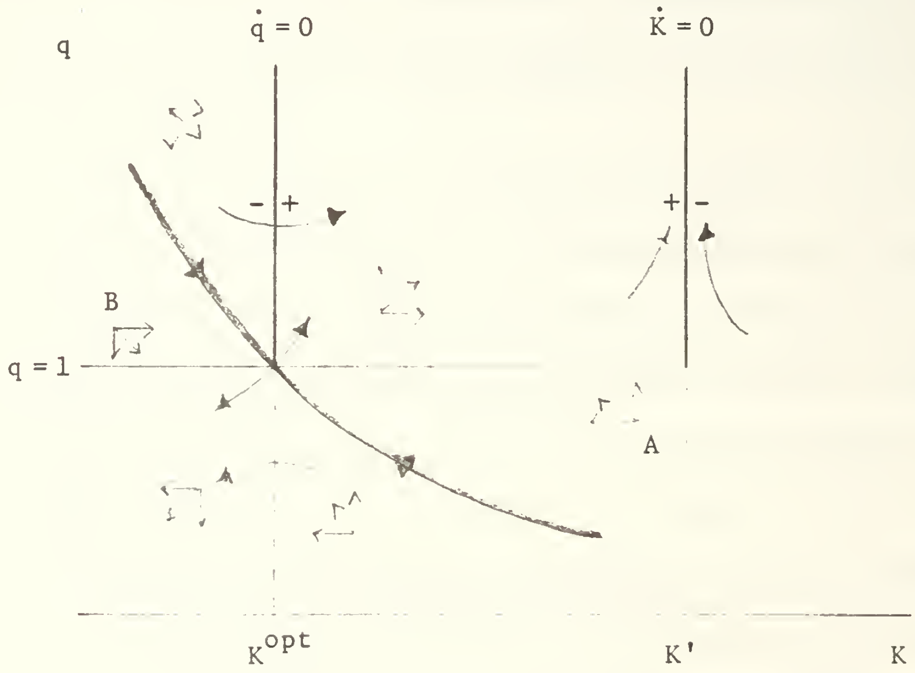


Figure II-27: Optimal Dynamic Paths of q and K[690]

It is clear from figure II-27 that the optimal path of K and q is the one converging to K^{opt}.[691] Along this optimal path the firm's dividend policy takes the following form, depending on the initial capital stock K_o :[692]

$$D^*(t) = \begin{cases} 0\ , & \text{if } K_o < K^{opt} \\ p(t)\cdot f(K^*(t),L^*(t)) - w(t)\cdot L^*(t)\ , & \text{if } K_o > K^{opt} \\ p(t)\cdot f(K^*(t),L^*(t)) - w(t)\cdot L^*(t) - \delta\cdot K^*(t)\ , & \text{if } K_o = K^{opt} \end{cases} \qquad (314)$$

The capital stock K^{opt} is called the long run desired stock of capital. It is the firm's equilibrium stock of capital. According to (314), if $K_o < K^{opt}$, the firm should not pay any dividends, but rather, should use all net receipts for investment purposes, and thereby grow — i.e. increase the capital stock — at the maximum rate consistent with exclusive reliance on the internal financing of investment. If $K_o > K^{opt}$, the firm should pay out all net receipts as dividends and not engage in any replacement investment. Thus, when $K_o \neq K^{opt}$, the firm's optimal dividend and investment policies are those that take its capital stock toward the long run desired stock of capital K^{opt} as rapidly as possible, i.e. along the heavy lines in figure II-27. When $K_o = K^{opt}$, the firm's capital stock is already at the long run desired level, i.e. the firm has attained its equilibrium capital stock, so that it should invest sufficient funds to maintain its equilibrium capital stock $K(t) = K^{opt}$ and pay out the remainder of net receipts as dividends. The policy implications of (314) are illustrated in figure II-28.

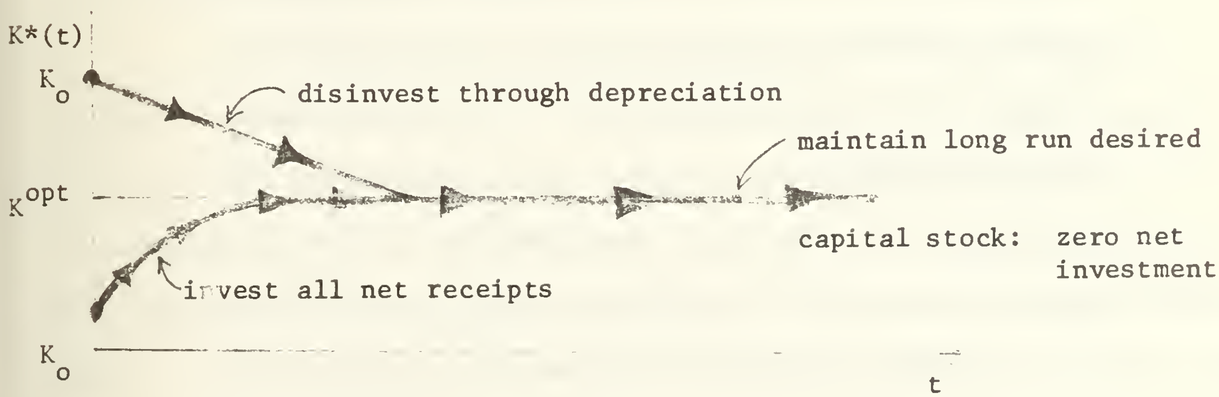


Figure II-28: The Time Path of K*(t)

Of particular interest is the path of the capital stock when $K_o < K^{opt}$ — i.e. the lower branch in figure II-28. The optimal path consists of the following two phases:

(i) The firm pays no dividends, and all net receipts are used to purchase additional capital. This could be interpreted as the 'growth maximization' phase. Anyone observing the behavior of the firm during the 'growth maximization' phase would find considerable support for the managerial models of the firm and would be likely to interpret the firm's objective as one of growth maximization, revenue maximization, or sales (i.e. quantity of units sold) maximization.[693]

(ii) Once the equilibrium stock of capital K^{opt} has been attained, net investment ceases. The firm invests just enough to maintain $K(t) = K^{opt}$ and distributes the remainder of net receipts as dividends. This phase could be interpreted as the 'profit maximization' phase. Anyone observing the behavior of the firm during the profit maximization phase would find considerable support for the traditional model of the firm and would be likely to interpret the firm's objective as one of profit maximization, dividend maximization, or share value maximization.[694]

It should be noted that the optimal path just described has the turnpike properties frequently discussed in the optimal growth theory literature.[695] Indeed, the profit maximization phase of the optimal path might be called the 'dividend' turnpike and during this phase the firm might be said to be operating at its 'golden stage' of production.[696]

During the golden stage of production $\dot{q}(t) = 0$, which implies, by (313), that

$$p \cdot \frac{\partial f(K,L)}{\partial K} = \delta + r \quad . \tag{315}$$

Condition (315) is just condition (283) arising out of the Jorgenson model.[697] Both (283) and (315) imply that the firm should expand its capital stock (or, if $K_o > K^{opt}$, contract its capital stock) until the marginal revenue product of the last unit 'hired' just equals the cost of capital, $\delta + r$. This is, of course, the optimality condition for long run profit maximization for the single period case discussed in section B of this chapter.

The value of the Wong model lies in its suggested reconciliation of the managerial theories with the traditional theory. The model suggests that at any point in time there will be a mixture of 'managerial' firms — i.e. those that are expanding as rapidly as possible toward their respective long run desired stocks of capital — and 'traditional' firms — i.e. those that have reached their respective long run desired stocks of capital.[698]

The Wong model, viewed as a modification of the Jorgenson model discussed above, is also of interest because of its more reasonable treatment of capital accumulation. In the Jorgenson model the firm could adjust to K^{opt} instantaneously, whereas in the Wong model the ability of the firm to accumulate capital is restricted by its need to generate funds with which to purchase capital goods.[699]

The main limitation of the Wong model is its assumption of an optimum firm size — the long run desired stock of capital — that remains constant over time. Under this assumption one should find a tendency for the growth rate of the firm to slow down as size increases, at least for very large firms (unless, of course, this optimum size is so great that very few firms have begun to approach it — in which case, however, most firms should behave in a manner more consistent with the managerial models than with the traditional models). Though the relationship between growth and size has been studied extensively, the results of these studies in toto have been inconclusive.[700] It does appear, however, that empirical support for the optimum size hypothesis and for the hypothesis that the growth rate of the firm diminishes, at least for firms in the largest size classes, as size increases is sufficiently weak as to bring into question the validity of any model based on either of these hypotheses.[701]

To summarize the discussion of the Wong model, the distinguishing features of the model are listed in table II-31.

b. The Leland Model

A second approach to reconciling the objective of profit maximization with alternative objectives, such as sales maximization, has been suggested by Leland.[702] By considering the dynamic role of profits, Leland has established conditions under which the firm's optimal current policies converge to profit maximization as the firm's planning horizon becomes infinite. Under the conditions set out by Leland, current profit maximization is necessary and sufficient for long run optimization with respect to alternative goals.[703,704] The

Table II-31 Summary of Wong Model

Class:	modern traditional (see (303) in text)
Firm's Objective:	maximize the stock market value of the firm (expressed as the present value of the future dividend stream)
Constraints:	technological constraint (embodied in the production function $(Q(t) = f(K(t),L(t))$; relationship between net investment, gross investment, and depreciation $(\dot{K} = p \cdot f(K,L) - w \cdot L - D - \delta \cdot K)$; and constraint on dividend stream $(0 \leq D \leq p \cdot f(K,L) - w \cdot L)$
Variables:	
Exogenous:	prices of output and labor at each point in time ($p(t)$ and $w(t)$, respectively), initial capital stock (K_o), market rate of discount (r), and rate of depreciation (δ)
Endogenous:	rates of flow of output and labor ($Q(t) = f(K(t),L(t))$ and $L(t)$, respectively), gross investment and capital stock at each point in time ($I(t) = p(t) \cdot f(K(t),L(t)) - w(t) \cdot L(t) - D(t)$ and $K(t)$, respectively), dividends paid at each point in time ($D(t)$), and current stock market value of the firm (V)
Decision:	streams of dividends and labor ($D(t)$ and $L(t)$, respectively)
Finance:	internal financing only
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the firm's operating and dividend policies that lead to the attainment of the equilibrium capital stock; at each instant, given its capital stock, the firm selects equilibrium operating policies, but the firm is in multiperiod equilibrium if and only if $K = K^{opt}$
Time:	multiperiod (continuous time)
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	maximum principle (continuous version)

implication of this result is that, while firms may pursue objectives other than profit maximization, the behavior they exhibit may not differ much from profit maximizing behavior. Leland develops two applications of his general result: a firm maximizing utility of sales and profits and a firm maximizing profit per worker.[705] Since under standard neoclassical assumptions myopic behavior will be optimal for the latter type of firm,[706] only the utility maximizing firm is discussed below.

The objective of the firm is to maximize the present value of utility expressed as a function of profits and sales (e.g. managerial utility), which is given by

$$\int_0^T e^{-rt} \cdot \mathcal{U}\{\pi[K(t),L(t),t] \ , \ \mathcal{S}[K(t),L(t),t]\} \, dt \ , \qquad (316)$$

where the firm's planning period extends from time $t = 0$ to time $t = T$ (and T is called the *planning horizon*); $K(t)$ is the firm's capital stock at time t ; L denotes a vector of decision variables, which includes those variables, such as labor and raw materials, that are variable in the short run and whose levels $L(t)$ the firm is able to control throughout the planning period; π is total profit and $\mathcal{S}$ is total sales, each of which is a function of the capital stock, the levels of the decision variables, and time; and r , which is assumed constant, is the rate at which future utility levels are discounted.[707] For each period t the choices for the decision variables must belong to $\Lambda(t) = [0,\overline{L}(t)]$, where $\overline{L}(t)$ is the vector of upper bounds on permissible values for the decision variables. These upper bounds may be established either by physical constraints or by institutional constraints, such as the need to maintain some minimum level of profitability or some minimum valuation ratio.

Net investment, $\dot{K}(t)$, is a function of current profits, the current capital stock, and time, with the inclusion of the time variable allowing for the possibility that the functional relationship between net investment on the one hand and current profits and the current capital stock on the other may shift over time.[708] At any point in time, with the capital stock given, it is assumed that net investment is an increasing function of current profits. In the simplest case, all investment is financed internally[709] and net investment is directly proportional to current profits:

$$\dot{K}(t) = m \cdot \pi(K(t),L(t),t) , \quad m > 0 . \qquad (317)$$

The firm's capital stock evolves according to (317), subject to the initial condition, $K(0) = K_o$.

If it is assumed that the total profit and total sales functions are time invariant, then the model of the firm can be formulated as the following optimal control problem:[710]

$$\underset{\{L(t)\varepsilon\Lambda(t)\}}{\text{maximize:}} \quad \int_o^T e^{-rt}\, \mathcal{U}[\pi(K(t),L(t)),\mathcal{S}(K(t),L(t))]\, dt$$

$$\text{subject to:} \quad \dot{K} = m \cdot \pi(K(t),L(t)) , \quad m > 0 \qquad (318)$$

$$K(0) = K_o$$

where

$$\pi = \pi(K(t),L(t)) = p \cdot f(K(t),L(t)) - w \cdot L(t) - i \cdot K(t) \qquad (319)$$

and

$$S = S(K(t),L(t)) = p \cdot f(K(t),L(t)) \tag{320}$$

are total profit and total sales, respectively, and where price, p, the wage rate, w, and the cost of capital, i, are all assumed to be constant through time. That is, product and factor markets are assumed to be perfectly competitive, and since the profit and sales functions are time invariant, the net investment function is also time invariant.[711] Following Leland,[712] it is further assumed that (i) given K_o, there exists an optimal policy function, $L^*_T(t)$, which holds for all $t \leq T$, where the subscript T denotes the function's dependence on the planning horizon; (ii) given $K(t)$, there exists a unique profit maximizing policy $L^P(t) \varepsilon \Lambda(t)$ that is independent of the planning horizon; and (iii) utility maximization and profit maximization are alternative goals in that if, given $K(t)$, $L^P(t)$ lies in the interior of $\Lambda(t)$, then $L^P(t)$ does not also maximize $U(\pi,S)$ for $L(t) \varepsilon \Lambda(t)$.

It follows from the maximum principle that the optimal path of the firm's operating policies, $L^*_T(t)$, must satisfy the following conditions:

$$H(K^*(t),L^*_T(t),\lambda(t),t) \geq H(K^*(t),L(t),\lambda(t),t) \tag{321}$$

$$\text{for all } L(t) \varepsilon \Lambda(t)$$

$$\dot{K}(t) = \frac{\partial H}{\partial \lambda} = m \cdot \pi(K(t),L(t)) \tag{322}$$

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial K} = -\{e^{-rt} \cdot \frac{\partial \mathcal{U}}{\partial K} + m \cdot \lambda(t) \cdot \frac{\partial \pi}{\partial K}\} \tag{323}$$

$$\lambda(T) = 0 , \tag{324}$$

where the Hamiltonian is

$$H(K(t),L(t),\lambda(t),t) = e^{-rt} \cdot \mathcal{U}[\pi(K(t),L(t)),\mathcal{S}(K(t),L(t))] \tag{325}$$

$$+ \lambda(t) \cdot m \cdot \pi(K(t),L(t)) .$$

Let $\lambda^*_T(t)$ denote the costate variable function that satisfies (323) and (324) when the firm adopts the operating policies $L^*_T(t)$.

Rather than solve conditions (321)-(324) explicitly for the optimal path of current operating policies, $L^*_T(t)$, what follows will focus on the behavior of the firm at an arbitrary time t $(0 < t < T)$. It will be shown that, as the planning horizon becomes infinite, i.e. as $T - t \to \infty$, the firm's optimal current operating policies will approach profit maximizing policies, i.e. $L^*_T(t) \to L^P(t)$.

From (321), $L^*_T(t)$ maximizes the Hamiltonian (325). Given the expressions for total profit and total sales, (319) and (320) respectively, and assuming that $L^*_T(t)$ lies in the interior of $\Lambda(t)$, it follows that $L^*_T(t)$ must satisfy the equation:

$$e^{-rt} \{\frac{\partial \mathcal{U}}{\partial \pi} (p \cdot \frac{\partial f}{\partial L} - w) + \frac{\partial \mathcal{U}}{\partial \mathcal{S}} (p \cdot \frac{\partial f}{\partial L})\} + m \cdot \lambda^*_T(t) \cdot$$

$$(p \cdot \frac{\partial f}{\partial L} - w) = 0 , \tag{326}$$

where $\frac{\partial f}{\partial L}$ is evaluated at $L^*_T(t)$. The profit maximizing policy is just that given by the familiar optimality condition:

$$p \cdot \frac{\partial f}{\partial L} - w = 0 , \tag{327}$$

where $\frac{\partial f}{\partial L}$ is evaluated at $L^P(t)$. For arbitrary t , if $L^P(t)$ satisfying (327) is substituted into the left-hand side of (326), the resulting expression simplifies to $e^{-rt} \cdot \frac{\partial U}{\partial S} \cdot p \cdot \frac{\partial f}{\partial L} = e^{-rt} \cdot \frac{\partial U}{\partial S} \cdot w > 0$ unless $\frac{\partial U}{\partial S} = 0$.[713] Thus, in general, $L^*_T(t) \neq L^P(t)$.[714] In general, (327) does not hold for $L^*_T(t)$. Thus, dividing each side of (326) by $m(p \cdot \frac{\partial f}{\partial L} - w)$ and solving for $\lambda^*_T(t)$ yields

$$\lambda^*_T(t) = \frac{-e^{-rt}\{\frac{\partial U}{\partial \pi}(p \cdot \frac{\partial f}{\partial L} - w) + \frac{\partial U}{\partial S}(p \cdot \frac{\partial f}{\partial L})\}}{m(p \cdot \frac{\partial f}{\partial L} - w)} . \tag{328}$$

The numerator of (328) is always finite. Therefore, if $\lambda^*_T(t)$ were to become infinitely large as the planning horizon became infinite, then optimal current operating policies would approach profit maximizing policies, and vice versa, or in symbols,[715]

$$\lim_{T-t\to\infty} \lambda^*_T(t) \to \infty \Leftrightarrow p \cdot \left.\frac{\partial f}{\partial L}\right|_{L^*_T(t)} - w \to 0$$
$$\Leftrightarrow L^*_T(t) \to L^P(t) , \tag{329}$$

where the second implication follows from the assumed uniqueness of profit maximizing policies.

The optimal value of the costate variable at time t, $\lambda^*_T(t)$, measures the marginal value of capital stock in terms of utility at time t. If the marginal value of capital stock becomes large as the planning horizon becomes infinite, it will be in the firm's best interests to sacrifice current performance (i.e. to accept a somewhat lower current level of utility) in order to build up capital stock. But since net investment is proportional to total profit, this building up of the capital stock is accomplished by increasing profits, so that in the short term the firm should not continue to expand sales at the expense of total profit to the extent it would if the planning horizon were shorter, but rather, should strive for greater total profit. In the limiting case indicated by (329), $\lambda^*_T(t)$ becomes unboundedly large as the planning horizon becomes infinite, implying that it is optimal to increase the capital stock as rapidly as possible, and therefore, to maximize total profit.

It remains to be shown that $\lim_{T-t\to\infty} \lambda^*_T(t) \to \infty$. From (323), the optimal path of the costate variable, $\lambda^*_T(t)$, must satisfy the first order linear differential equation,

$$\dot{\lambda}^*_T(t) = -[e^{-rt}\{\frac{\partial U}{\partial \pi}(p \cdot \frac{\partial f}{\partial K} - i) + \frac{\partial U}{\partial S}(p \cdot \frac{\partial f}{\partial K})\} + m \cdot \lambda^*_T(t) \cdot (p \cdot \frac{\partial f}{\partial K} - i)]$$

$$= \alpha \cdot \lambda^*_T(t) + \beta \qquad (330)$$

with boundary condition $\lambda^*_T(T) = 0$, where

$$\left.\begin{aligned} \alpha &= - m \cdot (p \frac{\partial f}{\partial K} - i) \\ \beta &= - e^{-rt}\{\frac{\partial U}{\partial \pi} (p \frac{\partial f}{\partial K} - i) + \frac{\partial U}{\partial S} (p \cdot \frac{\partial f}{\partial K}) \ . \end{aligned}\right\} \tag{331}$$

If $\alpha < 0$, the solution to (330) is given by

$$\lambda^*_T(t) = \frac{\beta}{\alpha} (e^{-\alpha(T - t)} - 1) \ , \tag{332}$$

and, if $\alpha = 0$, the solution is given by

$$\lambda^*_T(t) = - \beta(T - t) \ . \tag{333}$$

From (331), $\alpha < 0$ if and only if $p \frac{\partial f}{\partial K} - i > 0$ since, by assumption, $m > 0$. Also from (331), $\alpha = 0$ if and only if $p \frac{\partial f}{\partial K} - i = 0$, or equivalently, $p \cdot \frac{\partial f}{\partial K} = i > 0$. In either case, $\beta < 0$. Therefore, $\alpha \leq 0$, or equivalently,

$$p \frac{\partial f}{\partial K} \geq i \ , \tag{334}$$

implies, by (332) and (333) and the fact that $\beta < 0$, that $\lim_{T-t\to\infty} \lambda^*_T(t) = \infty$. Note, however, that when $\alpha > 0$, $\lambda^*_T(t)$ given by (332) is again the solution to (330), but $\lim_{T-t\to\infty} \lambda^*_T(t) = - \frac{\beta}{\alpha}$.[716] Thus, as long as the marginal revenue product of capital exceeds the cost of capital, i.e. as long as (334) holds,[717] the marginal value of capital stock will become unboundedly large as the planning horizon becomes infinite, optimal current policy will converge to profit maximization, and in the limit, the firm's equilibrium operating policy will be that which leads to maximum total profit.

In the Leland model, the long run objectives of the firm may be such that, in pursuing these objectives, the firm acts as a profit maximizer in order to generate funds to permit it to grow as rapidly as possible. The behavior of such a firm may be indistinguishable not only from profit maximizing behavior, but from growth maximizing behavior as well. Moreover, if the utility function U has a large number of arguments reflecting the wishes of various groups within the firm as interpreted by top management,[718] the firm's behavior may also be indistinguishable from behavior that would be predicted on the basis of behavioral models — the firm may not strive to maximize any single quantity, but rather, strives to seek a balance among competing alternatives and to maximize what may be thought of as 'collective utility'.[719] The essential point is that several different types of behavior may be virtually indistinguishable, in theory as well as in practice.

To summarize the discussion of Leland's managerial model of the firm, the distinguishing features of the model are presented in table II-32.

4. Optimal Investment, Dividends, and Growth: The Krouse Model

One of the main advantages of modeling the growth of the firm as an optimal control problem, as demonstrated by the Wong and Leland models of the preceding subsection, is that such models permit

Table II-32 Summary of Leland's Managerial Model

Class:	managerial (see (318) in text)
Firm's Objective:	maximize discounted utility (expressed as a function of total profit and total sales)
Constraints:	total profit function (see (319) in text), total sales function (see (320) in text), and net investment function ($\dot{K} = n \cdot \pi(K,L)$)
Variables:	
Exogenous:	price (p), wage rate (w), cost of capital (i), discount rate (r), and ratio of net investment to total profit (m) — all of which are assumed to remain constant over time — and initial capital stock (K_0)
Endogenous:	total profit, total sales, capital stock, and labor employed at each point in time ($\pi(K,L)$, $S(K,L)$, K, and L, respectively)
Decision:	labor stream ($L(t)$)
Parameter:	planning horizon (T)
Finance:	internal financing only
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	concerned with conditions under which optimal current policy will converge to profit maximization as the firm's long run equilibrium operating policy (i.e. model considers the stability of this long run equilibrium operating policy)
Time:	multiperiod (continuous time)
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	maximum principle (continuous version)

non-steady state growth. In the Krouse model discussed below,[720] the firm's optimal choice over time of investment and financing policies is modeled as an optimal control problem, and the rate of growth of dividends is free to vary over time. But unlike the continuous-time models discussed earlier in this section, Krouse treats the firm within a discrete-time framework. The application of Pontryagin's maximum principle within the discrete-time context is, however, analogous to its application within the continuous-time context.[721]

As in the models discussed earlier in this section, in the Krouse model the environment within which the firm operates is one of certainty. But in contrast to these models, the Krouse model gives explicit consideration to external financing of investment. In the models discussed earlier in this section, capital markets were (at least implicitly) assumed to be perfect. Since the firm's financial policies are irrelevant under certainty when capital markets are perfect, the restriction to internal financing does not affect the conclusions derived from these models. In his model Krouse allows for a specific form of capital market imperfection, and as a result, the method the firm chooses to finance its investment is no longer irrelevant to the firm's shareholders.

In his model Krouse permits external equity financing and introduces equity market imperfections in the form of transactions costs on new external equity issues. By permitting external equity financing,[722] Krouse's treatment of the firm's investment and financing decisions is more general than the treatment provided in models such as Marris's utility maximization model[723] in which the restriction to internal financing renders the firm's investment and dividend policies inextricable. By introducing equity market imperfections in the form of transactions costs on new external equity issues, Krouse is able to demonstrate that, in general, the firm's choice of dividend policy does affect its stock market value.[724]

More specifically, the Krouse model explores the intertemporal dependencies that relate the firm's stock market value to its investment and financing decisions. The objective of the firm is to maximize the wealth position of the firm's initial (i.e. as of time zero) shareholders, which is expressed as the present value of the future stream of dividends paid to the firm's initial shareholders. The solution to the model consists of programs of capital accumulation and financing that are optimal from the standpoint of the firm's shareholders. The model and its solution make clear that, while the firm's investment and financing *opportunities* are distinct, the firm's investment and finance *decisions* interact to determine earnings and dividend time sequences that ultimately determine the firm's stock market value.

First, define the following variables:

$D(t)$ = total dollar amount of dividends paid at time t .

$D_T(t)$ = portion (in dollars) of dividends paid at time t to shareholders of record at time T .

$$k(t) = \prod_{\tau=0}^{t} \{1/[1 + r(\tau)]\}$$

= present value of a dollar received at time t , where $r(\tau)$ is the discount rate for period τ .

$X(t)$ = net earnings (i.e. net income less depreciation) of the firm at time t .

$I(t)$ = dollar amount of equity funds raised 'internally' through retained earnings, when $0 \leq I(t) \leq X(t)$, or through retained earnings together with a pre-emptive rights offering, when $I(t) > X(t)$; in either case, involving zero transactions costs. It follows from this definition that

$I(t) \equiv X(t) - D(t)$.[725]

$E(t)$ = 'external' equity decision variable, representing the total dollar amount of equity funds raised through the sale of shares to the public, when $E(t) > 0$, and indicating the total dollar amount paid out to retire shares, when $E(t) < 0$; in each case, inclusive of transactions costs.

$\delta(t)$ = the ratio at time t of the market price of an equity share to its external issue price; $\delta(t) > 1$ indicating shares issued at a premium, and $\delta(t) < 1$ indicating shares issued at a discount.[726]

$V(t)$ = stock market value of the firm at the beginning of period t to shareholders of record at that time.

Figure II-29 depicts the timing of the firm's receipt of earnings and its dividend and equity decisions for a representative time period t . After the period's earnings, $X(t)$, have been realized, the dividend decision, $D(t)$, and the internal and external equity decisions, $I(t)$ and $E(t)$, respectively, are all made at the end of the period. The stock market value of the firm, $V(t)$, is established at the beginning of the period. Thus, external equity is raised *ex dividends* of that period.

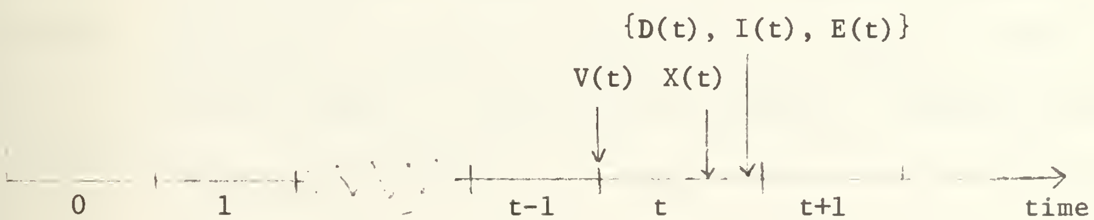


Figure II-29: A Representative Time Period

The stock market value of the firm at time $t = 0$, $V(0)$, or equivalently, the wealth position of the firm's initial shareholders, is equal to the present value of the future stream of dividends paid to the firm's initial shareholders,[727]

$$V(0) = \sum_{t=0}^{\infty} D_o(t) \cdot k(t) . \tag{335}$$

Since in practice $D_o(t)$ is not easy to observe, it is more convenient to reexpress (335) in terms of variables that are more readily observable. By separating each successive period from all later periods and employing the identity $D(t) \equiv X(t) - I(t)$, (335) can be reexpressed as[728]

$$V(0) = \sum_{t=0}^{\infty} [X(t) - I(t) - \delta(t) \cdot E(t)]\, k(t) . \tag{336}$$

The objective of the firm is to select time sequences $I(t)$ and $E(t)$ that maximize (336).

The ability of the firm to pay dividends is limited ultimately by its ability to generate earnings. The time path of earnings, $X(t)$, is dependent on the firm's investment budget and mode of financing at every point in time. Specifically, the change in earnings from period t to period $t + 1$ is a function of three factors: the firm's internal financing decision, $I(t)$; the firm's external financing decision, $E(t)$; and the rate of earnings adjustment attributable to technical progress, changes in the structure of product and/or factor markets, or to any other changes in the firm's environment, which are all embodied in some function $A(t)$. The change in earnings is given by

$$\Delta X(t) \equiv X(t+1) - X(t) = A(t)\,\phi[I(t),E(t)] \, . \qquad (337)$$

The earnings-possibilities function ϕ expresses the change in earnings owing not only to the size of the firm's capital budget, but also to the relative importance of the two sources of equity finance.[729] The relationship between the change in earnings and $I(t)$ and $E(t)$ embodied in ϕ assumes that each marginal dollar of investment expenditure is allocated optimally among both existing and potential new markets; and that during each period the firm's capital stock is employed optimally.[730]

Putting together (336) and (337), the Krouse model of the firm is formulated as the following optimal control problem:

$$\begin{aligned} &\underset{\{I(t),E(t)\}}{\text{maximize:}} && V(0) = \sum_{t=0}^{\infty} [X(t) - I(t) - \delta(t) \cdot E(t)]\, k(t) \\ &\text{subject to:} && \Delta X(t) = A(t) \cdot \phi(t) \qquad (338) \\ & && X(0) = X_o \, , \end{aligned}$$

where $\phi(t) \equiv \phi[I(t),E(t)]$ and where X_o is the given initial level of earnings. The solution to problem (338) can be characterized with the aid of the discrete version of the maximum principle.[731] The Hamiltonian for (338) is

$$\begin{aligned} H(X(t),I(t),E(t),\lambda(t+1),t) &= [X(t) - I(t) - \delta(t) \cdot E(t)]k(t) \qquad (339) \\ &\quad + \lambda(t+1)\,\{A(t) \cdot \phi(t)\} \, , \end{aligned}$$

where $\lambda(t+1)$ is the costate variable for period t . By the maximum principle, the optimal time paths $I^*(t)$ and $E^*(t)$ must satisfy the following conditions:[732]

$$H(X^*(t),I^*(t),E^*(t),\lambda(t+1),t) \geq H(X^*(t),I(t),E(t),\lambda(t+1),t) \quad (340)$$

$$\Delta X(t) = \frac{\partial H}{\partial \lambda} = A(t) \cdot \phi(t) \quad (341)$$

$$\Delta \lambda(t) = -\frac{\partial H}{\partial X} = -k(t) \quad (342)$$

$$\lim_{t\to\infty} \lambda(t) = 0 \quad (343)$$

From (342) and (343) it follows that[733]

$$\lambda(t+1) = \sum_{\tau=t+1}^{\infty} k(\tau) \quad . \quad (344)$$

The costate variable $\lambda(t+1)$, which represents the shadow price of an additional dollar of earnings measured in terms of the present value of dividends paid to initial shareholders in period t ,[734] is equal to the sum of the discount factors, $k(\tau)$, for periods $t+1$ and beyond. Since $k(\tau)$ represents the present value of a dollar received at the end of period τ , $\lambda(t+1)$ given by (344) can be interpreted as the present value of a perpetual annuity beginning in period $t+1$. With this interpretation of $\lambda(t+1)$, the Hamiltonian (339) can be given a richer meaning. An increase in the amount of equity finance raised in period t either by internal means or by external means, i.e. an increase in either $I(t)$ or $E(t)$, will have two opposing effects on the wealth position of initial shareholders: (i) a depressing effect, measured by $[X(t) - I(t) - \delta(t) \cdot E(t)] \cdot k(t)$, due to the increased number of shares, and hence, the increased number of claimants on the future pool of dividends, and (ii) an elevating effect, measured by $\lambda(t+1)\{A(t) \cdot \phi(t)\}$, due to the higher level of earnings (which, by assumption, is permanent unless the firm decides to disinvest) that makes possible an increase in the size of the dividend pool. Condition (340), which calls for the maximization of the Hamiltonian in each period with respect to $I(t)$ and $E(t)$, requires that these effects be balanced at the margin.

For $I^*(t)$ and $E^*(t)$ to maximize H it is necessary that[735]

$$\frac{\partial H}{\partial I} = -k(t) + \lambda(t+1) \cdot A(t) \cdot \frac{\partial \phi(t)}{\partial I(t)} = 0 \qquad (345)$$

$$\frac{\partial H}{\partial E} = -k(t) \cdot M(t) + \lambda(t+1) \cdot A(t) \cdot \frac{\partial \phi(t)}{\partial E(t)} = 0\,, \qquad (346)$$

where $M(t) \equiv \dfrac{\partial[\delta(t) \cdot E(t)]}{\partial E(t)}$ is called the 'marginal external issue market value.'[736] By defining $Z(t) = A(t) \sum_{\tau=t+1}^{\infty} \dfrac{k(\tau)}{k(t)}$,[737] the necessary conditions (345) and (346) can be rewritten as

$$Z(t) \cdot \frac{\partial \phi(t)}{\partial I(t)} = 1 \qquad (347)$$

$$Z(t) \cdot \frac{\partial \phi(t)}{\partial E(t)} = M(t)\,, \qquad (348)$$

where (347) and (348) must hold for each period t. Conditions (347) and (348) characterize the firm's equilibrium financial policies. Note that $Z(t)$ in (347) and (348) is a function of the discount factors, $k(\tau)$, for the current period and all future time periods. Thus, in contrast to the equilibrium decision rules that arise out of the Jorgenson model and similar models, the equilibrium decision rules given by (347) and (348) are not myopic, but rather, take into account future discount factors, which reflect the opportunity costs to the firm's shareholders of having their funds tied up in the firm's investment projects in future periods.

According to (347), in order for the firm to be in equilibrium, it should continue to raise additional equity funds through transaction costless internal means up to the point at which the value at time t of the resulting improvement in the future earnings stream, $Z(t)\cdot\frac{\partial\phi(t)}{\partial I(t)}$, just equals the stock market value of the new shares issued (i.e. one dollar at the margin). Similarly, according to (348), in order for the firm to be in equilibrium, it should continue to raise additional equity funds through external means up to the point at which the value at time t of the resulting improvement in the future earnings stream just equals the marginal external issue market value, $M(t)$, i.e. the stock market value of the new shares issued.[738] When adhered to, the equilibrium conditions (347) and (348) act as safeguards on the wealth positions of current shareholders by ensuring that the firm does not raise so much new equity — and in the process issue so many new shares — that the resulting earnings increase proves insufficient to justify the current share price.

If, as assumed in the traditional models, market discount rates are constant over time (i.e. $r(t) = \bar{r}$, a positive constant, for all t), the firm's production function and operating environment are unchanging (i.e. $A(t) = 1$), and equity markets are perfect, so that transactions are costless, all new shares are issued at prevailing market share prices (i.e. $M(t) = 1$), and Krouse's distinction between internal and external equity issues vanishes, then conditions (347) and (348) reduce to the classical investment criterion, namely, the firm should continue to invest up to the point at which the marginal internal rate of return, $\partial\phi(t)/\partial B(t)$, just equals the constant external market discount rate, $\bar{r}$, where $B(t) \equiv I(t) + E(t)$ is the aggregate amount of investment funds.[739] Under these same restrictive conditions, the Miller-Modigliani proposition on the irrelevancy of the firm's dividend policy also holds — the stock market value of the firm is dependent on the firm's investment plans (i.e. its operating decisions) only, and is independent of the dividend per share it sets (i.e. its dividend policy).[740] However, relaxing these assumptions will, in general, cause the firm's dividend policy to be of some consequence to the stock market value of the firm's shares.[741]

Krouse's model is noteworthy because of its general approach to the firm's investment and financing decisions. External financing is permitted, as are equity market imperfections, which render the firm's investment and financing decisions interdependent. The solution to the model generalizes the classical investment criterion and clearly indicates the impact of future opportunity costs on the current investment deci ion by the firm. Though Krouse considers only equity financing, the analysis is easily extended to permit debt financing[742] since in a worl' of certainty all methods of acquiring external finance — e.g. equi y shares and debt — reduce to the same thing.[743] The further extension to allow for uncertainty might, as Krouse suggests,[744] prove highly fruitful, although noone has yet done this.

To summarize the discussion in this subsection, the distinguishing features of the Krouse model are presented in table II-33 in terms of the same analytical framework used throughout this chapter.

Table II-33 Summary of Krouse Model

Class:	modern traditional (see (338) in text)
Firm's Objective:	maximize the stock market value of the firm (expressed as the present value of the future dividend stream)
Constraints:	change in earnings each period ($\Delta X(t) = A(t) \cdot \phi(t)$); implicitly, product demand, factor supply, technological, and financial conditions (embodied in $\phi(t) \equiv \phi[I(t), E(t)]$)
Variables:	
Exogenous:	present value factor for each period ($k(t)$), ratio of market price of an equity share to its external issue price for each period ($\delta(t)$), and initial level of earnings (X_o)
Endogenous:	current stock market value of the firm ($V(0)$) and net earnings, funds raised internally, funds raised externally, and dividends paid for each period ($X(t)$, $I(t)$, $E(t)$, and $D(t)$, respectively)
Decision:	internal funds stream ($I(t)$) and external funds stream ($E(t)$)
Finance:	external financing (equity only) permitted
Certainty/Uncertainty:	assumes certainty
Equilibrium/ Disequilibrium:	characterization of the equilibrium financial policies (with respect to internal equity issues versus external equity issues) of the firm)
Time:	multiperiod (discrete time)
Type of Model:	dynamic optimization (optimal control problem)
Solution Technique:	maximum principle (discrete version)

M. CHAPTER SUMMARY

This chapter has presented a survey of mathematical models of the firm. Thirty models (counting principal variations) representing a variety of mathematical formulations were detailed, and the economic implications of each model were carefully assessed. The models were presented in more or less chronological order, which also happens to correspond roughly to the order of increasing sophistication, in order to give the reader a better feel for the evolution of mathematical modeling in the theory of the firm.

Over the years, but particularly within the last decade, an almost bewildering array of models of the firm has appeared in the economic literature. This chapter has focused on those models that appear to this writer to be most representative of the different theories of the behavior of the firm and of the variety of mathematical structures selected by economic modelers. In addition, the models discussed above include those that have been cited most frequently by other writers in subsequent journal articles and books. The traditional models, which most writers, including this one, have taken as their point of departure, are, of course, the most well-known models of the firm. They are discussed at length in introductory texts as well as in more advanced works. But the Baumol, Marris, and O.E. Williamson models of section G, the behavioral models of section H, the Vickers model of section I, the Lintner model of section J, the Leland models of section K, and the Jorgenson, Arrow, and Krouse models of section L — virtually all the 'modern' models described in this chapter — are also very important. All have stimulated further theoretical and empirical work and all are generally recognized as significant contributions to the theory of the firm.

The chapter's discussion of the more important mathematical models of the firm has highlighted each model's essential features, and in particular, how each model treated the objectives of the firm, the role of financial considerations, uncertainty (or lack of it), the nature of equilibrium, and time. By way of summary, tables II-34 and II-35 briefly characterize the distinguishing features of each of the models included in this chapter. Table II-34 classifies the models according to the economic content of each, while table II-35 characterizes them on the basis of the mathematical form of each.

Table II-34 classifies the models discussed in this chapter as (i) traditional, (ii) managerial, (iii) behavioral, or (iv) modern traditional, primarily on the basis of the model's objective function (i.e. the firm's supposed objective). According to the traditional models, the firm's objective is to maximize total profit. The models are single period equilibrium models, the role of finance is ignored, and uncertainty is absent.

Unlike the traditional models, which assume the firm functions primarily for the benefit of the firm's owner-entrepreneurs, the managerial models assume the firm functions primarily for the benefit of the firm's managers. Citing the supposed separation of ownership from control, the managerialists argue that the firm seeks to maximize managerial utility, which different economists express as a function of sales only or the rate of growth of sales only (Baumol); the steady state rate of growth only or growth and the valuation ratio (Marris); staff, emoluments, and discretionary profit (Williamson); the rate of

MODEL	FIRM'S OBJECTIVE[1]	FINANCE[2]	CERTAINTY/ UNCERTAINTY[3]	EQUILIBRIUM/ DISEQUILIBRIUM[4]	TIME[5]
Traditional Models:[6]					
1. Perfect Competition (5, 6)[7] (1 output/2 inputs)	π	S	C	E	S
2. Perfect Competition (23) (n outputs/m inputs)	π	S	C	E	S
3. Perfect Competition (31)	π	S	C	E	S
4. Monopoly (40) (1 output, m inputs)	π	S	C	E	S
5. Duopoly (48)	π	S	C	E	S
6. Oligopoly (kinked demand curve)	π	S	C	E	S
7. Monopolistic Competition (57)	π	S	C	E	S

MODEL	FIRM'S OBJECTIVE	FINANCE	CERTAINTY/ UNCERTAINTY	EQUILIBRIUM/ DISEQUILIBRIUM	TIME
Managerial Models:					
1. Baumol sales (73)	R	S	C	E	S
2. Baumol growth (95)	g (sales)[8]	E	C	E	M
3. Marris (101-105) (growth max)	g[9]	E (debt only)	C	E	M
4. Marris (111) (utility max)	U (growth rate and valuation ratio)[10]	I	C	E	M
5. Williamson(122)	U (staff, emol., discret. profit)	S	C	E	S
6. Herendeen (169)	g (total assets)	E	C	E	M
7. Leland (318)	U (profit and sales)[11]	E	C	E/D	M

Behavioral Models:

In general:	intended as a supplement to, rather than as a substitute for, the conventional theory of the firm				

Modern Traditional Models:

1. Value Maximization (68)	V (dividends)[12]	I	C	E	M
2. Vickers (141) (profit max)	π	E (debt only)[13]	MV[14]	E	S
3. Vickers (151) (value max)	V (net income)	E (debt only)[13]	MV[15]	E	S
4. Lintner (181) (certainty)	P (dividends)[16]	I	MV	E	M
5. Lintner (185) (stable uncertainty)	P (dividends)	I	MV	E	M
6. Lintner (197) (incr. uncertainty)	P (dividends)	I	MV	E	M
7. Leland (213, 214) (quantity-setting)	E[U]	S	TSP	E	S

	MODEL	FIRM'S OBJECTIVE	FINANCE	CERTAINTY/ UNCERTAINTY	EQUILIBRIUM/ DISEQUILIBRIUM	TIME
8.	Leland (213, 227) (price-setting)	E[U]	S	TSP	E	S
9.	Meyer (240) (uncorr. demands)	TV	S	MV	E/MD	S
10.	Meyer (251) (corr. demands)	TV	S	MV	E/MD	S
11.	Leland (256, 259, 261)[17] (role of stock market)	E[U][18]	E (equity only)	TSP	E	S[19]
12.	Jorgenson (273)	V (cash flow stream)	S	C	E	M
13.	Arrow (287)	V (cash flow stream)	S	C	E	M
14.	Wong (303)	V (dividends)	I	C	E/D	M
15.	Krouse (338)	V (dividends)	E (equity only)	C	E	M

NOTES:

1. π = total profit, R = total sales revenue, g = rate of growth, U = managerial utility, V = stock market value of the firm, P = market price of a share of common stock, E[U] = expected utility of total profit, TV = total market value of the firm. With the exception of the behavioral models, all the models are optimization, and more specifically, maximization models. Therefore, only the objective functions, e.g. profits, growth, managerial utility, etc., have been listed.

3. C = certainty, TSP = time-state-preference framework, MV = mean-variance framework.

4. E = characterizes the equilibrium position of the firm, D = in addition to characterizing the short run equilibrium position of the firm, also explores convergence of optimal current operating policies to (long run equilibrium) profit maximization, MD = though the firm is in equilibrium, the market may be in disequilibrium.

5. Treatment of time within the model. S = single period and M = multiperiod.

6. Models are classified as traditional, managerial, behavioral, or modern traditional primarily on the basis of the firm's objective.

7. The number(s) in parentheses refer to the equation number(s) in the text where the model can be found.

8. The firm's objective is the maximization of the rate of growth of sales which, in the steady state situation assumed by Baumol, will also be the rate of growth of the firm's other nonfinancial economic quantities of interest, such as total assets, total profit, etc. However, the firm's money capital is permitted to grow at some other (constant) rate.

9. Steady state growth rate.

10. Managerial utility expressed as a function of the steady state growth rate and the firm's valuation ratio.

11. Actually, the firm maximizes discounted utility, where utility at each point in time is a function of total profit and sales.

12. The stock market value of the firm expressed as the present value of the flow of future dividends.

13. The amount of equity is assumed constant.

14. In the profit maximization version of the model this is subsumed in $r(D)$. See Vickers, The Theory of the Firm, op. cit., ch. 4.

15. In the value maximization version this is subsumed in both $r(D)$ and $\rho(D)$. Ibid., ch. 4.

16. The market price of a share of common stock expressed as the present value of the flow of future dividends (to the holder of that share).

17. Leland's model differs from the other models in that its primary focus is the equilibrium position of the stock market, rather than the equilibrium position of the individual firm in isolation. Out of the stock market equilibrium conditions come the conditions (265) and (266) for each firm's optimal output level.

18. The firm acts in a manner consistent with maximizing the expected utility of each shareholder.

19. Single period optimization.

growth of total assets (Herendeen); or both profit and sales (Leland). Where profit or the market value of the firm is absent from the objective function, it appears as a constraint; present and potential future owners force the firm to maintain some minimum level of profitability (or market value).

Like the traditional models, the managerial models (at least, the ones discussed in this chapter) abstract from uncertainty.[745] But unlike the traditional models, they generally treat finance explicitly and permit external financing of investment, and further they generally recognize the importance of growth to corporate managers and give the firm a dynamic character (even if it is only of a steady state character).

Unlike the other three classes of models listed in table II-34, the behavioral models are not optimization models. These models are intended as a supplement to, rather than as a substitute for, the traditional models. Although profits are important to the firm, total profit is just one of several goals, of which no one single goal is overriding. As indicated in section H, a great many of the behavioral models have been attempts to simulate the workings of real-world firms, and thus, these models have tended to be firm-specific, rather than generalizations of 'typical' firm behavior like the other models in table II-34.

The fourth class of models, the modern traditional models, have been so named because they share the traditional model's view of the firm as an economic entity that functions primarily for the benefit of the firm's owners (rather than its managers). But the modern traditional models differ from the traditional models in at least one of the following important respects:

- The role of the stock market is introduced; since the 'owners' of the firm do not own the firm's assets, but only the shares they hold, the objective of the firm becomes one of maximizing the share price, the stock market value of the firm (expressed alternatively in terms of the dividend stream, the net income stream, or the cash flow stream — all of which lead to the same result when certain conditions discussed in section I are satisfied), or the total market value of the firm.

- Financial considerations are introduced explicitly into the model (in the first version of the Vickers model, which retains the objective of profit maximization, the short term financial (i.e. working capital) needs of the firm are recognized.
- The existence of uncertainty is recognized and incorporated in one of two ways: via either the mean-variance framework or the time-state-preference framework. In the Leland (modern traditional) models the presence of uncertainty, coupled with risk aversion on the part of investors (and managers) leads to a unique formulation of the firm's objective function in terms of expected utility.
- The firm is permitted to grow, though, in contrast to the managerial models, the traditional objective of the firm appropriately modified) is retained.

The modern traditional models are 'modern', then, in the sense that they permit one or more of the restrictive assumptions underlying all the traditional models to be relaxed.

Table II-35 reexamines the models of the firm in a different light, focusing instead on the mathematical form of each. With the exception of the behavioral models, which are normally of the simulation variety and of which only certain component submodels were discussed in section H, each of the models was expressed in the form of either a mathematical programming problem (i.e. a static form) or an optimal control problem (i.e. a dynamic form). But within each of these problem types there are a variety of specific forms, according to the nature of the constraint set, and in the case of the mathematical programming formulation, according to whether the objective function and all the constraints are linear.

The survey of the literature dealing with the theory of the firm presented in this chapter has traced the evolution of that body of economic theory. As indicated by the range of models discussed in this chapter, the models of the firm that have been developed thus far have explored the behavioral implications of a fairly broad range of objectives — traditional, managerial, behavioral, and modern traditional. In the opinion of this writer, the major sources of satisfaction to shareholders, to managers, and to members of other groups within the firm are reflected in the variety of objective functions that have appeared in the literature. Though one may find it advantageous from a modeling standpoint to select a combination of specific objectives different from those already proposed, the need to find a 'more realistic' objective is not, in the opinion of this writer, likely to be one of the potentially more fruitful areas for research within the theory of the firm. Indeed, as suggested in several places in this chapter,[746] the behavioral implications of several different objectives may be virtually indistinguishable (theoretically as well as empirically) due to the contributions of profit toward meeting so many of the alternative objectives that have been proposed.

Table II-35. A Summary Look at the Mathematical Form of the Models of the Firm

MODEL	OPTIMIZATION[1]	LINEAR/ NONLINEAR[2]	CONSTRAINTS[3]	SOLUTION TECHNIQUE[4]
Traditional Models:[5]				
1. Perfect Competition (5, 6)[6] (1 output/2 inputs)	S	NL	N[7]	UN,[8] GLM
2. Perfect Competition (23) (n outputs / m inputs)	S	NL	E	CLM[9]
3. Perfect Competition (31)	S	L	I	S
4. Monopoly (40) (1 output, m inputs)	S	NL	E	CLM[9]
5. Duopoly (48)	S	NL	N[7]	UN[10]
6. Oligopoly (kinked demand curve)	graphical analysis - see Figure II-11			
7. Monopolistic Competition (57)	S	NL	N[7]	UN[11]
Managerial Models:				
1. Baumol sales (73)	S	NL	I	GLM
2. Baumol growth (95)	S	NL	N[12]	UN

3. Marris (101-105) (growth max)	S	NL	I	graphical
4. Marris (111) (utility max)	S	NL	E	UN[13]
5. Williamson (122)	S	NL	N[14]	UN
6. Herendeen (169)	S	NL	E	CLM
7. Leland (318)	D	NL	E	MPC
Behavioral Models:				
In general:	non-optimization simulation models that are directly concerned with the internal workings of the firm components of the models are generally of two types: learning process models (131) and rule-of-thumb models (134)			
Modern Traditional Models:				
1. Value Maximization (68)	D	NL	N	CCV
2. Vickers (141) (profit max)	S	NL	I	CLM[15]
3. Vickers (151) (value max)	S	NL	I	CLM[15]

MODEL	OPTIMIZATION[1]	LINEAR/ NONLINEAR[2]	CONSTRAINTS[3]	SOLUTION TECHNIQUE[4]
4. Lintner (181) (certainty)	S	NL	N	UN
5. Lintner (185) (stable uncertainty)	S	NL	N[16]	UN
6. Lintner (197) (incr. uncertainty)	S	NL	N	UN[17]
7. Leland (213, 214) (quantity-setting)	S	NL	N	UN
8. Leland (213, 227) (price-setting)	S	NL	N	UN
9. Meyer (240) (uncorr. demands)	S	NL	P[18]	CLM[19]
10. Meyer (251) (corr. demands)	S	NL	P[18]	CLM[19]
11. Leland (256, 259, 261) (role of stock market)	S	NL	E[20]	UN[21]
12. Jorgenson (273)	D	NL	E	CCV
13. Arrow (287)	D	NL	E	MPC
14. Wong (303)	D	NL	I	MPC
15. Krouse (338)	D	NL	E	MPD

NOTES:

1. S = static optimization, D = dynamic optimization.

2. L = linear, NL = nonlinear.

3. N = nonnegativity constraints only, E = all equality constraints, I = one or more inequality constraints, P = one or more probabilistic constraints. Since all the models have one or more nonnegativity constraints, the classifications E, I, and P pertain to constraints other than nonnegativity constraints.

4. UN = unconstrained optimization, S = simplex method, CLM = classical Lagrange multipliers, GLM = generalized Lagrange multipliers (Kuhn-Tucker), CCV = classical calculus of variations, MPC = maximum principle — continuous version, MPD = maximum principle — discrete version.

5. Models are classified as traditional, managerial, behavioral, or modern traditional, primarily on the basis of the firm's objective.

6. The number(s) in parentheses refer to the equation number(s) in the text where the model can be found.

7. Equality constraint in the form of the production function eliminated by substitution into the objective function.

8. The optimization is unconstrained when, as is normally assumed, all decision variables are strictly positive at optimality. The problem illustrated in the text employs generalized Lagrange multipliers in order to allow for corner solutions, i.e. the possibility that $L = 0$.

9. In the more general case, generalized Lagrange multipliers would be used if corner solutions were to be permitted.

10. Simultaneous solution of two unconstrained optimization problems. Implicity, the market demand curve for the two firms' product and the potential actions of the rival act as constraints, at least in the economic sense.

11. Unconstrained optimization for determining the short run equilibrium position of the individual firm in isolation; graphical analysis for determining the long run equilibrium of firms in the industry.

12. Equality constraints in the initial formulation (92) eliminated by introducing a constant retention ratio, r, and by substitution into the objective function.

13. This follows from converting problem (111) into an equivalent unconstrained optimization problem, as indicated in footnote 190.

14. Inequality constraint in the original formulation (116) eliminated by rewriting the constraint, by assuming that all decision variables are strictly positive at optimality, and by substituting into the objective function for discretionary profit.

15. The inequality constraint is assumed to hold as an equality at optimality to facilitate the use of classical Lagrange multipliers.

16. By substituting (188) into (186) and by using (186) to substitute for $\hat{g}$ in (185).

17. Solution not obtained directly; only the sensitivity of the share price, P_o, to changes in certain key parameters was discussed. For details of the calculations see Lintner, Maximum Corporate Growth Under Uncertainty, op. cit., pp. 203-210.

18. Probabilistic constraints converted to riskless equivalents by rewriting them in terms of the parameters of the probability distribution for quantity demanded.

19. Under the assumption that all the inequality constraints hold as equalities at optimality.

20. Equilibrium conditions in the portion of the model intended to characterize financial equilibrium.

21. Unconstrained optimization for characterizing production equilibrium — the equilibrium output for each firm. In addition, a system of $(N+1)(M+1)$ equations in $(N+1)(M+1)$ unknowns is set up for the characterization of financial equilibrium.

A potentially more fruitful area for research within the theory of the firm concerns the development of models of the firm under uncertainty — models that would lead, hopefully, to a better understanding of the relationship between the firm's operating decisions and its financial decisions. The traditional models and most of the modern revisions to the theory of the firm have assumed certainty and perfect capital markets. In such an environment the firm's financial policies are separable from its operating policies. Thus, financial considerations can be subsumed within the model, or if the question of financing is to be treated explicitly, new share issues can be treated like negative dividends, and the firm can be treated as if it relied exclusively on internal financing, without any loss of generality. However, as suggested by the Krouse model, the firm's financial policies are not irrelevant to shareholders when capital markets are imperfect. Also, as suggested by Leland's model of the firm in the context of stock market equilibrium, the firm's operating decisions are not separable from its financial decisions (under uncertainty) if markets are incomplete. With these notable exceptions,[747] modeling efforts within the theory of the firm have, in general, been focused on the firm's production and investment decisions. In chapter four of this thesis the relationship between the firm's optimal operating decisions - those relating to production and investment - and its optimal financial decisions - those involving its maintenance of cash balances, its choice of capital structure, and its choice of dividend policy - are examined more closely.

A second potentially fertile area for research within the theory of the firm involves modeling the firm under uncertainty in a multiperiod context. The modern revisions to the theory of the firm have, in general, either introduced uncertainty within a single period context or else extended the certainty model to a multiperiod context, but have not done both. One obstacle standing in the way of accomplishing this extension (of models of the firm) to the behavior of the firm under uncertainty over more than one period is the fact that, under uncertainty, maximizing the stock market value of the firm is not equivalent to maximizing shareholder utility, unless capital markets are both perfect and complete. Hence, specifying the appropriate objective function, at least in the case of the modern traditional models, appears to involve a conceptual problem that is nontrivial. Similarly, the multiperiod managerial models that have included growth as one of the firm's objectives have restricted their attention to steady state growth, probably due to the difficulty of explicitly allowing for non-steady state growth as an objective. Moreover, the managerial models have, in general, abstracted from uncertainty.[748] Thus, while much work has been done both in studying the impact of uncertainty on the behavior of the firm in the single period context and in studying the behavior of the firm under certainty in a multiperiod context, the behavior of the firm under uncertainty in a multiperiod context remains relatively virgin territory for economic modelers. The basic theoretical model developed in chapters three and four, as well as the representative airframe builder model formulated in chapter seven, are both dynamic and stochastic, in contrast to the models discussed in this chapter.

A third potentially productive area for research concerns the internal organization of the firm and the impact that organizational form and conflicts among the social groupings that comprise the firm may have on the firm's performance (relative to its objective(s)).

With the exception of the O.E. Williamson model, in which managers' preferences for staff could affect overall operating efficiency, and the behavioral models, which are mainly concerned with what goes on inside the firm, the models discussed in this chapter treat the firm like a black box, abstracting from what goes on inside the firm. With few exceptions, the models of the firm to date have focused their attention on external allocation questions - e.g. how much labor it is optimal for the firm to hire and how much output it is optimal for the firm to produce - to the exclusion of internal allocation questions - e.g. how should the firm's (limited) supply of money capital be (optimally) allocated among its operating divisions. Yet, these internal allocation questions may have important implications regarding how efficiently the economy, as well as the firm, functions.[749] The role of organizational factors, and in particular, how the degree of organizational slack may vary over the business cycle and how decentralization may adversely affect internal control and X-efficiency, are explored below in chapter five.

CHAPTER TWO FOOTNOTES

1. If one or more of the mathematical solution techniques listed in table II-35 are unfamiliar, the references given in footnotes 71 and 81 of chapter one will provide the necessary background.

2. See R. Frisch, "On the Notion of Equilibrium and Disequilibrium," Review of Economic Studies (vol. 3; 1935-36), pp. 100-105; Hicks, Value and Capital, op. cit., ch. 10; Samuelson, Foundations of Economic Analysis, op. cit., ch. 9; and J.S. Chipman, "The nature and meaning of equilibrium in economic theory," in D. Martindale, ed., Functionalism in the Social Sciences: The Strengths and Limits of Functionalism in Anthropology, Economics, Political Science and Sociology: A Symposium (American Academy of Political and Social Science; 1965), pp. 35-64.

3. In traditional models a distinction is made between the short run, in which the firm's capital stock is fixed, and the long run, in which the capital stock is variable. Whether the time span involved is the short run or the long run, the traditional model is still a single period model — even though the firm's capital stock is constant in the first case and variable in the second.

4. It is tempting to call single period models 'static' and multiperiod models 'dynamic'. However, the steady state growth models are multiperiod models in which all quantities grow at the same rate and hence remain in fixed relation to one another until the equilibrium is disturbed. For this reason such models can be formulated as static optimization problems, as pointed out in chapter one. Therefore, to avoid confusion, in this paper the terms static and dynamic will be reserved for describing mathematical optimization techniques.

5. See Hicks, Value and Capital, op. cit., ch. 10.

6. See Samuelson, Foundations of Economic Analysis, op. cit., ch. 9. An equilibrium is said to be stable if "a displacement from equilibrium is followed by a return to equilibrium." Ibid., pp. 261-262.

7. Marris, Theories of Corporate Growth, op. cit., p. 13.

8. There is a fourth financial statement, the statement of changes in financial position, that is a major source of financial information, though, for the purposes of this chapter, only the information provided in the three financial statements mentioned in the text will be needed.

9. A fuller discussion can be found in financial accounting textbooks, such as Welsch and Anthony, op. cit., ch. 3.

10. The financial statements illustrated in tables II-1, II-2, and II-3 are for typical manufacturing firms, since this is the type of business firm with which the theory of the firm is concerned.

11. For this reason, the balance sheet is also called the 'statement of financial position.'

12. It should be pointed out that there are several methods accepted by professional accountants for figuring depreciation and that the particular method selected by a firm often depends on tax considerations. For a comparative discussion of these methods see ibid., ch. 9.

13. The characteristics of preferred stock are described in ibid., ch. 12.

14. For this reason, the income statement is also called the 'profit and loss statement.'

15. This interest expense does not include dividends on preferred stock, since such dividends are not deductible for tax purposes.

16. A synonym for 'profit' that is widely used in the business literature is 'earnings.'

17. It is the author's conjecture that the balance sheet and income statement, as provided by the accountant, would be identical to those provided by the economist if the economy were in general equilibrium and if, in addition, markets were perfect (see footnote 126 for a definition of a perfect market) and complete (see footnote 430 and subsection 1a of section K for a definition and discussion of the meaning of complete markets) and the firm rented all its capital. One possible approach to demonstrating this result would be first to hypothesize a disequilibrium situation and to specify the accounting and economic balance sheets and income statements and then to demonstrate the 'convergence' of the two sets of financial statements as the economy approached general equilibrium. It is recognized by the author that it might be difficult to accomplish this without making the result dependent on just how the initial disequilibrium was specified.

18. Finance has no explicit role to play in the traditional models of the firm since, in a world of certainty, profit maximization provides a complete description of the behavior of the firm. Indeed traditional economic models of the firm are complete without acknowledging the existence of the stock market. See H.E. Leland, "Production Theory and the Stock Market," Bell Journal of Economics and Management Science (vol. 5, no. 1; Spring 1974), pp. 125-144. Though finance is not treated explicitly in these models, it is not ignored. Rather, it is subsumed within the general equilibrium analysis of a market economy since, in general equilibrium, there exist only those firms that are capable of financing their operations (with or without the stock market). See Arrow, The Firm in General Equilibrium Theory, op. cit., pp. 68-110.

19. Machlup, op. cit., p. 9.

20. A useful survey of the literature dealing with the price behavior of firms is provided in A. Silberston, "Price Behavior of Firms," Economic Journal (vol. 80; no. 319; September 1970), pp. 511-582.

21. See A.A. Walters, "Production and Cost Functions: An Econometric Survey," Econometrica (vol. 31; no. 1-2; January-April 1963), pp. 1-66, and D.W. Jorgenson, "The Theory of Investment Behavior," in R. Ferber, ed., Determinants of Investment Behavior (Columbia University Press; New York; 1967), pp. 133-135.

22. A now classic synthesis of the neoclassical (or traditional) theory of the firm was provided by Samuelson. See P.A. Samuelson, Foundations of Economic Analysis (Harvard University Press, Cambridge, Mass.; 1947), chs. 3-4,8.

23. Economists often distinguish pure competition from perfect competition. Of the following five conditions, (i) through (iv) define what is called *pure competition* and the addition of condition (v) gives *perfect competition*. The difference between the two is that 'pure' refers to the market structure, whereas 'perfect' refers to the nature of the information that is available to market participants. For the purposes of this paper this distinction is not of critical importance, though it should be remembered that perfect competition requires that all five conditions hold. Condition (v) is important because in the real world imperfect information is one of the major sources of uncertainty.

24. The stock market and certain agricultural markets are often cited as examples, but due to the increasing influence of large institutional investors in the stock market and the keen cost competition and widening role of cooperatives in agriculture, neither fits the competitive model exactly.

25. W.J. Baumol, "Models of Economic Competition," in P. Langhoff, ed., Models, Measurement and Marketing (Prentice-Hall, Englewood Cliffs, N.J.; 1965), pp. 143-168. An allocation of resources is said to be 'Pareto optimal' "if production and distribution cannot be

reorganized to increase the utility of one or more individuals without decreasing the utility of others." See J.M. Henderson and R.E. Quandt, Microeconomic Theory: A Mathematical Approach, 2nd ed. (McGraw-Hill; New York; 1971), ch. 7, for a mathematical statement of the conditions required for Pareto optimality, and in particular, p. 255, from which the quote was drawn.

26. Excellent mathematical expositions of the model of the firm under perfect competition can be found in Henderson and Quandt, op. cit., ch. 3, and in Intriligator, op. cit., ch. 8. An equally good, though geometric, presentation can be found in Herendeen, op. cit., ch. 4.

27. Note that the MP_L curve intersects the AP_L curve at the maximum point on the latter. This is easily proven. To find the maximum value of AP_L set

$$\frac{\partial}{\partial L}(AP_L) = \frac{L \cdot \partial f/\partial L - f(L,K)}{L^2} = \frac{L \cdot MP_L - Q}{L^2} = 0 .$$

Then $L \cdot MP_L - Q = 0$, or $MP_L = \frac{Q}{L} = AP_L$. Similarly, if K were varied while L were held fixed, the same argument would show that MP_L and AP_L could be replaced by MP_K and AP_K, respectively, and L could be replaced by K in figure II-1.

28. A stronger assumption is that the production function is concave. See footnote 30.

29. Some textbooks give the isoquants a shape different from that shown in figure II-2 in order to show that, when one input is fixed and the other is varied, eventually the law of diminishing marginal productivity will cause the marginal productivity of the variable input to become negative, in which case increased use of that input will cause the level of output to fall. Since a rational producer would stop adding amounts of the variable input once its marginal productivity fell to zero, figure II-2 has been drawn to show only the feasible region of production. An example of the more general shape is provided in Henderson and Quandt, op. cit., p. 60.

30. In addition, it is normally assumed that the production function is concave (and sometimes it is assumed to be strictly concave), at least over some portion of the domain of the function, which requires that $\partial^2 f/\partial L^2 \le 0$, $\partial^2 f/\partial K^2 \le 0$, and

$$\det\begin{pmatrix} \frac{\partial^2 f}{\partial L^2} & \frac{\partial^2 f}{\partial K \partial L} \\ \frac{\partial^2 f}{\partial L \partial K} & \frac{\partial^2 f}{\partial K^2} \end{pmatrix} = \frac{\partial^2 f}{\partial L^2} \cdot \frac{\partial^2 f}{\partial K^2} - \left(\frac{\partial^2 f}{\partial L \partial K}\right)^2 \ge 0 ,$$

with strict inequalities holding when strict concavity is assumed. It should be noted that (strict) concavity of the production function implies (strict) convexity of the isoquants.

31. The basic reference is H.W. Kuhn and A.W. Tucker, "Nonlinear Programming," in J. Neyman, ed., Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability (University of California Press; Berkeley,; 1951), pp. 481-492. The conditions can be found in any nonlinear programming text and in most mathematical economics texts. See, for example, Intriligator, *op. cit.*, ch. 4. The economic content of the Kuhn-Tucker theorem, and more generally, the economic interpretation of the dual to a nonlinear programming problem, are discussed in M.L. Balinski and W.J. Baumol, "The Dual in Nonlinear Programming and its Economic Interpretation," Review of Economic Studies (vol. 35, no. 103; July 1968), pp. 237-256. For problems (5) and (6) a simpler solution procedure is to assume $L > 0$ and $K > 0$, which is almost always the case, and solve these problems as unconstrained problems.

32. Since only the case in which $L > 0$ is of interest, in what follows it is assumed that $L > 0$, i.e. that the firm hires some labor.

33. Strictly speaking, equation (11) holds provided $MP_L \neq 0$. But note from (8) that, if L, p, and w are strictly positive, then $MP_L > 0$. When the optimal amounts of the variable inputs are used, it is generally the case that the marginal productivity of each is strictly positive. An exception to this rule would be the case in which the wage rate w were zero while the price p were positive. From (8), $MP_L = 0$, and the firm would hire labor until its marginal productivity fell to zero, i.e. with given capital stock, the firm would maximize total output. However, in the context of a general equilibrium model of a market economy, a zero price for any factor of production is highly unlikely. Therefore, in what follows qualifications like 'provided $MP_L \neq 0$' will be added only when the conditions underlying the model make it necessary to consider explicitly the possibility of the denominator being zero at optimality.

34. Problem (17) could also be solved with the aid of Lagrange multipliers to obtain the same result.

35. This result is easily demonstrated. Average total cost, AC, is given by $AC = \frac{C(Q)}{Q}$, and its derivative is $AC' = (Q \cdot C'(Q) - C(Q))/Q^2$. Since $Q^2 > 0$, it follows that the sign of AC' is determined by the sign of the numerator. Thus, $AC' < 0$ when $Q \cdot C'(Q) - C(Q) < 0$, which is equivalent to $C'(Q) < \frac{C(Q)}{Q}$, or $MC < AC$, and by the same argument, $AC' > 0$ when $MC > AC$. Also, $MC = AC$ when the latter is minimized.

36. In order that the firm earn maximum profit, it is necessary that $p = SRMC = LRMC$, for if $p \neq SRMC$, the firm can increase total profit by changing the level of output, and if $p \neq LRMC$, it can increase total profit by altering its scale of operations, K. In order that it earn zero profit, it is necessary that $p = SRAC = LRAC$ (since price equals average revenue and zero profit implies that average revenue equals average cost).

37. It is also normally assumed that the production function is written so that its partial derivatives for all outputs are usually positive and its partial derivatives for all inputs are usually negative, and these same assumptions are made here.

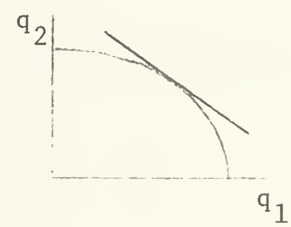
38. Note that this is really the long run problem since none of the inputs is fixed. For the short run problem, there would have to be an additional set of constraints on the inputs,

$$g_i(x_1,\ldots,x_m) \leq b_i \quad , \quad i = 1,\ldots,s \ ,$$

to reflect the s inputs that are in fixed supply. In solving this larger problem, one additional Lagrange multiplier would have to be introduced for each such constraint. An alternative exposition of the long run problem in terms of minimizing average total cost yields the same basic results. See E. Silberberg, "The Theory of the Firm in 'Long-Run' Equilibrium," American Economic Review (vol. 64; no. 4; September 1974), pp. 734-741.

39. It is also required that $q_i \geq 0$, for all i, and that $x_j \geq 0$, for all j, in which case the nonnegativity constraints $\overline{q} \geq \overline{0}$ and $\overline{x} \geq \overline{0}$ should be appended to problem (23). Solving this larger problem would require the application of the Kuhn-Tucker conditions. This is unnecessary, however, if attention is focused on nontrivial solutions (i.e. those in which all variables are strictly positive at optimality). It should be noted that the possibility of a zero value for one or more decision variables might have to be considered if the problem involves a company or one of its divisions that might decide not to use an input or not to produce an output — that it would, under different conditions, use or produce, respectively — during the particular period under consideration. For example, in the case of a seasonal good, it might prove convenient to leave such an output in the production function and to let the relevant decision variable assume a zero value when the good is 'out of season.'

40. In the simple case of two outputs, the production function in implicit form is $F(q_1,q_2,x_1,\ldots,x_m) = 0$. When all inputs are held fixed, the production function defines a locus of output combinations called a product transformation curve, the shape of which is bowed out from the origin, as in the figure to the right. The reason for this is that, when inputs are used with maximum technical efficiency, more of one output can be obtained only if



some of the other is sacrificed. As long as factor proportions are unequal for the two goods, obtaining more of q_1 required that successively larger amounts of q_2 be sacrificed. The slope of the tangent to the curve at any point is negative, and the negative of this slope is called the rate of product transformation. The macroeconomic analogue of the product transformation curve is the production possibilities curve, and in introductory economics textbooks this curve typically appears with guns in place of q_1 and butter in place of q_2, illustrating the familiar guns vs. butter choice that confronts society.

41. The second order conditions for profit maximization require that the bordered Hessian determinants

$$\begin{vmatrix} \lambda F_{11} & \lambda F_{12} & F_1 \\ \lambda F_{21} & \lambda F_{22} & F_2 \\ F_1 & F_2 & 0 \end{vmatrix}, \quad \cdots, \quad \begin{vmatrix} \lambda F_{11} & \cdots & \lambda F_{1,n+m} & F_1 \\ \vdots & & \vdots & \\ \lambda F_{n+m,1} & \cdots & \lambda F_{n+m,n+m} & F_{n+m} \\ F_1 & \cdots & F_{n+m} & 0 \end{vmatrix}$$

alternate in sign with the first determinant being positive, where $F_i = \partial F/\partial q_i$, $1 \leq i \leq n$; $F_{n+j} = \partial F/\partial x_j$, $1 \leq j \leq m$; and where the F_{ij} are the second order partial derivatives of F. See Henderson and Quandt, op. cit., pp. 96, 404-406. In problems such as (23), the second order conditions are seldom stated since only the first order conditions convey what is generally considered useful economic information.

42. A. Hunter, Competition and the Law (Allen & Unwin; London; 1966), ch. 2.

43. J.A. Schumpeter, Capitalism, Socialism and Democracy (Harper and Row; New York; 1942), chs. 7-8. Schumpeter offers an elegant argument against the acceptance of perfect competition as a goal of government antitrust policy — an argument that is as relevant today as it was when it was first presented.

44. See F.M. Scherer, "Market Structure and the Employment of Scientists and Engineers," American Economic Review (vol. 57; no. 3; June 1967), pp. 524-531, and W.S. Comanor, "Market Structure, Product Differentiation, and Industrial Research," Quarterly Journal of Economics (vol. 81; no. 4; November 1967), pp. 639-657.

45. Baumol, Models of Economic Competition, op. cit., p. 155.

46. The classic references for this topic are R. Dorfman, Application of Linear Programming to the Theory of the Firm (University of California Press; Berkeley; 1951); R. Dorfman, P.A. Samuelson, and R.M. Solow, Linear Programming and Economic Analysis (McGraw-Hill; New York; 1958); and K.E. Boulding and A.W. Spivey, eds., Linear Programming and the Theory of the Firm (Macmillan; New York;

1960). Two more recent texts are C. Kwang and Y. Wu, *Mathematical Programming and Economic Analysis of the Firm* (International Textbook Company; London; 1971) and D.C. Vandermeulen, *Linear Economic Theory* (Prentice-Hall; Englewood Cliffs, N.J.; 1971), both of which do an excellent job of explaining the mathematical and economic relationship between the linear and nonlinear formulations.

47. For a discussion of the more general case, in which there are n outputs and a choice of processes for producing each, see Vandermeulen, *op. cit.*, ch. 11.

48. The term profit is being used rather loosely here to mean what the accountant calls 'contribution margin', which is the difference between the price of the item and its variable costs. Since price is taken as given under perfect competition and since unit variable costs normally remain roughly constant over some range of output, the assumption that c_i is constant is reasonable. In a practical problem, if the c_i's were not known with certainty, then sensitivity analysis could be performed to deal with this uncertainty.

49. The theory of linear programming, and in particular, the extreme point theorem and its proof, can be found in G. Hadley, *Linear Programming* (Addison-Wesley; Reading, Mass.; 1962).

50. There may be more than $n-m$ q_i's at the zero level if the solution to problem (31) is degenerate.

51. Indeed, it can be shown that the Kuhn-Tucker conditions, when applied to problems (31) and (32), yield the simplex optimality conditions.

52. The notion of a process, or activity, is discussed in R. Dorfman, "Mathematical, or 'Linear' Programming: A Nonmathematical Exposition," *American Economic Review* (vol. 43; no. 5; December 1953), pp. 797-825.

53. To see this, note that, if process 1 is used to produce 5α units of output and process 2 is used to produce $5(1-\alpha)$ units of output, where $0 < \alpha < 1$, then process 1 requires 3α units of K and α units of L while process 2 requires $2(1-\alpha)$ units of K and $2(1-\alpha)$ units of L. But since the coordinates of point A are $\binom{1}{3}$ and the coordinates of B are $\binom{2}{2}$, the input combination $\alpha\binom{1}{3} + (1-\alpha)\binom{2}{2}$ lies on the line segment joining A and B. The validity of this simple linear relationship rests on the assumption that the processes are operated independently, so that two or more processes can be used simultaneously without any one process generating favorable or unfavorable effects on any other.

54. The vertical line beginning at A and the horizontal line beginning at C are technically part of the isoquant, though no rational producer would ever use any of those input combinations since in each case some amount of one of the inputs is being wasted. Thus, the feasible region of production lies between the rays extending from the origin through the points A and C .

55. Vandermeulen, op. cit., pp. 4, 171-172.

56. Ibid., pp. 171-172.

57. The solution to the linear programming problem is integer-valued. Were this not the case, integer programming could be employed to obtain the optimal integer-valued solution. See R.S. Garfinkel and G.L. Nemhauser, Integer Programming (Wiley; New York; 1972).

58. See W.J. Baumol and R.E. Quandt, "Dual Prices and Competition," in A.R. Oxenfeldt, ed., Models of Markets (Columbia University Press; New York; 1963), pp. 237-264.

59. The connection between 'monopoly power' and the term 'market power' defined earlier is the following. A firm may be able to influence the prices of its outputs or the prices it pays for its inputs (or it may be able to influence both output prices and input prices). Its ability to influence output prices is called monopoly power; its ability to influence input prices will be defined below (see footnote 63) as monopsony power; and monopoly power and monopsony power are referred to collectively as market power.

60. References for this section are Intriligator, op. cit., pp. 201-205, and Henderson and Quandt, op. cit., pp. 206-222, the latter of which deals with related issues such as the discriminating monopolist, who sells the same product at different prices in different markets, and the multiple-plant monopolist, who must decide how to allocate his production among his plants. It should be noted that factor markets, as well as product markets, can, in theory at least, be monopolistic, although this subsection is concerned with monopoly in a product market, and specifically, with the behavior of the single producer serving that market.

61. Note that (34), rather than the demand function (33), has been graphed in figure II-8. That is, the coordinate axes have been 'reversed'. This is the convention economists have adopted in order to be able to graph the demand function on the same set of axes as the marginal cost and average cost functions, in both of which quantity, q , is the independent variable.

62. The demand curve and the marginal revenue curve have the same p-intercept when the demand function is linear. This follows from the fact that $f(q)$ is of the form $p = a + bq$, $b < 0$, the total revenue function is $R(q) = p \cdot q = (a + bq) \cdot q = aq + bq^2$, and the marginal revenue function is $MR(q) = a + 2bq$. Note that the slope of MR is twice the slope of DD in absolute value.

63. When a firm has some influence over its input prices it is said to possess some degree of monopsony power, and if it is the sole buyer of some input, the market for that input is said to be a monopsony. In what follows it will be assumed that the monopolist is the sole purchaser of each input, so that the price of each can be expressed as a function of his output only. At the opposite extreme, the factor markets are perfectly competitive so that the price of each input is taken as given and r_j is constant, $j = 1,\ldots,m$. As a result, the optimality conditions for the case in which factor markets are perfectly competitive would be somewhat simpler in form than those presented below.

64. This is the long run problem since there are no restrictions placed on resource availabilities. The short run problem is only slightly more complicated, and so, is not treated separately here.

65. If the factor markets were perfectly competitive, then MC_j would equal r_j in equation (44).

66. Indeed, (46) is the general rule for profit maximization. Under perfect competition, $P = MR$ and, therefore, (12) is equivalent to (46).

67. If MC intersects MR in two places, then the optimum output is determined by the point of intersection between MR and MC at which the latter is rising. The easiest way to see this is to view problem (40) as the composite of two problems, the first of which provides a total cost function $C(q)$ (as described in subsection 1 of this section) and the second of which involves the unconstrained optimization problem: maximize $\pi = R(q) - C(q)$. The first order condition for this problem is $R'(q) - C'(q) = 0$, or $MR = MC$, and the second order condition is $R''(q) - C''(q) < 0$, or $R''(q) < C''(q)$, which is satisfied if MR is decreasing and MC is increasing.

68. For more on this second point see Schumpeter, op. cit., chs. 7-8. It should be noted that the development of a new product does not necessarily confer monopoly status on its developer. Whether or not it does depends on the extent to which existing products can serve as substitutes for the new product. Note that in both cases mentioned in the text a welfare argument is involved. In the first case, the welfare issue concerns the question of whether the improved productive efficiency that is possible under (regulated) monopoly outweighs (in the sense of producing a higher level of social welfare) the potential loss of operating efficiency that results from the removal of effective competition in the firm's product market. In the second case, the welfare issue concerns the question of whether temporary monopoly, as conferred by patents and copyrights, provides an incentive to innovate that causes (social) benefits to be generated that exceed the (social) costs associated with having temporarily just a single producer of the new product.

69. For example, see Galbraith, The New Industrial State; op. cit., pp. 181-183; Scherer, Industrial Market Structure and Economic Performance, op. cit., ch. 2; and P.A. Samuelson, Economics, 9th ed. (McGraw-Hill; New York; 1973), chs. 25-26.

70. See Scherer, Industrial Market Structure and Economic Performance, op. cit., ch. 20, and Steiner, op. cit., ch. 7.

71. The classic reference is A. Cournot, Researches into the Mathematical Principles of the Theory of Wealth (first published in 1838), translated by N.T. Bacon (A.M. Kelley; New York; 1960), which also represents one of the earliest applications of mathematics to economics. A brief history of the development of duopoly theory is given in J.R. Hicks, "Annual Survey of Economic Theory: The Theory of Monopoly," Econometrica (vol. 3; January 1935), pp. 12-16. An excellent treatment of the Cournot model is provided in Intriligator, op. cit., pp. 205-213.

72. This assumption requires that the factor markets be perfectly competitive. If the firms are able to influence input prices as well as output prices, then the problem must be reformulated to take this into account. See Intriligator, op. cit., pp. 205-208.

73. In the somewhat more general version of the model, in which input prices are a function of the combined amounts used by the two firms, there would also be a pair of conjectural variations for each input, which would be given interpretations similar to the interpretations given to dq_2/dq_1 and dq_1/dq_2 .

74. Similarly, the second order conditions would be of the same form as in the monopoly case, namely, that, at the optimum marginal revenue must be increasing less rapidly than marginal cost.

75. See J.W. Friedman, "Reaction Functions and the Theory of Duopoly," Review of Economic Studies (vol. 35; no. 3; July 1968), pp. 257-272, and Intriligator, op. cit., pp. 208-210. See also the additional references listed in footnote 87.

76. The details of the calculations are the following:

$$\pi_1 = q_1(a - b(q_1+q_2)) - cq_1 - d$$
$$\pi_2 = q_2(a - b(a_1+q_2)) - cq_2 - d$$
$$\left.\begin{aligned} d\pi_1/dq_1 &= a - b(q_1+q_2) - bq_1 - c = 0 \\ d\pi_2/dq_2 &= a - b(q_1+q_2) - bq_2 - c = 0 \end{aligned}\right\} \; q_1 = q_2 = \frac{a-c}{3b} \; .$$

Zero conjectural variations are reflected in the fact that $dq_2/dq_1 = 0$ in the calculation of $d\pi_1/dq_1$, and $dq_1/dq_2 = 0$ in the calculation of $d\pi_2/dq_2$.

77. See J.W. Friedman, op. cit., p. 259, for more on this point. A recent paper that permits learning in the context of duopoly is R.M. Cyert and M.H. DeGroot, "An Analysis of Cooperation and Learning in a Duopoly Context," American Economic Review (vol. 63; no. 1; March 1973), pp. 24-37.

78. J.W. Friedman, *op. cit.*, p. 259. See also W. Fellner, *Competition Among the Few* (Alfred A. Knopf; New York; 1949), p. 57, for a discussion of the contradiction implied by the behavioral assumption of zero conjectural variations.

79. *Ibid.*, pp. 259-261. As Friedman points out, the Cournot model shares a limitation common to all traditional models, namely, that the individual firm is assumed to maximize single period profits. In his paper Friedman makes the duopolist less — to use his words — 'short sighted' by having it maximize a discounted stream of profits.

80. H. von Stackelberg, *The Theory of the Market Economy*, translated by A.T. Peacock (Oxford University Press; New York; 1952).

81. For example, in the linear case if firm 2 is the leader and firm 1 is the follower, then

$$\left.\begin{aligned} d\pi_1/dq_1 &= a - b(q_1+q_2) - bq_1 - c = 0 \\ d\pi_2/dq_2 &= a - b(q_1+q_2) - bq_2 + \tfrac{1}{2}bq_2 - c = 0 \end{aligned}\right\} \quad \begin{aligned} q_1 &= \frac{a-c}{4b} \\ q_2 &= \frac{a-c}{2b} \end{aligned} ,$$

from which it follows that

$$\pi_1 = \frac{(a-c)^2}{16b} - d \quad \text{and} \quad \pi_2 = \frac{(a-c)^2}{8b} - d ,$$

as compared with $\pi_1 = \pi_2 = \dfrac{(a-c)^2}{9b} - d$ under the Cournot solution. If firm 1 is the leader and firm 2 is the follower, the results are reversed.

82. $$\left.\begin{aligned} d\pi_1/dq_1 &= a - b(q_1+q_2) - bq_1 + \tfrac{1}{2}bq_1 - c = 0 \\ d\pi_2/dq_2 &= a - b(q_1+q_2) - bq_2 + \tfrac{1}{2}bq_2 - c = 0 \end{aligned}\right\} \quad \begin{aligned} q_1 &= q_2 = \frac{2(a-c)}{5b} \\ \pi_1 &= \pi_2 = \frac{2(a-c)^2}{25b} - d . \end{aligned}$$

83. An alternative explanation of this situation is offered in the next section with the aid of game theory. Stated briefly, this situation illustrates the Prisoners' Dilemma game.

84. In the linear case considered above, conjectural variations in the von Stackelberg analysis are either 0 or $-\tfrac{1}{2}$.

85. In the linear case, joint profit $\pi_1 + \pi_2$ can be maximized even if the producers act independently of one another provided $dq_2/dq = dq_1/dq_2 = 1$, that is, provided each producer believes that its rival will match exactly its changes in output.

86. J.W. Friedman, *op. cit.*, pp. 259-261.

87. R.M. Cyert and M.H. DeGroot, "Multiperiod Decision Models with Alternating Choice as a Solution to the Duopoly Problem," *Quarterly Journal of Economics* (vol. 84; no. 3; August 1970), pp. 410-429, and J.W. Friedman, *op. cit.*, pp. 257-272, treat duopoly as a multiperiod maximization problem. Cyert and DeGroot,

An Analysis of Cooperation and Learning in a Duopoly Context, op. cit., pp. 24-37, extends the results of the two earlier papers by permitting the duopolists to alter their behavior and their strategies, in effect permitting duopolists to undergo a learning process through time that leads to increased cooperation and increased profits for both.

88. J.W. Friedman, op. cit., p. 259, elaborates on this point.

89. J. von Neumann and O. Morgenster, Theory of Games and Economic Behavior, 2nd ed (Princeton University Press; Princeton, N.J.; 1947). See also M. Shubik, Strategy and Market Structure, Competition, Oligopoly and the Theory of Games (Wiley; New York; 1959), which develops a theory of games of economic survival that the author applies to oligopoly. A convenient summary of game theory is given in Intriligator, op. cit., ch. 6.

90. This result is known as the minimax theorem and can be proved in several ways. See D. Gale, H.W. Kuhn, and A.W. Tucker, "Linear Programming and the Theory of Games," in T.C. Koopmans, ed., Activity Analysis of Production and Allocation, Cowles Monograph 13 (Wiley; New York; 1951). The theorem requires that players be rational in the sense that each tries to maximize the expected value of its payoffs. Then under this assumption and the two assumptions stated in the text, the minimax theorem asserts that there is a solution in the sense that even if one player knows beforehand that the other player will use its optimal strategy — and even if it knows what that other player's optimal strategy is — the first player cannot improve its position by altering its strategy (in the case of mixed strategies, knowledge of the other player's optimal probabilities would not help the first player, though, of course, knowing which pure strategy was going to be adopted on any one play might help the first player.

91. Two exceptions are the following: if the firms set market share as the sole objective or if the firms engage in games of ruin with the winner enjoying complete control of the market. Neither situation appears to be common, however. See Scherer, Industrial Market Structure and Economic Performance, op. cit., p. 141.

92. Strategies could also be specified in terms of output levels, or, if the products were somehow differentiated, so that the producers could charge different prices, the strategies could be defined in terms of prices. These other approaches might require that more than two strategies be specified.

93. The upper left-hand entry is the Cournot equilibrium and the lower right-hand entry is the von Stackelberg disequilibrium. The various expressions for each firm's profit level in each situation are given in footnotes 81 and 82 above.

94. This is not to imply that all nonzero-sum games lead to the Prisoners' Dilemma. Rather, it is merely intended to show what can happen in the context of a nonzero sum game, namely, that the strategies the two players will adopt are not uniquely determined, but depend on additional factors not included in the payoff matrix. The connections between duopoly and the Prisoners' Dilemma are described in more detail in F.T. Dolbear, et. al., "Collusion in the Prisoners' Dilemma: Number of Strategies," Journal of Conflict Resolution (vol. 13; June 1969), pp. 252-261.

95. If they were able to agree to maximize joint total profit, then they would earn combined profit of 8600 , and if this were divided equally between them, each would receive 4300 . In the linear case it is easy to show that this same result could be achieved if each conjectural variation were equal to one.

96. Note that while minimax is an unbeatable strategy for zero-sum games, it does not necessarily yield the optimal strategy in the nonzero-sum context. In figure II-10 the minimax strategy for each firm is to act as a leader, which leads to lower profit for both than could be achieved through cooperation.

97. In addition to direct evidence of cooperative behavior in the real business world (for example, see D. McClintick, "Gypsum Trial Shows How Price-Fix Plan Supposedly Operated," Wall Street Journal (October 3, 1975)), there is some evidence based on simulated oligopoly games played by humans that such a learning process does take place and that over time the players adjust their behavior toward the cooperative (i.e. joint profit maximizing) solution. See D.H. Stern, "Some Notes on Oligopoly Theory and Experiments", in M. Shubik, ed., Essays in Mathematical Economics (Princeton University Press; Princeton, N.J.; 1967), pp. 255-281.

98. As with these other analytical approaches, such as mathematical programming, generally applicable characterizations have been forthcoming only in the limiting cases of monopoly and perfect competition. This is not meant to imply that on these grounds the game theoretic approach should be eschewed. Not only has game theory provided useful insights, but the work of Shubik and Dolbear et. al., as well as Marris (see Marris, The Modern Corporation and Economic Theory, op. cit., pp. 283-304), could lead to further significant developments. Indeed, just the recognition that behavioral strategies can vary under oligopoly, thereby requiring a careful analysis of any particular oligopolistic situation before outcomes can be predicted, is, in the opinion of this writer, itself a significant step forward.

99. For example, see Galbraith, The New Industrial State, op. cit., ch. 16.

100. Ibid., p. 180.

101. On the first point, while it is recognized that these firms enjoy substantial power, this power is not without limits, as the major aluminum producers were recently forced to recognize. See "The price war in aluminum," Business Week (November 17, 1975) and J.W. Winski, "How a Long Price War Dragged On and Hurt Chicago Food Chains," Wall Street Journal (July 19, 1976). On the second point, there has been much debate as to whether the short run reactions of rivals are taken into account or whether producers are only concerned about long run implications. Baumol argues that firms ignore the day-to-day behavior of rivals and consider long run implications only. See W.J. Baumol, Business Behavior, Value and Growth, op. cit., pp. 27-33. This writer agrees with Baumol for the reasons that are discussed below in section H. Briefly, the time and expense that would be required to consider rivals' reactions on a day-to-day basis would be prohibitive, and such expense can only be justified on major (long run) decisions such as launching a new product or constructing a new plant.

102. The Cournot analysis is easily extended to n firms, $n > 2$, by first setting up the n profit equations, then by assuming zero conjectural variations for all firms and differentiating each profit equation with respect to each firm's level of output, and finally by solving the resulting n equations in n unknowns. For example, if there are n firms, each with cost function given by (53), and if the market demand function is linear as in (54), but with $\sum_{i=1}^{n} q_i$ in place of q_1+q_2, then the profit maximizing firms will each produce $q_i = \frac{a-c}{(n+1)b}$ units of output. Similarly, the von Stackelberg analysis could be generalized by allowing for various combinations of leaders and followers.

103. One of the classic works in this regard is Fellner's model, which followed along the lines originally suggested by E.H. Chamberlin, who, though he did not fully develop a theory of oligopoly in his classic The Theory of Monopolistic Competition, did recognize and appreciate the role of oligopolistic interdependence and who did suggest oligopoly as a separate significant type of market structure. See E.H. Chamberlin, The Theory of Monopolistic Competition, 8th ed. (Harvard University Press; Cambridge, Mass.; 1962), ch. 3. Fellner suggests that an oligopolistic group will attempt to maximize their joint profit, but that, due to each oligopolist's unwillingness to surrender complete sovereignty for the sake of joint profit maximization, the conflicting motives of group members will inhibit this collective effort. Fellner, op. cit. The difficulty of maintaining collusive agreements is also discussed in G.J. Stigler, "A Theory of Oligopoly," Journal of Political Economy (vol. 72; no. 1; February 1964), pp. 44-61, reprinted under the same title in G.J. Stigler, The Organization of Industry (Irwin; Homewood, Ill.; 1968), ch. 5, in which the importance of the number of buyers is brought out. Other noteworthy approaches to modeling oligopolistic behavior include 'dominant firm' price leadership,

in which the price leader alters the price when it judges the climate to be right (see J.W. Markham, "The Nature and Significance of Price Leadership," American Economic Review (vol. 41; no. 5; December 1951), pp. 891-905); 'barometric' price leadership, in which the price leader adjusts price when it considers the climate to be right and in which its competitors signal their approval or disapproval by following or not following, respectively (see G.J. Stigler, "The Kinky Oligopoly Demand Curve and Rigid Prices," Journal of Political Economy (vol. 55; no. 5; October 1947), pp. 432-449, reprinted under the same title in G.J. Stigler, The Organization of Industry, ch. 18); 'limit' pricing, in which prices are set just low enough to discourage potential entrants from entering the industry (see J.S. Bain, "A Note on Pricing in Monopoly and Oligopoly," American Economic Review (vol. 39; no. 2; March 1949), pp. 448-464, and P. Sylos-Labini, Oligopoly and Technical Progress, rev. ed. (Harvard University Press; Cambridge, Mass.; 1969)); and the independent maximization model, which is a variant of the limit pricing model (see D.A. Worcester, Jr., Monopoly, Big Business and Welfare in the Post-war United States (University of Washington Press; Seattle; 1967), pp. 83-105). In addition, there are several game theoretic approaches, such as M. Shubik, op. cit., and an experimental games approach adopted by Sherman. For the latter see R. Sherman, Oligopoly: An Empirical Approach (D.C. Heath; Lexington, Mass.; 1972). To say that no single model has gained universal acceptance would be more than a mild understatement. Moreover, it is not clear whether one all-encompassing model can ever be devised.

The kinked demand curve model treats the individual participant in an oligopolistic industry in isolation. Several models that purport to describe interfirm behavior should be noted. In addition to the game theoretic approaches of Shubik, Marris, and others (Shubik, op. cit., and Marris, Modern Corporation and Economic Theory, op. cit., pp. 283-304), Phillips and O.E. Williamson model interfirm behavior in oligopolistic industries around organizational as well as economic variables. See A. Phillips, "A Theory of Interfirm Organization," Quarterly Journal of Economics (vol. 74; no. 4; November 1960), pp. 602-613; A. Phillips, "Policy Implications of the Theory of Interfirm Organization," American Economic Review (vol. 51; no. 2; May 1961), pp. 245-254; A. Phillips, Market Structure, Organization, and Performance (Harvard University Press; Cambridge, Mass.; 1962); and O.E. Williamson, "A Dynamic Theory of Interfirm Behavior," Quarterly Journal of Economics (vol. 79; no. 4; November 1965), pp. 579-607. Phillips treats a wider range of oligopolistic behavior than Williamson, though within a static framework. Williamson's model is dynamic and examines a widely observed pattern of oligopolistic behavior, namely, alternating periods of collusiveness and competitiveness on the part of oligopolistic firms. See also K.E. Boulding, "The Uses of Price Theory," in A.R. Oxenfeldt, ed., Models of Markets (Columbia University Press; New York; 1963), pp. 146-162; Fellner, op. cit.; C. Kaysen, "A Dynamic Aspect of the Monopoly Problem," Review of Economics and Statistics (vol. 31; no. 2; May 1949), pp. 109-113; and J.S. Bain, Industrial Organization, op. cit., pp. 309-310.

104. The first discussions of the kinked oligopoly demand curve appeared in P.M. Sweezy, "Demand under Conditions of Oligopoly," Journal of Political Economy (vol. 47; no. 4; August 1939), pp. 568-573, and R.L. Hall and C.J. Hitch, "Price Theory and Business Behavior," Oxford Economic Papers (vol. 2; May 1939), pp. 12-45. A more recent (and critical) discussion of the model is provided by Stigler, The Kinky Oligopoly Demand Curve, op. cit.

105. Let the equation of DD be $p = a_1 - b_1 q$ and let the equation of dd be $p = a_2 - b_2 q$, where the a_i's and b_i's are positive and $a_1 > a_2$ and $b_1 > b_2$. Then marginal revenue corresponding to DD is given by $MR_{DD} = \frac{d}{dq}(a_1 - b_1 q)q = a_1 - 2b_1 q$, and similarly, $MR_{dd} = a_2 - 2b_2 q$. At q^*, DD and dd intersect, so that $a_1 - b_1 q^* = a_2 - b_2 q^*$. Thus, at q^* there is a jump discontinuity of height $MR_{dd}(q^*) - MR_{DD}(q^*) = (a_2 - 2b_2 q^*) - (a_1 - 2b_1 q^*) = (b_1 - b_2)q^* > 0$.

106. See R.M. Cyert, "Oligopoly Price Behavior and the Business Cycle," Journal of Political Economy (vol. 63; no. 1; February 1955), pp. 41-51; and W.J. Yordon, "Industrial Concentration and Price Flexibility in Inflation: Price Response Rates in Fourteen Industries, 1947-1958," Review of Economics and Statistics (vol. 43; no. 3; August 1961), pp. 287-294. For supporting evidence on the English experience, see R.R. Neild, Pricing and Employment in the Trade Cycle, N.I.E.S.R. Occasional Paper 21 (Cambridge University Press; Cambridge; 1963). For somewhat contradictory results see A.D. Brownlie, "Some Econometrics of Price Determination," Journal of Industrial Economics (vol. 13; no. 2; March 1965), pp. 116-121.

107. Stigler, The Kinky Oligopoly Demand Curve, op. cit., pp. 438-441. While Stigler's empirical results do not constitute a general disproof of the kinked oligopoly demand curve theory, they do imply that it cannot serve as a general theory of oligopoly. See also Cohen and Cyert, op. cit., p. 251, on this point.

108. Two cases in which it might prove acceptable have been suggested by Cohen and Cyert: (i) a new industry in its early stages of development and (ii) an older industry in which new and previously unrecognized rivals enter the market. In both cases existing producers will have had little opportunity to learn about their new competitors and how they are likely to react. Ibid., pp. 251-253.

109. The theory of monopolistic competition has its genesis in the works of Sraffa, Joan Robinson, and Chamberlin. See P. Sraffa, "The Laws of Returns under Competitive Conditions," Economic Journal (vol. 36; no. 144; December 1926), pp. 535-550. Sraffa's work helped inspire Mrs. Robinson's The Economics of Imperfect Competition, (Macmillan; London; 1933). The same year that Mrs.

Robinson's book was published, E.H. Chamberlin published the first edition of The Theory of Monopolistic Competition, op. cit. The works of Mrs. Robinson and Chamberlin were responsible for freeing the analysis of markets from the two extremes of monopoly and perfect competition. Where Chamberlin's work differed from Mrs. Robinson's was the former's emphasis on product differentiation and its important implications for public policy. See E.H. Chamberlin, Towards A More General Theory of Value (Oxford University Press; Oxford; 1957), ch. 5, for further discussion on this issue, and J.S. Bain, "Chamberlin's impact on microeconomic theory," in R.E. Kuenne, ed., Monopolistic Competition Theory: Studies in Impact (Wiley; New York; 1967), pp. 147-176, for an assessment of the importance of Chamberlin's contribution. A somewhat more critical view of Chamberlin's model and its implications is presented in G.C. Archibald, "Chamberlin versus Chicago," Review of Economic Studies (vol. 29; October 1961), pp. 2-28. The question of the optimality of product differentiation, and conditions under which product differentiation can be regarded as excessive (i.e. beyond the socially optimal degree of product differentiation), are discussed in K. Lancaster, "Socially Optimal Product Differentiation," American Economic Review (vol. 65; no. 4; September 1975), pp. 567-585.

110. As before, the second order condition requires that MC be rising faster than MR at the optimum.

111. Since the n firms have, by assumption, identical revenue functions and identical cost functions, they can be treated as if they were identical, and each firm's pro rata share of overall demand is 1/n .

112. The demand curves dd and DD in figure II-12 are analogous to dd and DD , respectively, in figure II-11. The two sets of curves are not identical, however, because of the different behavioral assumptions underlying each figure.

113. Chamberlin, Theory of Monopolistic Competition, op. cit., p. 81.

114. G.J. Stigler, "Monopolistic Competition in Retrospect," in G.J. Stigler, Five Lectures on Economic Problems (London School of Economics; London; 1949), reprinted in Stigler, The Organization of Industry, pp. 309-321.

115. Chamberlin, Theory of Monopolistic Competition, op. cit., p. 82.

116. Stigler, The Organization of Industry, p. 313.

117. Galbraith, The New Industrial State, op. cit., pp. 179-180.

118. See A. Henderson, "The Theory of Duopoly," Quarterly Journal of Economics (vol. 68; no. 4; November 1954), p. 565. Though the source is not current, its basic points, in the opinion of this writer, are still valid.

119. For example, see K.W. Rothschild, "Price Theory and Oligopoly," Economic Journal (vol. 57; no. 227; September 1947), pp. 299-320, and J. Lintner, Corporate Growth under Uncertainty, pp. 172-173.

120. Machlup, op. cit., p. 23; and Silberston, op. cit., pp. 530-533.

121. The rate of discount, ρ , measures the rate at which the firm is willing to trade off a dollar of profit this period for profits next period. As such, it is an interest rate, or equivalently, the price of waiting (for future profits). More specifically, according to formula (60), a dollar of profit earned in any period t has the same value to the firm as $1+\rho$ dollars of profit earned in period $t+1$, or equivalently, the firm is willing to sacrifice one dollar of profit this period only if by doing so it can increase next period's profit by (more than) $1+\rho$ dollars. A higher value for ρ implies a greater preference for profit this period rather than next, i.e. the sacrificing of profits this period requires the prospect of relatively greater future profit the greater is the value of ρ .

122. If instead, continuous compounding (as opposed to annual compounding) were used, then

$$V = \int_0^\infty \pi_t e^{-\rho t} dt = \int_0^\infty \pi_o e^{gt} e^{-\rho t} dt = \int_0^\infty \pi_o e^{-(\rho-g)t} dt = \frac{\pi_o}{\rho-g} \quad ,$$

again provided $g < \rho$; and if $g = 0$, then $V = \pi_o/\rho$. This (continuous) model of security valuation was originally proposed by Gordon and Shapiro. See M.J. Gordon and E. Shapiro, "Capital Equipment Analysis: The Required Rate of Profit," Management Science (vol. 3; no. 1; October 1956), pp. 102-110. Thus, in the continuous case, formula (61) must be modified slightly, but formula (62) holds exactly (with continuously compounded rates in place of annually compounded ones).

123. In terms of formulae (61) and (62), this means that $\rho > 0$; the firm will give up one dollar of current period profit only if offered the prospect of at least $1+\rho$ dollars of profit one period from now.

124. The trade off depends, of course, on the value of ρ . A low discount rate implies a relatively low time preference for profit, i.e. the firm would be almost as happy with one dollar of profit ten years from now as it would with one dollar of profit today. A high discount rate implies a relatively high time preference for profit, and in the extreme case of $\rho = \infty$, the firm would engage in short run (i.e. current period) profit maximization. If, as Rothschild and Galbraith have suggested, large firms are mainly interested in their long run survival (see K.W. Rothschild, "Price Theory and Oligopoly," Economic Journal (vol. 57; no. 227; September 1947), pp. 299-320, and

Galbraith, op. cit., p. 171,), then ceteris paribus one would expect larger firms to have lower discount rates than smaller firms.

125. For this reason, what appears to be 'satisficing' behavior may in actuality be perfectly consistent with long run profit maximization. See Silberston, op. cit., pp. 530-533.

126. Capital markets are said to be *perfect* when (i) all assets and goods are infinitely divisible; (ii) information is costless and available on the same basis to everyone; (iii) there are no brokerage fees or transactions costs of any kind; (iv) there are no taxes; (v) all individuals pay the same price for any particular asset; and (vi) no individual is wealthy enough and no firm is large enough to influence the market price of any asset. In perfect capital markets all firms and all individuals act as price takers. See E.F. Fama and M.H. Miller, The Theory of Finance (Holt, Rinehart and Winston, New York; 1972), p. 277, or M.H. Miller and F. Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business (vol. 34; no. 4; October 1961), p. 412. As will be shown below in section K, the irrelevancy of the debt-equity mix also requires that one additional condition be satisfied, namely, that the capital markets, i.e. the markets for stocks, for bonds, and for other financial securities, be complete with regard to the possible states of nature in each future time period. The explanation of the exact meaning of 'complete markets' is deferred to section K, where the appropriate conceptual framework is developed.

127. Modigliani and Miller, op. cit., pp. 261-296. Simpler expositions can be found in Mossin, op. cit., pp. 76-78, 86-89, and in G.C. Philippatos, Financial Management: Theory and Techniques (Holden-Day; San Francisco; 1973), pp. 292-296.

128. Modigliani and Miller divide firms into risk classes, and for each firm, ρ in formula (62) is the discount rate appropriate to the particular risk class to which the firm belongs. Ibid., pp. 266-267.

129. Ibid., p. 267.

130. The importance of capital markets being in equilibrium in order that formula (62) express the market value of the firm exactly in terms of the discounted flow of future profits needs emphasis. Formula (62) and the Modigliani and Miller results are equilibrium results; that is, they hold in equilibrium (provided the required assumptions are satisfied), but in disequilibrium there is no longer any assurance that they will hold.

131. The reason for this qualification is that the total market value of the firm will equal the discounted flow of profits as a consequence of general equilibrium in the capital markets. In the context of markets for the financial securities of all firms, the

expression 'maximize the total market value of the firm' only makes sense when either (i) the decisions of all other firms are taken as given or (ii) the maximization takes place relative to some index that reflects the market value of the shares of all firms. Most of the models that are discussed below take the first approach, i.e. they are partial equilibrium models. The second approach is discussed in Leland, Production Theory and the Stock Market, op. cit., pp. 136-138.

132. A capital gain is realized when a share is sold for more than the purchase price, while a capital loss results when the purchase price exceeds the selling price. Investor interest in capital gains is motivated by the differential tax treatment accorded long term capital gains (long term implying that the asset was held for at least nine months). However, due to the complexities associated with tax considerations, in what follows the role of income taxes will be ignored.

133. J. Lintner, "The Cost of Capital and Optimal Financing of Corporate Growth," Journal of Finance (vol. 18; no. 2; May 1963), p. 292.

134. See footnote 126 for the definition of a perfect market. See also Fama and Miller, op. cit., pp. 176-187, 300-301. Of necessity each firm must act as a price taker in the capital markets. See Mossin, Theory of Financial Markets, op. cit., pp. 154-155. In a world of uncertainty the objective of maximizing the current market value of the firm must be modified. Lintner suggests maximizing the expected current value of equity, given (or relative to) the level of some stock market index (e.g. Standard and Poor's or the Dow Jones Index). See J. Lintner, "Optimal Dividends and Corporate Growth under Uncertainty," Quarterly Journal of Economics (vol. 78; no. 1; February 1964), pp. 49-50. This modification alone is insufficient, for as Stiglitz, Jensen and Long, and Fama have shown, the stock market generally does not produce a Pareto optimal allocation of investment under the assumption that all firms maximize market value. See J. Stiglitz, "On the Optimality of the Stock Market Allocation of Investment," Quarterly Journal of Economics (vol. 86; no. 1; February 1972), pp. 25-60; M. Jensen and J. Long, "Corporate Investment under Uncertainty and Pareto Optimality in the Capital Markets," Bell Journal of Economics and Management Science (vol. 3; no. 1; Spring 1972), pp. 151-174; E.F. Fama, "Perfect Competition and Optimal Production Decisions under Uncertainty," Bell Journal of Economics and Management Science (vol. 3; no. 2; Autumn 1972), pp. 509-530. Wilson used a variant of Stiglitz's model to show that, unless certain assumptions are satisfied, every stockholder would recommend production decisions that do not maximize the market value of shares. See R. Wilson, "Comment on J. Stiglitz, 'On the Optimality of Stock Market Allocation of Investment'," Working Paper No. 8 (Institute for Mathematical Studies in the Social Sciences, Stanford University; Stanford, Calif.; 1972). LeRoy has shown that Stiglitz implicitly assumed

an environment of monopolistic competition and Wilson and Leland have shown that, in such an environment, if firms choose the level of investment so that shareholder utility is maximized, the resulting allocation of resources will be Pareto optimal from the point of view of the shareholders, though stock market values will not be maximized. See S.F. LeRoy, "Stock Market Optimality: Comment", Quarterly Journal of Economics (vol. 90; no. 1; February, 1976), pp. 150-155; Wilson, op. cit.; and Leland, op. cit., pp. 125-144. As Wilson and Leland conclude, "what Stiglitz brings into question is not the Pareto optimality of the stock market, but rather the value maximization criterion." Ibid., p. 137. Market value maximization implies a Pareto optimal allocation of investment — in the sense that no subsequent reallocation could make some share owner better off (i.e. increase his expected utility) without making another worse off (i.e. lowering someone else's expected utility) — provided there is perfect competition, i.e. provided firms in the same industry have perfectly correlated returns and provided each firm maximizes its relative value but considers the values of the other firms as given. A stronger condition for the Pareto optimality of the stock market allocation of investment is the requirement that markets be complete. See K.J. Arrow, "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies (vol. 31; no. 86; April 1964), pp. 91-96; and G. Debreu, The Theory of Value (Wiley; New York; 1959). Diamond, who assumed a perfectly competitive environment in the sense described above, showed that even if there are fewer firms than states of nature (see section K below), stock market value maximization leads to a "constrained Pareto-optimal resource allocation." See P.A. Diamond, "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty," American Economic Review (vol. 57; no. 4; September 1967), pp. 759-776. The results obtained by Diamond are consistent with those of Leland. See Leland, op. cit. To summarize, then, stock market value maximization is consistent (i) with the utility maximization of shareholders in a certain environment provided markets are perfect and (ii) with the expected utility maximization of shareholders (in the Pareto optimal sense defined above) in an uncertain environment provided markets are perfect and complete (and it is noted that, in light of the Leland and Diamond papers, the requirement that markets be complete can be relaxed somewhat).

135. Note that the objective of maximizing the stock market value of the firm is analogous to the static objective of profit maximization in that both objective functions imply the optimization of the economic position of the firm's owners. If the firm is run by an entrepreneur who receives total profit as its income, then maximizing total profit leads to maximum utility, provided of course that its marginal utility of income is always positive. In a dynamic context, the rational entrepreneur would, subject to the qualifications discussed in the previous footnote, maximize the total market value of the firm (i.e. the entrepreneur's current wealth). But where control and ownership are separate, optimizing the

economic position of the owners involves maximizing the stock market value of their shares (once again, subject to the qualifications stated in the previous footnote). See also J. Mossin, "Security Pricing and Investment Criteria in Competitive Markets," American Economic Review (vol. 59; no. 5; December 1969), p. 749.

136. See for example Lintner, Corporate Growth under Uncertainty, op. cit., pp. 181-187.

137. See for example Wong, op. cit., or Krouse, op. cit. Vickers adopts a similar objective function. In the Vickers's model the firm optimizes the economic position of the owners of the firm by maximizing the stock market value of the firm, which Vickers expresses as the "capitalized value of the residual equity income." See Vickers, op. cit., pp. 8, 162-169.

138. See Philippatos, op. cit., ch. 13, for a discussion of some of the existing share valuation formulas and Miller and Modigliani, op. cit., pp. 415-421, for a discussion of some of the existing approaches to determining the stock market value of the firm. The valuation formulas are based on present value calculations like those on which formulas (60), (61), and (62) are based. For example, substituting the dividends that will accrue in the future to shares currently outstanding for π_t in formula (60) and letting ρ be the appropriate discount rate, yields a formula for the market value of a share in terms of the future stream of dividends,

$$P = \sum_{t=1}^{\infty} \frac{D_t}{(1+\rho)^t} , \qquad (*)$$

where P is the share price and D_t are the dividends accruing at time t to shares currently outstanding. The expression (*) simplifies to

$$P = \frac{D_o(1+g)}{\rho-g}$$

if dividends per share are initially D_o and are growing at rate g.

139. Miller and Modigliani showed that, in a certain world with perfect capital markets, the discounted cash flow, the current earnings plus investment opportunities, the stream of dividends, and the stream of earnings approaches to share valuation, are all equivalent and that all are derived from the fundamental principle of share valuation (which characterizes general equilibrium in the capital markets) that "the price of each share must be such that the rate of return (dividends plus capital gains per dollar invested) on every share will be the same throughout the market over any given interval of time," provided of course that the approaches are adopted correctly and that the extreme case of a company being expected to never pay a dividend be excluded (for

in that special case the stream of dividends approach yields a share value of zero while the other approaches do not). Ibid., pp. 412-421. Hence some of these alternative formulations are different in form only. To quote Myers on the subject of share valuation: "Alternative forms ... are often seen ... Given a little algebraic ingenuity, the possible formats are endless. Consequently, it is pointless to say that any particular simplification is the correct way to represent present value." S.C. Myers, "A Time-State-Preference Model of Security Valuation," Journal of Financial and Quantitative Analysis (vol. 3; no. 1; March 1968), p. 20.

140. More generally, the two approaches are equivalent even when new equity issues are permitted, provided that there is perfect certainty and capital markets are perfect. This follows from Miller's and Modigliani's proof that in such an environment the firm's dividend policy is irrelevant. That is, given the firm's investment policy, whether the firm generates the needed finance internally or raises it externally through new equity issues is immaterial, and maximizing the share price leads to the same result as maximizing the market value of the firm's equity. Ibid., pp. 412-415. It needs to be emphasized, however, that only if markets are perfect will share owners be able to "correctly" comprehend the meaning of new share issues, and only then can the firm's method of raising finance with which to carry out the investment policy be irrelevant. Since in a certain world the distinction between debt and equity is irrelevant, the objective of maximizing the total market value of the firm can be re-expressed without loss of generality as maximizing the stock market value of the firm. Ibid., p. 412. When uncertainty is introduced, stronger conditions, such as the existence of complete markets, are needed. See Fama and Miller, op. cit., pp. 178-181.

141. Some conditions under which various pairs of objectives are interchangeable have been noted earlier in the subsection. To see why conditions (i)-(iii) make the four objectives interchangeable, note that, by condition (i), each term π_t in the numerator of formula (60) is independent of every other term. By (ii) there is a single riskless interest rate at which all individual companies and all individuals can borrow or lend freely. Thus $\rho = i \equiv$ constant in formula (60). Therefore, maximizing each π_t individually is equivalent to maximizing V. By (iii) the distinction between debt and equity is irrelevant (since perfect certainty implies complete assurance on the part of every investor as to each firm's future investment programs and future profit levels) so that maximizing the stock market value of the firm is formally equivalent to maximizing the total market value of the firm. By (ii) and (iii) and the Miller-Modigliani theorem on the irrelevance of the firm's dividend policy, maximizing the share value is interchangeable with the other three objectives. The interchangeability does not necessarily hold when uncertainty is introduced, for uncer uncertainty the firm may, by holding down

profit one period, reduce the variability in profit over time and thereby reduce ρ sufficiently to cause V to rise. Even when markets are complete, the interchangeability does not hold, since the basic Modigliani-Miller proposition on the irrelevance of the firm's capital structure implies that the stock market value of the firm's equity is maximized only if debt is reduced to zero (i.e. if the total market value of the firm, which equals the market value of debt plus the market value of equity, is constant, then the market value of equity is a maximum — equal to the total market value of the firm — when the market value of debt is zero).

142. Because of its simpler treatment of growth, the Lintner certainty model is a static optimization model, in contrast to the model of this subsection, which is a dynamic optimization model. The model discussed below is similar in form to the first of two certainty models presented by Lintner in his paper that appeared in the Marris and Wood collection. See Lintner, Corporate Growth under Uncertainty, op. cit., pp. 181-183. The model discussed below in the present subsection is also similar to, though somewhat simpler in form than, the Jorgenson model discussed below in section L.

143. In the model capital goods are treated as the numeraire. All prices, including the interest rate r introduced below, are expressed in terms of capital goods, rather than in terms of money. By selecting capital goods to be numeraire, the price of capital goods becomes, by definition, equal to one. Also, investment is measured in terms of capital goods directly, rather than in terms of some dollar-denominated measure. In addition, since the price of capital goods is constant (i.e. always equal to one), the effect that changing capital goods prices might have on the firm's investment decision can be subsumed within the model. The effect of selecting some other good to be numeraire, and permitting capital goods prices to fluctuate, is considered below in the discussion of the Jorgenson model in section L.

144. That is, $C[Q(t);K(t)]$ expresses actual cash outlays for the variable inputs as a function of the rate of output, $Q(t)$. Since the cost function excludes both depreciation and the opportunity cost of capital, it does not represent true cost in either the accounting sense (for that includes depreciation) or the economic sense (for that includes both depreciation and the opportunity cost of capital). These other costs are allowed for elsewhere in the model.

145. This also requires that the number of shares outstanding remain fixed over time. Otherwise, the current stock market value must be expressed in terms of the flow of future dividends to the shares held by shareholders of record at time zero. See C.G. Krouse, On the Theory of Optimal Investment, Dividends, and Growth in the Firm, op. cit., pp. 271-272. Other methods of valuation, namely, the discounted cash flow approach, the current earnings plus future investment opportunities approach, and the

stream of earnings approach, are possible, but Miller and Modigliani have shown that these other approaches are equivalent to the stream of dividends approach employed in (65). See Miller and Modigliani, op. cit., pp. 415-421.

146. Depreciation is the physical wearing out of the firm's capital stock and is measured in terms of the numeraire (see footnote 143).

147. This simplifying assumption will not affect the results, for Miller and Modigliani have shown that, in a certain world with perfect capital markets, the firm's dividend policy, given its investment policy, is irrelevant. Ibid., pp. 412-415. The irrelevance of the firm's dividend policy is evidence of "the general principle that there are no financial illusions in a rational and perfect economic environment." Ibid., p. 414. In such an environment, values are determined by the firm's investment policy (and the earning power of its assets) only.

148. The symbol $I(t)$ in equation (66) represents the gross increment to the firm's physical stock of capital. Since capital goods have been taken to be the numeraire, the accounting identities (63), (64), and (66) are all expressed in terms of units of capital. Therefore, $I(t)$, which appears in (63) and (64) as a financial flow and which appears in (66) as a physical flow, is the same (in terms of both units of measurement and magnitude) in all three equations due to the selection of capital goods as the numeraire.

149. For simplicity, it is assumed that each of the decision variables will be strictly postiive at each point in time along their respective optimal trajectories.

150. See Intriligator, op. cit., pp. 306-325.

151. For example, see K.J. Arrow, "Optimal Capital Policy with Irreversible Investment," in J.N. Wolfe, ed., Value, Capital and Growth (Aldine; Chicago; 1968), pp. 1-19.

152. Leland shows that, under certain conditions, the attainment of objectives such as maximizing the utility of sales and profits or maximizing profit per worker requires that the optimal current policies converge to profit maximization. See H.E. Leland, "Why Profit Maximization May Be A Better Assumption Than You Think," technical report no. 80 (Institute for Mathematical Studies in the Social Sciences, Stanford University; Stanford, Calif.; December 1972). Leland's model is discussed further in section L of this chapter.

153. Kuehn carried out a study in the United Kingdom to determine whether firms are growth maximizers or profit maximizers and found empirical support for the growth maximization hypothesis,

particularly among firms that were actively engaged in the takeover movement of the 1960's. See Kuehn, op. cit. Kuehn's analysis does not prove, however, that growth maximization is the universally adopted objective of modern corporate managers, and it remains to be seen whether it can be proved that a single objective applies universally.

154. A variation on this theme is that the behavior of the firm changes as the firm matures. See Wong, op. cit., pp. 689-694.

155. In an uncertain environment, π is interpreted as expected profit and ρ is the risk-adjusted rate of discount.

156. D.C. Mueller, "A Theory of Conglomerate Mergers," Quarterly Journal of Economics (vol. 83; no. 4; November 1969), p. 644.

157. D.C. Mueller, "A Theory of Conglomerate Mergers: Reply," Quarterly Journal of Economics (vol. 84; no. 4; November 1970), p. 675.

158. R.A. Gordon, Business Leadership in the Large Corporation, 2nd ed (University of California Press; Berkeley; 1961), pp. 305-306.

159. For a discussion of the close relationship between the size of the firm and the level of executive compensation see H.A. Simon, "The Compensation of Executives," Sociometry (vol. 20; no. 1; March 1957), pp. 32-35; Roberts, op. cit.; and McGuire, et. al., op. cit., pp. 753-761. Some companies have recently linked executive bonus schemes directly to measures of growth. See, "Is the Pepsi 'Generation' Getting Middle-Aged?," Forbes (September 15, 1975), p. 69.

160. Singh, op. cit., pp. 38-43.

161. Galbraith, The New Industrial State, op. cit., pp. 80-82, 167-169, and 161. E. Filippi and G. Zanetti, "Exogenous and Endogenous Factors in the Growth of Firms," in Marris and Wood, op. cit., pp. 168-171. For a discussion of some of the problems that confront small businesses see "Small Business: The maddening struggle to survive," Business Week (June 30, 1975). Large size is not, however, a guarantee of security, particularly when a firm that is large in absolute terms is small relative to other firms in the industry. For a discussion of the problems facing Chrysler Corp. due to its relatively small size see W.M. Bulkeley, "Chrysler's Riccardo Uses Tough Approach to Attack Firm's Ills," Wall Street Journal (July 7, 1976).

162. A. Silberston, "Economies of Scale in Theory and Practice," Economic Journal (vol. 82; no. 325s; March 1972 (supplement)), pp. 369-391; National Bureau of Economic Research, Cost Behavior and Price Policy (Columbia University Press; New York; 1943);

C.A. Smith, "Survey of the Empirical Evidence on Economies of Scale," in National Bureau of Economic Research, Business Concentration and Price Policy (Princeton University Press; Princeton; 1955), pp. 213-230; and J.S. Bain, "Economies of Scale, Concentration, and the Condition of Entry in Twenty Manufacturing Industries," American Economic Review (vol. 44; no. 1; March 1954), pp. 15-39. Generally, the empirical evidence pertaining to economies of scale is mixed, implying that the extent of economies of scale (both at the plant level and at the firm level) vary considerably from one industry to another. For a survey of the evidence, both pro and con, concerning economies of scale see A.A. Walters, "Production, and Cost Functions: An Econometric Survey," Econometrica (vol. 31; no. 1-2; January-April 1963), pp. 1-66.

163. Galbraith, The New Industrial State, op. cit., ch. 7.

164. J.K. Galbraith, American Capitalism, rev. ed. (Houghton Mifflin; Boston; 1957), chs. 9-10.

165. Marris, A Model of the 'Managerial' Enterprise, op. cit., pp. 187-188.

166. W.J. Baumol, "On the Theory of Expansion of the Firm," American Economic Review (vol. 52; no. 5; December 1962), p. 1085, and Marris, Managerial Capitalism, op. cit., pp. 101-102.

167. R.M. Cyert and K.D. George, "Competition, Growth, and Efficiency," Economic Journal (vol. 79; no. 313; March 1969), pp. 23-41. Of course, if growth is too rapid, the reverse may happen. See p. 72 and footnote 211.

168. Marris, Managerial Capitalism, op. cit., ch. 4.

169. Wood, Economic Analysis of the Corporate Economy, op. cit., p. 51.

170. Marris, Managerial Capitalism, op. cit., ch. 2. Marris argues that growth — the rate of change of size — rather than absolute size, is the primary objective of professional managers.

171. Baumol has devised two models, one in which the firm maximizes total revenue (i.e. size measured as total revenue) and one in which the firm maximizes the rate of growth of sales. These models are discussed in subsection 1 of this section and the Marris model is discussed in subsection 2. Like Baumol and Marris, Downie has also argued that the firm seeks to maximize its rate of growth. See Downie, op. cit.

172. Monsen and Downs, op. cit., pp. 226-227.

173. O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., pp. 1033-1036, and subsection 3 of this section.

174. R.A. Gordon, "Short-Period Price Determination in Theory and Practice," American Economic Review (vol. 38; no. 3; June 1948), pp. 270-271.

175. Marris, Managerial Capitalism, op. cit., p. 260.

176. A fuller discussion of the managerial models of the firm is presented in J.R. Wildsmith, Managerial Theories of the Firm (Dunellen; New York; 1974).

177. Baumol first described the essential features of his model in W.J. Baumol, "On the Theory of Oligopoly," Economica (new series, vol. 25; no. 99; August 1958), pp. 187-198. An expanded version of the model is provided in W.J. Baumol, Business Behavior, Value and Growth, op. cit., chs. 6-10.

178. Ibid., p. 46.

179. "New Spur for a Sluggish Giant," Business Week (March 17, 1975), p. 50.

180. Baumol, Business Behavior, Value and Growth, op. cit., pp. 48-50.

181. Ibid., ch. 4. Only on major decisions, such as the construction of a new plant or the launching of a new product, need the reactions of rivals be explicitly considered.

182. If $R(q)$ denotes total sales revenue, then revenue is maximized where $R'(q) = 0$ and $R''(q) < 0$. The statement in the text refers to the first order condition. Further, $R(q) = q \cdot p(q)$, where $p(q)$ is the inverse of the demand function, and $R'(q) = p(q) + q \cdot p'(q) = p(1 + \frac{q}{p} \cdot \frac{dp}{dq})$. The price elasticity of demand η, is defined by

$\eta = -\frac{p}{q} \cdot \frac{dq}{dp}$. Thus,

$$R'(q) = p(1 - \frac{1}{\eta}) \quad \text{and} \quad R'(q) = 0 \Longleftrightarrow \eta = 1 .$$

183. Note that the $AC + \pi_o/Q$ curve intersects DD at two points. Since marginal revenue is positive at both, the larger output brings larger sales revenue, and thus Q_2 is selected.

184. If $AC + \pi_o/Q$ were to lie above DD over the entire range of output, then it would be impossible for the firm to satisfy the profit constraint. That is, $(p_1 - AC(Q_1)) \cdot Q_1 < \pi_o$, where Q_1 leads to maximum total profit, and the firm's optimization problem would not have a feasible solution. In such a case, the best the firm could do would be to maximize total profit in order to come as close as possible to π_o. It should be noted that, while it has been assumed in the text that the profit constraint is binding at optimality, this need not be the case when advertising expenditure is excluded from the model. This

can be seen more clearly when the model of the firm is formulated without advertising expenditure as the following mathematical programming problem:

$$\begin{aligned} &\underset{\{Q\}}{\text{maximize}} && R(Q) \\ &\text{subject to} && R(Q) - C(Q) \geq \pi_o \\ &&& Q \geq 0 \end{aligned} \qquad (*)$$

where Q is total output, and $R(Q)$ and $C(Q)$ are total revenue and total cost, respectively. The Lagrangian for problem (*) is $L_\lambda = R(Q) + \lambda(\pi_o - [R(Q) - C(Q)]$, and assuming nontrivial Q at optimality, the Kuhn-Tucker conditions that must be satisfied by an optimal solution to (*) are the following:

$$\frac{\partial L_\lambda}{\partial Q} = R'(Q) - \lambda(R'(Q) - C'(Q)) = 0 \qquad (**)$$

$$(\pi_o - [R(Q) - C(Q)])\lambda = 0 \quad R(Q) - C(Q) \geq \pi_o \quad \lambda \leq 0 \qquad (***)$$

It follows from (***) either that $R(Q) - C(Q) = \pi_o$, in which case the minimum profit constraint is binding, as discussed in the text, or else that $\lambda = 0$, in which case (**) implies $R'(Q) = 0$. In this latter case, the minimum profit constraint is not binding. In practical terms, the minimum required level of total profit is low enough, and the firm's costs of production are also 'low enough', that, by maximizing total revenue, the firm is able to generate sufficient profits to satisfy the profit constraint. However, as discussed below, if advertising expenditure is introduced into the model, and if it is assumed that the marginal productivity of advertising expenditure is always positive, then the minimum profit constraint must be binding at optimality.

185. The rule for profit-maximizing advertising policy is derived easily from the following model:

$$\underset{\{Q,A\}}{\text{maximize:}} \quad \pi(Q,A) = R(Q,A) - C(Q) - A \qquad (*)$$

where Q is total output, A is advertising expenditure, π is total profit, and C is total production cost. The function $R(Q,A)$ expresses total revenue as a function of both the output level and the level of advertising expenditure. The necessary conditions for an optimal solution to (*) are the following:

$$\frac{\partial \pi}{\partial Q} = \frac{\partial R}{\partial Q} - \frac{dC}{dQ} = 0 \qquad (**)$$

$$\frac{\partial \pi}{\partial A} = \frac{\partial R}{\partial A} - 1 = 0 \qquad (***)$$

Equation (**) is interpreted as the familiar marginal revenue equals marginal cost rule for profit maximization, where marginal revenue is determined with the advertising level held fixed. Equation (***) is interpreted as the requirement that advertising expenditure be increased, for given output level, up to the point at which the addition to total revenue resulting from the last dollar spent on advertising just equal one dollar — the addition to total cost due to the increase in advertising expenditure. That is, (***) is also a marginal revenue equals marginal cost rule, but applied to advertising expenditure rather than to output level.

186. And, as demonstrated below, as long as the marginal productivity of advertising expenditure is always positive, any such increase in total profit above the required minimum will always be spent on advertising (and thereby lead to increased total revenue and to enhanced managerial utility).

187. The total revenue curve shifts upward from $R_1(Q)$ to $R_2(Q)$ in figure II-15 due to the increase in total revenue earned at each level of output, Q (one interpretation of this is that an increase in advertising expenditure causes product demand to increase so that any particular level of output can, due to the increase in advertising expenditure, be sold at a higher unit price), and the total cost curve shifts upward from $C_1(Q)$ to $C_2(Q)$ by the full amount of the additional advertising expenditure.

188. Note that $Q_2 < Q_3$, where the latter maximizes total revenue (given the current level of advertising expenditure).

189. What follows is an expanded and generalized version of the mathematical model of the firm provided in Baumol, Business Behavior, Value and Growth, op. cit., pp. 68-72. Specifically, the model (73) permits the firm to exercise choice regarding productive techniques and its input mix, thereby enabling several of Baumol's qualitative results to be demonstrated quantitatively.

190. The demand curve for the i-th good is of the form $q_i = D(p_i,A_i)$, or $f(q_i,p_i,A_i) = q_i - D(p_i,A_i) = 0$. Since the demand curve is downward-sloping, $\partial q_i/\partial p_i \neq 0$, and by the implicit function theorem, the original demand relation can be solved for p_i as a function of q_i and A_i, so that total revenue, R_i, is given by $R_i = p_i \cdot q_i = p_i(q_i,A_i) \cdot q_i = R_i(q_i,A_i)$.

191. The full statement of the Kuhn-Tucker conditions would include Lagrange multipliers for the decision variables. As before, for simplicity it is assumed that each of the decision variables appears in the optimal solution at a positive level. Generalizing to permit corner solutions is easy, and so, is not done here.

192. Note that even though the sales maximizer is carrying production beyond the most profitable level, it is still, according to (80) and (81), going to expand production in the relatively most profitable product lines using the relatively most profitable input combinations.

193. Ibid., ch. 7.

194. Ibid., pp. 50-53.

195. See D. Needham, Economic Analysis and Industrial Structure (Holt, Rinehart and Winston; New York; 1969), pp. 6-10, and J.H. Williamson, "Profit, Growth and Sales Maximization," Economica (vol. 33; no. 129; February 1966), pp. 13-15.

196. Needham, op. cit., pp. 8-10.

197. G.K. Yarrow, "Managerial Utility Maximization under Uncertainty," Economica (vol. 40; no. 158; May 1973), pp. 160-164.

198. Baumol, Theory of Expansion of the Firm, op. cit., pp. 1078, 1085.

199. Ibid., pp. 1078-1087.

200. Ibid., pp. 1085-1086.

201. But, of course, if utility is a monotonically increasing function of total revenue, then the two formulations are equivalent in terms of the firm's hypothesized performance, although formulating the firm's objective function as total revenue, rather than as the utility of total revenue, is simpler mathematically.

202. Ibid., p. 1078, 1084-1085.

203. It should be noted that Baumol's two models are not equivalent, nor is his growth maximization model a simple extension of his single period sales maximization model. Due to the intertemporal sales trade offs inherent in a growth maximization model, the latter is not derivable from the former, although as Williamson shows, the single period sales maximization model could be derived from a multiperiod sales maximization model. See J.H. Williamson, op. cit., pp. 13-14.

204. For simplicity, all flow variables are expressed 'per year', rather than per time period of arbitrary length, as in the original Baumol formulation.

205. Baumol actually defined I as the firm's "level of investment as a per cent of the value of current capital assets (the percentage rate of growth of the firm's money capital)." Baumol,

Theory of Expansion of the Firm, op. cit., p. 1086. This author has interpreted Baumol's definition in terms of the conventional economic definition of investment and capital assets, which were given in section C of chapter one of this paper. Note that, in terms of the firm's balance sheet given in table II-1, Baumol's definition of I, as interpreted by this author, implies a simplification of the assets side of the firm's balance sheet to include just capital assets, i.e. fixed assets and inventories. It should be emphasized that even under the author's interpretation of Baumol's definition, the variable I still represents the percentage rate of growth of the firm's money capital.

206. See table II-3.

207. Note that, because of the second constraint, any two variables from the set $\{\pi, D, E\}$ could serve as decision variables. Note also that the model could be expanded to take into account decision variables such as output levels, advertising outlays, etc., though as Baumol indicates, this is not critical to understanding the basic features of his model. Ibid., p. 1086.

208. Ibid., p. 1079-1081.

209. As before, $s < g$ is necessary in order that the integral converge.

210. The expansion costs term, $C(q)$, implicitly allows for the immediate investment, I, and also for all future related costs necessary to support the growth of sales.

211. See J.H. Williamson, op. cit., p. 4; Marris, A Model of the 'Managerial' Enterprise, op. cit., pp. 200-202; and Penrose, op. cit., chs. 9 and 11.

212. Various other static results are obtainable from the model. See Baumol, Theory of Expansion of the Firm, op. cit., pp. 1082-1085.

213. According to the conventional definition of steady state growth, it is only necessary that these quantities each grow at a constant rate; it is not necessary that they all grow at the same constant rate. Bannock, Baxter, and Rees, op. cit., pp. 386-387. Note that the Baumol growth maximization model permits sales and the stock of money capital to grow at different rates, although each growth rate must remain, by assumption, constant through time. The Marris model, which is discussed in the next subsection, makes the assumption that all quantities grow at the same constant rate (indeed, this is typically assumed in steady state growth models).

214. The linear homogeneity of the production function guarantees that output and input levels all grow at the same rate. The constant output price and constant input prices then ensure that total sales and total profit grow at the same rate, which is also the rate at which total retained earnings grow. If the amount of externally

raised finance grows at the same rate as retained earnings, then total money capital (which is the sum of the amounts raised externally and internally) must grow at the same rate, which, by the foregoing, also is the rate at which sales grow.

215. Baumol presented the model (92) of the growth maximizing firm without attempting to derive and interpret optimality conditions. What follows is this author's attempt to characterize the equilibrium rate of growth of sales of the growth maximizing firm, based on a modified version of Baumol's model.

216. The initial formulation of the model was presented first in Marris, A Model of the 'Managerial' Enterprise, op. cit., pp. 185-209. An expanded version of the initial formulation of the model subsequently appeared in Marris, Managerial Capitalism, op. cit., pp. 226-249. The second formulation of the model initially appeared in ibid., pp. 249-265. A subsequent statement of the second formulation is given in Marris, Theories of Corporate Growth, op. cit., pp. 1-36. The objective function referred to in the text belongs to the second formulation.

217. Marris, Managerial Capitalism, op. cit., pp. 101-107.

218. Ibid., pp. 18-22, and Marris, Theories of Corporate Growth, op. cit., p. 16.

219. For empirical evidence of an inverse relationship between the valuation ratio and the probability of takeover see D.A. Kuehn, "Stock Market Valuation and Acquisitions: An Empirical Test of One Component of Managerial Utility," Journal of Industrial Economics (vol. 17; no. 2; April 1969), pp. 132-144, and Kuehn, Takeovers and the Theory of the Firm, op. cit., ch. 3. In contrast, studies by Singh and Newbould found evidence that would support only a very weak inverse relationship between the valuation ratio and the probability of takeover, although Singh's results do show that taken-over firms do tend to have a smaller valuation ratio than non-taken-over firms. See Singh, op. cit., ch. 3; and G.D. Newbould, Management and Merger Activity (Guthstead; Liverpool; 1970). Excessive growth was one of the major factors that recently led a number of conglomerates into serious financial difficulties, among them American Standard, Inc. ("How American Standard cured its conglomeritis: A tough program to rid itself of nonprofitable acquisitions is paying off," Business Week (September 28, 1974)); Rucker Co. ("How Rucker cured its conglomerate fever," Business Week (April 7, 1975)); Gulf & Western Industries, Inc. ("Bluhdorn the raider as elder statesman," Business Week (January 20, 1975)); Westinghouse Electric Corp. (B.E. Calame and J.V. Conti, "Westinghouse Moves to Halt Old Drains and Avoid New Ones," Wall Street Journal (March 7, 1975)); Republic Corp. ("Republic Corp.", Forbes (March 1, 1976)); and the classic example, LTV "(Is There Life After Ling?" Forbes (April 15, 1975)). Excessive growth is also at least partly responsible for the recent W.T. Grant debacle ""Investigating the collapse of W.T. Grant," Business Week (July 19, 1976)).

220. Marris, Managerial Capitalism, op. cit., pp. xvii-xviii. In terms of the balance sheet illustrated in table II-1, the book value of 'net equity assets' is equal to stockholders' equity less the preferred stock portion of contributed capital. That is, 'net assets' is what was defined as 'equity' in section A of this chapter. It represents that portion of the firm's assets — with fixed assets valued at historic cost less depreciation — that are attributable to the firm's common stockholders. Given this definition of net equity assets, the valuation ratio can be interpreted as the ratio of the stock market valuation of the assets of the firm attributable to the firm's common stockholders to the value attached to those assets in the firm's balance sheet. It should be noted that, if fixed assets were valued at replacement costs, if the stock market and the markets for capital (goods) were perfect, and if uncertainty were absent, then, as a consequence of general equilibrium in these markets, the valuation ratios of all (publically held) firms would be equal to one.

221. Marris, Theories of Corporate Growth, op. cit., pp. 17-19.

222. Marris, A Model of the 'Managerial'Enterprise, op. cit.; pp. 185-209, and Marris, Managerial Capitalism, op. cit., pp. 226-249.

223. Marris, Theories of Corporate Growth, op. cit., p. 19.

224. This section is based primarily on Marris, A Model of the 'Managerial' Enterprise, op. cit., pp. 185-209. The symbolism used by Marris has been altered slightly — g is used in place of $\dot{C}$ — in order that g represent the growth rate of the firm throughout this entire subsection.

225. Ibid., p. 192. See also the discussion in Marris, An Introduction to Theories of Corporate Growth, op. cit., pp. 11-15. Steady state growth, which was assumed in the Baumol growth maximization model just discussed, is also assumed in the models presented in J.H. Williamson, op. cit.; R.M. Solow, op. cit.; J. Lintner, Optimum or Maximum Corporate Growth under Uncertainty, op. cit.; and Yarrow, op. cit.

226. Penrose, op. cit., pp. 212-214.

227. This assumption of steady state growth is, in the opinion of this writer, an enormous simplification of the actual growth process, but one that can be justified on analytical grounds. Clearly the firm's growth rate is not constant over time; it is affected by, among other factors, changing market conditions and the changing quality of management. Assuming steady state growth is tantamount to assuming that, if conditions external to the firm, its ability to influence its environment, and its selection of operating policies remain fixed, then the important quantifiable variables, such as total sales, the market value of the firm, and reported net income, would all grow at the same constant rate, and further, key ratios, such as the retention ratio, the leverage ratio, and

the profit rate, would remain constant. The assumption of steady state growth is not untenable, however, and is probably best thought of as the selection of a long run growth target. See Marris, An Introduction to Theories of Corporate Growth, op. cit., pp. 12-13, and R. Marris, "Some New Results on Growth and Profitability," in Marris and Wood, op. cit., pp. 422-427. By assuming steady state growth, the model of the firm can be formulated as a mathematical programming problem that can be solved with the aid of static optimization techniques, whereas a more realistic specification, with the growth rate variable, would require the application of optimal control theory and dynamic optimization techniques. The application of control theory to models of corporate growth is discussed below in section L of this chapter.

228. 'Productive assets', as defined by Marris, are equal to total assets less 'liquid assets', which in turn are defined by Marris to include cash in excess of the required minimum working balance. (i.e. some portion of what is listed as cash in the balance sheet in table II-1, is 'illiquid' in the sense that it satisfies the firm's recurring need for cash with which to make payments of various kinds.) Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 192, and Marris, Managerial Capitalism, op. cit., p. xvii. It should be noted that, in steady state, total assets, productive assets, and liquid assets all grow at the same constant rate, so that equation (103) is not affected by the presence of liquid assets in the model. Marris, A Model of the 'Managerial' Enterprise, op. cit., pp. 192-193.

229. J. Duesenberry, Income, Saving and the Theory of Consumer Behavior (Harvard University Press; Cambridge, Mass.; 1962), pp. 104-110. For a discussion of some empirical evidence concerning the relationship between corporate growth and diversification see C.H. Berry, Corporate Growth and Diversification (Princeton University Press; Princeton, N.J.; 1975). Berry found a high degree of association between corporate growth and the addition of new products among the largest firms in the economy.

230. More than 70 percent of all new consumer brands fail. This includes new products for which a profitable market fails to develop as well as unsuccessful attempts by producers to break into an established market by offering new brands. This second type of failure generally occurs because the new brand lacks a significant price or performance advantage over existing brands. See J.H. Davidson, "Why Most New Consumer Brands Fail," Harvard Business Review (March-April 1976), pp. 117-122.

231. See table II-2.

232. See tables II-1, II-2, and II-3.

233. In the second formulation of his model, Marris appeals to the work of Miller, Modigliani, Lintner, and J.H. Williamson to simplify the model by assuming that all finance is generated internally, and hence, that the constant a is simply the retention ratio. See

footnotes 140 and 237. At a more practical level, in the real world, where uncertainty exists and capital markets are imperfect, large established firms typically rely on new equity issues for only a small percentage of their expansion funds. Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 199. In recent years there has been a dearth of new equity issues, so that leaving new equity issues out of the model is not as unrealistic as it might first appear.

234. This terminology is Marris's. Equation (103) might also be called more descriptively the 'rate-of-growth-of-the-supply-of-finance' function. Following Marris, the shorter term will be used throughout this subsection.

235. These costs of growth were discussed previously in connection with the Baumol growth maximization model. See p. 221 and see also footnote 211 for references. The ill effects of an excessive rate of diversification can linger for a number of years. For an interesting discussion of the problems confronting the food processing industry see "The Hard Road of the Food Processors," Business Week (March 8, 1976). Most of the companies listed in footnote 219 are suffering from similar problems.

236. See Marris, Theories of Corporate Growth, op. cit., pp. 17-23.

237. As in the earlier formulation, it is assumed that there are no new equity shares issued. Thus, all finance is generated internally. Also as in the previous formulation, the firm grows in steady state. Since Lintner and Williamson have shown that the restriction to internal financing does not limit the set of attainable steady state growth rates, as long as capital markets are perfect, the assumptions of steady state growth and no external financing of investment are compatible. See J. Lintner, "The Cost of Capital and Optimal Financing of Corporate Growth," Journal of Finance (vol. 18; no. 2; May 1963), pp. 292-310; J. Lintner, "Corporate Finance: Risk and Investment," in R. Ferber, ed., Determinants of Investment Behavior (Columbia University Press; New York; 1967), pp. 215-254; and J.H. Williamson, op. cit., pp. 3, 6-7. Briefly, in a world of certainty with perfect capital markets, the alternative of issuing new equity shares does not enlarge the set of attainable steady state growth rates.

238. Marris, Theories of Corporate Growth, op. cit., p. 19.

239. Marris, Managerial Capitalism, op. cit., ch. 6. A second approach is suggested by Herendeen following J.H. Williamson. See Herendeen, op. cit., pp. 81-83, and J.H. Williamson, op. cit., pp. 9-13. This second approach assumes that the firm's profit rate p is constant, whereas the approach given here does not.

240. Since increasing r causes the supply-of-finance curves in figure II-17 to shift upward, the balanced growth curve must also shift upward, and g_r must increase. The reverse happens when r falls. Hence,

$$dg_r/dr > 0 .$$

In chapter six of The Economic Theory of 'Managerial' Capitalism Marris argues that $p'(g) > 0$ may hold for low rates of growth if the firm is able to exploit its temporary monopoly position in markets for new products that it has recently introduced. The temporary monopoly enables the firm to keep prices and profit margins high — but only until new competitors enter the market and force prices down. An example of this sort of phenomenon is the market for pocket calculators. See "A calculator war at the top of the line," Business Week (July 19, 1976). As the firm's growth rate increases, eventually the profit rate falls due to the Penrose effect. Higher rates of growth lead to internal inefficiencies, and in particular, to a less efficient utilization of the firm's capital. Evidence of this phenomenon is provided by firms that are slowing their growth in order to increase their profit rates. See R.E. Winter, "More Firms Slow Drive For Growth, Bid To Lift Return On Investment," Wall Street Journal (December 16, 1975). Allowing $p'(g) > 0$ over some range is not essential, so in "An Introduction to Theories of Corporate Growth" Marris assumes that $p'(g) < 0$ for all g.

To derive equation (108), let N denote the number of shares outstanding and let d_o denote the current dividend per share. Valuing each share of stock in terms of the present value of the future dividend stream, the market value of each share is equal to $d_o/(i-g)$, and thus $V = N \cdot d_o/(i-g)$. But $N \cdot d_o$ equals the sum paid out as dividends to shareholders, and thus, is equal to $p(1-r)K$, which leads by substitution to (108). This result follows directly from what Fama and Miller call the "equal rate of return principle" and what Miller and Modigliani call the "fundamental principle of valuation." See Fama and Miller, op. cit., pp. 36-37, 92-94, and Miller and Modigliani, op. cit., pp. 412, 422.

Note from (110) that if $p(g) = g$ (and $i \neq g$), then the valuation ratio is zero. From (103), $g = p$ implies that $r = 1$. If the firm tried to retain all profits, its valuation ratio would fall to zero. Thus, a management that values its security will set $r < 1$, so that $g < p$. Even if the firm were permitted to raise funds externally, $g \leq p$ must still be true in the long run. For if $g > p$ for the typical firm, then each year there is a net flow of funds from shareholders to the firm, which implies that shareholders on a net basis steadily increase their saving without any prospect of future reward. Similarly, $g \leq i$ in a general equilibrium setting. The possibility that g can exceed i forever leads to the well-known "growth stock paradox", which has no real economic significance since in a general equilibrium setting i is a variable, and if $g > i$ so that one or more shares did not have a finite value, then demand for that stock would necessarily cause v to rise until capital market equilibrium had been restored. See Miller and Modigliani, op. cit., p. 421 (footnote 14).

244. In actuality, however, firm's valuation ratios can fall below one. During the recent recession several firms had valuation ratios less than one but were not taken over, possibly because of the rather dismal business outlook and the inability of potential takeover raiders to raise the necessary funds — either internally or externally — with which to finance the acquisition.

245. See Marris, *Theories of Corporate Growth*, *op. cit.*, p. 18.

246. Since there is no debt, equity assets are the same as productive assets (if liquid assets are ignored), and $D(g)$ represents the dividend yield on the firm's equity assets. To see this, note that $p-g = (1-r)p = (1-r)pK/K = Nd_o/K$.

247. The dividend yield expressed in terms of the share price is $d_o/[d_o/(i-g)] = i-g = 1/Y(g)$. The quantity $Y(g) - 1/(i-g)$ is the present value of an income stream that is initially 1 and is growing at rate g , with constant rate of discount $i > g$. Note that $d_o \cdot Y(g)$ is the share price.

248. The percentage increase in the share price resulting from an increase in g is equal to

$$\frac{\frac{d}{dg}(d_o \cdot Y(g))}{d_o \cdot Y(g)} = \frac{d_o \cdot Y'(g)}{d_o \cdot Y(g)} = \frac{Y'(g)}{Y(g)} ,$$

the percentage increase in the present value function.

249. Marris, *Managerial Capitalism*, *op. cit.*, p. 255.

250. It is assumed that neither shareholders nor managers would ever find it to their advantage to select a negative steady state growth rate. Note that, if $v'(g) < 0$, for all g , condition (112) is not satisfied unless $v'(0) = 0$. However, it is reasonable to expect that in an expanding economy $v'(g) > 0$ for most firms.

251. This follows by substitution for v , using the constraint, into the objective function in (111). The resulting problem is a single variable unconstrained optimization problem. An alternative approach would be to use the method of Lagrange multipliers, which would be somewhat more cumbersome mathematically, but which would yield additional economic information, through the value of the Lagrange multiplier, concerning the value to the firm of a relaxation of the valuation constraint.

252. Note that condition (114) is not altered if $v'(g) < 0$, for all g , since the point of tangency must occur where $v(g)$ is downward-sloping.

253. These conclusions are consistent with the models of J.H. Williamson, Solow, and Lintner. See J.H. Williamson, op. cit.; Solow, op. cit.; and Lintner, Corporate Growth under Uncertainty, op. cit. Solow's analysis is noteworthy for two reasons: (i) his model determines the initial scale of the firm as well as its growth rate and (ii) he compares the behavior of growth maximizers with the behavior of value maximizers in response to various external stimuli, such as changes in factor prices and a change in the corporate profits tax rate.

254. See Singh, op. cit., p. 81. Singh finds that the empirical evidence relating to the relationship between the firm's valuation ratio and the probability of takeover supports the weak firm, but not the strong firm, of the valuation ratio constraint. The two models differ in that in (111) the probability of takeover increases as v decreases, but in (115) the firm is almost bound to be acquired if v falls below v_1. In problem (115) the managerial indifference curves would not be like those in figure II-18, but rather, would be L-shaped with a corner at $v = v_1$. For $v < v_1$ there would be only disutility, and managers would strive all out to attain v_1. Once v_1 had been reached, utility could only be increased by achieving faster growth. In short, there is a lexicographic ordering of the security and growth objectives. Both Borch and Rosenberg have questioned the appropriateness of lexicographic utility functions because such functions rule out a smooth trade off between the primary objective (e.g. growth) and security (i.e. the valuation ratio). See K.H. Borch, The Economics of Uncertainty (Princeton University Press; Princeton, N.J.; 1968), ch. 2, and R. Rosenberg, "Profit Constrained Revenue Maximization: Note," American Economic Review (vol. 61; no. 1; March 1971), pp. 208-209.

255. It should be noted that the issue concerning just what constitutes an empirical 'validation' and just what constitutes an empirical 'refutation' of an economic theory or of an economic model involves conceptual as well as practical difficulties. See A.G. Papandreou, Economics As A Science (Lippincott; New York; 1958). Unlike the physical sciences, theories and models in economics are, in general, not amenable to scientific testing under carefully controlled conditions. Models that are tested empirically must often be tested on the basis of historical data gathered for some other purpose. Since a model is, by definition, an abstraction, it is possible for two or more alternative models to be 'validated' using the same set of data, possibly because the data base was not broad enough to facilitate an adequate test of the theories, or possibly because the alternative theories implied patterns of behavior sufficiently similar that the statistical techniques utilized in the test were not powerful enough to distinguish between them. In addition, there is the conceptual problem associated with 'accepting' any theory, namely, that a different set of data — relating to a different population of, say, firms or to a different period of time — might lead to 'rejection' of the thoery. That is, there always remains the question concerning

how 'general' the theory is. As the discussion in this section indicates, these issues have been raised in connection with empirical results offered in support of the managerial theories (and in contradiction to the traditional theories).

256. D.A. Kuehn, Takeovers and the Theory of the Firm, op. cit. Chapter six, which offers very strong support for the growth maximization hypothesis, also appears in Cowling, Market Structure and Corporate Behavior, op. cit., pp. 21-37.

257. Moreover, the manufacturing sectors of the American and British economies are not so dissimilar, in the opinion of this writer, as to prevent inferences concerning the behavior of American firms from being drawn from Kuehn's findings.

258. I. Friend and M. Puckett, "Dividends and Stock Prices," American Economic Review (vol. 54; no. 5; September 1964), p. 657. There is also evidence that large institutional investors recently have been pressuring corporations to increase dividends and that increased dividends have helped increase share prices. For example, when Ford Motor Co. recently restored 20 cents of a previous 80 cent cut in its dividend (per share), its share price rose eight percent in five trading days. See "Dividend-hungry investors cry for more," Business Week (August 2, 1976).

259. Contrary evidence has been provided by Sorenson, whose study of 11 U.S. industries showed that management-controlled firms actually had a lower (though not significantly lower at the .05 level) retention ratio than owner-controlled firms. See Sorenson, op. cit., pp. 145-148.

260. S.R. Reid, Mergers, Managers, and the Economy (McGraw-Hill; New York; 1968); S.R. Reid, "A Reply to the Weston/Mansinghka Criticisms Dealing With Conglomerate Mergers," Journal of Finance (vol. 26; no. 4; September 1971), pp. 937-946; and J. Bossons, K.J. Cohen, and S.R. Reid, "Mergers for Whom — Managers or Stockholders," in Economic Concentration, Part 5, Hearings Before the Subcommittee on Antitrust and Monopoly, U.S. Senate, 89th Congress, 2nd Session (U.S. Government Printing Office; Washington, D.C., 1966). These studies conclude that the stockholders of firms that actively engage in growth by acquisition earn lower than normal returns. See also T.F. Hogarty, "The Profitability of Corporate Mergers," Journal of Business (vol. 43; no. 3; July 1970), pp. 317-327, for a survey of empirical research of the profitability of mergers, which concludes that, in general, mergers have a negative impact on the profitability of acquiring firms and a neutral impact on the profitability of the merged (i.e. acquiring and acquired firms combined) firm. For contrary results see E.M. Kelly, The Profitability of Growth through Mergers (Pennsylvania State University Press; University Park; 1967), in which the author concludes that active acquiring firms are neither more nor less profitable than other comparable firms in the same industry, and

G. Mandelker, "Risk and Return: The Case of Merging Firms," Journal of Financial Economics (vol. 1; pp. 303-335, in which the author concludes that, when the returns to shareholders are adjusted for risk, shareholders earn the same epxected returns from an acquisition as they do from any other investment or production activity involving similar risk.

261. Wood, Economic Analysis of the Corporate Economy, op. cit., p. 46. Wood also describes the practical difficulties one would encounter in trying to determine empirically the nature of the utility function and the growth-valuation function so that the model could be carefully tested against alternative models in order to investigate the significance of differences in managerial motivation. Ibid., pp. 43-48.

262. M. Nerlove, "Factors Affecting Differences Among Rates of Return on Investments in Individual Common Stocks," Review of Economics and Statistics (vol. 50; no. 3; August 1968), p. 328.

263. In particular, when capital markets are imperfect, shareholders are not, in general, indifferent between dividends and capital gains. The share valuation formula adopted by Marris expresses the share price in terms of the discounted stream of dividends. But when capital markets are imperfect, the share valuation formula adopted by Marris, and the alternative valuation procedures suggested by Miller and Modigliani, are not necessarily equivalent. See footnote 139. In view of the apparent imperfections in actual capital markets, there has been some disagreement as to the appropriate share valuation formula. In particular, there has been considerable disagreement as to the relative significance of dividends and retained earnings in determining share prices. See Friend and Puckett, op. cit.; Nerlove, op. cit.; Miller and Modigliani, op. cit.; M.J. Gordon, "Dividends, Earnings, and Stock Prices," Review of Economics and Statistics (vol. 41; no. 2; May 1959), pp. 99-105; J. Lintner, "Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations," Review of Economics and Statistics (vol. 44; no. 3; August 1962), pp. 243-269; and B. Graham, D.L. Dodd, and S. Cottle, Security Analysis, 4th ed. (McGraw-Hill; New York; 1962). Gordon and Graham et al., for example, argue that a dollar of dividends has four times the impact of a dollar of retained earnings on share prices.

264. Yarrow, Managerial Utility Maximization under Uncertainty, op. cit., pp. 164-170.

265. Ibid., pp. 158-160, 165.

266. It should be noted that the share valuation formula used by Marris is identical to the Sharpe-Lintner-Mossin valuation formula when expected returns in the latter are interpreted as the discounted stream of returns to each share (rather than as the current period returns to each share, as is done in the single period context to which the formula is normally applied). See W. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," Journal of Finance (vol. 19; no. 3; September 1964),

pp. 425-442; J. Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics (vol. 5, no. 1; February 1965), pp. 13-37; and J. Mossin, "Equilibrium in a Capital Asset Market," Econometrica (vol. 34; no. 5; October 1966), pp. 768-783. Their valuation model is of the form

$$V = \frac{\mu - Rb}{r} \tag{*}$$

where V is the stock market value of the security, μ is the expected returns to each share, R is the market price of risk, b is a measure of the riskiness of the security's returns (relative to some market-related index), and r is the riskless rate of interest. [Note that this differs from Modigliani's and Miller's valuation model in which the adjustment for risk is made in the denominator of the valuation formula. Modigliani and Miller, op. cit., pp. 267, 271. Under appropriate conditions the two approaches to handling risk can be made equivalent. See, for example, Mossin, Theory of Financial Markets, op. cit., p. 84, or Sherman, The Economics of Industry, op. cit., pp. 112-114.] Under certainty, risk is zero and $V = \mu/r$. But μ is the returns to each share, which consists of dividends plus capital gains, and under the assumption of perfect capital markets, r is the unique rate of interest. It follows then, from Miller's and Modigliani's fundamental principle of valuation, that $V = \mu/r$ with μ interpreted as the discounted stream of returns to each share and Marris's V (see equation (108) above) are the same. Equation (*) suggests that, to modify the Marris model to allow for uncertainty, the appropriate risk premium should be subtracted from the numerator of the valuation formula [or given the comments in brackets above, the discount rate in the denominator could be adjusted upward in a suitable manner to reflect risk]. This might also be accomplished by using the multiperiod version of the Sharpe-Lintner-Mossin valuation formula discussed in G.V.G. Stevens, "On the Impact of Uncertainty on the Value and Investment of the Neoclassical Firm," American Economic Review (vol. 64; no. 3; June 1974), pp. 319-336.

267. A model incorporating such arguments in the managerial utility function is discussed in the next subsection.

268. The issues involved in determining such a collective utility function for management are the same as those that have been discussed in connection with trying to specify a 'social welfare function' for making collective decisions. See G.M. Heal, The Theory of Economic Planning (North-Holland; Amsterdam; 1973), ch. 2. In essence, the problem involves determining how to weight the preferences of individuals — as embodied in each person's utility function — so as to arrive at a social utility function.

269. A theory of the firm that reflects the sociological nature of the firm is discussed below in section H of this chapter.

270. This means that marginally profitable or unprofitable divisions are shed, growth plans are shelved, and the company retrenches. M. Bralove, "A&P to Close Third of Its Stores, Set $195 Million Reserve," Wall Street Journal (March 14, 1975); R.E. Winter, "Firms Drop Operations To Lower Their Costs and Preserve Capital," Wall Street Journal (March 17, 1975); P.F. Drucker, "Aftermath of a Go-Go Decade," Wall Street Journal (March 25, 1975); and "Thinking Small," Newsweek (June 2, 1975). Once the upswing has begun, the emphasis switches back to growth. To quote Frank Beaudine, president of the Chicago-based executive search firm of Eastman & Beaudine, "Job searches a year ago were for cost-cutters. Now [private companies] want someone to set up a new marketing program ... companies are thinking growth." "A seller's market in executive talent," Business Week, (July 5, 1976).

271. A steady state growth model may also be criticized on three other grounds. First, firms may find it to their advantage to accept slower growth one period if that makes possible more rapid growth the next (and possibly higher overall growth as a result) or vice versa. Second, in reality, the firm's growth pattern may follow a 'growth' curve, with larger size implying a slower average growth rate. Marris recognizes these possiblities, but excludes them from his model. Marris, A Model of the 'Managerial' Enterprise, op. cit., p. 192. Third, as the firm expands, it may reach a 'critical phase' in which internal reorganization is needed before further expansion can take place efficiently. See Filippi and Zanetti, op. cit., pp. 163-171. However, this third point generally applies to small firms.

272. Williamson has provided several versions of his basic model. The first to appear in the literature was presented in O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., pp. 1032-1057. This version reappeared as the "staff and emoluments model" in O.E. Williamson, The Economics of Discretionary Behavior, op. cit., pp. 52-54. This subsection begins by describing this model, in which managerial utility is a function of the following three arguments: staff, managerial emoluments, and discretionary profit. A somewhat simpler version of the model, in which staff expenditure was defined to include managerial emoluments, appeared as the "staff model" in ibid., pp. 40-49, and as the "basic model" in O.E. Williamson, Corporate Control and Business Behavior, op. cit., pp. 54-63. To facilitate a geometric interpretation of the Williamson model, this second version is discussed later in the subsection. It should be noted that, in addition to the formulation of his basic model in The Economics of Discretionary Behavior and in Corporate Control and Business Behavior, Williamson applies the model to public utilities, though in a slightly different fashion in each book. Since this subsection is only concerned with Williamson's basic model, this particular application is not explored here. Finally, Williamson has also developed a managerial model that is similar to his basic model and that allows for the existence of uncertainty. See O.E. Williamson, "A Dynamic Stochastic Theory of Managerial Behavior," in A. Phillips and O.E. Williamson, eds., Prices: Issues in Theory, Practice, and Public Policy (University of Pennsylvania Press; Philadelphia; 1967), pp. 11-31.

273. In Corporate Control and Business Behavior, staff expenditure is defined to include managerial emoluments, thereby reducing the number of arguments in the managerial utility function from three to two. This simplification implies that the marginal rate of substitution between staff expenditures and managerial emoluments is constant and equal to one, i.e. adding a dollar's worth of staff expenditure has the same effect on managerial utility as adding a dollar's worth of managerial emoluments.

274. Simon, The Compensation of Executives, op. cit., pp. 32-35.

275. Emoluments are economic rents. Associated with them are zero productivites; that is, "they are not a return to entrepreneurial capacity but rather result from the strategic advantage that the management possesses in the distribution of the returns to monopoly power." See O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., p. 1035. While it might appear that managers would prefer to receive the cash equivalent of emoluments as salary, according to Williamson this is not the case. Managers find emoluments attractive for two reasons: there are tax advantages to receiving income in kind (e.g. meals bought on an expense account or company-provided limousine with chauffeur are not part of the individual's taxable income, whereas the equivalent sum as salary would be subject to personal income tax) and they are also less visible than salary to shareholders and unions, and when shareholders are dissatsified or unions are negotiating wages, the level of executive salaries is frequently brought up during the course of discussions. See Gordon, Business Leadership in the Large Corporation, op. cit., p. 164. In practice, determining what portion of executive compensation is discretionary would be very difficult, which is one of the reasons Williamson later lumped staff expenditures and emoluments together into a single variable.

276. Williamson's reported profit, π_r , corresponds to (though is not equal identically to) net operating income in the income statement illustrated in table II-2. According to (117), π_r understates net operating income to the extent of managerial emoluments, M , which serve to inflate administrative salaries as well as other components of general and administrative expenses listed in the income statement. Williamson's discretionary profit, π_d , corresponds to (though, on account of π_r , is not equal identically to) that portion of net income in excess of the required minimum, π_o .

277. For simplicity it is assumed that there is a single output. The demand function for this good can be expressed as $P = P(Y,S)$, where P is the price of the good. To appreciate the dependence of P on S , recall that S includes advertising outlays. Then $R = P(Y,S) \cdot Y = R(Y,S)$.

278. This follows the approach adopted by Williamson. If corner solutions are to be permitted, then the Kuhn-Tucker conditions for the inequality-constrained problem can be written down and solved. At the corner solution, discretionary profit is zero. The Lagrange multiplier measures the marginal utility of a reduction of one unit (i.e. one dollar) in the profit constraint π_o. The decision variables must satisfy one of two sets of conditions. (i) $\partial U/\partial S = \partial U/\partial M = 0$ and $\lambda = \partial U/\partial \pi_o = - \partial U/\partial \pi_d$, i.e. the firm's managers are satiated with respect to staff and also with respect to emoluments, which is probably a rare occurrence (and which is rendered impossible by the usual assumption that all first partial derivatives of the utility function are strictly positive, i.e. that satiation is impossible) (ii) or $\partial R/\partial Y = dC/dY$,

$$\partial R/\partial S = 1 - \frac{1}{(1-t)} \frac{\partial U/\partial S}{[(\partial U/\partial \pi_d)+\lambda]}, \quad \text{and} \quad \partial U/\partial M = (1-t)[\frac{U}{\partial \pi_d} + \lambda] ,$$

which are identical to (126)-(128) when $\lambda = 0$. In this case, the managers' freedom to trade off π_d for S or M is constrained. That is, if the firm is in equilibrium and the constraint $\pi_d \geq 0$ is binding, then any attempt to raise S or M (or both) by reducing π_r would cause not only the loss in utility due to the reduction in π_d but also the implied loss in utility due to violation of the constraint. This secondary effect is allowed for by adjusting $\partial U/\partial \pi_d$ by λ.

279. Equations (123)-(125) are also satisfied when $\frac{\partial U}{\partial \pi_d} = \frac{\partial U}{\partial S} = \frac{\partial U}{\partial M} = 0$, which implies satiation with respect to each of the arguments of the managerial utility function. If satiation is ruled out, then this solution is no longer possible.

280. Not shown in figure II-20 are the positions of the profit maximizer's revenue and cost functions. Since the profit maximizer would spend less on staff ($\overline{S}$ rather than S^* in figure II-19), the demand and marginal revenue curves for a profit maximizer would lie below those of the Williamson-type firm (i.e. the Williamson-type firm would spend an additional $S^*-\overline{S}$ on staff (including advertising), thereby causing its demand and marginal revenue curves to lie to the right of the profit maximizer's). In addition, if nonadvertising staff expenditures are pushed beyond the profit maximizing level, then, as long as these expenditures (i.e. the additional staff hired) have a positive marginal product, the Williamson-type firm's marginal production cost curve will lie to the right of the profit-maximizer's. Thus the effects of the increases in advertising and nonadvertising staff expenditures reinforce one another and cause the Williamson-type firm's output to be higher than the short run profit maximizer's.

281. See O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., p. 1052. While the empirical results are more than ten years old, Williamson's conclusions remain valid. During the recent recession the business literature — Business Week, Forbes, Fortune, the Wall Street Journal, etc. — abounded with articles describing companys' efforts to cut discretionary spending of the

type described by Williamson. For example, see "Charging less to expense accounts," Business Week (January 27, 1975), and G. Bronson, "Profit-Pinched Firms Extend Cost Cutting Down to Paper Clips," Wall Street Journal (January 30, 1975). Profits were so pinched at Playboy Enterprises Inc. that the company sold off the corporate jet (sans bunnies, however).

282. Ibid., p. 1052.

283. Though retained earnings do not appear in his model, Williamson is very much aware of their significance to managers. An important influence in this regard is the extent of internal representation on the board of directors. Williamson's empirical results imply that the higher the proportion of internal representation, the higher the level of executive compensation and the higher the earnings-retention ratio, which increases the amount of funds "available to the management for the pursuit of expansionary objectives, and the resulting investment, being based on a combination of profit and expansionary goals, will exceed the amount dictated by profit considerations alone." Ibid., p. 1051. Thus, the implications of the Williamson model are not inconsistent with those of the Marris model, though the Marris model treats the growth of the firm explicitly, whereas the Williamson model does not.

284. J.H. Williamson, op. cit., pp. 3, 9-14. In the J.H. Williamson model the firm sacrifices profit by pushing the growth rate beyond the optimum, rather than by pushing the sales level beyond the optimum. See also Needham, op. cit., pp. 6-8.

285. S. Peterson, "Corporate Control and Capitalism," Quarterly Journal of Economics (vol. 79; no. 1; February 1965), pp. 1-24. This point is disputed by Galbraith. See Galbraith, The New Industrial State, op. cit., p. 172.

286. See page 242 of this chapter.

287. Solow, op. cit., pp. 341-342.

288. It is, however, an important question. For if non-profit-maximizing behavior is "close" to profit-maximizing behavior, it may prove simpler, as Machlup argues, to stick with the profit maximization assumption. See Machlup, op. cit., p. 5.

289. Ibid., p. 5.

290. A study conducted by Monsen, Chiu, and Cooley found that owner-controlled firms earned a significantly higher rate of return on equity than manager-controlled firms. Subsequent studies by Kamerschen, Larner, Elliott, Sorenson, and Kania and McKean found no consistent differences in profitability or the returns to shareholders between these two types of firms. A more recent study by Stano supports Monsen et. al. A study by Palmer falls

between the two extremes by showing that the extent of managerial control affects profit rates only for firms that enjoy a high degree of monopoly power. In addition, Stano's results indicate that manager-controlled firms take greater risks, although the opposite conclusion was reached in a study by McEachern. See R.J. Monsen, J.S. Chiu, and D.E. Cooley, "The Effect of Separation of Ownership and Control on the Performance of the Large Firm," Quarterly Journal of Economics (vol. 82; no. 3; August 1968), pp. 435-451; D.R. Kamerschen, "The Influence of Ownership and Control on Profit Rates," American Economic Review (vol. 58; no. 3; June 1968), pp. 432-447; Larner, op. cit., ch. 3; J.W. Elliott, "Control, Size, Growth, and Financial Performance in the Firm," Journal of Financial and Quantitative Analysis (vol. 7; no. 1; January 1972), pp. 1309-1320; R. Sorenson, "The Separation of Ownership and Control and Firm Performance: An Empirical Analysis," Southern Economic Journal (vol. 41; no. 1; July 1974), pp. 145-148; J.J. Kania and J.R. McKean, "Ownership, Control, and the Contemporary Corporation: A General Behavior Analysis," Kyklos (vol. 29; no. 2; 1976), pp. 272-291; M. Stano, "Monopoly Power, Ownership Control, and Corporate Performance," Bell Journal of Economics (vol. 7; no. 2; Autumn 1976), pp. 672-679; J.P. Palmer, "The Profit-Performance Effects of the Separation of Ownership from Control in Large U.S. Industrial Corporations," Bell Journal of Economics and Management Science (vol. 4; no. 1; Spring 1973), pp. 293-303; and W.A. McEachern, "Corporate Control and Risk," Economic Inquiry (vol. 14; no. 2; June 1976), pp. 270-278. In summary, then, the empirical evidence to date is not conclusive with regard to how sensitive the firm's profitability and the rate of return earned by its shareholders are with respect to the degree of managerial control.

291. The basic references are Cyert and March, op. cit., Cohen and Cyert, op. cit., ch. 16; and Monsen and Downs, op. cit. A critical appraisal of the behavioral theory, and in particular, the mark-up pricing models of Cyert and March, is provided in Baumol and Stewart, op. cit. An interesting discussion of the behavioral theory in relation to the traditional and managerial theories can be found in Machlup, op. cit., and also in Silberston, Price Behavior of Firms, op. cit., pp. 534-541. An attempt to achieve a synthesis of the three theories is provided in A. Phillips, "An Attempt to Synthesize Some Theories of the Firm," in Phillips and Williamson, op. cit., pp. 32-44.

292. Cohen and Cyert, op. cit., p. 330; and Baumol and Stewart, op. cit., p. 118. This line of argument reflects the view that one of the criteria by which a model should be judged is the empirical testability of its implications. See Papandreou, op. cit., chs. 1,7.

293. Cyert and March, op. cit., ch. 3, and Monsen and Downs, op. cit.

294. Cyert and March, op. cit., pp. 8-10; H.A. Simon, Administrative Behavior, 2nd ed. (Macmillan; New York; 1957), pp. xxiv-xxvi; and H.A. Simon, "Theories of Decision-Making in Economics and Behavioral Science," American Economic Review (vol. 49; no. 3; June 1959), pp. 262-265. See also H. Leibenstein, "Allocative Efficiency vs. 'X-Efficiency'," American Economic Review (vol. 56; no. 3; June 1966), pp. 406-409.

295. Cyert and March, op. cit., pp. 19-21; Cohen and Cyert, op. cit., p. 330.

296. H.A. Simon, "A Behavioral Model of Rational Choice," Quarterly Journal of Economics (vol. 69; no. 1; February 1955), pp. 99-118; Simon, Theories of Decision-Making in Economics and Behavioral Science, op. cit.; Cyert and March, op. cit.; and Cohen and Cyert, op. cit.

297. Cyert and March, op. cit., pp. 1-3. As argued below, their theories have not as yet achieved such general validity. For an interesting discussion of some of the difficulties that the behavioralist researchers have encountered see H.A. Simon, "New Developments in the Theory of the Firm," American Economic Review (vol. 52; no. 2; May 1962), pp. 1-15.

298. For example, the Cyert and March study of department store pricing, which is discussed below. Cyert and March reported that some department store mark-ups had been stable for nearly half a century. See Cyert and March, op. cit., p. 138.

299. Baumol and Stewart, op. cit.

300. Ibid., p. 119.

301. Ibid., p. 119. In testing the mark-up pricing models of Cyert and March, Baumol and Stewart found that the rules of thumb did change over time. In particular, mark-ups increased in response to increasing costs, though the Cyert and March model "gives us no clue as to the response in the values of its parameters to such exogenous changes in the economic data, and this clearly limits the analytic power of the construct." Ibid., p. 133.

302. Cohen and Cyert, op. cit., p. 330.

303. Ibid., p. 330.

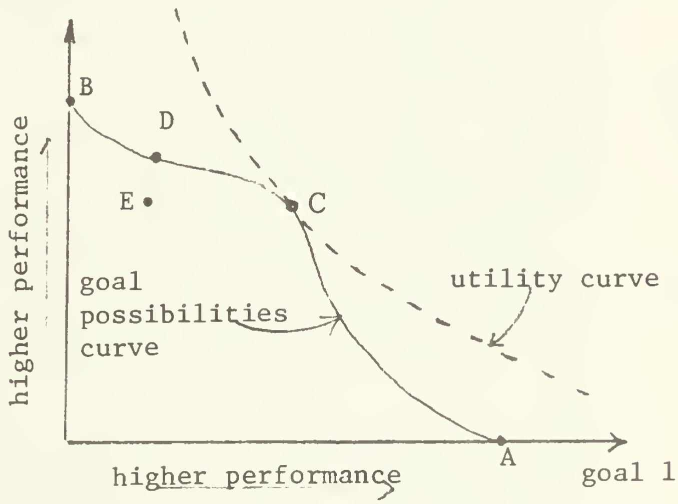
304. Cyert and March, op. cit., p. 21.

305. Ibid., p. 26.

306. Cyert and March acknowledge that, in reality, there may be more than five goals, but argue that the five goals they list are the most representative and that listing additional goals would quickly involve diminishing returns. Ibid., pp. 40-41.

307. By way of contrast, in the traditional theory of the firm, the goal of the owners — maximum total profit — is the goal of the firm. In the managerial models, the goal of the managers (whose goals are either identical or else somehow incorporated into one grand managerial utility function) is the goal of the firm. In each case conformance to the single goal was purchased by payments of wages to workers, interest to sources of capital, dividends to stockholders, etc. The behavioral theory can be regarded as more general in that it is consistent with these extreme cases of one ascendant subcoalition, while also allowing for interactions among the various competing interest groups. It should be noted that the aggregation of the goals of the members of the coalition into a set of corporate goals requires the adoption of a social choice rule for this purpose and that the selection of a 'best' social choice rule involves theoretical as well as practical difficulties. See Heal, op. cit., ch. 2.

308. The outcome of the goal-setting process can be illustrated with the aid of a goal possibilities curve, such as the one in the figure below (where the number of goals has been reduced to two to permit geometric representation). The goal possibilities curve shows for each level of attainment of one goal, the highest possible level of attainment of the other. The pair of goals set through the bargaining process may lie anywhere on or inside this curve, but not outside it. If subcoalition 1 were ascendant, then it would set the corporate goal(s), and the resulting goal pair would be represented by A (i.e. the highest possible performance with respect to that goal). Similarly, if



Figure

subcoalition 2 were ascendant, the goal pair would be represented by B . If the preferences of the subcoalitions could be aggregated into a grand utility function (see the preceding footnote), such as the one shown in the figure, collective utility would be maximized by adopting the pair of goals represented by C , and the associated performance levels would represent 'satisficing', rather than maximizing (with respect to the goal(s) of any one subcoalition) behavior. However, the outcome of the bargaining process depends on the relative bargaining strengths of the subcoalitions, and depending on the social choice rule adopted, the established goal pair may lie somewhere else on the goal possibilities curve (e.g. at D), or possibly even inside the curve (e.g. at E). In this latter case the goal-setting process is said to be 'goal inefficient.'

309. This second example suggests a correspondence between organizational slack and the managerial emoluments and staff components of Oliver Williamson's managerial model discussed in the previous section.

310. Note the contrast with the traditional theory, in which it is assumed that, for any given output level, the firm is producing at minimum cost.

311. Cyert and March, op. cit., pp. 36-38.

312. Ibid., pp. 36-38.

313. Ibid., pp. 146-147.

314. Cohen and Cyert, op. cit., pp. 339-350.

315. For example, Minnesota Mining & Manufacturing Co. produces 35,000 different products. See "Minnesota Mining — A New Ball Game?", Forbes (July 1, 1976), p. 34.

316. Cyert and March, op. cit., ch. 7. See also W.J. Baumol and R.E. Quandt, "Rules of Thumb and Optimally Imperfect Decisions," American Economic Review (vol. 54; no. 2; March 1964), pp. 23-46, and R.M. Cyert and M.I. Kamien, "Behavioral Rules and the Theory of the Firm," in Phillips and Williamson, op. cit., pp. 1-10.

317. Baumol, Business Behavior, Value and Growth, op. cit., p. 29. Since decision-making is not costless — the information needed to make a decision must be gathered and processed — reliance on simple rules of thumb can be seen as an attempt to minimize the costs associated with the decision-making process. When a decision is not crucial, the relatively inexpensive approximate solution reached with the aid of some rule of thumb may be more cost effective than one obtained by means of a more costly optimizing procedure. See Baumol and Quandt, Rules of Thumb and Optimally Imperfect Decisions, op. cit., p. 23, and S.G. Winter, Jr., "Satisficing, Selection, and the Innovating Remnant," Quarterly Journal of Economics (vol. 85; no. 2; May 1971), p. 242.

318. Cyert and March, op. cit., p. 102, and Herendeen, op. cit., pp. 57-58. Indeed, in an uncertain world, mark-up pricing may even be consistent with 'profit-maximizing' behavior. See Sylos-Labini, op. cit., pp. 26-32.

319. Herendeen, op. cit., p. 58.

320. Cyert and March, op. cit., pp. 138, 147. Cyert and March actually used three pricing models: a mark-up model for normal items, a sale-pricing model, and a mark-down model that was used by the store for special sales or when inventory levels of some items proved unsatisfactory. The mark-up model distinguished three classes of goods: 'standard items', 'exclusive items' that were not carried by competitors, and 'import items'. The test results cited in the text refer to the mark-up model applied to 'standard items', i.e. non-import items carried also by competitors. The mark-up policy for these items is called a 40 percent mark-up by Cyert and March; though it is not a 40 percent mark-up over average variable cost, but rather, the mark-up represents 40 percent of the retail price (before rounding).

321. Baumol and Stewart, op. cit., pp. 124-127, 133.

322. This is not meant to imply that mark-up pricing, or more generally, the use of rules of thumb, actually is perfectly consistent with profit-maximizing behavior. Although most economists would argue that it is not strictly consistent, conclusive empirical evidence is lacking. Moreover, there exists some doubt as to whether it is even possible to subject this question to a meaningful empirical test. Ibid., p. 136.

323. Equation (134) requires that $\eta > 1$, which will hold if the firm is maximizing profit since $MR = MC$ and $MC > 0$ imply that $\eta > 1$. See footnote 182.

324. Herendeen presents a somewhat more complex model that makes the quantity of output sold a function of advertising outlays, the mark-up, the state of demand, and the average price charged by other firms. In his model, as in (134), the optimal mark-up is independent of the average variable cost of production. See Herendeen, op. cit., pp. 159-162.

325. The model (134) is one of the simplest of a great variety of mark-up pricing models that have appeared in the literature. For others see A.D.H. Kaplan, J.B. Dirlam, and R.F. Lanzillotti, Pricing in Big Business — A Case Approach (Brookings Institution; Washington, D.C.; 1958); W.A.H. Godley and W.D. Nordhaus, "Pricing in the Trade Cycle," Economic Journal (vol. 82; no. 327; September 1972), pp. 853-882; and A.S. Eichner, "A Theory of the Determination of the Mark-up under Oligopoly," Economic Journal (vol. 83; no. 332; December 1973), pp. 1184-1200. For evidence to the contrary — that firms tend to follow marginalist principles in setting prices — see J.S. Earley, "Marginal Policies of 'Excellently Managed' Companies," American Economic Review (vol. 46; no. 1; March 1956), pp. 44-70, which, unfortunately, is somewhat dated. Since, as (134) implies, mark-up pricing and marginalist pricing are not wholly inconsistent, statistical tests might be unable to distinguish between the two approaches to pricing.

326. O.E. Williamson, The Economics of Discretionary Behavior, op. cit., pp. 10-11.

327. See Silberston, op. cit., p. 536, and Baumol and Stewart, op. cit., p. 133. Cyert and March were not unaware of this problem. See Cyert and March, op. cit., ch. 8. The comments in the text are not meant to imply that a model cannot be considered a worthy contribution to the theory of the firm unless it has been tested empirically. Where the purpose of the model is to explain, rather than to predict, as in the case of the behavioral models, setting up an empirical test is inherently more difficult. There still remains, however, a general feeling that "meaningful theory ... is theory capable of being refuted by reference to empirical data." Papandreou, op. cit., p. 7. See also Samuelson, Foundations of Economic Analysis, op. cit., p. 4, and footnote 255 of this chapter.

328. Machlup, op. cit., p. 26.

329. Baumol and Quandt, Rules of Thumb and Optimally Imperfect Decisions, op. cit.

330. The obvious example is the recent improvements in the state of inventory theory, which, when coupled with the enhanced data handling capabilities provided by computer-based information systems, have led to a major improvement in industry's inventory management.

331. However, it could be argued that, if these models are viewed in the context of a general equilibrium model of capital markets, the role of external finance is subsumed. That is, the stock market and the bond market will permit to come into existence only those firms that are sufficiently attractive to potential investors that the necessary funds can be raised through the issuance of equity shares and bonds.

332. See section C of chapter one of this paper. In what follows the terms 'money capital' and 'financial capital' are used synonomously. The distinction between financial (or money) capital and real capital is drawn nicely in Vickers, op. cit., pp. 105-106.

333. See Harcourt, op. cit., especially, ch. 1.

334. The securities that are issued may take any one of several forms, although those models of the firm that incorporate finance typically consider the following two classes of securities: (i) debt instruments (i.e. bonds, notes, and other fixed interest securities) and (ii) equity instruments (i.e. common stock). Note that bank loans are typically treated in the same manner as bonds, rather than as a separate mode of financing.

335. Leland, Production Theory and the Stock Market, op. cit., p. 125. See also Vickers, op. cit., pp. 108-109.

336. For example, Veblen, whose The Theory of Business Enterprise was first published in 1904, recognized the importance of financial capital, arguing that through borrowing the firm could increase the rate of return on equity as long as the rate of return on invested capital exceeded the rate of interest on the borrowed funds. T. Veblen, The Theory of Business Enterprise (Mentor; New York; 1958), ch. 3. See also O. Lange, "The Place of Interest in the Theory of Production," Review of Economic Studies (vol. 3; June 1936), pp. 159-192; J.A. Schumpeter, The Theory of Economic Development (Oxford University Press; Fair Lawn, N.J.; 1961), ch. 3 [first published in 1911]; and T. Scitovsky, Welfare and Competition, rev. ed. (Allen & Unwin; London; 1971), ch. 9.

337. Vickers, op. cit., pp. 108-109.

338. See Leland, Production Theory and the Stock Market, op. cit., p. 126, on this point.

339. See footnote 376 and the references listed therein.

340. See footnote 430 and subsection 1 of section K of this chapter.

341. See C.G. Krouse, "On the Theory of Optimal Investment, Dividends, and Growth in the Firm," American Economic Review (vol. 63; no. 3; June 1973), pp. 269-279; D. Durand, "The Cost of Capital, Corporation Finance, and the Theory of Investment: Comment," American Economic Review (vol. 49; no. 4; September 1959), pp. 639-655; R.C. Stapleton, "Taxes, the Cost of Capital and the Theory of Investment," Economic Journal (vol. 82; no. 328; December 1972), pp. 1273-1292; and A.J. Senchack, Jr., "The Firm's Optimal Financial Decisions: An Integration of Corporate Financial Theory Under Certainty," unpublished doctoral dissertation (University of California; Los Angeles; 1973).

342. J. Hirshleifer, "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," Quarterly Journal of Economics (vol. 80; no. 2; May 1966), pp. 264-268; Hirshleifer, Investment, Interest, and Capital, op. cit., ch. 9; Diamond, op. cit.; and Leland, Production Theory and the Stock Market, op. cit. A general discussion of the impact of uncertainty (when risk aversion is present) can be found in J.J. McCall, "Probabilistic Microeconomics," Bell Journal of Economics and Management Science (vol. 2; no. 2; Autumn 1971), pp. 403-433.

343. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, op. cit.; Lintner, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, op. cit.; and Mossin, Equilibrium in a Capital Asset Market, op. cit..

344. Diamond, op. cit.; Stiglitz, On the Optimality of the Stock Market Allocation of Investment, op. cit.; Jensen and Long, op. cit.; Fama, Perfect Competition and Optimal Production Decisions under Uncertainty, op. cit.; and Leland, Production Theory and the Stock Market, op. cit. It should also be noted that a recent paper by Stevens employs a revised version of the Sharpe-Lintner-Mossin valuation equation to derive optimal investment and production policies for the firm in a multiperiod setting. See Stevens, op. cit.

345. See J.E. Walter, "Dividend Policies and Common Stock Prices," Journal of Finance (vol. 11; no. 1; March 1956), pp. 29-41; J.E. Walter, "Dividend Policy: Its Influence on the Value of the Enterprise," Journal of Finance (vol. 18; no. 2; May 1963), pp. 280-291; Gordon and Shapiro, op. cit.; M.J. Gordon, Dividends, Earnings, and Stock Prices, op. cit.; and M.J. Gordon, The Investment, Financing, and Valuation of the Corporation (Irwin; Homewood, Ill.; 1962). The Gordon model has been criticized by Modigliani and Miller on the grounds that the restriction to internal financing in the model makes the firm's production (or investment) and dividend (or financing) decisions indistinguishable. Miller and Modigliani, op. cit., p. 425.

346. Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., p. 292.

347. Senchack, op. cit., pp. 1-2.

348. E.M. Lerner and W.T. Carleton, "The Integration of Capital Budgeting and Stock Valuation," American Economic Review (vol. 54; no. 5; September 1964), pp. 683-702; and E.M. Lerner and W.T. Carleton, A Theory of Financial Analysis (Harcourt, Brace & World; New York; 1966).

349. B. Davis, "Investment and Rate of Return for the Regulated Firm," Bell Journal of Economics and Management Science (vol. 1; no. 2; Autumn 1970), pp. 245-270; and B. Davis and D.J. Elzinga, "The Solution of an Optimal Control Problem in Financial Modeling," Operations Research (vol. 19; no. 6; October 1971), pp. 1419-1433.

350. Krouse, op. cit. The Krouse model is discussed in section L of this chapter. In an earlier paper Krouse developed a linear programming model that permitted both debt and equity financing. See C.G. Krouse, "Optimal Financing and Capital Structure Programs for the Firm," Journal of Finance (vol. 27; no. 5; December 1972), pp. 1057-1071. Charnes and others have also formulated linear programming models with a view toward helping the firm solve its financial and capital allocation problems. See A. Charnes, W.W. Cooper, and M.H. Miller, "Application of Linear Programming to Financial Budgeting and the Costing of Funds," Journal of Business (vol. 32; no. 1; January 1959), pp. 20-46; and W.T. Carleton, "Linear Programming and Capital Budgeting Models: A New Interpretation," Journal of Finance (vol. 24; no. 5; December 1969), pp. 825-833.

351. Senchack, op. cit.

352. The cost of capital may be defined as the current-market-value weighted average cost of debt and equity capital. See H. Benishay, "Variability in Earnings-Price Ratios of Corporate Equities," American Economic Review (vol. 51; no. 1; March 1961), pp. 81-94; J.F. Weston, Managerial Finance (Holt, Rinehart & Winston; New York; 1962); H. Bierman, Jr., Financial Policy Decisions (Macmillan; London; 1970), ch. 5. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit.; and F. Modigliani and M. Miller, "'The Cost of Capital, Corporation Finance, and the Theory of Investment': Reply," American Economic Review (vol. 49; no. 4; September 1959), pp. 655-669. The cost of capital represents the "minimum rate of return required for an investment to be advantageous to the stockholders." Ibid., p. 659. There appears to be some disagreement on this point. For example, Bierman argues that it is incorrect to use the cost of capital to evaluate proposed investments. Bierman, op. cit., ch. 4. The source of disagreement is the following. The cost of capital, as defined above, is an *average* cost. The appropriate cost to use to evaluate a proposed investment — an increment to the firm's capital stock — is the *marginal* cost of capital. Thus, using the average cost of capital is correct only if the marginal cost is equal to the average cost, which requires that the firm's current capital structure be optimal (so that the average cost of capital is a minimum) and that the increment to the capital stock be small enough that the marginal cost of capital does not increase. It should be noted that, in the case of certainty and perfect capital markets, as in the traditional theory of the firm, every firm's cost of capital is the prevailing market rate of interest. Thus, average cost of capital = marginal cost of capital ≡ constant, and the cost of capital defined above can be used to evaluate proposed investments. See Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit., p. 262. However, when uncertainty exists, the cost of capital becomes more difficult to define precisely and to measure. See Philippatos, op. cit., chs. 8-9.

353. For example, see H.G. Guthmann and H.E. Dougall, Corporate Financial Policy, 3rd ed. (Prentice-Hall; Englewood Cliffs, N.J.; 1955); E. Solomon, The Theory of Financial Management (Columbia University Press; New York; 1963), pp. 92-99; and Philippatos, op. cit., pp. 288-292.

354. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit. As will be noted below, Modigliani's and Miller's result is dependent on a number of critical assumptions they make. There are, of course, a number of intermediate views, such as Durand's view that changes in the capital

structure do not increase the overall level of risk, but merely redistribute risk among the existing holders of the firm's securities. Hence, there is no leverage effect (though there is an optimal debt-equity mix due to the existence of market imperfections. See D. Durand, "Costs of Debt and Equity Funds for Business: Trends and Problems of Measurement," in Conference on Research in Business Finance (National Bureau of Economic Research; New York; 1952), pp. 215-247, and Durand, The Cost of Capital, Corporation Finance, and the Theory of Investment: Comment, op. cit.

355. In this section no attempt will be made to cover what has become a very extensive literature. Rather, the purpose of this section is to consider the implications of Modigliani's and Miller's important work, which is often cited by economists building models of the firm in order to justify some of their simplifying assumptions.

356. What follows is based on Philippatos, op. cit., pp. 288-292.

357. See Stapleton, op. cit., on this point.

358. In this case there exists a range of optimal capital structures, and within this range, modest changes in the firm's capital structure will not alter the firm's market value. In the event there is a lone stationary value, then at that point occurs the firm's unique optimum capital structure.

359. The firm's cost of capital, C, is given by

$$C = \alpha K_d + (1-\alpha)K_e ,$$

where α is the proportion of debt, K_d is the cost of debt capital and K_e is the cost of equity capital. The market value of the firm, V, is given by

$$V = \frac{\pi}{C} ,$$

where π is net income and C is the cost of capital, so that V and C vary inversely.

360. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit. (and in particular p. 275).

361. See footnote 126 for the definition of a perfect capital market. One type of market imperfection is due to corporate profit taxes, the effect of which is discussed in F. Modigliani and M.H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," American Economic Review (vol. 53; no. 3; June 1963), pp. 433-443; and D.P. Baron, "Firm Valuation, Corporate Taxes, and Default Risk," Journal of Finance (vol. 30; no. 5; December 1975), pp. 1251-1264.

362. _Ibid._, p. 292. They do explicitly recognize, however, that there may be other grounds, such as concern over terms that might be imposed on managers' freedom of action by creditors were the firm to increase its debt-equity mix, on which the firm might prefer one form of financing over another. _Ibid._, pp. 292-293.

363. For a detailed discussion of the Modigliani-Miller model, see Fama and Miller, _op. cit._, ch. 4.

364. One apparent limitation of the original development was Modigliani's and Miller's assumption that firms could be divided into risk classes, where two firms belong to the same risk class provided the returns to the assets of each always occur in the same proportion, e.g. $X_j = \lambda X_i$, where X_j and X_i are the returns to firms j and i, respectively, and λ is the constant of proportionality. Modigliani and Miller, _The Cost of Capital, Corporate Finance and the Theory of Investment_, _op. cit._, p. 266, and Modigliani and Miller, _'The Cost of Capital, Corporation Finance, and the Theory of Investment': Reply_, _op. cit._, p. 655. But Stiglitz later showed that Modigliani's and Miller's result is not dependent on their assumption of risk classes. J.E. Stiglitz, "A Re-Examination of the Modigliani-Miller Theorem," _American Economic Review_ (vol. 59; no. 5; December 1969), pp. 784-793. Stiglitz has also generalized the Modigliani-Miller model further to a multiperiod setting. J.E. Stiglitz, "On the Irrevelance of Corporate Financial Policy," _American Economic Review_ (vol. 64; no. 6; December 1974), pp. 851-866.

365. Modigliani and Miller, _The Cost of Capital, Corporation Finance and the Theory of Investment_, _op. cit._, p. 268. See also Mossin, _Theory of Financial Markets_, _op. cit._, p. 87, on this point.

366. _Ibid._, ch. 5, which also demonstrates that the Modigliani-Miller result holds even when investor preferences are arbitrary and when investor beliefs regarding future returns from securities are heterogeneous; and D.P. Baron, "Default Risk and the Modigliani-Miller Theorem: A Synthesis," _American Economic Review_ (vol. 66; no. 1; March 1976), pp. 204-212. It should be noted that Baron claims to have shown in an earlier paper the indeterminancy of the debt-equity mix even when default risk is present, but Hagen disputes this earlier result. See D.P. Baron, "Default Risk, Homemade Leverage, and the Modigliani-Miller Theorem," _American Economic Review_ (vol. 64; no. 1; March 1974), pp. 176-182; and K.P. Hagen, "Default Risk, Homemade Leverage, and the Modigliani-Miller Theorem: Note," _American Economic Review_ (vol. 66; no. 1; March 1976), pp. 199-203.

367. Baron, _Default Risk and the Modigliani-Miller Theorem: A Synthesis_, _op. cit._; Hirshleifer, _Investment Decision Under Uncertainty: Applications of the State-Preference Approach_, _op. cit._, pp. 264-268; and Hirshleifer, _Investment, Interest, and Capital_, _op. cit._,

pp. 263-264, 271-272. The importance of complete markets is demonstrated below in section K of this chapter. It should be noted that the above papers also demonstrate that the Modigliani-Miller theorem fails to hold when markets are imperfect. See also Mossin, Security Pricing and Investment Criteria in Competitive Markets, op. cit.; Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit. ; Lintner, Corporation Finance: Risk and Investment, op. cit., and Senchack, op. cit.

368. See Baron, Default Risk and the Modigliani-Miller Theorem: A Synthesis, op. cit.

369. Mossin, Theory of Financial Markets, op. cit., p. 78.

370. Robert M. Baylis, senior vice-president of First Boston Corp., quoted in "The Big Squeeze On U.S. Companies," Business Week (September 22, 1975), p. 51. See also T.E. Fandell, "TWA Tries to Restructure Its Heavy Debt In Effort to Avert Potential Bankruptcy," Wall Street Journal (August 18, 1975).

371. F. Andrews, "Stock-Market Surge Beckons Firms Back To Equity Financings After Long Hiatus," Wall Street Journal (February 23, 1976).

372. "Capital Crisis," Business Week (September 22, 1975).

373. See Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 272-273, for discussion on this point.

374. Bierman, op. cit., p. 134. Of course, this avoids the question of how significant an impact the firm's capital structure has on its cost of capital. Even according to the traditional view, there may be a range of capital structures over which the impact is insignificant. Unfortunately, the empirical evidence to date is not conclusive. See Philippatos, op. cit., ch. 10 (particularly appendix A).

375. F. Modigliani and M.H. Miller, 'The Cost of Capital, Corporation Finance and the Theory of Investment': Reply, op. cit., pp. 655-669; and Miller and Modigliani, op. cit. See also Fama and Miller, op. cit., pp. 78-85.

376. Miller and Modigliani, op. cit., pp. 414, 430. Putting this result together with their earlier result leads to the implication that, under certain conditions, the firm's investment policy and its financial policies are separable; that is, given the firm's investment policy, how the firm chooses to finance that investment — new debt issue (i.e. borrowing), new equity issue, retained earnings, or some combination of these — will not affect either the cost of capital or the total market value of the firm. See also Lintner, The Cost of Capital and Optimal Financing of Corporate

Growth, op. cit., p. 295, on this point. It should be noted that, though Miller and Modigliani speak only of the firm's investment policy as being given, they are also assuming that the firm's production policy (and hence, the firm's operating policies, which include both the investment and the production policies) is given. See Fama and Miller, op. cit., pp. 80-81, or Mossin, Theory of Financial Markets, op. cit., p. 141, for more on this point.

377. What follows is based on Miller and Modigliani, op. cit., pp. 412-414.

378. The firm's dividend policy is also irrelevant under uncertainty, provided capital markets are complete. See M. Rubinstein, "The Irrelevancy of Dividned Policy in an Arrow-Debreu Economy," Journal of Finance (vol. 31; no. 4; September 1976), pp. 1229-1230, for a proof of the irrelevancy of the firm's dividend policy when financial markets are perfect and complete.

379. This is what Miller and Modigliani call 'the fundamental principle of valuation.' Miller and Modigliani, op. cit., p. 412. Note that, since (136) must hold for the securities of all firms, it is not necessary to use subscripts to distinguish among firms.

380. Conceivably the firm's dividend policy could also affect $V(t)$ indirectly through $V(t+1)$ if the current dividend distribution served to convey what would otherwise be unavailble information concerning future dividend policy. Miller and Modigliani initially assumed the firm's future dividend policy was known, although they explicitly recognized the informational content of dividends. Ibid., pp. 413, 430. See also "Chrysler Declares 15-Cent Payout, First in 1½ Years," Wall Street Journal (August 6, 1976).

381. Miller and Modigliani, op. cit., p. 413. See also Mossin, Theory of Financial Markets, op. cit., p. 141, on this point.

382. Note that from equation (139)

$$D(t) - m(t+1)\cdot p(t+1) = X(t) - I(t) \qquad (*)$$

holds identically. Each side of (*) gives an expression for net dividends. That is, under the assumptions of certainty and perfect capital markets, the receipts from new share issues can be interpreted as negative dividend flows, i.e. a dollar raised through a new equity issue 'cancels out' an additional dollar paid out as dividends in the follwing sense. Given the firm's investment policy, an additional dollar of dividends requires that an additional dollar be raised through a new equity issue, and the opposing effects of these actions on the firm's share value cancel each other out. This is the reason the firm's dividend policy is irrelevant.

383. In an uncertain world the additional assumption of complete markets is needed in order that individuals be free to achieve their most desired patterns of claims through the market place (the reasoning here is perfectly analogous to that involved in establishing the indeterminancy of the debt-equity mix, i.e. in effect, the existence of complete markets for contingent claims reduces everything to the certainty case). Rubinstein, The Irrelevancy of Dividend Policy in an Arrow-Debreu Economy, op. cit.

384. Note that this independence does *not* follow from the frequently used Gordon share valuation model,

$$v = d_o/(i-g) \quad ,$$

where v is the share price, d_o is the initial dividend payout, i is the constant rate of interest, g is the constant steady state rate of growth, and all growth is financed internally. In this case the firm's investment and dividend policies are inextricably bound together. Letting r denote the constant retention ratio, p the rate of profit, K the initial capital stock, and N the number of shares outstanding, the share value can be expressed as a function of the retention ratio,

$$v(r) = \frac{(1-r)pK}{N(i-g)} \quad ,$$

and by differentiation,

$$v'(r) = - \frac{pK(i-p)}{N(i-g)^2} \quad ,$$

which equals zero only if $i = p$. Thus, changes in dividend policy (reflected in changes in r) are irrelevant only if the internal rate of return on investment, p , equals the market rate of interest, i , for then investors would be indifferent between the firm's investing a sum at rate of return p or distributing an equal sum to them which they could invest at rate $i = p$.

385. Since its investment policy is not affected, nor will its growth possibilities be influenced. See Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., p. 303.

386. Miller and Modigliani, op. cit., p. 414.

387. Gordon, The Investment, Financing, and Valuation of the Corporation, op. cit., pp. 55-66; Gordon, Dividends, Earnings and Stock Prices, op. cit.; Lintner, Dividends, Earnings, Leverage, Stock Prices and the Supply of Capital to Corporations, op. cit.; and E.F. Brigham and M.J. Gordon, "Leverage, Dividend Policy, and the Cost of Capital," Journal of Finance (vol. 23; no. 1; March 1968), pp. 85-103. For empirical evidence indicating that investors' relative preferences for dividends as opposed to retained earnings

may be less in growth than in non-growth industries, see Friend and Puckett, op. cit. Recently shareholders have been pressuring companies to raise dividends. For example, see "Wall Street: Back to Dividends," Newsweek (August 16, 1976).

388. See Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., p. 304; Lintner, Corporation Finance: Risk and Investment, op. cit., pp. 230-231; J.R. Meyer and E. Kuh, The Investment Decision (Harvard University Press; Cambridge; 1957). See also J.V. Conti, "More Companies Cut or Omit Dividends As Earnings Decline," Wall Street Journal (May 16, 1975).

389. See Lintner, Corporation Finance: Risk and Investment, op. cit.; and for empirical evidence see P. Dhrymes and M. Kurz, "Investment, Dividend, and External Finance Behavior of Firms," in R. Ferber, ed., Determinants of Investment Behavior (Columbia University Press; New York; 1967), pp. 427-467.

390. Ibid. offers supporting empirical evidence.

391. Reinforcing the effects of the above factors is the possibility that security-minded corporate managers may favor retained earnings in order to reduce the chances of financial ruin. See K.H. Borch, The Economics of Uncertainty (Princeton University Press; Princeton; 1968).

392. E.F. Fama, "The Empirical Relationships Between the Dividend and Investment Decisions of Firms," American Economic Review (vol. 64; no. 3; June 1974), pp. 304-318. This disagreement as to the relevance of the firm's dividend policy — with each side marshaling empirical support for its position — is indicative of the difficulties arising out of the economist's inability to design controlled scientific experiments to test propositions. See footnote 255.

393. Vickers, The Theory of the Firm; and D. Vickers, "The Cost of Capital and the Structure of the Firm," Journal of Finance (vol. 25; no. 1; March 1970), pp. 35-46.

394. Business risk is measured by the coefficient of variation of net operating income and financial risk is measured by the coefficient of variation of net income, where net operating income equals revenue minus operating costs (and thus business risk reflects decisions made with regard to the assets side of the firm's balance sheet) and net income is net operating income less interest on debt capital and corporate taxes(and thus financial risk mainly reflects decisions made with regard to the liabilities side by the balance sheet). See tables II-1 and II-2 in section A. It is financial risk that measures the degree of risk in the owners' economic position. See Vickers, The Theory of the Firm, op. cit., chs. 3-4. See also J.C. Van Horne, Financial Management

and Policy, 2nd ed. (Prentice-Hall; Englewood Cliffs, N.J.; 1971), chs. 3,7, and P.A. Tinsley, "Capital Structure, Precautionary Balances, and Valuation of the Firm: The Problem of Financial Risk," Journal of Financial and Quantitative Analysis (vol. 5; no. 2; March 1970), pp. 33-62.

395. This enables Vickers to express the interest rate on debt and the capitalization rate on equity each as a function of leverage only (or when the amount of equity is fixed, as a function of the amount of debt only). Vickers, The Theory of the Firm, op. cit., pp. 64-69, 133-137, 162-167.

396. Turnovsky considered the effects of changing business risk, but treated business risk as exogenously determined. Arzac generalized further by deriving a criterion for investment decisions under general uncertainty when business risk is controllable. See S.J. Turnovsky, "Financial Structure and the Theory of Production," Journal of Finance (vol. 25; no. 5; December 1970), pp. 1061-1080, and Arzac, op. cit. In contrast to Vickers, Arzac must make the interest rate on debt and the capitalization rate on equity each a function of output as well as of the amounts of debt and equity. Ibid., p. 1229.

397. In reality, one or both types of risk may vary systematically with either the size of the company or with the degree of management control. See J.P. Palmer, "The Profit Variability Effects of the Managerial Enterprise," Western Economic Journal (vol. 11; no. 2; June 1973), pp. 228-231.

398. In this paper the terms risk and uncertainty are used synonomously to mean what Frank Knight defines as risk. See subsection 7 of section C of chapter one of this paper.

399. See Hirshleifer, Investment, Interest, and Capital, op. cit., chs. 8-10.

400. See footnote 394.

401. Vickers, The Theory of the Firm, op. cit., chs. 6-10.

402. Following Vickers, taxes are ignored, there is a single output and two inputs, and the firm's equity is assumed constant.

403. Ibid., p. 128.

404. According to this definition of r, the different debt securities the firm may have issued have been combined into a single composite security and r is the interest rate on this security, computed as the weighted average of the interest rates on the different debt securities comprising the composite security.

405. _Ibid._, chs. 5,7. The net working capital requirement, $g(Q)$, reflects the required investment in current operating assets by the firm, e.g. inventories of raw materials, semi-finished goods, and finished goods, trade credit extended, receivables awaiting collection, etc. See table II-1 in section A. These requirements are directly related to the level of output. The coefficients α and β reflect money capital requirements directly related to the amounts used of the inputs, e.g. for labor there is a money capital requirement when workers are hired and paid before a project has been completed (i.e. the output sold) and for real capital there is also a money capital requirement in that each unit of X_2 requires the investment in fixed assets of β dollars of money capital.

406. The units of measurement employed in (141) deserve comment. X_1 is a flow variable and is expressed in units (of current factor) per period. Its direct unit cost, e.g. the wage rate when X_1 represents labor, is expressed in dollars per unit (of current input), so that w_1X_1 also measures a flow. X_2, on the other hand, is a stock variable and is measured in capacity units (e.g. the capital stock measured in constant dollars). _Ibid._, ch. 7. Its direct unit cost w_2 is expressed in percentage terms, so that w_2X_2 measures a flow. Similarly, $r(D)$ is measured in percentage terms and D is a stock variable, so that $r(D) \cdot D$ measures a flow. The objective function in (141) is, then, a flow equation. The constraint in (141) is a stock inequality. Both E and D represent stocks of money capital, as shown in the firm's balance sheet (see table II-1). The function $g(Q)$ gives the stock of money capital required to support a flow of Q units of output per period, while α gives the stock of money capital required to support each unit flow of X_1 and β gives the stock of money capital required to support each (physical) unit of productive capacity, X_2.

407. These conditions follow from taking the appropriate first partial derivatives of the Lagrangian:

$$L_\lambda = p(Q) \cdot f(X_1, X_2) - w_1X_1 - w_2X_2 - r(D) \cdot D$$
$$+ \lambda[E + D - g(Q) - \alpha X_1 - \beta X_2] \quad ,$$

where the Lagrange multiplier λ measures the effect on net income of varying the equity capital availability. If the amount of equity capital available should exceed the firm's needs, then of course $\lambda = 0$.

408. _Ibid._, p. 162. The interpretation of the Lagrange multiplier is made difficult because of the form of the constraint in (141). The right-hand side is not constant, only the amount of equity, E, is constant. The interpretation becomes more difficult yet when both E and D are permitted to vary. See C.R. Jones and J.D. Finnerty, "Structural Planning under Controllable Business

Risk: Comment," unpublished paper (Naval Postgraduate School; Monterey, Calif.; November 1976). Thus, it is necessary to exercise care in interpreting λ . To develop the correct interpretation, note first that π in (141) represents 'net income' whereas $p(Q)f(X_1,X_2) - w_1X_1 - w_2X_2$ represents 'net operating income'. See table II-2. Denote net income by π_o . Taking the total differential of the money capital constraint (but holding E constant) gives

$$g'(Q)[\frac{\partial f}{\partial X_1} dX_1 + \frac{\partial f}{\partial X_2} dX_2] + \alpha dX_1 + \beta dX_2 = dD . \qquad (*)$$

Then taking the total differential of the net operating income equation yields

$$\begin{aligned} d\pi_o &= MR(Q)[\frac{\partial f}{\partial X_1} dX_1 + \frac{\partial f}{\partial X_2} dX_2] - w_1 dX_1 - w_2 dX_2 \\ &= [MR(Q)\frac{\partial f}{\partial X_1} - w_1]dX_1 + [MR(Q)\frac{\partial f}{\partial X_2} - w_2]dX_2 \\ &= \lambda[g'(Q)\frac{\partial f}{\partial X_1} + \alpha]dX_1 + \lambda[g'(Q)\frac{\partial f}{\partial X_2} + \beta]dX_2 , \qquad (**) \end{aligned}$$

where the last equality follows by substitution using (142) and (143). Then dividing (**) by (*) gives

$$\lambda = \frac{d\pi_o}{dD} .$$

Proceeding along more conventional lines, the money capital constraint can be written as

$$g(Q) + \alpha X_1 + \beta X_2 - D = E ,$$

where E is constant, and it can be shown that $\lambda = d\pi/dE$. To see this, write the total differential of the money capital constraint (letting E vary) as follows:

$$g'(Q)[\frac{\partial f}{\partial X_1} dX_1 + \frac{\partial f}{\partial X_2} dX_2] + \alpha dX_1 + \beta dX_2 = dE + dD ,$$

or equivalently,

$$[g'(Q)\frac{\partial f}{\partial X_1} + \alpha]dX_1 + [g'(Q)\frac{\partial f}{\partial X_2} + \beta]dX_2 - dD = dE . \qquad (***)$$

Then taking the total differential of the net income equation yields:

$$\begin{aligned} d\pi &= MR(Q)[\frac{\partial f}{\partial X_1} dX_1 + \frac{\partial f}{\partial X_2} dX_2] - w_1 dX_1 - w_2 dX_2 - [r(D) + \frac{\partial r}{\partial D} D]dD \\ &= [MR(Q)\frac{\partial f}{\partial X_1} - w_1]dX_1 + [MR(Q)\frac{\partial f}{\partial X_2} - w_2]dX_2 - [r(D) + \frac{\partial r}{\partial D} D]dD \\ &= \lambda[g'(Q)\frac{\partial f}{\partial X_1} + \alpha]dX_1 + \lambda[g'(Q)\frac{\partial f}{\partial X_2} + \beta]dX_2 - \lambda dD , \qquad (****) \end{aligned}$$

where the last equality follows by substitution using (142), (143) and (144). Then dividing (****) by (***) gives

$$\lambda = \frac{d\pi}{dE} \ .$$

This interpretation of λ as the instantaneous rate of change of the optimal value of the objective function (maximum net operating income) with respect to the constraint (the amount of equity capital) agrees with the more conventional interpretation of the Lagrange multiplier.

Thus, λ can be interpreted as the marginal contribution of debt capital to net operating income or as the marginal contribution of equity capital to net income (which is equal to the marginal contribution of equity capital to net operating income). The distinction between net income and net operating income is important because the marginal contribution of debt capital to net income is, by (144), equal to zero at the optimum. Thus, Vickers's marginal profit productivity of money capital, (Vickers, The Theory of the Firm, op. cit., p. 162) is unambiguous only if interpreted in terms of net operating income. The distinction between equity capital and money capital is important here too, for it is not money capital that is in limited supply, but rather, it is equity capital that is scarce.

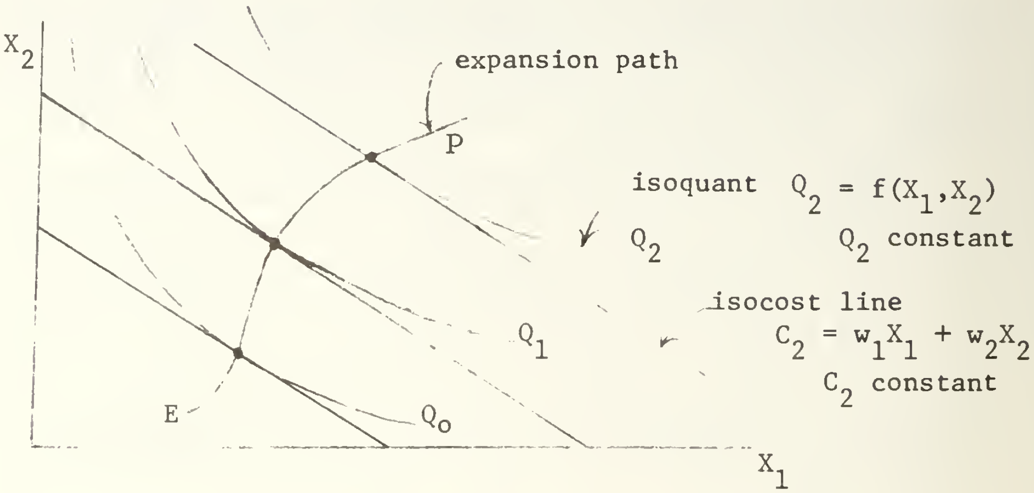
409. See footnote 408. It should be noted that, since the amount of debt capital is optimal, given the current amount of equity capital, any increase in money capital would have to be equity capital (if π is to increase). Thus, the appropriate price of money capital is λ . Note that $\lambda = d\pi/dE$ has the same units as r (percent per period), i.e. λ represents an imputed interest rate. Vickers calls λ the 'marginal profit productivity' of money capital. Ibid., p. 153. The use of 'productivity' has been eschewed here because money capital per se is nonproductive since it is not a factor of production.

410. For purposes of comparison with the conventional neoclassical model, see equations (14). In the development of (14) it was assumed that price is constant, so that $p = MR(Q)$, where the equilibrium condition requires that the use of each factor be increased until the marginal revenue product of that factor just equals its marginal cost w_i .

411. See equation (16). Note that the conventional result holds when there is no equity capital constraint (i.e. when $\lambda = 0$).

412. For the single-output-two-input firm under discussion, the expansion path is the locus of points at each of which the marginal rate of technical substitution (i.e. the negative of the slope of an isoquant in the figure below) is equal to the ratio of factor prices (i.e. the negative of the slope of an isocost line in the figure below). Since at optimality the input mix must be such that

the marginal rate of technical substitution equals the ratio of input prices, the rational entrepreneur will select only input combinations that lie on the expansion path EP in the figure below.



Figure

When $\frac{\alpha}{\beta} \neq \frac{w_1}{w_2}$ and $\lambda \neq 0$, the common slope of the isocost lines changes, thereby altering the firm's expansion path. This causes the equilibrium input combinations employed by the firm as it expands its output to change.

413. Note also that, if factor prices are defined to include the appropriate cost of money capital, the Vickers model becomes the neoclassical model. That is, if the price of money capital λ is known by the firm, and if the unit price of X_1 is $w_1 + \lambda\alpha$ and the unit price of X_2 is $w_2 + \lambda\beta$, then the model of the firm can be reformulated as the mathematical programming problem:

$$\text{maximize: } \pi = p(Q)\cdot f(X_1,X_2) - (w_1 + \lambda\alpha)X_1 - (w_2 + \lambda\beta X_2 - \lambda g(Q) ,$$

where $\lambda \cdot g(Q)$ measures the cost of meeting the firm's net working capital requirements. Differentiating first with respect to X_1 and then with respect to X_2 leads to two equations that can be manipulated to yield the equilibrium conditions (146)-(150). Thus, the Vickers model yields results consistent with those of the neoclassical model, provided the marginal cost of each factor is properly defined, although the Vickers model has an important advantage in that it determines the firm's (imputed) marginal cost of money capital, λ .

414. _Ibid._, chs. 9-10.

415. The Lagrangian for problem (151) is the following:

$$L_\lambda = \frac{1}{\rho(D)} [p(Q)\cdot f(X_1,X_2) - w_1X_1 - w_2X_2 - r(D)\cdot D]$$
$$+ \lambda'[E + D - g(Q) - \alpha X_1 - \beta X_2] ,$$

where λ' is used in place of λ to distinguish the Lagrange multiplier for problem (151) from the Lagrange multiplier for problem (141).

416. Vickers calls λ' the 'marginal value productivity of money capital'. _Ibid._, p. 164. As in the case of λ, the use of 'productivity' has been eschewed here since money capital is non-productive. See footnote 409. Actually, λ' measures the marginal value of equity capital, dV/dE, since it is equity that is scarce. The argument required to show this is the same as that used in footnote 408. Similarly, it can be shown that λ' measures the marginal value of debt capital in terms of the capitalized value of net operating income.

417. To further generalize the Vickers model in the manner suggested by Arzac, both the average rate of interest on debt and the owners' capitalization rate can be made functions of output, q, the debt level, D, and the amount of equity capital, E; that is, $r(q,D,E)$ and $\rho(q,D,E)$, respectively, where q is subject to random disturbances. Then $V-E$ is maximized with respect to the inputs, X_i, and the amounts of debt and equity, D and E, respectively, subject to the money capital constraint. Arzac, _op. cit._

418. This section is based on the growth model described in Herendeen, _op. cit._, pp. 113-119. Herendeen's symbolism has been changed slightly in order that it be made consistent with the symbolism used throughout this paper.

419. As indicated in section A of this chapter, the 'book value' of an item is the value at which the item is listed in the firm's balance sheet. The book values of equity, debt, and total assets are the amounts listed on the firm's balance sheet and bear no direct relationship to the market values of these items. See section A and table II-1. For example, the book value of equity is equal to the book value of total assets less the book value of total debt. The market value of equity is the stock market value of the firm's outstanding shares and reflects the market's assessment of the firm's future prospects as well as its assessment of how well the firm's assets are currently being employed, and is therefore unlikely to equal the book value of equity.

420. Herendeen measured net income as income before taxes but after deducting interest on debt. Ibid., p. 113. Hence, his definition of net income falls somewhere between what are conventionally defined as 'net operating income' and 'net income'. See section A and table II-2.

421. The leverage ratio is the ratio of the book value of debt to the book value of equity. Vickers and Turnovsky also measure the leverage ratio in terms of book values (rather than in terms of market values). Vickers, The Theory of the Firm, op. cit., p. 41, and Turnovsky, op. cit., p. 1065. The reason for using book values rather than market values is briefly the following: (i) the book value of debt represents a contractual obligation to the firm's creditors, and similarly, the book value of equity represents a quasi-contractual obligation to the firm's shareholders, whereas the market values of debt and equity represent the current market appraisals of these obligations, and (ii) in setting its financial policy, the firm can choose a ratio of book debt to book equity, but it cannot select a ratio of market (or book) debt to market equity since the market values reflect the market's appraisal of the firm's operating decisions as well as of its financial policy. See Herendeen, op. cit., pp. 125-126.

422. As in the Vickers model, the different debt securities the firm may have issued are aggregated into a single composite debt security and i is the interest rate on this composite security, computed as the weighted average of the interest rates on the different debt securities comprising the composite security.

423. Herendeen actually defined the interest rate as a function of leverage and the firm's risk class, and he defined the owners' capitalization rate as a function of leverage, risk class, the dividend rate, and the state of the equity market. In specifying the model, however, he took the firm's risk class and dividend rate and the state of the equity market as given, so that the interest rate and the owners' capitalization rate each varied as a function of L only.

424. Ibid., p. 117.

425. Ibid., p. 127, footnote 17.

426. Note that under the assumptions of certainty and perfect markets, it follows that, when the economy is in general equilibrium, $p = i(L) = e$.

427. To see this, note that, when all net income is reinvested, the rate of growth of equity, dE/E , is equal to the rate of return on equity since $dE = \pi$ and $dE/E = \pi/E = e$. Since, by definition, $L = D/E$, it follows that $D = LE$ and $dD = LdE$ for

given L . The rate of growth of total assets is given by

$$\begin{aligned} g &= (dE + dD)/(E + D) \\ &= (dE + LdE)/(E + LE) \\ &= dE/E = e \ . \end{aligned}$$

428. Net dividend payments are equal to the total dollar value of dividends paid less the total dollar receipts from new equity issues, i.e., receipts from new equity issues are treated as negative dividends. This is the same treatment given the proceeds of new equity issues in subsection 1 of this section.

429. The argument is the same as that used in footnote 427 to show that $g = e$, but with π_r in place of π and e_r in place of e .

430. These costs of growth were discussed above in section G in connection with the Baumol growth maximization and Marris models. See pages 72 and 80 of this chapter, and in particular, footnote 211 for references. These costs may be referred to as the 'real' costs of growth in order to distinguish them from the financial costs of growth discussed below.

431. <u>Ibid.</u>, p. 115.

432. <u>Ibid.</u>, p. 115.

433. See tables II-2 and II-3.

434. Once again, this can be shown directly by following the line of argument given in footnote 427 and by recognizing that $dE = \pi_r - B$.

435. As in the Vickers model, total production costs include the cost of labor and raw materials and the cost of depreciation, obsolescence, and maintenance of real capital exclusive of interest on debt.

436. The variable S includes only those selling costs necessary to maintain current profitability p with zero growth. Additional selling outlays — those that promote growth — are included in B .

437. The terminology used is that of Herendeen. <u>Ibid.</u>, p. 116. Though this terminology is a bit awkward, the author has decided not to change it. It should be emphasized, however, that the 'rate' is computed with respect to the book value of total assets, A , rather than with respect to time.

438. This treatment of revenue, production costs, and selling outlays differs from the conventional treatment only in that these variables are normalized by dividing each by the book value of total assets.

439. Since growth by diversification is permitted, the firm may produce more than one output, in which case Y could be an aggregate measure of output given in constant dollars. Since the demand curve for each good will be downward-sloping (ruling out the possibility of perfectly competitive markets), it is still reasonable to assume that $\partial^2 r/\partial y^2 < 0$, i.e. that 'weighted' marginal revenue is declining.

440. As y increases, Y grows faster than A , that is, output expands relative to capacity (i.e. total assets). For the case of a single output, $r = r(y,s)$ is analogous to the single period relationship between total revenue on the one hand and total output and total advertising expenditure on the other, as in the Baumol sales maximization model (73). The only difference is that r , y , and s are normalized (by the book value of total assets) variables. Utilizing this analogy, $\partial r/\partial y > 0$ and $\partial^2 r/\partial y^2 < 0$ are interpreted to mean that the marginal rate of revenue in terms of the rate of output is positive and falling, respectively, and $\partial r/\partial s > 0$ and $\partial^2 r/\partial s^2 < 0$ are interpreted to mean that the marginal rate of revenue in terms of the rate of selling outlays is positive and falling, respectively. Similarly, c(y) is analogous to the single period total cost function and $dc/dy > 0$ and $d^2c/dy^2 < 0$ are interpreted to mean that the marginal rate of cost is positive and rising. That is, the signs of the derivatives are the same as they would be if r , y , s , and c represented non-normalized variables in a single period model. It should be noted that the reason for the normalization is that the Herendeen model is a steady state growth model. Revenue, output, selling outlays, and total cost are permitted to grow over time, though, as a percentage of total assets, each remains fixed over time.

441. Gordon, The Investment, Financing, and Valuation of the Corporation, op. cit., ch. 4; Marris, An Introduction to Theories of Corporate Growth, op. cit., p. 18; and Herendeen, op. cit., p. 115. Note that (167) holds as an equilibrium condition under the assumptions of certainty and perfect capital markets. See footnotes 139, 140, and 242.

442. Merely dividing each side of equation (167) by E yields the equation $v = k/(\rho-g)$, where the argument L has been suppressed. The reason for treating new equity issues simply as negative dividends may be demonstrated with the aid of this expression for v . Let k_1 be the dividend rate in the absence of new equity issues expressed as the ratio of the market value of new issues to the current book value of equity. Without new equity issues

$$v = k_1/(\rho-g) \quad (*)$$

With new equity issues the growth equation (162) becomes

$$g = (1-b)[(1-t)e - k_1 + n] \ ,$$

which may be solved for k_1 to yield

$$k_1 = (1-t)e - g/(1-b) + n \ . \qquad (**)$$

The rate of growth of dividends per share when there are new issues is no longer g, but is $g_k = g - n/v$, where n/v gives the percentage increase in the number of shares outstanding. Then (*) may be written as

$$v = \frac{(1-t)e - g/(1-b) + n}{\rho - (g - n/v)} \ ,$$

which may be solved for v to obtain

$$v = \frac{(1-t) - g/(1-b)}{\rho - g} = \frac{k_1 - n}{\rho - g} \ , \qquad (***)$$

where equation (**) was used to substitute $k_1 - n$ for $(1-t)e - g/(1-b)$. In equation (***) $k_1 - n$ is the net dividend rate k appearing in equations (161), (162), and (163) above and equation (168) below. Also note that, using equation (160), v can be reexpressed as follows:

$$v = \frac{(1-t)[L(p - i(L)) + p] - g/(1-b)}{\rho - g} \ . \qquad (****)$$

In footnote 423 it was stated that ρ generally depends on the net dividend rate k. Note in equation (****) that, if $\partial\rho/\partial k = 0$, the valuation ratio is independent of the firm's dividend policy. This was also the case in the Marris growth model. See Marris, An Introduction to Theories of Corporate Growth, op. cit., pp. 22-23. However, unlike the Marris growth model, the presence of L in (****) indicates that the firm's financial policy — as reflected in its selection of an optimal leverage ratio L^* — does matter. Note in addition that even if $\partial\rho/\partial k \neq 0$, expressing the share value as the present value of the future flow of dividends and treating new equity issues as negative dividends has the effect of making current shareholders indifferent between equity funds raised through retained earnings and those raised through new equity issues. This would not be the case if there were transactions costs associated with new issues or if current shareholders failed to fully comprehend the implications of the firm's policies (i.e. if capital markets were not perfect).

443. The requirement $v = v_o$ is not stated explicitly by Herendeen in Herendeen, op. cit., ch. 7, where the model is developed. Yet it seems implicit in his treatment of equation (168) as the 'valuation constraint.'

444. Like (167), from which this condition follows, the expression for $\rho(L)$ characterizes equilibrium under the assumptions stated in footnote 441. Otherwise, like (167), it holds only as a definition.

445. Herendeen, op. cit., p. 117.

446. Ibid., ch. 8. As demonstrated in subsection 1 of this section, the firm's dividend policy would not matter to shareholders if the future were known with certainty and capital markets were perfect. The meaning of $\partial\rho/\partial k < 0$ is simply that shareholders prefer some portion of their yield in the form of dividends, rather than in the form of future, less certain capital gains, and that, beyond some point, further reductions in the net dividend rate will cause the capitalization rate, ρ, to increase. See also Gordon, The Investment, Financing, and Valuation of the Corporation, op. cit., pp. 55-66. Hence, the modification of the model suggested by this writer — letting k become a decision variable — permits the requirement that capital markets be perfect to be relaxed.

447. It is assumed for simplicity that conditions in the product and financial markets are such that when $v > v_o$ it is still profitable for the firm to increase its growth rate. Then at optimality $v = v_o$ must hold. For somewhat greater generality, the constraint in (169) could be rewritten with v in place of v_o and the constraint $v \geq v_o$ could be added. For an intuitive argument as to why v should decline as g increases see footnote 454 below.

448. The Lagrangian is

$$L_\lambda = (1-b)[(1-t)\{(L+1)[r(y,s)-c(y)-s] - i(L)\cdot L\} - k]$$
$$+ \lambda\{(1-b)[(1-t)\{(L+1)[r(y,s)-c(y)-s] - i(L)\cdot L\}-k]-\rho(L,k)+\frac{k}{v_o}\}$$

449. The first part of this result was also obtained by J.H. Williamson. See J.H. Williamson, op. cit., pp. 11-12.

450. By the same argument employed in footnote 408, it can be shown that $\lambda = -\frac{v_o^2}{k^*}\frac{dg^*}{dv_o}$, or equivalently, $\lambda = dg^*/d(k^*/v_o)$, where the * denotes the variable's equilibrium (i.e. optimal) value. The Lagrange multiplier λ is interpreted as the instantaneous rate of change of the optimal growth rate of total assets with respect to the optimal dividend yield. Thus, the product $\frac{\lambda}{1+\lambda}\frac{\partial\rho}{\partial L}$ measures the cost of a change in the leverage ratio in terms of the change in the growth rate necessary to ensure that $v = v_o$ remains satisfied (where the change in g induced by the change in L is transmitted through a change in ρ in the constraint equation).

451. Recall that equation (160) states that e is identical to $L(\rho - i(L)) + p$, so that

$$\frac{\partial e}{\partial L} = p - \frac{di}{dL} L - i(L) .$$

Rearranging terms in (177) and making the appropriate substitution leads to (178).

452. This can be seen clearly when the objective function in (169) is rewritten as

$$g(y,s,L) = (1-b)[(1-t)e(y,s,L) - k] .$$

453. Note the contrast with the unconstrained case where maximum growth is achieved by setting $k = 0$, i.e. by not paying any dividends, or if dividends are paid to current shareholders, then by raising an equivalent sum through new equity issues (or possibly, by paying dividends to existing shareholders by means of a new share distribution). Comparing the constrained with the unconstrained case also reveals that, since (174) and (175) hold even in the absence of the valuation constraint, the valuation constraint in (169) affects only the firm's financial policy, i.e. its selection of L and k , but not its selection of y and s . In footnote 447 it was pointed out that raising g tends to lower v , so that the firm would, in equilibrium, find its valuation ratio at the lower bound, $v = v_o$. This can be argued as follows: (i) provided $\rho > i$, raising g by adjusting leverage only, requires that L be increased, which eventually could be expected to cause ρ to rise faster than g , and hence, eventually to cause v to fall; thereafter, increases in L would cause v to fall steadily until finally $v = v_o$; and (ii) raising g by adjusting dividend policy only requires that k be decreased, which causes the numerator in the expression for v to fall; ρ and g both increase, so that the denominator in the expression for v may also decrease if g increases faster than ρ . However, one would expect that eventually ρ would increase faster than g , and that sometime before this point had been reached, $\rho-g$ would begin to decrease more slowly than v . Thereafter, v would decrease steadily until finally $v = v_o$. Note that the above arguments imply that $dg^*/dv_o \leq 0$, and hence, that

$$\lambda = - \frac{v_o^2}{k^*} \frac{dg^*}{dv_o} \geq 0 .$$

454. In the event capital markets were assumed to be perfect, the Marris managerial utility maximization model and the Herendeen managerial utility maximization model would be virtually identical, with the only difference being Herendeen's more detailed expression for the firm's rate of growth. In particular, under the assumptions of certainty and perfect capital markets, the firm's financial policies are irrelevant, so that both models would give the role of finance equal treatment.

455. See section F earlier in this chapter for a general discussion of this class of models.

456. Herendeen, op. cit., p. 118.

457. Ibid., ch. 8.

458. This section is based on the following three papers written by Lintner: J. Lintner, "The Cost of Capital and Optimal Financing of Corporate Growth," Journal of Finance (vol. 18; no. 2; May 1963), pp. 292-310; J. Lintner, "Optimal Dividends and Corporate Growth under Uncertainty," Quarterly Journal of Economics (vol. 78; no. 1; February 1964), pp. 49-95; and J. Lintner, "Optimum or Maximum Corporate Growth under Uncertainty," in Marris and Wood, op. cit., pp. 172-241. However, the discussion draws most heavily from the third paper.

459. In fairness to Marris, he does, in light of Lintner's treatment of uncertainty, suggest how the effects of uncertainty can be incorporated in his own model. See Marris, Theories of Corporate Growth, op. cit., pp. 23-26, and Marris, Modern Corporation and Economic Theory, op. cit., pp. 308-314.

460. Lintner, Maximum Corporate Growth under Uncertainty, op. cit., p. 194, and Lintner, Optimal Dividends and Corporate Growth under Uncertainty, op. cit., p. 68. The *certainty equivalent* of a random dividend receipt is defined by Lintner to be "that single value which, if certain, would be equivalent in the decision-maker's mind to the uncertain prospect represented by the full distribution of the random element." Ibid., p. 68. For example, in terms of figure I-1 (see chapter one of this paper), the certainty equivalent of the risk-return combination represented by the point B is the point C since the latter point represents the same (expected) utility level as B, but with zero risk (and under risk aversion this decrease in risk is accompanied by a sacrifice of a portion of expected return, so that $\mu_C < \mu_B$).

461. More specifically, the firm maximizes the current equilibrium stock market value of its equity, where the stock market is assumed to be perfectly competitive, investors are risk averse, investors' assessments of returns from shareholding are assumed to be lognormally distributed, and, for the case in which uncertainties increase with futurity, it is further assumed that the movement of share prices follows a random walk. Lintner, Maximum Corporate Growth under Uncertainty, op. cit., p. 179. Empirical support for the lognormality assumption is provided in J. Lintner, "Equilibrium in a Random Walk and Lognormal Securities Market," Discussion Paper No. 235 (Harvard Institute of Economic Research, Harvard University; Cambridge, Mass.; July 1972), and in R.C. Blattberg and N.J. Gonedes, "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices,"

Journal of Business (vol. 47; no. 2; April 1974), pp. 244-280. The random walk hypothesis also seems justified in light of the amount of empirical evidence that has been accumulated in support of it. For example, see C.W. Granger and O. Morgenstern, "Spectral Analysis of New York Stock Market Prices," Kyklos (vol. 16; no. 1; 1963), pp. 1-27; P.H. Cootner, ed., The Random Character of Stock Market Prices (MIT Press; Cambridge, Mass.; 1964); E.F. Fama, "The Behavior of Stock Market Prices," Journal of Business (vol. 38; no. 1; January 1965), pp. 34-105; and E.F. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance (vol. 25; no. 2; May 1970), pp. 383-417.

462. The results obtained by Lintner are consistent with those obtained by Marris, John Williamson, and Herendeen, all of which were discussed earlier in this paper.

463. Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 173-176.

464. Ibid., p. 190. That is, the expected growth rate is a function of the retention ratio and of the basic risk variable. The point is that a variable reflecting risk belongs to the set of decision variables. In the special case of certainty there is no risk and the retention ratio is the only decision variable.

465. Ibid., p. 173.

466. Ibid., pp. 177-178.

467. For example, firms appear to set target payout ratios, and the target payout ratio for a firm — based on the expected long run average profitability of investments — often remains reasonably stable over a considerable period of time. See J. Lintner, "Distribution of Incomes of Corporations among Dividends, Retained Earnings, and Taxes," American Economic Review (vol. 46; no. 2; May 1956), pp. 97-113; E.F. Fama and H. Babiak, "Dividend Policy: An Empirical Analysis," Journal of the American Statistical Association (vol. 63; no. 324; December 1968), pp. 1132-1161; and J.A. Brittain, Corporate Dividend Policy (Brookings Institution; Washington, D.C.; 1966). In addition, firms in many industries appear to work toward target debt-equity ratios, with a firm deviating from its own target ratio only temporarily and only when justified by conditions in the financial markets. See Donaldson, op. cit.

468. This section is based on Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., pp. 297-304; Lintner, Optimal Dividends and Corporate Growth under Uncertainty, op. cit., pp. 58-65; and Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 181-187.

469. This is Fama's and Miller's 'equal rate of return principle.' See footnote 242 for references.

470. The restriction to internal financing does not alter the results provided there is certainty and capital markets are perfect. In the first version of his certainty model Lintner did permit new equity issues. As demonstrated in section I in connection with the Herendeen model, the receipts from new share issues are tantamount to negative dividends provided the stock market is perfect. See Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., pp. 300-301.

471. The retention ratio, r, is the proportion of net income not paid out in the form of dividends. In terms of table II-3, r is the ratio of retained earnings to net income. Equivalently, the models presented below could be expressed in terms of the payout ratio, $x \equiv 1-r$, which is the proportion of net income paid out in dividends. See, for example, ibid., p. 301. It should be noted that, in the initial development of the certainty model in the 1971 paper, r is defined differently as the proportion of gross revenue not available for dividends and x_o in equation (181), below represents current gross revenue per share. Lintner's conclusions still follow, however, when r represents the retention ratio in its conventional sense, for as he notes, the two approaches are equivalent as long as r is "suitably denominated." Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 184-186.

472. Net income (or 'total profit') divided by the number of shares outstanding.

473. This equation corresponds to equation (108), which gives the market value of the firm's equity. Letting N denote the number of shares outstanding, the exact correspondence is given by $pK = N \cdot X_o$ and $P_o = V/N$. In (181), as in (108), it is assumed that $g < i$ so that the indefinite integral exists. It should be noted that, if new equity issues are permitted, equation (181) is only slightly modified, becoming

$$p_o = \frac{(1-r)X_o}{i-(g-n)} ,$$

where $n \equiv d\log N/dt$ is the relative rate at which new shares are issued.

474. Ibid., p. 185.

475. This interpretation of $(1-r)dg/dr$ is Lintner's. Ibid., p. 186.

476. When the value maximizing firm is in equilibrium, i.e. growing along its equilibrium steady state growth path, the inequalities in (183) become equalities. Since it is shown below that in equilibrium dg/dr equals the current earnings yield on a share of stock, (183) written as an equality merely states that, when the firm is in equilibrium, the equal rate of return principle must hold. Hence, the economic interpretation of the characterization

of disequilibrium provided by the inequality (183) is somewhat richer than the interpretation of (182).

477. The remainder of the discussion follows Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 187-197. In the two earlier papers indicated in footnote 468 Lintner first defined the profit rate as a random variable, and then expressed the variance of the growth rate in terms of the variance of the profit rate. Since his later approach is somewhat simpler, it is the one followed here. See Lintner, The Cost of Capital and Optimal Financing of Corporate Growth, op. cit., pp. 305-306, and Lintner, Optimal Dividends and Corporate Growth under Uncertainty, op. cit., pp. 66-67. It should be noted that the probability distribution of growth rates is subjective in the sense that it is based on management's assessment of the future. The reason for requiring fixed duration is, of course, that comparisons are only meaningful when the length of the interval is fixed (though the particular length is irrelevant).

478. It is the stationarity of the probability distribution, and in particular, the constancy of the variance, that constitutes 'stable uncertainty'. In the next subsection the variance will increase with futurity, and that version of the model will be said to involve 'increasing uncertainty'.

479. It is understood that $\bar{g}$ and σ_g^2 are both functions of the length of the time interval over which growth is measured. That is, the mean and variance should be expressed as $\bar{g}(\tau)$ and $\sigma_g^2)$, where τ is the period over which growth is measured. For simplicity, it is assumed that the length of the interval has already been specified, so that τ may be suppressed.

480. For any variable X with initial value X_o and with exponential growth rate g, the distribution of which is stationary over time, the value of the variable t periods into the future is

$$X_t = X_o e^{\sum_{i=1}^{t} g_i} ,$$

where g_i is the actual growth rate during the i-th period and where it has been assumed for simplicity that each time period is of unit length (i.e. $\tau = 1$). Then

$$\ln X_t = \ln X_o + \sum_{i=1}^{t} g_i \quad .$$

Since the g_i's are normally distributed, since the sum of any finite number of independent normally distributed random variables is normally distributed, and since adding a constant will not affect the normality, it follows that $\ln X_t$ is normally distributed.

481. The model is a continuous time model. The use of the term 'period' is meant merely to indicate that the time unit (which must be specified in order for the distribution of g to be specified) may be chosen arbitrarily.

482. It is not necessary that investors' expectations be homogeneous; the same conclusions are drawn if investors' assessments are permitted to differ. See Lintner, Equilibrium in a Random Walk and Lognormal Securities Market, op. cit. See also J.A. Ohlson and W.T. Ziemba, "Portfolio Selection in a Lognormal Market When the Investor Has a Power Utility Function," Journal of Financial and Quantitative Analysis (vol. 11; no. 1; March 1976), pp. 57-71.

483. The utility function is of the firm $U(x) = -(x)^{-a}$, where $a > 0$ measures the degree of risk aversion and where x is the uncertain return. Ibid., pp. 69-70. See also J. Tinbergen, "The Optimum Rate of Saving," Economic Journal (vol. 66; no. 264; December 1956), pp. 603-609.

484. See footnote 126.

485. For the derivation of equation (185) see Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 192-194 and section B of the appendix.

486. Ibid., p. 229. In the portfolio selection problem that underlies the determination of the equilibrium vector of stock prices, each investor maximizes its expected utility of end-of-period wealth subject to a constraint expressed in terms of its initial wealth endowment. The general equilibrium model set up to solve this problem generates a shadow price for each such wealth constraint, and ω is the weighted average of these shadow prices, where the weight associated with each investor is equal to the fraction of the market's total wealth in the hands of the individual investor divided by the elasticity of that investor's utility function (i.e. its degree of risk aversion as embodied in the constant a — see footnote 483).

487. Ibid., pp. 192-193, 196, 229.

488. Ibid., p. 194. See also Lintner, Optimal Dividends and Corporate Growth under Uncertainty, op. cit., p. 69.

489. Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 194, 232. The function $f(\sigma_{ij})$ is a weighted sum of the covariances of the firm's share price with other share prices.

490. Note that this term may be negative (so that $-\alpha f(\sigma_{ij})$ is positive) when the price of the stock moves in a countercyclical pattern with regard to the market average. This raises $\hat{g}$ and thereby increases the share price P_o. The reason for this is

that countercyclical stocks have the effect of reducing the overall variability of returns — i.e. portfolio risk — when included in a portfolio. When investors are risk averse, the contribution to risk reduction causes investors to bid up the share price.

491. Yet another interpretation of (185) is possible. Since

$$\omega - \hat{g} = \omega - (\overline{g} - \alpha C\sigma_g^{\ 2} - \alpha f(\sigma_{ij}))$$

$$= [\omega + \alpha C\sigma_g^{\ 2} + \alpha f(\sigma_{ij})] - \overline{g} \ ,$$

the expression in brackets can be interpreted as a risk-adjusted discount factor (or interest rate), where ω is the riskless discount factor and $\alpha C\sigma_g^2 + \alpha f(\sigma_{ij})$ is the 'risk premium'. Then the current share price P_o is equal to the present value of the future dividend stream, where the appropriate discount rate is the risk-adjusted one.

492. Additionally it is assumed that f in (187) is twice differentiable with

$$\frac{\partial \overline{g}}{\partial r} > 0 \qquad \frac{\partial^2 \overline{g}}{\partial r^2} < 0 \qquad \frac{\partial^2 \overline{g}}{\partial r^2} \cdot \frac{\partial^2 \overline{g}}{\partial (\sigma_g^{\ 2})^2} - \frac{\partial^2 \overline{g}}{\partial r \partial \sigma_g^{\ 2}} > 0 \ ,$$

$$\frac{\partial \overline{g}}{\partial \sigma_g^{\ 2}} > 0 \qquad \frac{\partial^2 \overline{g}}{\partial (\sigma_g^{\ 2})^2} < 0$$

which imply that f is a concave function.

493. The efficient set possesses the following properties:
(a) for all feasible policy mixes with the same $\overline{g}$ and r, only the one with minimum σ_g^2 is admitted;
(b) for all feasible policy mixes with the same σ_g^2 and r, only the one with maximum $\overline{g}$ is admitted; and
(c) for all feasible policy mixes with the same $\overline{g}$ and σ_g^2, only the one with minimum r is admitted.
Thus, in figure II-23 the shaded area consists of feasible policy mixes, but those lying to the right of and below the solid line do not belong to the efficient set.

494. Ibid., p. 190.

495. Ibid., p. 190.

496. It follows from the concavity of $\overline{g}$ (see footnote 492) that these conditions are also sufficient for a global maximum.

497. See figure II-23. At all points above the σ_g^2 axis the slope of the contour, $\partial\bar{g}/\partial\sigma_g^2 = \partial\bar{g}/\partial v_o$, is finite. Only where the efficient contour touches the axis ($\bar{g} = 0$ and σ_g^2 a minimum) might the slope become infinite.

498. In fact, it is clear that (193) reduces to (183) when all uncertainty is removed, for then $\bar{g} = g$, $\partial\bar{g}/\partial r = \partial g/\partial r$, $\hat{g} = \bar{g} = g$, and therefore, $\omega = i$. This correspondence between ω and i justifies calling ω the 'riskless rate of interest' in the uncertainty case.

499. See subsection 7 of section C of chapter one of this paper, and in particular, figure I-1.

500. Lintner argues that "the best dividend payout x^* will vary inversely — and the best retention ratio will vary directly — with the level of the risks being borne by the firm." Ibid., p. 197. Lintner's statement appears inconsistent with his finding that "the optimal dividend payout ratio x^* will be higher (and the best retention ratio r^* will be lower) [under uncertainty] than under certainty." Ibid., p. 197. In the opinion of this writer, Lintner's second statement is correct and his first statement is backwards.

501. Under uncertainty a growth maximizer would maximize the *expected* rate of growth.

502. See (184) above.

503. Ibid., p. 202.

504. The discussion follows ibid., pp. 203-213. An earlier version of the increasing uncertainty model (but with the disturbances built directly into the profit rate rather than directly into the growth rate, as in the later version) can be found in Lintner, Optimal Dividends and Corporate Growth under Uncertainty, op. cit., pp. 76-91.

505. Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 203-204.

506. By this assumption the sequence of assessed future growth rates $\{\tilde{g}(\tau)\}$ beginning at any particular point in time forms a random walk.

507. Ibid., p. 204.

508. See footnote 243.

509. Ibid., p. 207.

510. Ibid., p. 208. It should be noted that all but (v) carry over directly from the 'stable uncertainty' version of the model.

511. Ibid., pp. 209-213.

512. Ibid., p. 210.

513. Ibid., pp. 211-212. That is, (191) still holds when σ_u^2 is independent of the firm's decisions (which makes sense intuitively because setting σ_u^2 equal to any particular value, say $\sigma_u^2 = 0$, as in the 'stable uncertainty' case, will not affect the firm's choice of v_o). But making σ_u^2 dependent on v_o will affect the firm's selection of v_o, and when the firm is in equilibrium (in the second case considered by Lintner), $\partial\hat{g}^o/\partial v_o = \partial\overline{g}/\partial v_o - \alpha C > 0$, where $\hat{g}^o$ is the initial certainty equivalent of the firm's growth rate.

514. Ibid., p. 180.

515. Ibid., pp. 173-177.

516. See Hirshleifer, Investment, Interest, and Capital, op. cit., ch. 10.

517. The time-state-preference approach was developed by Arrow and has been extended by Debreu and Hirshleifer. See K.J. Arrow, "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies (vol. 31; no. 86; April 1964), pp. 91-96; G. Debreu, The Theory of Value (Wiley; New York; 1959); J. Hirshleifer, "Efficient Allocation of Capital in an Uncertain World," American Economic Review (vol. 54; no. 3; May 1964), pp. 77-85; J. Hirshleifer, "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," Quarterly Journal of Economics (vol. 79; no. 4; November 1960), pp. 509-536; and J. Hirshleifer, "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," Quarterly Journal of Economics (vol. 80; no. 2; May 1966), pp. 252-277. Related work includes Diamond, op. cit.; Myers, op. cit.; and Leland, Production Theory and the Stock Market, op. cit. The mean-variance and time-state-preference approaches to the portfolio selection problem are compared in both Hirshleifer, Investment Decision Under Uncertainty: Choice-Theoretic Approaches, op. cit., and in Hirshleifer, Investment, Interest, and Capital, op. cit. The latter also summarizes the important results contained in the three Hirshleifer papers cited earlier in this footnote.

518. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit.

519. The notion of a state of nature is somewhat imprecise. A state of nature is some configuration of the individual decision-maker's choice environment. Yet, just how the set of distinct possible states is to be specified is unclear. Because there does not exist a natural, generally agreed-upon, and manageably small set of state definitions readily applicable to the real world, it would be very difficult to test empirically a time-state-preference model, and as yet, no one has accomplished this. See Hirshleifer, Investment, Interest, and Capital, op. cit., p. 277, on this point. Nevertheless, it is typically assumed that a finite set of distinct possible states of nature exists for each future period. Ibid., pp. 216-217.

520. Ibid., p. 217.

521. Strictly speaking, the complex entitlement is called an asset, and assets that are tradable are called securities. Ibid., p. 216. For the limited purposes of this brief introduction, this distinction is not important.

522. In this subsection the role of the firm, i.e. production, is abstracted from in order to focus the discussion on pure exchange. Once the basic time-state-preference framework has been presented, the form will be brought into the discussion.

523. What follows is based on ibid., pp. 231-235, 244-248, and Hirshleifer, Investment Decision Under Uncertainty: Choice-Theoretic Approaches, op. cit., pp. 523-530.

524. Continuing the previous examples, state a might represent war and state b peace, or state a might represent prosperity and state b depression.

525. The justification for this shape is provided by Hirshleifer. Ibid., pp. 525-526. Put simply, a concave shape would imply that, when utility is maximized subject to a wealth constraint, such as that embodied in the line W in figure II-25(a), a corner solution would result. But since a corner solution would imply that the individual had staked literally everything on the occurrence of a particular state (and absolute impoverishment if the other state obtained), concavity must be rejected as a possible shape. Though other shapes somewhere between the extremes of convexity and concavity are not excluded by this argument, it can be shown that, under the von Neumann-Morgenstern postulates of rational choice, there exists a unique preference-scaling function $v(c)$ for contingent incomes c that holds over all possible states and that, if $v(c)$ is a concave function (indicative of risk aversion), the indifference curves in figure II-25 must be convex to the origin. Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 218-221, 233-234. It should be noted that Hirshleifer's argument requires the use of a cardinal measure of utility. Recently, Mishan has suggested an approach that would remove this restriction. See E.J. Mishan, "Choices Involving Risk: Simple Steps Toward an Ordinalist Analysis," Economic Journal (vol. 86; no. 344; December 1976), pp. 759-777.

526. As in the ordinary case, the convex shape of each isoquant is interpreted economically to mean that the individual has a diminishing marginal rate of substitution of one type of claim for the other. That is, along any isoquant, as contingent claims c_{1a} are substituted for contingent claims c_{1b} , as for example when the individual's pair of contingent claims (c_{1a}, c_{1b}) moves from point A to point B in figure II-25(a), say through an exchange of claims with some other individual, the rate at which he is willing to

sacrifice additional amounts of c_{1b} in return for additional amounts of c_{1a} (i.e. $-dc_{1b}/dc_{1a}$) diminishes as c_{1a} increases (and as c_{1b} decreases).

527. For example, y_o might be thought of as the individual's current stock of liquid assets (e.g. cash) and y_{1a} and y_{1b} might be thought of as the future returns to be received from investments to which the individual is unalterably committed, where the exact amount to be received is dependent on the future state of nature (e.g. war versus peace).

528. See footnote 126 for a precise definition of a perfect market.

529. A market for contingent claims is *complete* provided there are securities sufficient in number and characteristics to permit individuals to carry out exchanges among all the objects of choice that enter their individual preference functions. A mathematical interpretation of this concept might be helpful. Denote a security X by $X = (x_o, x_{1a}, x_{1b})$, where the x's denote flows of the generalized commodity at different dates and in different states. In order that the market for contingent claims be complete, it is necessary and sufficient that at least three securities exist such that the associated three-component vectors span E^3. Any three vectors that form a basis will do, and the simplest of these is the natural basis (see the next footnote).

530. These securities are called *elementary securities* and correspond to the natural basis for E^3. Note that $X_1 = (3,2,1)$, $X_2 = (1,0,4)$, and $X_3 = (0,3,0)$ would also make the market for contingent claims complete since, for any feasible desired consumption set (c_o, c_{1a}, c_{1b}), it is possible to find amounts of these securities, a_1, a_2, and a_3, respectively, such that

$$(c_o, c_{1a}, c_{1b}) = a_1(3,2,1) + a_2(1,0,4) + a_3(0,3,0) ,$$

i.e. such that the most desired consumption set can be achieved through exchange. If only two securities exist, say X_1 and X_2, then some consumption sets, such as $(2,2,2)$, are unobtainable. In this case the market for contingent claims is said to be *incomplete*.

531. Then ϕ_{1a} represents the amount of c_o that must be sacrificed to obtain a unit claim to consumption at time $t = 1$ if and only if state a obtains, and ϕ_{1b} is interpreted similarly.

532. For c_o fixed, equation (198) corresponds to the line W in figure II-25(a). Equation (198) also corresponds to the "budget line" in the traditional analysis of the consumer's budget allocation problem.

533. An economy consists of a number of individuals, each of whom solves a problem of the form (199). The exact form of the utility function may vary from one individual to another, although, by the perfect markets assumption, each individual faces (and takes as given) the prices ϕ_{1a} and ϕ_{1b}. Indeed, problem (199), when solved for all individuals simultaneously subject to the conservation relations, yields the general equilibrium prices ϕ_{1a} and ϕ_{1b} (although, it should be emphasized, these are not absolute prices, but are only relative prices in terms of c_o).

534. Conditions (201) are perfectly analogous to the equilibrium conditions in the three-good-single period-certainty case, namely, that the consumer will allocate his budget in such a way that, when he is in equilibrium, the marginal rate of substitution between each pair of goods must equal the ratio of the market prices of those goods (recall that $\phi_o \equiv 1$ since c_o has been chosen to serve as the numeraire).

535. I. Fisher, The Theory of Interest (Macmillan; New York; 1930; reprinted, Augustus M. Kelley, 1955), and Hirshleifer, Investment Decision Under Uncertainty: Applications of the State-Preference Approach, op. cit., pp. 252-253.

536. If any individual engages in production on his own, say farming, such a situation is formally equivalent to one in which that individual is the sole owner of a firm that produces the good in question. In other words, the use of the term "firm" is meant to include all forms of productive organization.

537. Note that the form in which the production function is expressed implies that there may be technological uncertainty. That is, if the states of nature are defined in terms of different states of technology (e.g., in the two-state case, according to whether or not a major invention coupled with innovation takes place), q_{1a} could denote production in one state of technology while q_{1b} could denote production in the other.

538. That is, the investment opportunity set for each investor when every firm maximizes its equilibrium market value includes the investor's opportunity set (under equilibrium market prices) that would exist if one or more firms deviated from value maximization.

539. When markets are incomplete, the set of available securities fails to provide a set of consumption sets that forms a basis for E^n (in this case E^3). As a result, trading in contingent claims is severely restricted and a single unambiguous market-determined value W_o^f for each firm does not exist, i.e. it is impossible to obtain a net present certainty-equivalent value for each stream of contingent claims, and the above analysis breaks down. See Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 264-272.

540. These equilibrium conditions are analogous to the equilibrium conditions (25), (26), and (27) obtained above in section B for the single period case under certainty.

541. In the pure exchange and the production and exchange models just considered, there was just one future period. For extensions of the time-state-preference approach to more than one future period, see Myers, op. cit., and J.H. Drèze and F. Modigliani, "Epargne et consommationen avenir a lé atoire," Cahiers du seminaire d'économetrie (1966). Neither of these permits trading in any but the current period. For a recent multiperiod model that does allow trading to take place in intermediate periods, see A. Kraus and R.H. Litzenberger, "Market Equilibrium in a Multiperiod State Preference Model with Logarithmic Utility," Journal of Finance (vol. 30; no. 5; December 1975), pp. 1213-1227.

542. See footnote 126 and subsection 1 of section I.

543. What follows is based on Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 263-264.

544. It is assumed here that the firm raises all funds externally. While actual firms do raise a substantial portion of their investment funds internally, here it is assumed that all earning are fully distributed to shareholders, who then decide how to allocate these funds among firms.

545. In effect, the firm is liquidated at the end of the period and bondholders receive principal plus interest and shareholders receive the value of their shares plus dividends.

546. One potential source of confusion concerning equations (208) should be noted. Shares are generally viewed as riskier, and hence less desirable, than bonds since, in the event of bankruptcy, the bondholders have first claim on the firm's assets, whereas the shareholders are residual claimants only. Thus, it might be thought that e_{1a} is "riskier" than d_{1a}, and hence, that e_{1a} should sell at a lower price than d_{1a}. But this "risk" has already been taken into account in the quantification of e_{1a} and d_{1a}. If $q_{1a} < d_o(1+r)$, where r is the rate of interest on bonds, then $d_{1a} = q_{1a}$ and $e_{1a} = 0$. In other words, conditional upon the occurrence of the states a and b, the returns d_{1a}, d_{1b}, e_{1a}, and e_{1b} all become certainties.

547. A fuller discussion than the one provided here is given in ibid., pp. 264, 271-272, which also includes a numerical example to illustrate that the financing decision is of consequence to the market value of the firm when markets are incomplete.

548. A fuller discussion of the effects of market imperfections can be found in Hirshleifer, Investment Decision Under Uncertainty: Applications of the State-Preference Approach, op. cit.,

pp. 267-268. More specifically, it has been shown that the question of the firm's optimal capital structure is not irrelevant when (i) investors incur transactions costs when trading securities (W.J. Baumol and B. Malkiel, "The Firm's Optimal Debt-Equity Combination and the Cost of Capital," Quarterly Journal of Economics (vol. 91; no. 4; November 1967), pp. 547-578); (ii) securities markets are partially segmented and debt is traded in a separate market where investors are either more pessimistic about the firm or more risk averse than equity holders (J.E. Stiglitz, "Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs," Bell Journal of Economics and Mangagement Science (vol. 3; no. 2; Autumn 1972), pp. 458-482, and M. Rubinstein, "Corporate Financial Policy in Segmented Markets," Journal of Financial and Quantitative Analysis (vol. 8; no. 5; December 1973), pp. 749-761); (iii) corporations can borrow at a lower rate of interest than can investors (Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit.); and (iv) the probability of bankruptcy is nonzero and there are costs associated with bankruptcy and reorganization (H. Bierman, Jr., and J. Thomas, "Ruin Considerations and Debt Issuance," Journal of Financial and Quantitative Analysis (vol. 7; no. 1; January 1972), pp. 1361-1378; A. Kraus and R.H. Litzenberger, "A State-Preference Model of Optimal Financial Leverage," Journal of Finance (vol. 28; no. 4; September 1973), pp. 911-922; A. Robichek and S. Myers, "Problems in the Theory of Optimal Capital Structure," Journal of Financial and Quantitative Analysis (vol. 1; no. 2; June 1966), pp. 1-35; V.L. Smith, "Corporate Financial Theory under Uncertainty," Quarterly Journal of Economics (vol. 84; no. 3; August 1970), pp. 451-471; V.L. Smith, "Default Risk, Scale, and the Homemade Leverage Theorem," American Economic Review (vol. 62; no. 1; March 1972), pp. 66-76; and J.H. Scott, Jr., "A Theory of Optimal Capital Structure," Bell Journal of Economics (vol. 7; no. 1; Spring 1976), pp. 33-54).

549. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit., pp. 268-271.

550. The literature is extensive. The more often cited papers include the following: D.P. Baron, "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition," International Economic Review (vol. 11; no. 3; October 1970), pp. 463-480; D.P. Baron, "Demand Uncertainty in Imperfect Competition," International Economic Review (vol. 12; no. 2; June 1971), pp. 196-208; P.J. Dhrymes, "On the Theory of the Monopolistic Multiproduct Firm under Uncertainty," International Economic Review (vol. 5; no. 3; September 1964), pp. 239-257; J.H. Drèze and J.J. Gabszewicz, "Demand Fluctuations, Capacity Utilization and Prices," Operations Research Verfahren (vol. 3; 1967), pp. 119-141; E. Sheshinski and J.H. Drèze, "Demand Fluctuations, Capacity Utilization, and Costs," American Economic Review (vol. 66; no. 5; December 1976), pp. 731-742; H.E. Leland, "Theory of the Firm Facing Uncertain

Demand," American Economic Review (vol. 62; no. 3; June 1972), pp. 278-291; J.J. McCall, "Competitive Production for Constant Risk Utility Functions," Review of Economic Studies (vol. 34; no. 4; October 1967), pp. 417-420; E.S. Mills, "Uncertainty and Price Theory," Quarterly Journal of Economics (vol. 73; no. 1; February 1959), pp. 116-130; R.R. Nelson, "Uncertainty, Prediction, and Competitive Equilibrium," Quarterly Journal of Economics (vol. 75; no. 1; February 1961), pp. 41-62; W.Y. Oi, "The Desirability of Price Instability under Perfect Competition," Econometrica (vol. 29; no. 1; January 1961), pp. 58-64; A. Sandmo, "On the Theory of the Competitive Firm under Price Uncertainty," American Economic Review (vol. 61; no. 1; March 1971), pp. 65-73; C.A. Tisdell, The Theory of Price Uncertainty, Production, and Profit (Princeton University Press; Princeton, N.J.; 1968); E. Zabel, "A Dynamic Model of the Competitive Firm," International Economic Review (vol. 8; no. 2; June 1967), pp. 194-208; and E. Zabel, "Monopoly and Uncertainty," Review of Economic Studies (vol. 37; no. 2; April 1970), pp. 205-220. A summary of the contents of most of these papers is given in Baron, Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition, op. cit. This subsection is concerned primarily with the model presented in the Leland paper.

551. See, for example, Dhrymes, op. cit., Drèze and Gabszewicz, op. cit., Mills, op. cit., Sandmo, op. cit., Tisdell, op. cit., and Zabel, Monopoly and Uncertainty, op. cit.

552. See, for example, Drèze and Gabszewicz, op. cit., Tisdell, op. cit., and Zabel, A Dynamic Model of the Competitive Firm, op. cit.

553. Either the notion that risk average investors control the firm or the notion that "security" is one of management's goals may be invoked to justify a risk averse utility function for the firm.

554. See, for example, Dhrymes, op. cit., and Sandmo, op. cit.

555. This still leaves open the question of whose utility is being maximized: the utility of managers or the utility of shareholders. Even if this issue could be resolved, there still remains the question of whose utility is being maximized when individuals within the dominant group have different utility functions, for Arrow has shown that there does not exist a social choice rule for aggregating individual preferences that possesses all the properties one would like such a rule to possess. See K.J. Arrow, Social Choice and Individual Values, 2nd ed. (Wiley; New York; 1963). Moreover, it is not clear just how the requirements should be weakened so that an acceptable rule may be obtained. For a discussion of these points see Heal, op. cit., ch. 2. One way around the problem is to assume, as Sandmo suggests, that decisions are made either by a single individual or by a small group of individuals whose preferences are sufficiently similar to permit the existence of a group utility function. Sandmo, op. cit., pp. 65-66.

556. Actually, there are three different quantity variables: quantity demanded, quantity produced, and quantity supplied to the market. Following Leland, it will be assumed that the product market is in equilibrium and that there are no advantages to inventory holding, so that all three quantities are equal and may be represented by a single variable, q .

557. That is, firms are usually treated as quantity setters, rather than as price setters. See, for example, Baron, Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition, op. cit.; Dhrymes, op. cit.; Nelson, op. cit.; Oi, op. cit.; and Sandmo, op. cit. Two exceptions to this statement are Drèze and Gabszewicz, op. cit., in which quantity demanded is a random variable and firms reach their output decisions after observing market demand; and Mills, op. cit., in which the firm sets both price and quantity of output and in which shortages or surpluses develop when quantity demanded does not equal quantity supplied.

558. Baron, Demand Uncertainty in Imperfect Competition, op. cit., also contrasts these modes of behavior. Baron's treatment of uncertainty differs from Leland's in that Baron begins by assuming that price and quantity are jointly random, whereas Leland makes the demand relation random and dependent on the state of nature that obtains when the firm tries to sell its output. However, each ends up working with conditional probability distributions, where the distribution of price (quantity) is conditional upon the value for quantity (price) selected by the firm.

559. The remainder of this subsection follows Leland, Theory of the Firm Facing Uncertain Demand, op. cit.

560. An obvious example of this type of firm is an agricultural firm. It should be noted that the perfectly competitive firm facing a random price independent of its output level is but a special case of the firm considered in this subsection.

561. Assuming the attitude of the firm (i.e. of its shareholders or of its managers or of some other group whose utility is embodied in U) toward risk never changes, risk aversion is implied by $U''(\pi) < 0$; risk neutrality is implied by $U''(\pi) = 0$; and risk preference is implied by $U''(\pi) > 0$.

562. Ibid., p. 281. A fuller discussion of the sufficiency condition is provided in H.E. Leland, "On the Existence of Optimal Policies under Uncertainty," Journal of Economic Theory (vol. 4; no. 1; February 1972), pp. 35-44.

563. Leland, Theory of the Firm Facing Uncertain Demand, op. cit., p. 281.

564. It is the demand curve that would result if the firm knew with certainty that price would equal its expected value for all values of q .

565. Ibid., p. 279. The principle of increasing uncertainty states that, as total expected revenue increases (due to changes in either p or q), the 'riskiness', or dispersion of the probability distribution, of total revenue also increases. Leland shows that the principle of increasing uncertainty is equivalent to the requirements that:
(i) sign $E[MR(q,u)]$ = sign $\partial[MR(q,u)]/\partial u$ when the firm is a quantity-setter; and
(ii) sign $E[MR(p,u)]$ = sign $\partial[MR(p,u)]/\partial u$ when the firm is a price-setter.

566. See footnote 553.

567. This result is a generalization of results obtained by other economists, such as Baron and Sandmo, for the purely competitive case. See Baron, Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition, op. cit., and Sandmo, op. cit.

568. Assuming that, in the case of an increase in fixed costs, the increase did not cause the firm to go bankrupt (in which case the firm's output level would fall to zero).

569. The analysis of the effect on the behavior of the firm of a change in fixed costs is similar to the analysis of the effect of a change in initial wealth on risky investment in portfolio theory. In both cases the effect depends not only on the degree of risk aversion, but also on the change in the degree of risk aversion as profit (or initial wealth) increases. See, for example, K.J. Arrow, Aspects of the Theory of Risk-Bearing (Yojö Johnssonin Säätiö; Helsinki; 1965).

570. The measure of absolute risk aversion used by Leland — and throughout much of the financial literature as well — is that attributable to Pratt. The measure is $-U''(\pi)/U'(\pi)$; and decreasing absolute risk aversion implies that $d[-U''(\pi)/U'(\pi)]/d\pi < 0$. See J.W. Pratt, "Risk Aversion in the Small and in the Large," Econometrica (vol. 32; no. 1-2; January-April 1964), pp. 122-136.

571. Leland, Theory of the Firm Facing Uncertain Demand, op. cit., p. 283. Briefly, the argument is the following. To find the effect on q of a small change dF in fixed costs, form the total differential of (215) to obtain:

$$D\,dq - E\{[MR(q_a,u) - MC(q_a)]U''(\pi)\}\,dF = 0 \ ,$$

where $D = \partial[E\{(MR - MC)U'(\pi)\}]/\partial q$ at $q = q_a$, which must be negative due to (216). Solving for dq/dF gives

$$dq/dF = E[(MR - MC)U''(\pi)]/D \ . \qquad (*)$$

Since $D < 0$, dq/dF is opposite in sign from the numerator in (*), which in turn can be shown to be opposite in sign from $d[-U''(\pi)/U'(\pi)]/d\pi$. Hence, decreasing absolute risk aversion implies $dq/dF < 0$, so that output decreases if fixed costs increase.

572. Recall that the principle of increasing uncertainty implies that the level of risk falls as expected total revenue falls. As long as expected marginal revenue is positive, the fall in the level of output will lead to a reduction in expected total revenue, and hence, to a reduction in the level of risk.

573. Ibid., p. 283.

574. Ibid., p. 284.

575. See footnotes 570 and 571.

576. Ibid., p. 284. The terminology employed to describe the two effects is motivated by the analogy that can be drawn between the effect of a change in a on optimal q and the "substitution" and "income" effects of a change in market price on quantity demanded in traditional price theory.

577. This corresponds to the Giffen good of traditional price theory — a rather special case in which an increase in the price of the good leads to an increase in quantity demanded.

578. An obvious example of this type of behavior is provided by firms in the electric power industry.

579. Mills, op. cit.; Leland, Theory of the Firm Facing Uncertain Demand, op. cit., pp. 286-288; R.A. Meyer, "Monopoly Pricing and Capacity Choice Under Uncertainty," American Economic Review (vol. 65; no. 3; June 1975), pp. 326-337; and R.A. Meyer, "Risk-Efficient Monopoly Pricing for the Multiproduct Firm," Quarterly Journal of Economics (vol. 90; no. 3; August 1976), pp. 461-474. The model presented in the second Meyer paper is discussed in the next subsection.

580. It is still assumed that the total variable cost function $C(q)$ is known with certainty. However, whereas the quantity-setting firm knew exactly where on the total variable cost curve its output-total variable cost combination would be once it had set its output level, the price-setting firm does not know where on this curve its output-total variable cost combination will lie because output is random. Hence, the shape of the total variable cost curve is likely to have an important bearing on how the existence of uncertainty affects the firm's selection of optimal price.

581. Risk neutrality implies $U'(\pi) = k$, for all π for some constant k. Constant marginal cost implies $C'[q(p,u)] = M$, for all q for some constant M. The necessary condition for an optimum becomes

$$k\, E\{q(p,u) + p[\partial q(p,u)/\partial p] - M[\partial q(p,u)/\partial p]\} = 0 ,$$

which, by using (230), can be rewritten as

$$k\{h(p) + p[dh(p)/dp] - M[dh(p)/dp]\} = 0 ,$$

which, from (231) is satisfied when $p = p_c$. Thus, the introduction of uncertainty has no effect on optimal p — provided the firm is risk neutral and marginal cost is constant.

582. It can be shown that, if marginal cost is rising at a nondecreasing rate $[C''(q) > 0 \ , \ C'''(q) \geq 0]$, the risk neutral firm will charge a higher price in the presence of uncertainty than it would under certainty, and that the opposite is true when marginal cost is falling at a nonincreasing rate $[C''(q) < 0 \ , \ C'''(q) \leq 0]$. Ibid., p. 285, and H.E. Leland, "Theory of the Firm Facing Uncertain Demand," technical report no. 24 (Institute for Mathematical Studies in the Social Sciences; Stanford University; Stanford, Calif.; January 1970).

583. Leland, Theory of the Firm Facing Uncertain Demand (1972), op. cit., p. 285.

584. The partial derivative $\partial\pi(p,u)/\partial p$ monotonically increasing with u implies that, for $p = p_n$, a value u_n can be found such that

$$E[\partial\pi(p_n,u)/\partial p] = \partial\pi(p_n,u_n)/\partial p \ , \qquad (*)$$

and also that, for $u > u_n$,

$$\partial\pi(p_n,u)/\partial p > \partial\pi(p_n,u_n)/\partial p \ . \qquad (**)$$

Risk aversion implies that, for $u > u_n$,

$$U'[\pi(p_n,u_n)] > U'[\pi(p_n,u)] \ , \qquad (***)$$

so that, for $u > u_n$,

$$[U'[\pi(p_n,u_n)] - U'[\pi(p_n,u)]]\{\partial\pi(p_n,u)/\partial p - \partial\pi(p_n,u_n)/\partial p\} > 0 \ .$$

This last inequality also holds for $u < u_n$, since both (**) and (***) are reversed when $u < u_n$, so that the expected value of the expression on the left must be positive. Taking the expectation and simplifying using (*) leads to

$$E\{U'(\pi)[\partial\pi(p_n,u)/\partial p]\} < 0 \ .$$

585. If $\partial\pi(p,u)/\partial p$ is not even monotonic — and there do not appear to be any strong reasons to expect that it should be — the foregoing analysis becomes even more unsettled.

586. Leland also demonstrates that this statement is equally valid for the analysis of the effect on optimal price of a change in the price-setting firm's fixed costs. Ibid., p. 286.

587. Baron, Demand Uncertainty in Imperfect Competition, op. cit., reaches the same conclusion, though by a somewhat different approach. See footnote 558.

588. Zabel, Monopoly and Uncertainty, op. cit. While more general in its treatment of time than the Leland model, the Zabel model is somewhat less general in that it assumes the multiplicative form of random demand and also assumes risk neutrality. Both the Zabel model and the Leland model are single period models, but recently Zabel has generalized his model to permit any finite number of time periods. E. Zabel, "Multiperiod Monopoly under Uncertainty," Journal of Economic Theory (vol. 5; no. 3; November 1972), pp. 524-536.

589. Dhrymes, op. cit.; Meyer, Monopoly Pricing and Capacity Choice Under Uncertainty, op. cit.; and Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit. Like the Leland model, these models are static.

590. D.M. Holthausen, "Input Choices and Uncertain Demand," American Economic Review (vol. 66; no. 1; March 1976), pp. 94-103; R.N. Batra and A. Ullah, "Competitive Firm and the Theory of Input Demand under Price Uncertainty," Journal of Political Economy (vol. 82; no. 3; May/June 1974), pp. 537-548; and R. Hartman, "Factor Demand with Output Price Uncertainty," American Economic Review (vol. 66; no. 4; September 1976), pp. 675-681.

591. Holthausen, op. cit., p. 102.

592. This subsection is based on Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit. In some cases the symbolism adopted by Meyer has been changed in order to make it consistent with the notation used throughout this paper.

593. In developing his model, Meyer had two particular types of problems in mind, one single period and the other multiperiod, but in both of which the firm produces a single good. In the single period setting, the firm acts as a discriminating monopolist, and the products are distinguished on the basis of the n different markets in which they are sold. So, for example, q_i is the amount produced for the i-th market which is distinct from the amount q_j produced for the j-th market when $i \neq j$. Ibid., pp. 461-462. See also Henderson and Quandt, op. cit., pp. 170-172. In the multiperiod setting, the prices, outputs, and demands are arrayed over time, and the products are distinguished on the basis of the n different time periods in which they are sold. So, for example, an amount q_1 is produced the first period, an amount q_2 the second, and so on. Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit., pp. 461-462. This corresponds to the well-known peak-load pricing model. See O.E. Williamson, "Peak-Load Pricing and Optimal Capacity under Indivisibility Constraints," American Economic Review (vol. 56; no. 4; September 1966), pp. 810-827, and O.E. Williamson, "Peak-Load Pricing: Some Further Remarks," Bell Journal of Economics and Management Science (vol. 5; no. 1; Spring 1974), pp. 223-228. In other words, the

prefix 'multi' in multiproduct results from distinguishing commodities on the basis of date and place of sale, as for example, an electric power company selling electricity at different prices to various domestic and commercial users or selling at different prices at different times of the day (e.g. 'peak' and 'off-peak' prices). Rather than merely repeat Meyer's development, what follows generalizes the discriminating monopolist version of Meyer's model to permit the firm to produce as many as n distinct goods. This necessitates a reformulation of one of the original constraints, but does not otherwise alter the model or its interpretation.

594. Note that the demand curves defined here represent the logical extension of Leland's demand curve (212) to the multiproduct case.

595. Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit., p. 464.

596. Ibid., p. 467.

597. See section B of this chapter.

598. See O.E. Williamson, Peak-Load Pricing and Optimal Capacity under Indivisibility Constraints, op. cit.

599. In the discriminating monopolist version of Meyer's model, $g(q_1,\dots,q_n)$ in equation (233) is of the form $g(q_1,\dots,q_n) = \sum_{i=1}^{n} q_i$. Equation (233) is more general. For example, a firm might produce two distinct goods, selling one good in m markets and the other in $n - m$ markets (where $0 < m < n$), or the firm might produce n distinct goods.

600. Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit., p. 463. This treatment of risk differs somewhat from the expected utility approach of the previous subsection. Under the expected utility formulation, only those aspects of risk borne by the firm (or more specifically, by that individual or group whose expected utility is being maximized) are incorporated in the model. Thus, there is an implied shifting of the risk burden to consumers and to suppliers of inputs (i.e. labor) built into such models. The constraint (234) alleviates at least part of this problem by allowing for consumer risk — the probability that the number of units demanded will exceed the number of units supplied, so that some portion of total demand goes unsatisfied. As Meyer suggests, welfare considerations could help determine the optimal value for each ε_i in a larger model explicitly incorporating consumers. Ibid., p. 464.

601. Ibid., p. 465. That is, given ε_i and the probability distribution for D_i , N_i is the smallest number such that

$$\text{Prob}\left\{D_i > q_i\right\} \leq \varepsilon_i \Longleftrightarrow \text{Prob}\left\{\frac{D_i - D_i^*}{\sigma_i} > \frac{q_i - D_i^*}{\sigma_i}\right\} \leq \varepsilon_i$$

$$\Longleftrightarrow q_i \geq D_i^* + N_i\sigma_i \quad .$$

Note that under either of two conditions N_i can be obtained as a function of ε_i only. First, if the probability distribution of D_i is approximately normal, N_i can be estimated independently of the parameters of the distribution. Second, for an arbitrary probability distribution for D_i , Chebyshev's inequality may be invoked to determine an upper bound on N_i independently of the probability distribution. See E. Parzen, Modern Probability Theory and Its Applications (Wiley; New York; 1960), pp. 226-227.

602. Meyer, Risk-Efficient Monopoly Pricing for the Multiproduct Firm, op. cit., p. 462. This is just the Sharpe-Lintner-Mossin equation for the equilibrium market value of the firm under uncertainty. See Sharpe, op. cit.; Lintner, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, op. cit.; and Mossin, Equilibrium in a Capital Asset Market, op. cit. Equation (236) arises out of a mean-variance model of risky choice under uncertainty. The placement of such a model in this section, which deals with the time-state-preference model of risky choice under uncertainty, might seem puzzling. Actually, the mean-variance framework can be regarded as a special case of the time-state-preference framework. Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 308-310. In addition, as already pointed out, the Meyer paper's approach is similar to that of the Leland paper discussed in the previous subsection.

603. Note the contrast between the valuation equation (236) and the valuation equation in problem (151) of the Vickers model. In (151) allowance is made for risk by adjusting the rate of discount, whereas in (236) the rate of discount ρ is riskless, and allowance is made for risk by adjusting the numerator. The expression in braces in (236) can be interpreted as the certainty equivalent of expected total profit, much in the spirit of John Lintner's valuation equation (185), which expresses the stock market value of the firm as the present value of the certainty equivalents of the dividend stream. As suggested in footnote 491, these two approaches to dealing with uncertainty — adjusting the denominator by forming a risk-adjusted discount rate or adjusting the numerator by forming the certainty equivalent of expected returns — are equivalent (provided, of course, the adjustments are made correctly).

604. Meyer, <u>Risk-Efficient Monopoly Pricing for the Multiproduct Firm</u>, <u>op. cit.</u>, p. 464, especially footnote 9.

605. The Lagrangian is

$$L_\lambda = V + \lambda[Q - g(q_1,\ldots,q_n)] + \sum_{i=1}^{n} \mu_i(q_i - D_i^* - N_i\sigma_i) \quad .$$

The necessary conditions (241)-(243) follow from the assumption that each of the decision variables appears in the optimal solution at a positive level. It is interesting to consider, however, the short run problem in which Q is fixed and in which (233) might be satisfied as a strict inequality. Again assume that all prices and output levels are positive at optimality and that (235) is again satisfied at optimality as an equality. Then, in addition to (233), (235), (241), and (242), the Kuhn-Tucker conditions for an optimum require that the constraints

$$\lambda[Q - g(q_1,\ldots,q_n)] = 0 \quad , \quad g(q_1,\ldots,q_n) \leq Q \quad , \quad \lambda \geq 0 \qquad (*)$$

be added. Since, in the short run, capacity is fixed, Q is no longer a decision variable, and (243) disappears. The significance of the conditions (*) is considered below in footnote 609.

For the peak-load pricing problem, the necessary conditions take a slightly different form. The constraint $g(q_1,\ldots,q_n) \leq Q$ in problem (240) must be replaced by the set of constraints $q_i \leq Q$, $i = 1,\ldots,n$, and, as a consequence, a set of n Lagrange multipliers λ_i , $i = 1,\ldots,n$, must be introduced, and (242) becomes

$$\frac{\partial L_\lambda}{\partial q_i} = -\frac{1}{\rho}\frac{\partial C}{\partial q_i} - \lambda_i + \mu_i = 0 \quad , \quad i = 1,\ldots,n \quad ,$$

and (243) becomes

$$\frac{\partial L_\lambda}{\partial Q} = -\frac{1}{\rho}\frac{\partial C}{\partial Q} + \sum_{i=1}^{n} \lambda_i = 0 \quad .$$

Since the analysis of the peak-load pricing problem is discussed at length by Meyer, and since the equilibrium conditions are similar in interpretation and form to (244)-(246) given below, the peak-load pricing problem will not be considered further here.

606. The risk adjustment here is determined by the stock market since λ is expressed in terms of V , which in turn depends on the market-determined price of risk, R .

607. Riskless marginal revenue MR_i^* is evaluated along the expected quantity certainty demand curve (232) at the optimal (risky) price.

608. The terminology is Meyer's. Ibid., p. 466. It would be more descriptive to refer to the expression in brackets in (245) as *risk-adjusted* marginal revenue, though Meyer's terminology will continue to be used.

609. This is, of course, perfectly analogous to the equilibrium condition under certainty that requires that, given optimal prices and capacity, the firm should continue to expand the output of each good up to the point at which marginal revenue just equals the sum of marginal production cost and marginal capacity cost. For the short run problem considered in footnote 499: when there is spare capacity, $g(q_1,\ldots,q_n) < Q$, and by (*) of footnote 605, $\lambda = 0$. Thus, (242) yields the familiar equilibrium condition,

$$\mu_i = \frac{1}{\rho}\frac{\partial C}{\partial q_i} ,$$

or marginal revenue equals marginal production cost. When there is spare capacity, its marginal cost is zero, and capacity considerations do not affect the firm's pricing and output decisions.

610. The interpretation of equation (247) is given in the preceding footnote. Once ε_i has been made sufficiently small, the effect of $\beta = 0$ is to shield the firm's total market value from any further effects of uncertainty. As a result, the firm's optimal investment, pricing, and production decision rules are the same as they would be in the absence of uncertainty. Put slightly differently, according to the valuation equation (239), the stock market evaluates the riskiness of a firm's profit stream in relation to overall market returns, and when a firm's profits and overall market returns are uncorrelated, the stock market values the firm's shares as it would a riskless investment since, in principle, a portfolio made up of a sufficiently large number of such securities would enable its holder to diversify away virtually all risk, and in the limit, would enable him to eliminate risk entirely.

611. That is, the relevant demand curve for comparison with the certainty case is the expected quantity certainty demand curve given by (232). When the firm maximizes its total market value in a riskless setting, it maximizes

$$V = \frac{1}{\rho}\sum_{i=1}^{n} p_i q_i - C(q_1,\ldots,q_n,Q) , \qquad (*)$$

which is like (239) with $\beta = 0$ and q_i in place of $D_i{*}$. In (*) $q_i = D_i{*} = h(p_1,\ldots,p_n)$ given by (232). That is, quantity demanded equals quantity sold with certainty. In (239) $D_i{*}$ represents quantity sold only approximately (and the approximation is close only if ε_i is chosen to be sufficiently small). Adding the capacity constraint (233), forming the Lagrangian, and differentiating, yields optimal decision rules for the certainty case that are identical with (244) and (247), thus bearing out the statements made in the text and in the preceding footnote concerning the implication of $\beta = 0$. Note also that, even though expected quantity sold is the same regardless of the existence of uncertainty, achieving this requires that ε_i be made small, which implies that the expected quantity of output will exceed expected quantity sold by an amount $E[q_i] - D_i^* = q_i - D_i^* = N_i\sigma_i$, regardless of the value of β. Hence, even when $\beta = 0$, it cannot be said that the presence of uncertainty has absolutely no effect on the firm's behavior.

612. So that vector-matrix multiplication is defined, D, q, and D^* must be n by 1 vectors. They have been written horizontally to save space. The matrix Ω is the variance-covariance matrix and its dimension is n by n.

613. Note that when demands are uncorrelated, Ω is a diagonal matrix, and (250) reduces to the n constraints

$$q_i \geq D_i^* + N_i\sigma_i \quad , \quad i = 1,\ldots,n \ ,$$

as in the special case considered in the previous subsection.

614. The Lagrangian is

$$L_\lambda = V + \lambda[Q - g(q_1,\ldots,q_n)] + \sum_{i=1}^{n} \mu_i(q_i - D_i^* - S_i) \ ,$$

which is the same as the Lagrangian in footnote 605 except that V is of a somewhat different form and S_i appears in place of $N_i\sigma_i$, with each change necessitated by the generalization permitting nonzero correlations between demands.

615. If $\beta = 0$ in (253), the necessary conditions for an optimal solution to problem (251) simplify, as before, to the optimality conditions under certainty. In particular, relation (247) would again hold.

616. In the more general case of the multiproduct firm considered above, this statement is false. When there are strong complementarities among the goods produced by the multiproduct firm, the firm may find it profitable to sell one or more goods at a price below marginal production cost (i.e. as a 'loss leader'). For example, when the firm produces razors and razor blades, the firm may find it profitable to give away the razors so that it can increase the demand for its razor blades and thereby increase its total profit. The argument given below also applies to such firms, but is stated in terms of the discriminating monopolist in order to demonstrate more clearly that under uncertainty there is an additional justification for selling at a price below marginal production cost.

617. As Meyer suggests, this defines sets of risk-efficient prices which are analogous to risk-efficient sets of securities in portfolio theory. Ibid., p. 471. A set of prices is risk-efficient if it minimizes risk for a given level of expected total profit, just as a portfolio of securities lies on the efficiency frontier if it minimizes risk for any given level of expected portfolio returns. See Mossin, Theory of Financial Markets, op. cit., ch. 3.

618. This suggests that, under risk, optimal pricing is more dependent on the customer's risk-class than on the price elasticity of demand, which plays such a critical role in the riskless setting. Meyer, Risk-Efficient Monopoly Pricing for the Multi-Product Firm, op. cit., p. 470. See also Henderson and Quandt, op. cit., pp. 215-216, for a discussion of the optimal pricing rule for the discriminating monopolist in a riskless setting, namely, that the ratio of the market prices charged any two groups of customers should be dependent on the two price elasticities, η_1 and η_2, according to the relation

$$\frac{p_1}{p_2} = \frac{1 - 1/\eta_2}{1 - 1/\eta_1} \quad .$$

619. This is analogous to the percentage yield for a security lying below the riskless rate of interest when that security's returns are sufficiently negatively correlated with the returns of the other securities in the portfolio. See Mossin, Theory of Financial Markets, op. cit., ch. 4.

620. Several papers have assumed that firms maximize their stock market value, and upon finding that the allocation of investment is not Pareto optimal, have suggested that the stock market is not Pareto optimal. See J.E. Stiglitz, On the Optimality of the Stock Market Allocation of Investment, op. cit.; M. Jensen and J. Long, Corporate Investment under Uncertainty and Pareto Optimality in the Capital Markets, op. cit.; and E.F. Fama, Perfect Competition and Optimal Production Decisions under Uncertainty, op. cit. What these papers really bring into question, however, is the presumption of value maximization, rather than the optimality of stock markets. See R. Wilson, Comment on J. Stiglitz, 'On the Optimality of Stock Market Allocation of Investment', op. cit., and S.F. LeRoy, Stock Market Optimality: Comment, op. cit.

621. Using a variant of Stiglitz's model, Wilson shows that stockholders would unanimously recommend production decisions that do not maximize the firm's total market value. R. Wilson, Comment on J. Stiglitz, 'On the Optimality of Stock Market Allocation of Investment', op. cit.

622. The Leland model is a generalization of a model due to Diamond. See Diamond, op. cit. In the Leland and Diamond models, unanimity is assured by the assumption that the technology of the firm is such that every alternative open to the firm would not alter the set of available state-distributions of returns. The question of unanimity of shareholder preferences is further explored and more general conditions leading to shareholder unanimity are developed in R.C. Merton and M.G. Subrahmanyam, "The Optimality of a Competitive Stock Market," Bell Journal of Economics and Management Science (vol. 5; no. 1; Spring 1974), pp. 145-170; S. Ekern and R. Wilson, "On the Theory of the Firm in an Economy with Incomplete Markets," Bell Journal of Economics and Management Science (vol. 5; no. 1; Spring 1974), pp. 171-180; R. Radner, "A Note on Unanimity of Stockholders' Preferences Among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model," Bell Journal of Economics and Management Science (vol. 5; no. 1; Spring 1974), pp. 181-184; S. Ekern, "On the Theory of the Firm in an Economy with Incomplete Markets: An Addendum," Bell Journal of Economics (vol. 6; no. 1; Spring 1975), pp. 388-393; and R. Forsythe, "On the Theory of the Firm under Uncertainty," Social Science Working Paper No. 90 (California Institute of Technology; Pasadena, Calif.; July 1975).

623. The Leland model employs a time-state-preference framework is demonstrating this result. In a similar manner, Long employs a mean-variance framework to show that maximizing the stock market value of the firm does not imply that shareholder welfare has been maximized. J. Long, "Wealth, Welfare, and the Price of Risk," Journal of Finance (vol. 27; no. 2; May 1972), pp. 419-433.

624. The remainder of this subsection discusses the model of the firm under uncertainty presented in Leland, Production Theory and the Stock Market, op. cit.

625. Baron, Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition, op. cit., and Sandmo, op. cit. The model of the firm under either pure or perfect competition can be treated as a special case of the model of the quantity-setting firm developed by Leland, which was discussed above in subsection 2b of this section.

626. This has the effect of making the zeroth firm's securities riskless. As noted by Leland, the same effect could be achieved by assuming the existence of a riskless bond paying a fixed rate of interest.

627. Leland, Production Theory and the Stock Market, op. cit., p. 130.

628. Note that since $r \equiv \pi^o(q^o)/V^o$ the system of equations is homogeneous of degree zero in $\hat{V}$, so that only relative prices can be determined. By selecting the securities of the zeroth firm as numeraire, $\hat{V}^o = 1$ and all other security prices are normalized in terms of the zeroth firm's share price. Then r becomes simply $r = \pi^o(q^o)/\hat{V}^o = \pi^o(q^o)$.

629. Ibid., p. 131.

630. Ibid., p. 131.

631. Ibid., pp. 131-133.

632. In (257) $r = \pi^o(q^o)$, since $\hat{V}^o = 1$, and $\bar{s}_i = \hat{s}_i$, which must hold at the financial equilibrium.

633. The importance of this result, as Leland suggests, is that it depends only on observable variables, i.e. cost functions and current market values. Ibid., p. 133.

634. Note that even though (262) and (263) are similar in form, one cannot use (263) to derive a marginal cost pricing rule analogous to (265). The reason is simply that, since price $p^k(\theta)$ is random (and dependent on the state of nature θ), the firm cannot equate $p^k(\theta)$ and $MC^k(q^k)$. Under uncertainty both profit maximization and "p = MC" are meaningless.

635. However, Leland does show that, when all firms operate within a competitive environment, facing the same random price and having identical strictly convex cost functions, the firm will maximize its stock market value relative to the market value of other firms by selecting $q^j = \hat{q}^j$ unanimously supported by the firm's shareholders. Ibid., pp. 137-138.

636. This is a straightforward generalization of Leland's result. Leland considers the simple case in which there is only one risky firm in addition to the riskless firm. Ibid., pp. 136-137. The derivation of (267), which permits the number of risky firms to be arbitrary, is as follows. Differentiating (260) with respect to q^j and evaluating at $q^j = \hat{q}^j$ gives

$$E\{U_i'(R_i,\theta)[\frac{d\pi^j}{dq^j} - r\frac{\partial\hat{V}^j}{\partial q^j}]\} + E_i\{U_i''(R_i,\theta)(\pi^j - r\hat{V}^j)$$

$$[r\sum_{k=1}^{N}\frac{\partial\hat{V}^k}{\partial q^j}\bar{s}_i^k + \sum_{k=1}^{N}\hat{s}_i^k(-r\frac{\partial\hat{V}^k}{\partial q^j}) + \hat{s}_i^j\frac{d\pi^j}{dq^j} + \sum_{k=1}^{N}(\pi^k - r\hat{V}^k)\frac{\partial\hat{s}_i^k}{\partial q^j}]\}$$

$$= -E\{U_i'(R_i,\theta)\}(r\frac{\partial\hat{V}^j}{\partial q^j}) \tag{*}$$

$$+ E_i\{U_i''(R_i,\theta)(\pi^j - r\hat{V}^j)[\hat{s}_i^j\frac{d\pi^j}{dq^j} + \sum_{k=1}^{N}(\pi^k - r\hat{V}^k)\frac{\partial\hat{s}_i^k}{\partial q^j}]\} = 0$$

since in equilibrium $\bar{s}_i^k = \hat{s}_i^k$ for all k and $E\{U_i'(R_i,\theta)\frac{d\pi^j}{dq^j}\} = 0$. Defining $D_i \equiv E_i\{U_i''(R_i,\theta)[\pi^j - r\hat{V}^j]^2\}$ and dividing each side of (*) by D_i gives equation (267) in the text. Note that $D_i < 0$ since, by assumption, $U_i''(R_i,\theta) < 0$ while $[\pi^j - r\hat{V}^j]^2 > 0$.

637. For the case of one risky firm (considered by Leland), the expression

$$\left(\hat{s}_i^j \frac{d\pi^j}{dq^j} + \sum_{k \neq j} (\pi^k - r\hat{V}^k) \frac{\partial \hat{s}_i^k}{\partial q^j}\right)$$

in (268) is replaced by

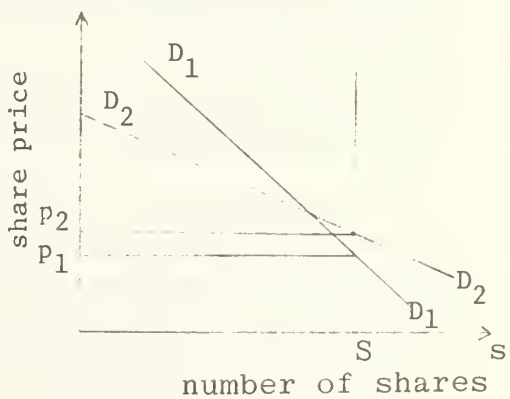
$$\hat{s}_i^1 \frac{d\pi'}{dq'} .$$

Ibid., p. 136. In this special case it can be shown that for the single risky firm,

$$\left.\frac{\partial V^1}{\partial q^1}\right|_{q^1=\hat{q}^1} < 0 .$$

Ibid., p. 137, footnote 28.

638. This result seems paradoxical for two reasons. First, a higher share value would appear to make all shareholders better off since it would increase their wealth. Second, unless the firm maximizes its share value, someone else will take over the firm, adopt policies that lead to maximum share value, and sell out at a sure profit. However, as Leland argues, both lines of reasoning are incorrect. Ibid., p. 127. To illustrate why, assume there are N identical investors each holding S shares in the firm. Two possible demand curves for the individual investor are shown in the figure to the right. Under policy (1) the demand curve for shares is D_1D_1 and the equilibrium share price is p_1, while under policy (2) the demand curve is D_2D_2 and the equilibrium share price is p_2. Under policy (2) the share price, and hence the total stock market value of the firm, is greater. But the shareholders would be better off under policy (1) since the total value investors attach to their shares — measured as the sum of current market value and consumers' surplus, i.e. the area under the demand curve



Figure

between $s = 0$ and $s = S$ — is greater under policy (1). Also, if someone were to buy up all the firm's shares, he would have to pay an amount equal to the area under D_1D_1 (since to buy up the shares he would have to bid up the price as the number of shares not in his possession decreased). But changing to policy (2) and then selling the shares would yield him an amount no greater than the area under D_2D_2 — an amount less than he originally paid for the shares. Thus, both of the arguments that appear to support value maximization are incorrect. Value maximization need not maximize (total) shareholder welfare. Note that what is involved here is just the well-known paradox of value.

639. If securities markets were perfect and complete, then $\left.\frac{\partial \hat{V}^j}{\partial q^j}\right|_{q^j=\hat{q}^j} = 0$ for all firms, and the firms could achieve a Pareto optimal choice of outputs by maximizing the firm's market value. Rather than attempting to show this using equation (268), it might be simpler to use an arbitrage argument of the type suggested by Leland. Ibid., p. 133, footnote 22.

640. As well as several other models, some of which are noted in footnote 225.

641. An exception to this statement is the Baumol growth maximization model, in which sales and the firm's quantity of money capital were permitted to grow at different (though constant) rates. See footnote 213.

642. Therefore, what is really a dynamic (i.e. multiperiod) problem from an economic standpoint can be solved by employing static (in the mathematical sense) optimization techniques.

643. Marris, Theories of Corporate Growth, op. cit., pp. 12-13, and Lintner, Maximum Corporate Growth under Uncertainty, op. cit., pp. 174-176.

644. Krouse, On the Theory of Optimal Investment, Dividends, and Growth in the Firm, op. cit., p. 269. Krouse's model is discussed below in subsection 4.

645. The first effect — the impact of the business cycle on the firm's capital investment — can be explained by the Arrow model, which is discussed below in subsection 2, and the second effect — a declining growth rate as the firm matures — can be explained by the Wong model which is discussed below in subsection 3.

646. The basic references for the calculus of variations are G.A. Bliss, Lectures on the Calculus of Variations (University of Chicago Press; Chicago; 1946); I.M. Gelfand and S.V. Fomin, Calculus of Variations, trans. by R.A. Silverman (Prentice-Hall; Englewood Cliffs, N.J., 1963); and M.R. Hestenes, Calculus of Variations and Optimal Control Theory (Wiley; New York; 1966). A particularly good reference for economists is G. Hadley and M.C. Kemp, Variational Methods in Economics (North-Holland; Amsterdam; 1971). The basic references for the maximum principle are L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, and E.F. Mishchenko, The Mathematical Theory of Optimal Processes, trans. by K.N. Trirogoff (Wiley Interscience; New York; 1962); M. Athans and P.L. Falb, Optimal Control (McGraw-Hill; New York; 1966); Hestenes, op. cit.; and E.B. Lee and L. Markus, Foundations of Optimal Control Theory (Wiley; New York; 1967). Less mathematically sophisticated approaches to the calculus of variations and optimal control theory that emphasize economic applications include Intriligator, op. cit., chs. 12,14; Takayama, op. cit., chs. 5,8; and A. Bensoussan, E.G. Hurst, Jr., and B. Näslund, Management Applications of Modern Control Theory (North-Holland; Amsterdam; 1974), which is particularly noteworthy in that it covers stochastic control theory as well as deterministic control theory.

647. The interpretation of the costate variables is discussed at length in Intriligator, op. cit., pp. 351-353; M. Albouy and A. Breton, "Interprétation Economique du Principe du Maximum," Revenue Française de Recherche Opérationnelle (vol. 14; 1968); and R. Dorfman, "An Economic Interpretation of Optimal Control Theory," American Economic Review (vol. 59; no. 5; December 1969), pp. 817-831.

648. In addition to models of the firm, several other economic and management applications of optimal control theory have been suggested. For two surveys see A.P. Jacquemin and J. Thisse, "Strategy of the firm and market structure: an application of optimal control theory," in Cowling, ed., Market Structure and Corporate Behavior, op. cit., pp. 61-84, and Bensoussan, Hurst, and Näslund, op. cit.

649. This subsection is based on Jorgenson, op. cit., pp. 140-147.

650. Thus, $Q(t)$ is the rate at which output is produced at time t, $L(t)$ is the rate at which labor services are applied to production at time t, etc.

651. Note that the price of capital goods is permitted to vary. That is, unlike the multiperiod model of the firm discussed in section F in which capital goods were selected as the numeraire, in the Jorgenson model some other good (possibly money) serves as the numeraire.

652. Note that, according to (271), the production function — the technological relationship between output and inputs — remains constant over time. In the next subsection a model that permits technological progress (by making t itself a separate argument of the production function) is discussed. In addition, note that in (271) $K(t)$ represents the physical stock of capital at time t, whereas $Q(t)$ and $L(t)$ represent instantaneous rates of flow of output and labor services, respectively, at time t. Shephard and Färe have explored the conceptual difficulties inherent in specifying the production function implicitly as in (271). See Shephard, op. cit., and Shephard and Färe, op. cit.

653. The constant δ in (272) is intended to measure depreciation in physical units (i.e. in the same units in which physical capital is measured) per unit of time. Thus, δ must convert from stock units (in which K is measured) into flow units (in which $\dot{K}$ and I are measured). Note, however, that if $K(t)$ in the production function (271) were interpreted as a flow of capital services (e.g. machine hours per period) so that Q, L, and K all represent flows, then $K(t)$ on the right-hand side of (272) would also measure this flow of capital services. But K on the left-hand side of (272) must represent a physical stock of capital. Thus, in this case the constant δ would also have to reflect the conversion from one flow unit (e.g. machine hours per unit time period) to another (e.g. machines per unit time period) in order for the units of measurement in (272) to be in agreement.

654. Note that it is not necessary to list K as a separate decision variable, because, given the initial capital stock, K_0, specifying the investment stream $I(t)$ automatically specifies the capital stock at each point in time, $K(t)$, because of (272).

655. In setting out the necessary conditions it has been assumed that, at optimality, Q, L, I, and K are strictly positive for all t.

656. The justification for giving i this interpretation is the following, which is based on the relationship between the price of capital goods and the price of capital services. Jorgenson, op. cit., pp. 143-144. For a unit investment in capital goods at time s, the flow of capital services received between time t ($>s$) and time $t+dt$ is given by

$$e^{-\delta(t-s)}\,dt \quad . \qquad (*)$$

If $i(t)$ is the unit price of capital services at time t, then the discounted price is $i(t)\cdot e^{-rt}$ and the present value of the stream of capital services (*) is given by

$$i(t)\cdot e^{-rt}\cdot e^{-\delta(t-s)}\,dt \quad .$$

If the unit price of capital goods at time s is $q(s)$, then the present value of a unit of investment goods at time s is

$$e^{-rs}\cdot q(s) \quad .$$

If capital markets are perfect, as Jorgenson assumes, then the present value of the unit investment at time s must equal the present value of the future flow of capital services, so that

$$e^{-rs}q(s) = \int_s^\infty i(t)\cdot e^{-rt}\cdot e^{-\delta(t-s)}\,dt ,$$

or upon solving for the unit price of investment goods,

$$q(s) = \int_s^\infty i(t)\cdot e^{-(r+\delta)(t-s)}\,dt . \qquad (**)$$

By differentiating (**) with respect to s, an expression for the unit price of capital services implicit in (**) can be obtained:

$$\dot{q}(s) = [r+\delta]q(s) - i(s) ,$$

or

$$i = q(r+\delta) - \dot{q} ,$$

which is just equation (281). It should be noted that, if problem (273) is restated so that the objective of the firm is to maximize the present value of the future profit stream, i.e. $\int_0^\infty P(t)e^{-rt}\,dt$, where $P(t) = p(t)\cdot Q(t) - w(t)\cdot L(t) - i(t)\cdot K(t)$ is total profit in the economic sense, then the solution is identical to the solution obtained from (275)-(278) together with the contraints in (273). Ibid., pp. 144-145. That is, the cost of capital, $i(t)$, is the same as the price of a unit of capital, i, introduced in section B. Also, as pionted out in section F, when there is certainty and markets are perfect, maximizing economic profit in the single period sense is equivalent first, to maximizing the present value of the profit stream, and second, to maximizing the present value of the cash flow stream.

657. Equation (281) might be interpreted more easily if rewritten as

$$i/q = r + \delta - \dot{q}/q . \qquad (*)$$

Equation (*) reexpresses the cost of capital as an interest rate. According to (*), the cost of capital is equal to the discount rate (which is equal to the market rate of interest under perfect markets), r, (the opportunity cost associated with utilizing the capital in production), plus the rate of depreciation, δ (the rate at which physical capital wears out through use), minus the percentage rate of change of capital goods prices (the percentage rate at which the market value of capital goods (relative to the numeraire) appreciates). In other words, the owner of a unit of capital must be compensated for the opportunity cost of his capital, r, and for the depreciation of the unit of capital through use, δ, but if capital goods prices are rising (relative to the numeraire), these costs are at least partially offset by the percentage increase in the market value of the undepreciated balance.

658. In this regard the Jorgenson model is in agreement with the simple value maximization model discussed in section F. Indeed, with capital goods as numeraire, $q = 1$, $\dot{q} = 0$, and (283) reduces to (72).

659. For more on this point see J. Tobin, "Comment," in Ferber, op. cit., pp. 156-160. Also, because of the assumptions of certainty and perfect capital markets, the firm's financial policies are irrelevant. A model somewhat similar to the Jorgenson model, but one that allows for the risk of default, has been devised by Inselbag. See I. Inselbag, "Financing Decisions and the Theory of the Firm," Journal of Financial and Quantitative Analysis (vol. 8; no. 5; December 1973), pp. 763-776. Inselbag introduces uncertainty into the model by allowing for the possibility of bankruptcy and by making the implicit cost of debt an increasing function of both the firm's leverage ratio and the amount borrowed. Ibid., p. 768. This treatment of the cost of debt parallels Vicker's approach. See section I of this chapter. Because of this treatment of uncertainty and because Inselbag employs a deterministic objective functional (like Jorgenson's, but with dividends, $D(t)$, in place of net cash flow, $R(t)$), the Inselbag model is essentially a deterministic model (which, like the Jorgenson model, is easily solved with the aid of the classical calculus of variations).

660. By dividing each side of equation (281) by q , the expression for the cost of capital becomes

$$\frac{i}{q} = r + \delta - \frac{\dot{q}}{q} , \tag{*}$$

where i/q is the cost of capital expressed as a percentage of the price of capital goods (i.e. as an interest rate), r is the short term rate of interest, δ is the rate of depreciation, and $\dot{q}/q$ is the percentage rate of change of capital goods prices. Dividing each side of (283) by q gives

$$\frac{p}{q}\frac{\partial Q}{\partial K} = \frac{i}{q} , \tag{**}$$

in which p/q expresses the price of output in terms of the price of capital goods. Using (*) to substitute for i/q and defining $p' \equiv p/q$, (**) becomes

$$p' \frac{\partial Q}{\partial K} = i/q = r + \delta - \dot{q}/q ,$$

which is the equilibrium condition stated in the text and which is one of the standard equations of neoclassical capital theory. See, for example, K.J. Arrow, "Optimal Capital Policy, the Cost of Capital, and Myopic Decision Rules," Annals of the Institute of Statistical Mathematics (vol. 16; 1964), pp. 21-30, or S.A. Marglin, Approaches to Dynamic Investment Planning (North-Holland; Amsterdam; 1963). See also M. Nerlove and K.J. Arrow, "Optimal Advertising Policy Under Dynamic Conditions," Economica (vol. 29; no. 114; May 1962), pp. 129-142, which derives results that are formally equivalent, but for advertising, rather than capital, policy.

661. The remainder of this subsection is based on the model developed in Arrow, Optimal Capital Policy with Irreversible Investment, op. cit., pp. 1-19. The model is a generalization of several earlier studies of the firm's optimal capital policy over time conducted by Arrow and others, among them K.J. Arrow, M. Beckmann, and S. Karlin, "The Optimal Expansion of the Capacity of a Firm," In K.J. Arrow, S. Karlin, and H. Scarf, eds., Studies in the Mathematical Theory of Inventory and Production (Stanford University Press; Stanford, Calif.; 1958), ch. 7; K.J. Arrow, "Optimal Capital Adjustment," in K.J. Arrow, S. Karlin, and H. Scarf, eds., Studies in Applied Probability and Management Science (Stanford University Press; Stanford, Calif.; 1962), ch. 1; and Arrow, Optimal Capital Policy, the Cost of Capital, and Myopic Decision Rules, op. cit.

662. Note that these costs could be recovered if the market for capital goods were perfect.

663. Arrow, Optimal Capital Policy with Irreversible Investment, op. cit., pp. 2-3.

664. Ibid., p. 3.

665. As a second possibility, the firm may postpone investment because it expects technological improvements to be made in the very near future that will cause the capital goods (e.g. machines) currently available to become technologically obsolete.

666. To see why (285) holds, note that if the short term rate of interest at each time t is $\rho(t)$, then a sum $R(t)$ received at time t has present value

$$V = R(t)\, e^{-\int_0^t \rho(s)ds} ,$$

which, by the definition of $\alpha(t)$, is also given by

$$V = R(t)\, \alpha(t) .$$

Then

$$\alpha(t) = e^{-\int_0^t \rho(s)ds} , \qquad (*)$$

and differentiating each side of (*) with respect to t gives

$$\dot{\alpha}(t) = -\rho(t)\, e^{-\int_0^t \rho(s)ds} = -\rho(t)\cdot\alpha(t) ,$$

from which (285) immediately follows. For the special case in which $\rho(t) \equiv \rho$, a constant, for all t, $\alpha(t) = e^{-\rho t}$.

667. Ibid., pp. 5-6. Define the following variables:

$$x(t) = K(t)e^{\delta t} \qquad y(t) = I(t)e^{\delta t}$$

$$\beta(t) = \alpha(t)e^{-\delta t} \qquad P^*(x,t) = e^{\delta t}P(xe^{-\delta t},t) \quad .$$

Then

$$\dot{x} = (\dot{K} + \delta K)e^{\delta t} = I(t)e^{\delta t} = y$$

and

$$\beta(t)[P^*(x,t)-y(t)] = \alpha(t)[P(k(t),t) - I(t)] \ .$$

Then problem (286) becomes

$$\underset{\{y\}}{\text{maximize:}} \quad \int_o^\infty \beta(t)[P^*(x,t) - y]\, dt$$

$$\text{subject to:} \quad \dot{x} = y$$

$$x(0) = K_o$$

$$y \geq 0 \ .$$

Substituting α for β , P for P^* , I for y , and K for x gives problem (287), which is just problem (286) with $\delta \equiv 0$.

668. Ibid., p. 6.

669. Ibid., pp. 6-7.

670. Since $p(t)$ is measured in terms of the change in the present value of the cash flow stream (i.e. the objective functional in (287)) with respect to a unit change in the capital stock, equation (293) is really just the familiar condition that capital should be accumulated up to the point at which the improvement in the present value of the cash flow stream due to the additional returns resulting from a unit increase in the capital stock just equals the fall in the present value of the cash flow stream due to the fall in $p(t)$ induced by the increase in the capital stock — i.e. until 'marginal benefit' just equals 'marginal cost', where each is expressed in terms of the present value of the cash flow stream.

671. The initial value of $q(t)$ — namely, $q(0)$ — has not been specified. However, it must be such that the necessary conditions can all be satisfied simultaneously, and in particular, it must be chosen small enough that $q(t) \leq 0$ for all $t > 0$.

672. The terminology 'free intervals' and 'blocked intervals' is Arrow's. Ibid., p. 9.

673. See footnotes 660 and 678.

674. Ibid., p. 9.

675. Where the two differ, however, is during periods when capacity is not expanding.

676. For technical reasons, the analysis of blocked intervals for which $t_o = 0$ or for which t becomes infinite, or both, must be carried out separately, although the results obtained are similar to those obtained for the case $t_o > 0$ and $t_1 < \infty$. Details are provided in ibid., pp. 10-11.

677. Ibid., p. 8.

678. Recall that money capital markets have been assumed perfect, and further, that $\rho(t)$ is expressed in terms of capital goods prices and the rate of depreciation is zero, so that $\rho(t)$ is the cost of capital (expressed as an interest rate).

679. The irreversibility of investment may also mean that, at the time a recession ends, firms find that they have substantial excess capacity, so that increases in investment spending will lag behind the upturn in the level of economic activity. See, for example, D.P. Garino, "Firms Like Monsanto Give Capital Projects Tough Second Looks," Wall Street Journal (December 30, 1976).

680. See section G of this chapter.

681. O.E. Williamson, A Dynamic Stochastic Theory of Managerial Behavior, op. cit., pp. 11-31.

682. This subsection is based on Wong, op. cit., pp. 689-694. A similar model is presented in Takayama, op. cit., pp. 688-697.

683. This is more easily appreciated by referring to the typical firm's statement of retained earnings illustrated in table II-3. In (301), $p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t)$ corresponds to net income, $D(t)$ corresponds to dividends paid, and $I(t)$ corresponds to retained earnings (under the implicit assumption that all retained earnings are reinvested).

684. More importantly, the constraint on investment (301) restricts the speed of adjustment of the firm's capital stock to the desired capital stock. In the Jorgenson model the firm can adjust its capital stock to the most desired level instantaneously, i.e. investment is reversible. Note that (301) together with the constraint $D(t) \leq p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t)$ imply that $I(t) \geq 0$. That is, investment is irreversible. The maximum rate at which the firm's capital stock can be reduced is the rate of depreciation.

685. The derivation of (310) follows. When $q > 1$, $1-q < 0$, so that dividing each side of (309) by $1-q$ reverses the sense of the inequality, giving $D^*(t) \leq D(t)$, which implies $D^*(t) = 0$, the minimum permissible value. When $q < 1$, dividing each side of (309) by $1-q$ preserves the sense of the inequality, giving $D^*(t) \geq D(t)$, which implies $D^*(t) = p(t) \cdot f(K^*(t), L^*(t)) - w(t) \cdot L^*(t)$, the maximum permissible value. When $q = 1$, $1-q = 0$, so that division by $1-q$ is impossible. For $q = 1$, the optimal dividend policy must be determined by further considerations, which are set out below.

686. As usual, it is assumed that $L^*(t) > 0$. If the constraint $L(t) \geq 0$ were binding, then at optimality $p \cdot \frac{\partial f(K,L)}{\partial L} \leq w$, as in condition (7) in section B of this chapter, so that it does not pay the firm to hire any labor.

687. The condition (308) may be thought of as the 'transversality condition' for the infinite horizon problem. Difficulties associated with extending the transversality condition to the infinite horizon case are discussed briefly in Takayama, op. cit., pp. 623-625, and more fully in K.J. Arrow, "Applications of Control Theory to Economic Growth," in G.B. Dantzig and A.F. Veinott Jr., eds., Mathematics of the Decision Sciences, Part 2 (American Mathematical Society; Providence, R.I.; 1968), pp. 85-119.

688. For a discussion of sufficiency see O.L. Mangasarian, "Sufficient Conditions for the Optimal Control of Nonlinear Systems," SIAM Journal of Control (vol. 4; no. 1; February 1966), pp. 139-152.

689. From (312), if $q > 1$, then $\dot{K} = 0$ implies that

$$p(t) \cdot f(K(t), L(t)) - w(t) \cdot L(t) - \delta \cdot K(t) = 0 . \qquad (*)$$

If $L(t)$ in (*) satisfies (311) and if the production function $f(K,L)$ is concave, then (*) is solved by a unique level of the capital stock, K' in figure II-27. From (313), $\dot{q} = 0$ implies that

$$p \cdot \frac{\partial f(K,L)}{\partial K} = \delta + r \quad . \qquad (**)$$

If $L(t)$ is (**) satisfies (311) and if the production function $f(K.L)$ is concave, then (**) is solved by a unique level of the capital stock, K^{opt} in figure II-27.

690. In the figure the arrows indicate the direction of change of q and K , depending on the sign of $\dot{q}$, the sign of $\dot{K}$, and whether q is less than, equal to, or greater than one. For example, at point A, $\dot{q} > 0$ and $q < 1$, so that $\dot{K} = -\delta \cdot K < 0$. Therefore, q is increasing and K is decreasing, so that (K,q) moves in the northwest direction. At point B, $\dot{q} < 0$, and $q > 1$, so that $\dot{K} > 0$, and therefore, (K,q) moves in the southeast direction.

691. Actually there are three possible paths: (i) K and q converging to zero, (ii) $q(t) \to \infty$ with
$\dot{q}(t) = q(t)[(\delta + r) - p \cdot \frac{\partial f(K,L)}{\partial K}]$, and (iii) K converging to K^{opt} and q converging ot one. Along path (i) the firm eventually goes out of business. Along path (ii) necessary condition (308) is violated. Hence, only path (iii) could be optimal for a viable firm.

692. The justification for the third condition is the following. If $K_o = K^{opt}$, then $\dot{K} = 0$. From (302), $I(t) = \delta \cdot K(t)$, and then from (301),

$$D^*(t) = p(t) \cdot f(K^*(t), L^*(t)) - w(t) \cdot L^*(t) - \delta \cdot K^*(t) .$$

693. Wong, op. cit., p. 692. It should be noted, however, that the managerial models could not, in general, be used to replicate this phase exactly. In particular, the multiperiod managerial models, such as the Marris model, typically involve some sort of valuation constraint that prevents the firm's retention ratio from falling to zero.

694. Ibid., p. 692.

695. For example, see R. Radner, "Paths of Economic Growth that are Optimal with Regard only to Final States: A Turnpike Theorem," Review of Economic Studies (vol. 28; February 1961), pp. 98-104; D. Cass, "Optimum Growth in an Aggregative Model of Capital Accumulation," Review of Economic Studies (vol. 32; no. 91; July 1965), pp. 233-240; and S.J. Turnovsky, "Turnpike Theorems and Efficient Economic Growth," in E. Burmeister and A.R. Dobell, Mathematical Theories of Economic Growth (Macmillan; New York; 1970), ch. 10.

696. Wong, op. cit., p. 691.

697. In Wong's version of the model, as in the Arrow model, capital goods are taken to be the numeraire, so that in (281) $q = 1$ and $\dot{q} = 0$. Thus, $i = r + \delta$, so that (283) is identical to (315).

698. The existence of what are commonly known as 'mature industries' and 'growth industries' would appear to lend at least partial support to Wong's thesis. For example, see "New Leaders in Semiconductors," Business Week (March 1, 1976), which describes the fast growth of new companies in a rapidly expanding industry, and "LTV: Weak growth in mature industries," Business Week (April 5, 1976), which describes the problems confronting a firm whose major product lines are in mature industries. But in these cases, the growth of the firm is most heavily influenced by demand factors, rather than by the cost factors implied in the Wong model. As Marris and Wood argue, the existence of constant

or increasing returns to scale implies that there is nothing "to prevent an individual firm from growing continuously and indefinitely." Marris and Wood, op. cit., p. xvii. Of course, this does not mean that firms will always continue to grow. Due either to the maturity of traditional markets and an unwillingness to diversify or to an increased relative preference for profits rather than growth, a firm's growth rate may fall, and concomitantly, its profitability may improve — causing the firm to exhibit behavior consistent with the Wong model — even though there is no optimum size, K^{opt} , that causes it to behave in this manner. See, for example, "His Master's New Voice," Newsweek (November 17, 1975), p. 81, which describes RCA Corp. as "a mature, not a growth, company," and "Bringing Order to a Billion-Dollar Empire," Business Week (September 8, 1975), which describes a shift in management's preferences away from growth and toward increasing profits.

699. However, as noted earlier in this subsection, in the Wong model investment is irreversible (i.e. capital goods cannot be sold), which represents the antithesis of the instantaneous downward adjustments in the size of the firm's capital stock that are possible in the Jorgenson model. Ideally, one would hope for a more realistic treatment of capital decumulation — one that falls somewhere between these two extremes.

700. See Eatwell, op. cit., and the discussion of this point in section D of chapter one. In addition, the evidence supporting the existence of either constant or increasing returns to scale in manufacturing industries also casts considerable doubt on the existence of optimum firm sizes. For example, see J. Johnston, Statistical Cost Analysis (McGraw-Hill; New York; 1960), and A.A. Walters, op. cit. For results of an earlier comprehensive study generally supportive of decreasing average cost for large firms in heavy manufacturing industries, see National Bureau of Economic Research, Cost Behavior and Price Policy, op. cit. Similar results have been obtained in studies of manufacturing industries in other countries. For example, see C.F. Pratten, Economies of Scale in Manufacturing Industries (Cambridge University Press; Cambridge; 1971), and A. Silberston, Economies of Scale in Theory and Practice, op. cit.

701. Once again, a warning on the interpretation of empirical evidence in economics appears in order. The empirical 'proof' that, in general, returns to scale are constant or increasing is conditional on the data used in these various empirical tests. It is possible that firms have not yet reached the scale of operations beyond which returns to scale would decrease. If this were the case, empirical tests would not indicate the existence of decreasing returns to scale. (Whether or not this is actually true is a moot point — one that will be settled only if firms become large enough that diminishing returns to scale set in.)

702. This subsection is based on H.E. Leland, "Why Profit Maximization May Be A Better Assumption Than You Think," technical report no. 80 (Institute for Mathematical Studies in the Social Sciences, Stanford University; Stanford, Calif.; December 1972).

703. In particular, see the corollary to Theorem I. Ibid., pp. 7-8.

704. Leland is not the only economist who has recognized that profit maximization may be consistent with other goals. Edith Penrose has also argued that profit maximization, though in the long run, rather than in the short run, may be consistent with growth maximization: "managers of firms wish to maximize long-run profits...[since] to increase total long-run profits of the enterprise...[is] equivalent to increasing the long-run rate of growth." Penrose, op. cit., pp. 29-30.

705. Leland, Why Profit Maximization May Be A Better Assumption Than You Think, op. cit., pp. 17-22.

706. In particular, the production function $f(K,L)$ is linear homogeneous and product markets are perfectly competitive. Ibid., p. 21.

707. In the more general formulation of his model, Leland expressed the objective functional as

$$\int_0^T F[K(t),L(t),t]\,dt \quad , \qquad (*)$$

without restrictions on how the future is discounted. Ibid., p. 4.

708. In the general formulation of the model,

$$\dot{K}(t) = I[\pi\{K(t),L(t),t\},K(t),t] \quad ,$$

where Leland uses P in place of π . Ibid., p. 5. As in the Arrow model, the separate argument t can be interpreted as a surrogate for technical progress.

709. Leland does show, however, that many of the basic results he establishes still hold when external finance is permitted. Ibid., appendix B.

710. Note that the extreme cases $U(\pi,S) = \pi$ and $U(\pi,s) = S$ correspond to pure profit maximization and to pure sales maximization, respectively.

711. For example, total profit, π , in (318) is a composite function of time. In contrast to (317) where time t is an argument of the profit function, thereby implying that the functional relationship between π and its arguments K and L may shift over time, time affects total profit, and hence net investment, only indirectly — through its impact on K and L — in (318). Thus, $\dot{K}$ may change over time, but the functional relationship between $\dot{K}$ and its arguments K and L will not change over time in (318).

712. Ibid., p. 5.

713. The inequality $e^{-rt} \frac{\partial U}{\partial S} \cdot w > 0$ follows from the assumption of nonsatiability, $\frac{\partial U}{\partial S} > 0$ (i.e. more sales are always preferred to less), and the fact that $e^{-rt} > 0$ and $w > 0$ (i.e. in general, labor is not a free good).

714. Since the left-hand side of (326) is positive when evaluated at $L^p(t)$, it may be concluded that a utility maximizing firm, where utility is a function of total profit and total sales, will, in general, produce more output than a pure profit maximizer. This result is in agreement with the results obtained from the Baumol sales maximization model, which was discussed in section G of this chapter. The more interesting question, however, is 'how different' the policies of these two types of firms are.

715. Leland establishes a more general result for arbitrary objective function $F[K(t),L(t),t]$ — see footnote 707 — and for a more general net investment function. Ibid., pp. 7-8. He also goes on to establish two simple tests for convergence of optimal current operating policies to profit maximization. Ibid., pp. 14-17.

716. In this case $\lambda^*_T(t)$ approaches a finite limit as the planning horizon becomes infinite, but as long as $\beta < 0$, $\lambda^*_T(t)$ is an increasing function of $T - t$, implying, as Leland suggests, that optimal current policy moves closer to profit maximization, even though profit maximization is not reached in the limit. Ibid., pp. 8-9.

717. A somewhat weaker necessary condition for $\lambda^*_T(t)$ to become unboundedly large would result if the model (318) were modified to permit π, S, and $\dot{K}$ to depend directly on t, as in (317). Then the functional relationships embodied in (318) could be made to shift outward over time and a condition analogous to (334) could hold as a result of these shifting relationships, rather than because of the (stronger) requirement that the shape of the time invariant functional relationship $\pi(K,L)$ be such that (334) holds for all K attainable through internally financed investment.

718. In the model (318) U may be thought of as top management's utility function, where the inclusion of the profit argument reflects the wishes of the financial executives and shareholders and the inclusion of the sales argument reflects the wishes of the production and marketing staffs.

719. In view of the issues raised in the literature dealing with social welfare functions and social choice rules, it is recognized that the notion of a 'collective utility' function is somewhat imprecise. For example, see Heal, op. cit., ch. 2. It is this writer's opinion that one way around these criticisms is to let this utility function be formed by top management, say the chairman of the board of directors (without any claim being made as to the 'optimality' or even the 'desirability' of this procedure).

720. This subsection is based on C.G. Krouse, "On the Theory of Optimal Investment, Dividends, and Growth in the Firm," American Economic Review (vol. 63; no. 3; June 1973), pp. 269-279.

721. See S. Katz, "A Discrete Version of Pontryagin's Maximum Principle," Journal of Electronics and Control (vol. 13; no. 2; August 1962), pp. 179-184, and H. Halkin, "A Maximum Principle of the Pontryagin Type for Systems Described by Nonlinear Difference Equations," SIAM Journal of Control (vol. 4; no. 1; February 1966), pp. 90-111. See also Takayama, op. cit., pp. 468-485; and Bensoussan, Hurst, and Näslund, op. cit., pp. 27-30.

722. Krouse distinguishes between 'external' equity financing, in which shares are sold to the public and in which transactions costs are incurred, and 'internal' equity financing, in which shares are sold through a preemptive rights offering to existing shareholders in a manner that does not involve transactions costs. Thus, internal equity financing is tantamount to paying 'negative dividends'. Krouse, On the Theory of Optimal Investment, Dividends, and Growth in the Firm, op. cit., p. 271. Krouse considers only equity financing, i.e. the firm issues no debt. The extension of Krouse's results by permitting debt financing has been accomplished by Senchack, op. cit.

723. See section G of this chapter.

724. See the discussion in section I of this chapter of Miller's and Modigliani's proposition concerning the irrelevance of the firm's dividend policy, and in particular, the remarks concerning the critically important assumption of perfect equity markets.

725. In words, equity funds raised internally are equivalent to negative dividends.

726. Expectations or information assymmetry between existing shareholders and potential shareholders is held to account when $\delta(t) \neq 1$. Krouse, On the Theory of Optimal Investment, Dividends, and Growth in the Firm, op. cit., p. 271.

727. This includes, of course, dividends for the initial period, which are paid at the end of that period.

728. The derivation of (336) from (335) follows. Separating the initial period from later periods, (335) becomes

$$V(0) = k(0)\cdot D(0) + k(0) \sum_{t=1}^{\infty} D_o(t)\cdot \hat{k}(t) , \qquad (*)$$

where $\hat{k}(t)$ discounts back to the beginning of period 1 (in contrast to $k(t)$, which discounts to the beginning of period 0). Since any new equity shares issued during period 0 are issued *ex dividends*, the entire dividend payout of the initial period is received by initial shareholders, so that $D_o(0) = D(0)$. However, in later periods the initial shareholders receive only a pro rata portion of the total dividends paid. In particular, they receive a portion $1 - \delta(0)\cdot E(0)/V(1)$ of the dividends paid to shareholders of record at the beginning of period 1 . For example, if the current market share price is \$80 , the issue price is \$100 , 100 shares are outstanding, and 10 new shares are issued, then $\delta(0) = 4/5$, $E(0) = \$1000$, and $V(1) = 8800$, so that $1 - \delta(0)\cdot E(0)/V(1) = 10/11$, which is the proportion of the firm's outstanding shares owned by initial shareholders.

Thus, (*) may be rewritten as

$$V(0) = k(0)\{D(0) + [1 - \delta(0)\cdot E(0)/V(1)] \sum_{t=1}^{\infty} D_1(t)\cdot\hat{k}(t)\} \quad , \qquad (**)$$

where the sum represents the present value of dividends paid to shareholders of record at time 1 (i.e. at the beginning of period 1), and this sum multiplied by the term in brackets gives the portion of this sum paid to initial shareholders. But, by definition,

$V(1) = \sum_{t=1}^{\infty} D_1(t)\cdot\hat{k}(t)$, so that (**) may be rewritten as

$$V(0) = k(0)[D(0) - \delta(0)\cdot E(0)] + \sum_{t=1}^{\infty} D_1(t)\cdot k(t) \quad . \qquad (***)$$

By proceeding in this same manner iteratively through periods 1 , 2 , etc., and by using the identity $D(t) \equiv X(t) - I(t)$ to substitute for $D(t)$, (***) is transformed into (336).

729. In particular, note the difference between $\phi[I(t),E(t)]$ in (337) and the alternative form, $\phi[I(t) + E(t)]$, which allows only for the size of the firm's capital budget. By distinguishing between the two sources of finance, the former can be made to reflect fully the transactions costs associated with external equity financing. It is assumed that $\phi[0,0] = 0$; that ϕ has a full set of continuous second partial derivatives; and that, beyond some point, the firm experiences diminishing returns to investment.

730. Ibid., pp. 272-273. Of somewhat less importance, it is also assumed that the new level of earnings could be maintained throughout all future time periods provided net investment were nonnegative in each future time period. That is, any level of earnings, once attained, can be maintained simply by making gross investment sufficient to offset depreciation. Should depreciation exceed gross investment, disinvestment occurs and 'negative' retained earnings (i.e. total dividends paid exceed earnings) may result.

731. See footnote 721 for references.

732. The necessary conditions set out below are written somewhat differently from the way they are presented in Krouse. Ibid., pp. 274-275. This is done in order to bring out more clearly the

analogy that exists between the continuous and the discrete versions of the maximum principle (under the appropriate assumptions). This difference is, however, one of form only, and the solution to the model obtained here is identical to the solution furnished by Krouse.

733. The derivation of (344) follows. Using (342) to write expressions for $\Delta\lambda(t+1)$, $\Delta\lambda(t+2)$, ... ,$\Delta\lambda(t+n-1)$, and then summing, yields,

$$\begin{aligned}
\lambda(t+2) - \lambda(t+1) &= -k(t+1) \\
\lambda(t+3) - \lambda(t+2) &= -k(t+2) \\
\lambda(t+4) - \lambda(t+3) &= -k(t+3) \\
&\vdots \\
\lambda(t+n) - \lambda(t+n-1) &= -k(t+n-1) \\
\hline
\lambda(t+n) - \lambda(t+1) &= -\sum_{\tau=t+1}^{t+n-1} k(\tau) \qquad (*)
\end{aligned}$$

Taking the limit of each side of (*) as $n \to \infty$ and then applying (343) yields

$$-\lambda(t+1) = -\sum_{\tau=t+1}^{\infty} k(\tau) , \qquad (**)$$

which is equvalent to (344). The infinite series in (**) converges, provided the discount rate each period is strictly positive, because the infinite series is bounded from above by the convergent geometric series $\sum_{\tau=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{\tau}$, where r is the greatest lower bound for the set $\{r(t+1), r(t+2), \ldots , r(t+n), \ldots\}$.

734. That is, $\lambda(t+1)$ is measured in terms of the units in which the term $D_o(t)\cdot k(t)$ in (335) is denominated.

735. These conditions are also sufficient for a unique maximum to the optimal control problem (338) if H is strictly concave. Strict concavity can be attained by imposing the appropriate restrictions on $\phi(t)$ and $M(t)$.

736. Ibid., p. 275. The product $\delta(t)\cdot E(t)$ represents the incremental stock market value of the firm when an amount $E(t)$ is raised through an external equity issue (assuming $E(t) > 0$). The partial derivative $M(t)$ measures the instantaneous rate of change of this incremental stock market value with respect to $E(t)$.

737. Note that

$$\sum_{\tau=t+1}^{\infty} \frac{k(\tau)}{k(t)} = \frac{1}{1 + r(t+1)} + \frac{1}{[1 + r(t+1)]\ [1 + r(t+2)]} + \cdots$$

represents the value as of the beginning of period t of a perpetual annutiy beginning that period. Thus $Z(t)$ is equivalent in value to a perpetual annuity beginning in period t that pays an amount $A(t)$ per period (forever).

738. Note that since the structure of ϕ takes into account transactions costs on external equity issues, the necessary condition (348) also takes these costs into account. In particular, if $\delta(t) \equiv 1$ and if the transactions costs associated with any external issue are strictly positive, then $E(t) = 0$, i.e. external sources of equity will not be tapped. For external equity financing to be employed, conditions (347) and (348) require that, over some range of $E(t)$, the marginal external issue market value be less than the marginal contribution of an external issue to investable funds (i.e. $E(t)$ less transactions costs). Ibid., p. 277. To see this, let J represent externally acquired funds net of transactions costs. Then $\frac{\partial\phi}{\partial E} = \frac{\partial\phi}{\partial J} \cdot \frac{\partial J}{\partial E}$ and, by definition, $\frac{\partial\phi}{\partial J} \equiv \frac{\partial\phi}{\partial I}$. From (347) and (348), at optimality $M(t) = \frac{\partial J(t)}{\partial E(t)}$, i.e. it is necessary that the marginal external issue market value just equal the marginal contribution of an external issue to investable funds.

739. To see why this is true, note that, under the assumptions just stated, $Z(t)$ becomes

$$Z(t) = \sum_{t=1}^{\infty} \left(\frac{1}{1+\bar{r}}\right)^{t} = \frac{1}{\bar{r}} \quad .$$

Since, by assumption, $A(t) = M(t) = 1$ and I and E are indistinguishable, (347) and (348) become simply

$$\frac{1}{\bar{r}} \frac{\partial\phi(t)}{\partial B(t)} = 1 \quad \text{or} \quad \frac{\partial\phi(t)}{\partial B(t)} = \bar{r}$$

740. Ibid., pp. 277-278.

741. Ibid., p. 278.

742. This extension is carried out in Senchack, op. cit.

743. See Hirshleifer, Investment, Interest, and Capital, op. cit., p. 261, on this point. The argument is, in essence, that the distinction between debt and equity really becomes important only when there is uncertainty because then the difference in the relative riskiness of the two types of securities is what distinguishes them.

744. Krouse, On the Theory of Optimal Investment, Dividends, and Growth in the Firm, op. cit., p. 279.

745. For a reformulation of the managerial theories in terms of managerial utility maximization under uncertainty, see G.K. Yarrow, Managerial Utility Maximization under Uncertainty, op. cit. Also, O.E. Williamson has modified his basic managerial model of the firm to incorporate uncertainty. See O.E. Williamson, A Dynamic Stochastic Theory of Managerial Behavior, op. cit., pp. 11-31.

746. See subsection 4 of section G, subsection 3 of section H, and subsection 3 of section L.

747. The Vickers and Herendeen models discussed in section I also explore the relationship between the firm's operating decisions and its financial decisions. However, by making the average rate of interest and the owners' capitalization rate functions of the level of debt (Vickers) and the leverage ratio (Herendeen), they assume (rather than demonstrate) that a significant relationship exists. That is, the Vickers and Herendeen models abstract from the factors, i.e. market imperfections and the incompleteness of markets, that may cause the interest rate and the owners' capitalization rate to vary in the manner they suggest.

748. However, Yarrow has shown how to extend Baumol's sales maximization model and Marris's utility maximization model to permit uncertainty. Yarrow, Managerial Utility Maximization under Uncertainty, op. cit.

749. Leibenstein, op. cit.

III. THE BEHAVIOR OF THE FIRM OVER THE BUSINESS CYCLE

A. INTRODUCTION[1]

The traditional theory of the firm[2] postulates that the firm functions primarily for the economic benefit of its owners. Depending on the treatments given time and uncertainty, the objective function in traditional models has been formulated variously in terms of maximizing total profit;[3] the current stock market value of the firm;[4] the current share price;[5] the total market value of the firm;[6] the expected utility of total profit;[7] or expected shareholder utility.[8] An alternative view of the objectives of the firm has been put forward by the managerialists. Citing the apparent separation of ownership from control in large corporations,[9] they argue that corporate managers are able to pursue their own objectives to the detriment of the owners' objectives. In the managerial models of the firm, the objective function has been formulated variously in terms of maximizing total revenue;[10] the steady state rate of growth of the firm;[11] managerial utility expressed as a function of the steady state growth rate and the valuation ratio;[12] managerial utility expressed as a function of staff, managerial emoluments, and discretionary profit;[13] or the discounted value of managerial utility expressed as a function of total profit and total sales.[14]

An extensive literature has accumulated as a result of managerial criticisms of the traditional theory and attempts by proponents of the two views to defend their positions with empirical evidence.[15] With some exceptions, the debate has tended to focus attention on the differences between traditional firms and managerial firms, with the implication - either stated or implied - that firms always behave either one way (e.g. always maximizing profit) or the other (e.g. always maximizing growth).[16] One such exception is a paper by O.E. Williamson that suggests that the state of the firm's immediate environment may condition the firm's behavior and cause it to alternate between periods of profit maximization and periods of managerial utility maximization.[17] More recently, Solow has suggested that both types of firms would be expected to exhibit very similar responses to many forms of external stimuli,[18] and Leland has stated conditions under which the optimal current operating policies of a managerial firm would converge to profit maximization as the firm's long run equilibrium operating policy.[19] A somewhat different approach to reconciling the traditional and managerial theories is due to Wong, whose model suggests that growth maximization and profit maximization may correspond to different phases in the firm's life cycle.[20] In the opinion of this writer, the Williamson, Solow, Leland, and Wong studies suggest that, even at a theoretical level, the line of demarcation between traditional firms and managerial firms may be poorly marked.

In addition to the traditional and managerial theories of the firm, there are the behavioral theories,[21] which also suggest to this writer that the behavior of actual firms over time may at times appear more consistent with the traditional models and may at other times appear more consistent with the managerial models. According to the behavioral view, the firm does not seek to maximize any single objective. The firm sets various goals that are mutually satisfactory to the different social groupings that comprise the firm, and the actual behavior of the firm may change over time as a result of power shifts among the groupings that bring about a realignment of the firm's goals. For example, during certain periods the firm may appear to pursue traditional objectives as it sells off unprofitable divisions and reduces staffs and managerial emoluments in an effort to raise profitability. During other periods it may appear to pursue managerial objectives as it actively seeks out other firms to take over and increases advertising and promotional expenditures dramatically in order to increase sales. The behavioral theories explain such changes in behavior in terms of factors internal to the firm. An alternative explanation is that changes in the firm's operating environment are responsible.

One of the main differences between the traditional and managerial models of the firm on the one hand and the behavioral models on the other is that the former are more concerned with modeling phenomena external to the firm - e.g. modeling the firm's choice of price and output level subject to given market demand conditions and a given market structure or modeling the growth of the firm subject to a stock market-imposed valuation constraint - while the behavioral models are mainly concerned with modeling phenomena internal to the firm - e.g. modeling the actual decision-making processes of firms. The modeling requirements in the two cases differ. In the first case, if only external phenomena were of interest, a profit maximization model might serve as a reasonable 'if ... then' type of model to test, say, the impact of a change in the corporate profit tax rate on the behavior of the firm, even if actual firms do not try to maximize profit (or stock market value).[22] In the second case, if only internal phenomena were of interest, a more detailed, and possibly even firm-specific, model, such as a model of the sociological process at work within the firm, might be needed. In this thesis, chapters three and four are mainly concerned with external phenomena - the business cycle in chapter three and the financial markets in chapter four - while chapter five is mainly concerned with internal phenomena - specifically, internal resource allocation.

It is this writer's belief that actual firms do have multiple objectives and that the relative weights attached to these objectives vary over time, depending largely on the state of the firm's immediate operating environment. One strongly influential factor is the state of demand for the firm's products, and in particular, the state of the business cycle.[23]

During the upswing overall demand is increasing and individual markets are expanding.[24] Sales and profits increase and the outlook for investment is relatively favorable.[25] Firms grow internally by expanding sales of existing products and by introducing and developing markets for new products, and they grow externally by taking over other firms.[26]

During the downswing, however, overall demand is falling and individual markets are contracting. Profits fall and the outlook for investment is relatively unfavorable.[27] Plans for introducing new products or for capital spending may be postponed.[28] Rather than actively seeking takeover candidates, firms may try to sell off unprofitable or marginally profitable subsidiary companies.[29] During the upswing firms tend to be more growth-oriented[30] - indeed, they often feel they must grow in order to protect their market shares - while during the downswing they tend to be more profit-oriented.[31] In recent years the business literature has contained many stories of companies that grew very rapidly by acquisition during the economic upsurge of the 1960s and that were forced to sell off many of their acquisitions during the recession of the early 1970s.[32]

In the opinion of this writer, the impact of the business cycle on the behavior of the firm needs to be explored. There have been several papers that have either dealt explicitly with some facet of the firm's behavior over the business cycle[33] or explored a pattern of behavior that could be interpreted in terms of the business cycle.[34] In addition, other studies have dealt with the dynamics of demand and of costs,[35] while others have explored the dynamics of the firm's inventory policy.[36] A third set of studies has explored the impact of uncertainty on the behavior of the firm.[37] But no one has yet provided an integrated model - managerial or otherwise - of the behavior of the firm over the business cycle.

The purpose of this chapter and the two that follow is to develop such a model. In the first stage of the model's development, which is presented in this chapter, financial considerations are subsumed within the model. Attention is focused on real (as opposed to financial) factors that affect the behavior of the firm. In chapter four the role of finance is introduced into the model and in chapter five the role of factors internal to the firm, such as the internal allocation of physical capital and human capital and internal planning and control, are brought into the model.

The major purpose of this chapter is to explain how changes in the level of demand over the business cycle may, by alternatively loosening and tightening the constraints on managerial discretion, cause the observed pattern of the firm's behavior to (appear at least to) alternate between 'profit maximization' (i.e. the mode of behavior postulated in the traditional models) and 'growth maximization' (i.e. the mode of behavior conjectured in the managerial models). A model of the firm is developed in sections B and C, first for the certainty case and then for the uncertainty case. The behavior of the firm over the business cycle is explored in sections D and E, first for the simple case of two possible states of the firm's operating environment and then for the more general case.

B. THE MODEL UNDER CERTAINTY

1. The Objectives of the Firm

The debate between the traditionalists and the managerialists has focused on the different objectives of shareholders (or loosely, the 'owners') and managers and the extent to which the supposed separation of ownership from control in large corporations has permitted professional managers to pursue their own objectives to the detriment of the objectives of the firm's shareholders. In contrast, the behavioralists have argued that, in general, the firm does not pursue the goals of one social grouping to the mutual exclusion of the goals of the other social groupings that comprise the firm. In this subsection it is conjectured that the firm does not pursue exclusively the objectives of either shareholders or managers, but rather, that it seeks to maximize the collective utility of shareholders and managers, as interpreted by the corporate board of directors.[38] This collective utility is embodied in a multivariate utility function, the arguments of which reflect the sources of shareholder as well as managerial satisfaction.

In this thesis it is assumed that corporate managers derive utility, or satisfaction, from three main sources. The first source is total revenue, which is denoted by R(t).[39] Total revenue is a direct source of satisfaction to managers directly involved with sales and an indirect source of satisfaction to all other managers because, ceteris paribus, greater size means larger salaries, greater promotional opportunities, etc.[40] The second source is managerial emoluments, which are denoted by M(t).[41] Managerial emoluments, which are defined here in the O.E. Williamson sense to include managerial salaries and perquisites in excess of the opportunity cost of the managers receiving them, are a source of satisfaction to managers directly because of the utility derived from direct compensation and also indirectly because they contribute to the status and prestige of managers.[42] The third source is growth,[43] which will be allowed for in the model developed below. Growth is valued directly because it lends the impression that the firm is 'progressive' and indirectly because it creates more opportunities for the internal promotion of lower and middle level managers.[44]

In this thesis it is also assumed that shareholders derive utility from two main sources. The first source is the dividends they receive, which add to their immediate income.[45] Hereafter total dividends paid are denoted by D(t). The second source is the earning power of the firm's capital assets, which affects the dividends shareholders can expect to receive in the future (if they continue to hold their shares) and the price per share they can expect to receive when they sell their shares.[46]

While dividends are not generally regarded as a source of direct satisfaction to managers, nor are they necessarily a source of disutility.[47] Managers are held to value security.[48] In a publicly held company that security depends on the attitude of shareholders, who could vote out management at the annual meeting or sell their shares to a takeover raider if they are dissatisfied with the policies of current management.[49] Higher dividends, to the extent that they increase shareholder satisfaction directly or else lead to an increase in the share price[50] and thereby improve shareholder satisfaction, can indirectly benefit managers by making their position relatively more secure.[51]

Having described the sources of shareholder and managerial satisfaction, the objective function of the model of the firm developed in the next subsection may now be formulated. In this thesis it is assumed that the firm has a finite planning horizon T periods into the future, where time is measured continuously in units of arbitrary length (e.g. periods $\equiv$ years).

In the certainty model without finance, which is developed in the next subsection, the firm is assumed to select operating policies that maximize the discounted utility over the planning period (t = o to t = T) plus the discounted utility of the firm's capital stock at the planning horizon,

$$\int_o^T U_1(R(t), D(t), M(t))e^{-rt}dt + U_2(K(T))e^{-rT} , \qquad (1)$$

where K(T) denotes the firm's capital stock at the planning horizon and where r is the collective time rate of discount, $0 < r < 1$, which is assumed constant (at least over the planning period). In the certainty version of the model, if capital markets are assumed to be perfect, then r can be taken to be the exogenously determined market rate of interest. In the uncertainty versions, in which there may be bonds with different rates of interest, r can be taken to be the board of directors' (collective) subjective time rate of discount (determined as of time t = 0). In subsection 2, which immediately follows this subsection, the model of the firm is formulated under the assumption that the firm seeks to maximize (1) subject to certain constraints specified in that subsection. The purpose of the remainder of this subsection is to explain the interpretation of (1) and to indicate how (1) is to be modified when uncertainty is permitted.

The utility functions U_1 and U_2 in (1) are interpreted as collective utility functions.[52] The meaning of such functions deserves comment. As proved by Arrow,[53] there does not, in general, exist a social choice rule - a process by which individual preferences can be aggregated into a social preference scaling - that exhibits transitivity[54] and the four other properties generally considered desirable.[55] However, the smaller is the number of individuals whose preferences are being aggregated and the more closely in agreement are these individuals' preferences regarding corporate priorities, the more likely it is that an acceptable social choice rule could be found.[56]

It should be emphasized that the models developed in this thesis are planning models. Accordingly, the firm is assumed to set its operating policies at time $t = o$ for the time period spanning $t = o$ to $t = T$.[57] Since all decisions are made at time $t = o$, it is reasonable to assume that the functional forms U_1 and U_2 do not vary with t within each planning problem. The functional forms are set at time $t = o$ when the planning operation is carried out. The functional forms U_1 and U_2 may change in a real time sense,[58] however, as the firm repeats the planning cycle, sets a new time $t = o$, and reformulates the planning problem.

Following Sandmo,[59] it is assumed that within the firm there is a relatively small group of key decision makers whose preferences are sufficiently similar as to justify aggregating their preferences into a collective utility function. Specifically, it may be argued that the corporate board of directors, which includes members of top management and which is elected by the shareholders, sets corporate policy.[60] Continuing this line of argument, it is the board of directors who, in setting corporate policy, interpret the wishes of shareholders and of managers at all levels and who, in setting corporate dividend and investment policies, at least implicitly make trade offs of the kind embodied in U_1 and U_2 in (1). In the remainder of this thesis, U_1 and U_2 are interpreted as corporate utility functions that reflect the trade offs between shareholder objectives and managerial objectives, as determined by the corporate board of directors.

The interpretation of U_1 in (1) in terms of the sources of shareholder and managerial satisfaction is straightforward. The interpretation of U_2 is less clear, and so is discussed further here. As stated above, it is assumed that managers derive utility from the growth of their firm. Given any initial size of the capital stock, K(0), and the terminal size of the capital stock, K(T), the average compound rate of growth (per period), g, of the firm is given by

$$g = \log_T \left[\frac{K(T)}{K(0)} \right] - 1 , \qquad (2)$$

where $\log_T$ denotes the logarithm to the base T.[61] Since K(0) is fixed, by assumption, and since $\log_T$ is a monotonically increasing function of K(T)/K(0), any preference ordering over K(T) yields a preference ordering over the average growth rate g. Hence, managers' desires for growth can be (and hereafter are assumed to be) embodied in U_2. In addition, the terminal capital stock is also a source of satisfaction to shareholders. If the firm's shareholders are interested in having the firm remain a functioning enterprise beyond the planning horizon, then they will associate a positive utility level with a nonzero terminal capital stock. It is assumed, then, that U_2 embodies both managers' desires for growth and shareholders' desires for post-planning horizon dividends.

Before concluding this subsection at least some mention should be made of the objectives of the firm under uncertainty and as to how (1) will have to be modified when uncertainty is permitted. As modeled in section C of this chapter, under uncertainty R(t) and D(t) in (1) become random variables. Therefore, the level of utility (though not the function itself) U_1 also becomes random. It is generally accepted that individuals, and in particular, shareholders and managers, are risk averse.[62] If it is assumed that U_1 is consistent with the von Neumann-Morgenstern 'postulates of rational choice',[63] then the appropriate modification of (1) is the following:

$$\int_0^T E\{U_1(\tilde{R}(t), \tilde{D}(t), M(t))\}e^{-rt}dt + U_2(K(T))e^{-rT}, \qquad (3)$$

where E denotes mathematical expectation (taken over possible states of the firm's environment) and the tilde over R and D denotes a random variable. Provided U_1 is concave in R and D, the risk aversion of shareholders and managers will be taken into account.[64] In section C of this chapter the model of the firm will be formulated under the assumption that the firm seeks to maximize (3) subject to certain constraints specified in that section.

In concluding this subsection, it should be noted that the treatment of the firm's objectives just described shares something in common with the traditional, managerial, and behavioral views of the firm. Both (1) and (3) reflect shareholder as well as managerial sources of satisfaction. But by incorporating both types of sources, (1) and (3) differ from the traditional and managerial models that include either one or the other but not both. The advantage of (1) and (3) is that trade offs between shareholder and managerial sources of satisfaction can be explored. In addition, the inclusion of objectives of both shareholders and managers is suggestive of the behavioral approach. Where (1) and (3) differ from the behavioral approach is in first, the assumption of stable (over the planning period) functional forms U_1 and U_2,[65] and second, the assumption of maximizing behavior on the part of the firm.

2. The Model

The preceding subsection described the objective functional (1) of the certainty model to be formulated in this subsection. To complete the certainty model it is necessary to specify the constraint set, the decision variables, and exogenously determined variables.

It is assumed that the firm uses two inputs, capital and labor, the amounts of which are denoted by K(t) and L(t), respectively, to produce a single output, the amount of which is denoted by Q(t). The technological relationship between the amounts of the inputs applied to production and the maximum quantity of output obtainable is embodied in the firm's production function,

$$Q(t) = f(K(t), L(t)) , \tag{4}$$

which is assumed to have a full set of continuous second partial derivatives.[66] Both product and factor markets are assumed to be perfectly competitive, so that the firm takes the price of output, the price of labor, and the price of capital goods, all of which are strictly positive at each time t, as given.

These prices at time t are denoted by $p(t)$, $w(t)$, and $q(t)$, respectively, where all prices are expressed in terms of some arbitrarily selected numeraire good.

The variable $K(t)$ measures the firm's physical stock of capital at time t. The variable $I(t)$ denotes the firm's gross investment at time t, i.e. the rate at which its physical stock of capital is augmented before allowing for depreciation. It is assumed that physical capital wears out at a constant percentage rate δ, so that net investment, the net rate at which the firm's physical stock of capital is augmented, satisfies the identity

$$\dot{K}(t) = I(t) - \delta \cdot K(t) \ , \qquad (5)$$

where the dot denotes differentiation with respect to time.

At each time t the firm earns sales revenue of $p(t) \cdot Q(t)$ and incurs a cost of labor of $w(t) \cdot L(t)$. In addition, the firm pays out a sum $M(t)$ for managerial emoluments. These are payments to managers in excess of their opportunity cost.[67] In determining the firm's net income, allowance must also be made for taxes and for depreciation, which is a noncash outlay that is deductible for tax purposes. It is assumed that the firm pays an exogenously determined proportional tax rate τ, where $0 < \tau < 1$, and that the depreciation expense is figured for tax purposes on a replacement cost basis.[68] A typical firm's income statement constructed under the above assumptions is shown in table III-1. Net income, $\pi(t)$, is equal to[69]

$$\pi(t) = (1-\tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - q(t) \cdot [\delta \cdot K(t)]\} \ . \qquad (6)$$

It is assumed that the firm must generate sufficient net income to meet some exogenously determined minimum required net income level, π_o. That is, $\pi(t)$ in (6) must satisfy

$$\pi(t) \geq \pi_o , \tag{7}$$

where it is assumed that π_o is the same for each time t. The minimum net income level π_o is assumed to be determined by financial factors that in this chapter are subsumed within the model, such as the need to generate sufficient funds to pay some unspecified minimum level of dividends.[70]

It should be noted that π_o serves as a modeling device that is intended to take into account the existence of financial factors that can influence the firm's behavior. This frequently used device[71] has the limitation that it obscures the separate influences of individual financial factors, as well as their interaction. While such a modeling device is, in the opinion of this writer, adequate for building a model that is intended to demonstrate the systematic variation in the behavior of the firm in response to changes in its operating environment (i.e. the business cycle), its shortcomings become apparent when an attempt is made to model the interaction between the firm's operating decisions and these financial factors. For this reason, the next chapter modifies the model developed in this chapter to permit the firm's financial decisions to be examined more closely.

Through its operations the firm generates cash, which it uses to pay dividends and to add to its capital stock.

Table III-1 Typical Firm's Income Statement With Depreciation Figured on a Replacement Cost Basis

Sales revenue		$p(t)\cdot Q(t)$
Expenses:		
Labor	$w(t)\cdot L(t)$	
Emoluments	$M(t)$	
Depreciation	$q(t)\cdot[\delta\cdot K(t)]$	
Total Expenses		$w(t)\cdot L(t) + M(t) + q(t)\cdot[\delta\cdot K(t)]$
Pretax Income		$p(t)\cdot Q(t) - w(t)\cdot L(t) - M(t) - q(t)[\delta\cdot K(t)]$
Income tax		$\tau\{p(t)\cdot Q(t) - w(t)\cdot L(t) - M(t) - q(t)[\delta\cdot K(t)]\}$
Net Income		$\pi(t) = (1 - \tau)\{p(t)\cdot Q(t) - w(t)\cdot L(t) - M(t) - q(t)[\delta\cdot K(t)]\}$

Since depreciation is a noncash expense, total cash generated is equal to net income plus the amount of depreciation expense, $\pi(t) + q(t)\cdot[\delta\cdot K(t)]$. Since under complete certainty (including certainty with regard to the timing and size of all future transactions) the firm would have no use for cash other than what was used to pay dividends or to purchase capital goods,[72] the following identity between sources of cash and uses of cash must always be satisfied:

$$\pi(t) + q(t)\cdot [\delta\cdot K(t)] = D(t) + q(t)\cdot I(t) . \tag{8}$$

The derivation of this identity is illustrated in table III-2. The identity (8) can be reexpressed to yield the following expression for total dividends paid at time t:

$$D(t) = \pi(t) - q(t)\cdot[I(t) - \delta\cdot K(t)] . \tag{9}$$

Note that D(t) in (9) is dependent on the tax rate since $\pi(t)$ is net of tax. Note that when $M(t) = 0$ and $\tau = 0$ (9) simplifies to Jorgenson's expression for net cash flow (equation (269) in chapter two).

Collecting (1), (4), (5), (6), (7), and (9), the model of the firm is expressed as the following optimal control problem:

$$\underset{\{L(t),I(t),M(t)\}}{\text{maximize}} \int_0^T U_1(R(t),D(t),M(t))e^{-rt}dt + U_2(K(T))e^{-rT}$$

$$\begin{aligned} \text{subject to} \quad & Q(t) = f(K(t),L(t)) \ , \ 0 \le t \le T \\ & \dot{K}(t) = I(t) - \delta \cdot K(t) \ , \ 0 \le t \le T \ , \ K(0) \text{ given} \\ & \pi(t) = (1-\tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) \\ & \qquad - q(t) \cdot [\delta \cdot K(t)]\} \ge \pi_o \ , \ 0 \le t \le T \\ & D(t) = \pi(t) - q(t) \cdot [I(t) - \delta \cdot K(t)] \ , \ 0 \le t \le T \\ & L(t) \ , \ K(t), \ Q(t), \ M(t) \ge 0 \ , \ 0 \le t \le T \end{aligned} \tag{10}$$

where K(0) is the firm's initial capital stock, which is assumed given. In words, the optimal control problem (10) states that the objective of the firm is to select the time paths of labor, L(t), investment, I(t), and managerial emoluments, M(t), that maximize discounted collective utility over the time period extending to the firm's planning horizon T, subject to a technological constraint, a net investment constraint (identity), a minimum profitability constraint, and a dividend payout constraint (identity).

The important role that profits play in the model (10) should be noted. Net income is constrained to satisfy some exogenously determined minimum level π_o. Yet, beyond this minimum, additional profits contribute to collective utility by permitting the firm to pay additional dividends. In other words, the relationship between profits and the sources of managerial utility is not lexicographic - as was the case in the managerial models discussed in section G of chapter two - but rather, there is a smooth trade off between profits and the managerial objectives[73] that is implied in the form of the objective functional in (10).

Table III-2 Typical Firm's Sources and Uses of Cash under Certainty

Sources of cash:		
Sales revenue		$p(t)\cdot Q(t)$
Total Expenses and Taxes	$(1-\tau)\{w(t)\cdot L(t) + M(t) + q(t)\cdot[\delta\cdot K(t)]\} + \tau\cdot p(t)\cdot Q(t)$	
Adjustment for noncash outlay	$q(t)\cdot[\delta\cdot K(t)]$	
Cash outflow for expenses		$(1-\tau)\{w(t)\cdot L(t) + M(t)\} + \tau\{p(t)\cdot Q(t) - q(t)\cdot[\delta\cdot K(t)]\}$
Total cash generated		$(1-\tau)\{p(t)\cdot Q(t) - w(t)\cdot L(t) - M(t)\} + \tau\cdot q(t)\cdot[\delta\cdot K(t)] = \pi(t) + q(t)\cdot[\delta\cdot K(t)]$
Uses of cash:		
To pay dividends	$D(t)$	
To purchase capital goods	$q(t)\cdot I(t)$	
Total cash applied		$D(t) + q(t)\cdot I(t) \equiv \pi(t) + q(t)\cdot[\delta\cdot K(t)]$
Increase in stock of cash		0

The next subsection explores the implications of the model (10) for the behavior of the firm under certainty.

3. The Firm's Optimal Operating Policies under Certainty

The implications of the model of the firm (10) formulated in the previous subsection can be explored with the aid of Pontryagin's maximum principle. First, use the first and fourth constraints in (1) to reformulate the model as the following optimal control problem:

$$\begin{aligned}
\underset{\{L(t),I(t),M(t)\}}{\text{maximize}} \quad & \int_0^T U_1[p(t)\cdot f(K(t),L(t));\ (1-\tau)\{p(t)\cdot f(K(t),L(t)) \\
& - w(t)\cdot L(t) - M(t)\} + \tau\cdot q(t)\cdot[\delta\cdot K(t)] - q(t)\cdot I(t); \\
& M(t)]\ e^{-rt}dt + U_2(K(T))e^{-rT}
\end{aligned} \tag{11}$$

$$\begin{aligned}
\text{subject to} \quad & \dot{K}(t) = I(t) - \delta\cdot K(t)\ ,\ 0 \leq t \leq T\ ,\ K(0) \text{ given} \\
& (1-\tau)\{p(t)\cdot f(K(t),\ L(t)) - w(t)\cdot L(t) - M(t) \\
& \qquad -q(t)\cdot[\delta\cdot K(t)]\} \geq \pi_0\ ,\ 0 \leq t \leq T \\
& L(t),\ K(t),\ M(t) \geq 0\ ,\ 0 \leq t \leq T
\end{aligned}$$

The Hamiltonian for the optimal control problem (11) is

$$H[K,\ L,\ I,\ M,\ \lambda,\ t] = U_1[\quad]\ e^{-rt} + \lambda(t)[I(t) - \delta\cdot K(t)]\ , \tag{12}$$

where $U_1[\ \ ]$ stands for the function U_1 in the objective functional of problem (11) and where the argument t of the functions K, L, I, M, and λ has been omitted for notational convenience. In (12) λ denotes the costate variable. At each time t, $0 \leq t \leq T$, the value of the costate variable $\lambda(t)$ measures the shadow price of capital in terms of the discounted value of collective utility.

The Hamiltonian (12) for each time t represents the sum of the discounted collective utility of revenue earned and dividends and managerial emoluments received at time t, $U_1[\quad]e^{-rt}$, and the discounted collective utility of an increase in the capital stock at time t, $\lambda(t)[I(t) - \delta \cdot K(t)]$. As in section L of chapter two, the Hamiltonian embodies the intertemporal trade off that confronts the firm at each time t. The firm can increase revenue,[74] dividends, and managerial emoluments and thereby reach a higher level of current utility, or it can accept lower revenue and pay lower dividends and managerial emoluments in order to be able to increase further the size of the capital stock, which will make possible higher revenue, dividend payments, and managerial emoluments in future time periods.

In the model of the firm (11) there are two constraints on the firm's capital stock at each time t that require special treatment. The nonnegativity constraint on capital at each time t and the constraint on profits at each time t constrain the values that may be assumed by the state variable K(t) in the optimal control problem formulation of the model. However, in order for the firm to remain viable, its capital stock must be strictly positive at each time t. Therefore, the nonnegativity constraint on K(t) will not be considered here explicitly, although it is noted that if it were included,[75] it would be treated in the same manner mathematically as the minimum profit constraint. Allowing for the minimum profit constraint requires that a generalized version of the maximum principle - analogous to the extension of Lagrange multipliers to static optimization problems containing inequality constraints -

be employed.[76] For this purpose define the "multiplier" $\mu_1(t)$ and form the Lagrangian (or generalized Hamiltonian)

$$L_\mu[K, L, I, M, \lambda, \mu_1, t] =$$
$$H[K, L, I, M, \lambda, t] + \mu_1(t)\cdot[(1-\tau)\{p(t)\cdot f(K(t), L(t)) - w(t)\cdot L(t) - M(t) - q(t)\cdot[\delta\cdot K(t)]\} - \pi_o] , \tag{13}$$

where the subscript μ of the Lagrangian is used to distinguish the Lagrangian from the time path of labor, $L(t)$.

In order that the time paths $L^*(t)$, $I^*(t)$, and $M^*(t)$ provide an optimal solution to problem (11), it is necessary that they satisfy the following conditions:[77]

$\{L^*(t), I^*(t), M^*(t)\}$ maximize $H(K, L, I, M, \lambda, t)$ subject to the minimum net income constraint and the nonnegativity constraints in (11), $0 \leq t \leq T$ (14)

$$\dot{K}(t) = I^*(t) - \delta\cdot K(t),\ 0 \leq t \leq T,\ K(0) \text{ given} \tag{15}$$

$$\dot{\lambda}^*(t) = -\frac{\partial L_\mu}{\partial K} , \quad \lambda^*(T) = \frac{\partial U_2(K(T))}{\partial K(T)} e^{-rT} \tag{16}$$

Since (15) merely repeats the net investment constraint, it only needs to be noted that its presence among the necessary conditions requires that the optimal investment time path, $I(t)^*$, satisfy this first order differential equation with boundary condition. The two remaining necessary conditions do require interpretation.

The necessary condition (14) requires that the time paths $L(t)$, $I(t)$, and $M(t)$ be selected so as to balance at each time t the immediate impact on collective utility and the impact on future collective utility levels of a marginal change in the value of each of these policy variables.[78] It is noted that in order for the firm to remain viable, the amount of labor used will be strictly positive at each time t, $0 \leq t \leq T$. Hence, the possibility that $L(t) = 0$ is not economically interesting, and the nonnegativity constraint on $L(t)$ is not treated explicitly in the discussion below. However, the possibility that $M(t) = 0$, which would result if the firm were to behave like a short run profit maximizer, is economically interesting, and hence, is considered explicitly in the discussion below.

To begin, define the Lagrange multiplier μ_2 and modify the Lagrangian (13) to take into account the nonnegativity constraint, $M(t) \geq 0$. This gives [79]

$$L_{\mu,t} = H(K, L, I, M, \lambda, t) + \mu_1[(1-\tau)\{p(t)\cdot f(K(t), L(t)) - w(t)\cdot L(t) - M(t) - q(t)\cdot[\delta\cdot K(t)]\} - \pi_o] + \mu_2[M(t)] \quad , \tag{17}$$

where a problem of the type (17) must be solved for each time t in order to satisfy necessary condition (14) and where it is understood that, since H is expressed in terms of a present value, μ_1 and μ_2 contain implicitly discount factors of the form e^{-rt} in order that $L_{\mu,t}$ be expressed in the same units as H.

The Kuhn-Tucker conditions necessary for an optimal solution to (14) are the following:

$$\frac{\partial L_{\mu,t}}{\partial L} = \{ \frac{\partial U_1}{\partial R} \cdot p \cdot \frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p \cdot \frac{\partial f}{\partial L} - w)\} e^{-rt} + \mu_1 \{(1-\tau)(p \cdot \frac{\partial f}{\partial L} - w)\} = 0 \tag{18}$$

$$\frac{\partial L_{\mu,t}}{\partial I} = \{ \frac{\partial U_1}{\partial D}(-q(t))\} e^{-rt} + \lambda(t) = 0 \tag{19}$$

$$\frac{\partial L_{\mu,t}}{\partial M} = \{ \frac{\partial U_1}{\partial D}(1-\tau)(-1) + \frac{\partial U_1}{\partial M}\} e^{-rt} + \mu_1 (1-\tau)(-1) + \mu_2 = 0 \tag{20}$$

$$\left.\begin{aligned} &(1-\tau)\{p \cdot f(K, L) - wL - M - q \cdot \delta \cdot K\} \geq \pi_o \qquad \mu_1 \geq 0 \\ &\mu_1 \cdot [(1-\tau)\{p \cdot f(K, L) - wL - M - q \cdot \delta \cdot K\} - \pi_o] = 0 \end{aligned}\right\} \tag{21}$$

$$M(t) \geq 0 \qquad \mu_2 \geq 0 \qquad \mu_2 \cdot M = 0 \tag{22}$$

To interpret the necessary conditions (18) - (22), four cases are considered below, depending on whether μ_1 and μ_2 are zero or positive. To ease the exposition, it is noted that μ_1 and μ_2 do not appear in (19). Hence, (19) must be satisfied in each of the four cases. Condition (19) requires that, in order for the firm to be in equilibrium, the following condition must be satisfied at each point in time:

$$\frac{\lambda(t)}{(\partial U_1/\partial D)e^{-rt}} = q(t) \ . \tag{23}$$

The costate variable $\lambda(t)$ is interpreted as the shadow price of physical capital, expressed in terms of discounted collective utility. The ratio $\lambda(t)/(\partial U_1/\partial D)e^{-rt}$ can be interpreted as the firm's (internal) marginal rate of substitution between physical capital and dividends. The right-hand side of (23) is the price of capital goods, which is also the market-determined (i.e. external) rate at which capital goods and dividends can be traded off. According to (23) the firm should continue to invest in physical capital (i.e. purchase capital goods) up to the point at which its marginal rate of substitution between physical capital and dividends just equals the price of capital goods - at which point the internal and external trade offs between physical capital and dividends will be equated. It should be noted that (23) is analogous to equation (277) of chapter two that arose out of the Jorgenson model, with the difference being that values are measured in (23) in terms of discounted collective utility, while in (277) of chapter two they are measured in terms of the stock market value of the firm (i.e. discounted cash flow).

The four cases are the following:

<u>case (i): $\mu_1 = \mu_2 = 0$.</u>

In this case neither the minimum profit constraint nor the managerial emoluments nonnegativity constraint are necessarily binding at optimality. Conditions (18) and (20) simplify to the following:

$$\frac{\partial U_1}{\partial R} p \frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w) = 0 \tag{24}$$

$$-\frac{\partial U_1}{\partial D}(1-\tau) + \frac{\partial U_1}{\partial M} = 0 \ . \tag{25}$$

If it is assumed that the firm can never be satiated with respect to revenue, dividends, or managerial emoluments, then $\frac{\partial U_1}{\partial R} > 0$, $\frac{\partial U_1}{\partial D} > 0$, and $\frac{\partial U_1}{\partial M} > 0$. Since p and w are both positive and since $0 < \tau < 1$, condition (24) can be satisfied only if

$$0 < p \cdot \frac{\partial f}{\partial L} < w \; . \tag{26}$$

As was pointed out in section B of chapter two, a short run profit maximizer would continue to hire labor up to the point at which the marginal revenue product of the last unit employed just equals its wage. As in the case of Baumol's sales maximizer, the firm modeled in (11) will, according to (26), continue to hire labor beyond this point, so that if in equilibrium $\mu_1 = \mu_2 = 0$, then the wage rate will exceed the marginal revenue product of labor. This implies that the firm modeled in (11), like the Baumol sales maximizer of subsection 1 of section G in chapter two, will produce more output and earn greater sales revenue than a short run profit maximizer. Another way to see this same result is to convert (26), which characterizes equilibrium in terms of the market for labor, into the equivalent expression that characterizes equilibrium in terms of the product market,

$$p < \frac{w}{\partial f/\partial L} \; , \tag{27}$$

which is interpreted to mean that, when the firm is in equilibrium, marginal cost, $\frac{w}{\partial f/\partial L}$, exceeds price ($\equiv$ marginal revenue under perfect competition). Since in equilibrium the marginal returns to labor will be diminishing,[80] (27) implies that the firm modeled in (11) will produce more output than a short run profit maximizer.

Both (26) and (27) continue to hold in cases (ii) - (iv) discussed below. For this reason, the important results (26) and (27) are stated as the following lemma.

<u>Lemma III-1</u>

When the firm modeled in (11) is in equilibrium,

$$0 < p \cdot {}^{\partial f}/\partial L < w \text{ , or equivalently, } p < \frac{w}{\partial f/\partial L} \text{ ,}$$

and

$$Q^* > Q_p \text{ ,}$$

where Q^* is the equilibrium output level for the firm in (11) and Q_p is the equilibrium output level for the short run profit maximizer.

Condition (25) implies that if in equilibrium $\mu_1 = \mu_2 = 0$, then the firm's marginal rate of substitution between dividends and managerial emoluments must equal one minus the tax rate, or in symbols,

$$\frac{\partial U_1/\partial M}{\partial U_1/\partial D} = (1 - \tau) \text{ ,} \tag{28}$$

which is similar in form to O.E. Williamson's equilibrium condition for the optimal amounts of discretionary profit and managerial emoluments (see equation (128) of chapter two.) The main difference between (28) and Williamson's result is that (28) reflects a collective utility trade off, whereas Williamson's result reflects a purely managerial trade off. In (28) the ratio $\frac{\partial U_1/\partial M}{\partial U_1/\partial D}$ is interpreted as the rate at which the board of directors perceives that managerial emoluments and dividends can be traded off within the collective utility function, i.e. it is a subjective rate of trade off, whereas

$(1 - \tau)$ represents the objective rate at which one can be traded off for the other within the firm's income statement. According to (28) the two rates of trade off must be equal at each point in time for the firm to be in equilibrium.

case (ii): $\mu_1 = 0$, $\mu_2 > 0$.

In this case the managerial emoluments nonnegativity constraint is necessarily binding, whereas the minimum profit constraint is not necessarily binding, at optimality. Condition (18) simplifies to (24), which is interpreted in case (i). Since $\mu_2 > 0$, it follows from (22) that $M = 0$. That is, managerial emoluments are zero. It follows that

$$\frac{\partial H}{\partial M} = \{ \frac{\partial U_1}{\partial D}(1-\tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} < 0 . \qquad (29)$$

According to (29) the Hamiltonian is a constrained maximum with respect to M. Due to its relatively high preference for paying dividends, the firm would like to reduce managerial emoluments (i.e. cut salaries) further, but is constrained from doing so by the fact that, in a perfectly competitive market for executive talent, any further cut in salaries (i.e. $M < 0$) would induce managers to leave the firm. Also from (20),

$$\frac{\partial U_1/\partial M}{\partial U_1/\partial D} < \frac{\partial U_1/\partial M + \mu_2 e^{rt}}{\partial U_1/\partial D} = (1 - \tau) , \qquad (30)$$

since $\mu_2 > 0$. The interpretation of (30) is that the marginal rate of substitution between dividends and managerial emoluments is less than $(1 - \tau)$, the rate at which they can be traded off within the firm's income statement.[81] As a result of (29) and (30) the firm eschews paying managerial emoluments, and instead, pays higher dividends (after paying the increase in taxes). One possible explanation for this result is that the board of directors has responded to shareholders' demands for higher dividends and lower managerial salaries by raising dividends at the expense of emoluments. Another possible explanation is that the firm has been threatened with a takeover, and that top management has responded to this threat to its own security by increasing dividends and decreasing managerial salaries by the maximum tolerable amount (in the sense that the new salary levels will not cause a mass exodus from the ranks of management). In either case, the implicit ranking of objectives is such that dividends rank ahead of managerial emoluments at the margin.

<u>case (iii): $\mu_1 > 0$, $\mu_2 = 0$.</u>

In this case the minimum profit constraint is necessarily binding, whereas the managerial emoluments nonnegativity constraint is not necessarily binding, at optimality. Condition (18) can be reexpressed as

$$\frac{\partial U_1}{\partial R} p\cdot\frac{\partial f}{\partial L} e^{-rt} + (1 - \tau)(p\cdot\frac{\partial f}{\partial L} - w)[\mu_1 + \frac{\partial U_1}{\partial D} e^{-rt}] = 0 . \qquad (31)$$

Since all terms in (31) must be positive, with the possible exception of $(p\cdot\frac{\partial f}{\partial L} - w)$, it follows that, in order for (31) to be satisfied, condition (26), and by implication, condition (27), must hold. As in cases (i) and (ii), the firm produces more output and earns greater sales revenue than a short run profit maximizer would. The difference between cases (i) (or (ii)) and (iii) is that the minimum profit constraint is binding in the latter. From (18), $\mu_1 > 0$ and $p \cdot \frac{\partial f}{\partial L} - w < 0$ imply that

$$\frac{\partial H}{\partial L} = \{ \frac{\partial U_1}{\partial R} p \cdot \frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p \cdot \frac{\partial f}{\partial L} - w)\} e^{-rt} > 0 . \qquad (32)$$

According to (32) the Hamiltonian is a constrained maximum with respect to L. The firm could reach a higher level of discounted utility if it could increase its use of labor (and thereby increase its sales revenue). But, as in the Baumol sales maximization model,[82] this would necessitate lower profits (since $p \cdot \frac{\partial f}{\partial L} - w < 0$ and since, under diminishing returns, $\frac{\partial f}{\partial L}$ is falling), which would violate the minimum profit constraint. The implication of (32) is that the profit constraint restricts the firm's choice of output level.[83] As demonstrated later in this section, whether or not the minimum profit constraint is binding has other important policy implications as well.

Turning next to condition (20), since $\mu_1 > 0$ and $0 < \tau < 1$, it follows from (20) that

$$\frac{\partial H}{\partial M} = \{-(1-\tau) \frac{\partial U_1}{\partial D} + \frac{\partial U_1}{\partial M}\} e^{-rt} > 0 . \qquad (33)$$

According to (33) the Hamiltonian is a constrained maximum with respect to M, in this case because attempts to pay higher managerial emoluments would cause the minimum profit constraint to be violated. Thus, in this case there exists an implicit ranking of objectives at the margin according to which profits (and thereby dividends) rank ahead of both managerial emoluments and total revenue.

case (iv): $\mu_1 > 0$, $\mu_2 > 0$.

In this case both the minimum profit constraint and the managerial emoluments nonnegativity constraint are necessarily binding at optimality. Condition (18) can be reexpressed as (31), which leads to (32). As in case (iii), $0 < p \cdot \frac{\partial f}{\partial L} < w$ and also the Hamiltonian is a constrained maximum with respect to L (the interpretation of which was given under case (iii)). Condition (20), however, is more difficult to interpret than in cases (i) - (iii) since both $\mu_1 > 0$ and $\mu_2 > 0$. Since $0 < \tau < 1$, it follows from (20) that

$$\frac{\partial H}{\partial M}\{\gtrless\}0 \text{ depending on } \frac{\mu_2}{\mu_1}\{\lessgtr\}(1 - \tau) , \tag{34}$$

where $\frac{\partial H}{\partial M}$ is given in (29) and (33) and where the sign of $\frac{\partial H}{\partial M}$ in (34) depends on the relative values of the ratio of implicit prices, μ_2/μ_1 , which is determined internally, and one minus the tax rate, $1 - \tau$, which is determined externally. Therefore, the sign of $\frac{\partial H}{\partial M}$ is determined by the relationship between the implicit internal trade off between net income and managerial emoluments, as measured at the margin by $\frac{\mu_2}{\mu_1}$, and the externally imposed trade off between these variables, as measured by $1 - \tau$. If $\frac{\mu_2}{\mu_1} < (1 - \tau)$ in (34), then $\frac{\partial H}{\partial M} > 0$ as in case (iii), and the firm could increase the discounted value of collective utility by increasing emoluments, though at the cost of violating the minimum profit constraint. If $\frac{\mu_2}{\mu_1} > (1 - \tau)$ in (34), then $\frac{\partial H}{\partial M} < 0$ as in case (ii), and the firm could increase the discounted value of collective utility if it were able to pay negative managerial emoluments (i.e. lower managerial salaries). Since both constraints are binding, the firm is unable to alter M, regardless of the sign of $\frac{\partial H}{\partial M}$.[84] Note that, as in cases (ii) and (iii), at optimality there exists an implicit ranking of objectives at the margin according to which dividends rank ahead of managerial emoluments, and further, as in case (iii), there exists an implicit ranking of objectives at the margin according to which dividends rank ahead of revenue.

In the above four cases reference was made to implicit rankings of objectives at the margin when the firm is in equilibrium. These results are summarized and proved as the following theorem:

<u>Theorem III-1</u>

If the individual firm modeled in (11) is in multiperiod equilibrium, then at each time t, $0 \leq t \leq T$, at which the inequality

$$\frac{-\left(p\cdot\frac{\partial f}{\partial L} - w\right)}{p\cdot\frac{\partial f}{\partial L}} < 1 \qquad (35)$$

holds the firm will prefer:

(a) a marginal increase in dividends (net income) to a marginal increase in managerial emoluments;

(b) a marginal increase in dividends (net income) to a marginal increase in total sales revenue;

(c) a marginal increase in managerial emoluments to a marginal increase in total sales revenue;

where relative preferences are determined ordinally according to the relative marginal utilities of total sales revenue, dividends, and managerial emoluments.

Remark 1

Since $0 < p\cdot\frac{\partial f}{\partial L} < w$ in each of the four cases, it follows that

$$\frac{-\,(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}} > 0\ . \tag{36}$$

The ratio on the left-hand side of (35) and (36) is interpreted as follows. The denominator is interpreted as the marginal revenue product of labor. The difference between the marginal revenue product and the wage (i.e. the marginal cost of labor) is interpreted as labor's marginal contribution to pretax income. The ratio on the left-hand side of (35) and (36) is interpreted as the negative of the ratio of labor's marginal contribution to pretax income to labor's marginal revenue product. Accordingly, inequality (35) can be interpreted as requiring that, when the firm is in equilibrium, the marginal decrement to pretax income resulting from the application of an additional unit of labor not exceed in absolute value the marginal revenue product of labor.

Remark 2

In the unconstrained case, the relative preferences are reflected in the pure marginal utilities, $\frac{\partial U_1}{\partial R}$, $\frac{\partial U_1}{\partial D}$, and $\frac{\partial U_1}{\partial M}$. However, when the minimum profit constraint is binding, additional net income that is used to pay dividends also causes the constraint to become nonbinding. Hence, when this constraint is binding a marginal increase in net income has a direct impact on the utility derived from dividends, $\frac{\partial U_1}{\partial D}$, and an indirect impact in terms of rendering the constraint nonbinding. Since this latter effect in the current period is measured by $\mu_1 e^{rt}$,[85] when the minimum profit constraint is binding the adjusted marginal utility $\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$ must be used. Similarly, when the nonnegativity constraint on managerial emoluments is binding, the appropriate measure of marginal utility is the adjusted one, $\frac{\partial U_1}{\partial M} + \mu_2 e^{rt}$.

Proof of Theorem III-1

The proof proceeds by reconsidering the four cases.

case (i): From (25), $\frac{\partial U_1}{\partial M} = (1-\tau)\frac{\partial U_1}{\partial D} < \frac{\partial U_1}{\partial D}$, so that (a) is satisfied. From (24), $\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}}(1-\tau)\frac{\partial U_1}{\partial D} < \frac{\partial U_1}{\partial D}$,

so that (b) is satisfied. Using (25) to substitute for

$\frac{\partial U_1}{\partial D}(1-\tau)$ in (24) yields $\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}}\frac{\partial U_1}{\partial M} < \frac{\partial U_1}{\partial M}$,

so that (c) is satisfied. Hence, $\frac{\partial U_1}{\partial R} < \frac{\partial U_1}{\partial M} < \frac{\partial U_1}{\partial D}$.[86]

case (ii): From (30), $\frac{\partial U_1}{\partial M} + \mu_2 e^{rt} < (1-\tau)\frac{\partial U_1}{\partial D} < \frac{\partial U_1}{\partial D}$, so that (a) is satisfied. Since (18) simplifies to (24), $\frac{\partial U_1}{\partial R} < \frac{\partial U_1}{\partial D}$ follows as in case (i), and (b) is satisfied. Using (30) to substitute for $\frac{\partial U_1}{\partial D}(1-\tau)$ in (24) yields

$\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}}\left[\frac{\partial U_1}{\partial M} + \mu_2 e^{rt}\right] < \frac{\partial U_1}{\partial M} + \mu_2 e^{rt}$, so that (c) is satisfied. Hence, $\frac{\partial U_1}{\partial R} < \frac{\partial U_1}{\partial M} + \mu_2 e^{rt} < \frac{\partial U_1}{\partial D}$.

case (iii): From (31), $\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}}(1-\tau)\left[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\right]$

$< \frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$, so that (b) is satisfied. From (20),

$\frac{\partial U_1}{\partial M} = (1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}] < \frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$, so that (a) is satisfied. Using (20) to substitute for $(1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}]$

in (31) yields $\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}} \cdot \frac{\partial U_1}{\partial M} < \frac{\partial U_1}{\partial M}$, so that

(c) is satisfied. Hence, $\frac{\partial U_1}{\partial R} < \frac{\partial U_1}{\partial M} < \frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$.

<u>case (iv):</u> As in case (iii), (31) holds so that (b) is satisfied. From (20), $\frac{\partial U_1}{\partial M} + \mu_2 e^{rt} = (1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}] < \frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$,

so that (a) is satisfied. Substituting for $(1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}]$

in (31) yields $\frac{\partial U_1}{\partial R} = \frac{-(p\cdot\frac{\partial f}{\partial L} - w)}{p\cdot\frac{\partial f}{\partial L}}[\frac{\partial U_1}{\partial M} + \mu_2 e^{rt}] < \frac{\partial U_1}{\partial M} + \mu_2 e^{rt}$.

Hence, $\frac{\partial U_1}{\partial R} < \frac{\partial U_1}{\partial M} + \mu_2 e^{rt} < \frac{\partial U_1}{\partial D} + \mu_1 e^{rt}$. Q.E.D.

In view of the fact that ∂U_1 is ordinal in nature, the marginal utilities $\frac{\partial U_1}{\partial R}$, $\frac{\partial U_1}{\partial M}$, and $\frac{\partial U_1}{\partial D}$ could not be observed directly. However, marginal rates of substitution, such as the marginal rate of substitution between dividends and managerial emoluments, $\frac{\partial U_1/\partial D}{\partial U_1/\partial M}$, which measure the rate at which measurable quantities are traded off, are, in theory at least, observable.[87] Hence, theorem III-1 could be restated equivalently in terms of observable rates of trade off:

(a) the marginal rate of substitution between dividends and total sales revenue exceeds the marginal rate of substitution between managerial emoluments and total sales revenue (i.e. $\frac{\partial U_1/\partial D + \mu_1 e^{rt}}{\partial U_1/\partial R} > \frac{\partial U_1/\partial M + \mu_2 e^{rt}}{\partial U_1/\partial R}$).

(b) the marginal rate of substitution between dividends and managerial emoluments exceeds the marginal rate of substitution between total sales revenue and managerial emoluments

(i.e. $\dfrac{\partial U_1/\partial D + \mu_1 e^{rt}}{\partial U_1/\partial M + \mu_2 e^{rt}} > \dfrac{\partial U_1/\partial R}{\partial U_1/\partial M + \mu_2 e^{rt}}$).

(c) the marginal rate of substitution between managerial emoluments and dividends exceeds the marginal rate of substitution between total sales revenue and dividends

(i.e. $\dfrac{\partial U_1/\partial M + \mu_2 e^{rt}}{\partial U_1/\partial D + \mu_1 e^{rt}} > \dfrac{\partial U_1/\partial R}{\partial U_1/\partial D + \mu_1 e^{rt}}$).

Similarly, the four corollaries and theorem III-2 stated below can also be restated equivalently in terms of marginal rates of substitution.

By following the proof of theorem III-1, and reversing the appropriate inequalities, the following corollaries are easily proved:

<u>Corollary III-1-1</u>

If the individual firm modeled in (11) is in multiperiod equilibrium, then at each time t, $0 \leq t \leq T$, at which the inequality

$$\frac{1}{1 - \tau} > \frac{-\left(p \cdot \frac{\partial f}{\partial L} - w\right)}{p \cdot \frac{\partial f}{\partial L}} > 1$$

holds the firm will exhibit marginal preferences (a) and (b) stated in theorem III-1, but marginal preference (c) will be reversed.

Corollary III-1-2

If the individual firm modeled in (11) is in multiperiod equilibrium, then at each time t, $0 \leq t \leq T$, at which the inequality

$$\frac{-\left(p\cdot\frac{\partial f}{\partial L} - w\right)(1 - \tau)}{p\cdot\frac{\partial f}{\partial L}} > 1 \tag{37}$$

holds the firm will exhibit marginal preference (a) stated in theorem III-1, but marginal preferences (b) and (c) will be reversed.

Corollary III-1-3

If the inequality in (35) is replaced by an equality, then at each time t, $0 \leq t \leq T$, at which the equality holds the firm will exhibit marginal preferences (a) and (b) stated in theorem III-1, but (c) will become an indifference relation (i.e. the firm will be indifferent between a marginal increase in managerial emoluments and a marginal increase in total sales revenue).

Corollary III-1-4

If the inequality in (37) is replaced by an equality, then at each time t, $0 \leq t \leq T$, at which the equality holds the firm will exhibit marginal preference (a) in theorem III-1, but (b) will become an indifference relation and (c) will be reversed.

Theorem III-1 and its four corollaries can be summarized in the following theorem, which establishes the firm's equilibrium hierarchy of objectives at each time t, $0 \leq t \leq T$.

Theorem III-2

When the individual firm modeled in (11) is in multiperiod equilibrium, the firm will always prefer a marginal increase in dividends (net income) to a marginal increase in managerial emoluments. Denote by $x \prec y$ the preference relation 'y is preferred to x' and by $x \sim y$ the indifference relation, and let R, D, and M denote a marginal increase in total sales revenue, dividends, and managerial emoluments, respectively. Then at each time t the firm in equilibrium will exhibit one of the following five hierarchies of objectives at the margin, depending on the relative value of labor's marginal decrement to pretax income $(-(p\cdot\frac{\partial f}{\partial L} - w))$:[88]

(i) $R \prec M \prec D$, if $-(p\cdot\frac{\partial f}{\partial L} - w) < p\cdot\frac{\partial f}{\partial L}$

(ii) $R \sim M \prec D$, if $-(p\cdot\frac{\partial f}{\partial L} - w) = p\cdot\frac{\partial f}{\partial L}$

(iii) $M \prec R \prec D$, if $p\cdot\frac{\partial f}{\partial L} < -(p\cdot\frac{\partial f}{\partial L} - w) < (\frac{1}{1-\tau})p\cdot\frac{\partial f}{\partial L}$

(iv) $M \prec R \sim D$, if $-(p\cdot\frac{\partial f}{\partial L} - w) = (\frac{1}{1-\tau})p\cdot\frac{\partial f}{\partial L}$

(v) $M \prec D \prec R$, if $(\frac{1}{1-\tau})p\cdot\frac{\partial f}{\partial L} < -(p\cdot\frac{\partial f}{\partial L} - w)$

Proof

Hierarchy (i) is a restatement of theorem III-1. Similarly, hierarchies (ii) - (v) are restatements of corollary III-1-3, corollary III-1-1, corollary III-1-4, and corollary III-1-2, respectively. This proves the second part of the theorem. The first part follows since $M \prec D$ in each of the five cases. Q.E.D.

Both parts of theorem III-2 reflect the important role played by corporate taxes in the model (11). In particular, managerial emoluments are a tax deductible expense, whereas dividends are paid out of net income. Thus, a dollar decrease in the amount of managerial emoluments, which increases pretax income by one dollar, permits an increase of only $(1 - \tau)$ dollars in dividends, with the remaining τ dollars being paid in taxes. Thus, the existence of corporate taxes modifies the rate at which dividends and managerial emoluments can be traded off within the firm's income statement. This modification takes the form of a bias in favor of (tax deductible) managerial emoluments. Therefore, when the firm is in equilibrium, a marginal increase in dividends will be preferred to a marginal increase in managerial emoluments, though the existence of a positive corporate tax rate will inhibit the firm from paying higher dividends in line with this relative preference.

To interpret the second part of theorem III-2, first note that $p\cdot\frac{\partial f}{\partial L}$ can be interpreted as labor's marginal revenue product; that $(p\cdot\frac{\partial f}{\partial L} - w)$ can be interpreted as labor's marginal decrement to pretax income;[89] and that $(1 - \tau)(p\cdot\frac{\partial f}{\partial L} - w)$ can be interpreted as labor's marginal decrement to net income.[90] In (i) - (iii) in theorem III-2 the direction of the preference relation between R and M is determined by the relationship between labor's marginal revenue product, $p\cdot\frac{\partial f}{\partial L}$, which determines how increasing the labor input will affect total sales revenue, and the absolute value of labor's marginal decrement to pretax income, $-(p\cdot\frac{\partial f}{\partial L} - w)$, which determines how increasing the labor input will force managerial emoluments to decrease in order to keep dividends constant. With a nonzero wage rate, the rate of trade off between total sales revenue and managerial emoluments within the firm's income statement may be greater than, less than, or equal to one, which gives rise to the three possible preference relations between R and M stated in theorem III-2. So, for example, if the behavior of the marginal revenue product of labor is such that $-(p\cdot\frac{\partial f}{\partial L} - w) < p\cdot\frac{\partial f}{\partial L}$ holds in equilibrium,[91] then an increase in the labor input would lead to an increase in total sales revenue that exceeds in absolute value the corresponding decrement to managerial emoluments. As a result, $\frac{\partial U_1}{\partial M} > \frac{\partial U_1}{\partial R}$ (in the unconstrained case at least), but the firm cannot decrease revenue and increase managerial emoluments because of the asymmetrical effect these changes would have on the firm's income statement.

In (iii) - (v) in theorem III-2 the direction of the preference relation between R and D is determined by the relationship between the absolute value of labor's marginal decrement to net income, $-(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)$, which determines how increasing the labor input will affect net income and dividends, and labor's marginal revenue product, $p\cdot\frac{\partial f}{\partial L}$.[92] With a positive wage rate and a positive tax rate, total sales revenue and dividends cannot be traded off one-for-one within the firm's income statement. The actual rate of trade off may be greater than, less than, or equal to one, which gives rise to the three possible preference relations between R and D stated in theorem III-2.

The foregoing results were all based on an analysis of the implications of the necessary condition (14) for the equilibrium time paths for labor, investment, and managerial emoluments. The remainder of this subsection explores the implications of necessary condition (16) for the optimal time path of the firm's capital stock.

Necessary condition (16) consists of a first order differential equation with boundary condition. The latter requires that when capital grows along its optimal time path the value of the costate variable at the planning horizon must equal the discounted (to the present) marginal collective utility of the terminal capital stock, K(T). As long as $\frac{\partial U_2(K(T))}{\partial K(T)} > 0$, the firm places a positive value on having nonzero terminal capital stock, and in equilibrium, $K(T) > 0$. Note, however, that if $U_2(K(T)) \equiv 0$, then in equilibrium $\lambda^*(T) \geq 0$ and $\lambda^*(T) \cdot K(T) = 0$.[93] Thus, it is possible that along its equilibrium time path the capital stock would approach zero as $t \to T$. That is, if terminal capital stock were not a direct source of utility, the firm might find it to its advantage to exhaust its capital stock by time T. By assuming that managers value growth and that stockholders value an ongoing enterprise, this possibility has been ruled out.

To characterize the solution to the first order differential equation in (16), first evaluate $-\frac{\partial L_\mu}{\partial K}$, where L_μ is given by (13). This yields

$$\dot{\lambda} = -\frac{\partial L_\mu}{\partial K} = -\Big[\Big(\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial f}{\partial K} + \frac{\partial U_1}{\partial D}\cdot[(1-\tau)\cdot p\cdot\frac{\partial f}{\partial K} + \tau\cdot q\cdot\delta]\Big)e^{-rt} - \lambda\delta + \mu_1\cdot(1-\tau)\{p\cdot\frac{\partial f}{\partial K} - q\cdot\delta\}\Big] . \tag{38}$$

To simplify the exposition, the case in which the minimum profit constraint is not binding (i.e. $\mu_1 = 0$) is considered first. Next, solve (19) for $\lambda(t)$ and differentiate with respect to t to obtain

$$\dot{\lambda} = \frac{\partial U_1}{\partial D}[-r \cdot q \cdot e^{-rt} + \dot{q} \cdot e^{-rt}] \quad . \tag{39}$$

Using (19) to substitute for λ and (39) to substitute for $\dot{\lambda}$ in (38) yields, after simplifying by multiplying through by e^{rt},

$$\frac{\partial U_1}{\partial D}[-r \cdot q + \dot{q}] = -[(\frac{\partial U_1}{\partial R} \cdot p \cdot \frac{\partial f}{\partial K} + \frac{\partial U_1}{\partial D} \cdot [(1-\tau)p \cdot \frac{\partial f}{\partial K} + \tau \cdot q \cdot \delta]) - \delta \cdot q \cdot \frac{\partial U_1}{\partial D}] \ .$$

Rearranging terms yields

$$q[r + (1-\tau)\delta] - \dot{q} = p \cdot \frac{\partial f}{\partial K}(1-\tau) + p \cdot \frac{\partial f}{\partial K}(\frac{\partial U_1/\partial R}{\partial U_1/\partial D}) \ . \tag{40}$$

Note that when $\tau = 0$ the expression on the left-hand side of (40) becomes Jorgenson's expression for the firm's cost of capital.[94] Note also that since depreciation is a tax deductible expense, $(1-\tau)\delta q$ represents the cost of depreciation figured net of tax. Hence, the left-hand side of (40), which below is denoted by i, represents the firm's cost of capital when there are taxes and when depreciation is figured on a replacement cost basis. Note further that $p \cdot \frac{\partial f}{\partial K}(1-\tau)$ on the right-hand side of (40) represents the marginal revenue product of capital, once again figured net of tax. Hence, the traditional (i.e. long run profit maximizing or value maximizing) firm would expand its capital stock up to the point at which,

$$i = p \cdot \frac{\partial f}{\partial K}(1 - \tau) \tag{41}$$

i.e. up to the point at which the marginal revenue product of capital (net of tax) just equals its marginal cost (net of tax). But since p, $\partial U_1/\partial R$, and $\partial U_1/\partial D$ are positive by assumption, and since i > 0 (which must hold in general equilibrium, unless capital is a free good) implies, by (40), that $\frac{\partial f}{\partial K} > 0$,[95] it follows from (40) that

$$0 < p \cdot \frac{\partial f}{\partial K}(1 - \tau) < i \; . \tag{42}$$

According to (42) the firm modeled in (11) will tend to employ 'too much' capital at each point in time in the sense that it will employ more capital than a short run profit maximizing firm would. Just how much more capital it would employ than a traditional firm at each time t depends on the magnitude of

$$p \cdot \frac{\partial f}{\partial K} \cdot \left(\frac{\partial U_1/\partial R}{\partial U_1/\partial D}\right) \tag{43}$$

in (40) at time t. Note that $\frac{\partial U_1}{\partial \pi} = \frac{\partial U_1}{\partial R}\frac{\partial R}{\partial \pi} + \frac{\partial U_1}{\partial D}\frac{\partial D}{\partial \pi} + \frac{\partial U_1}{\partial M}\frac{\partial M}{\partial \pi}$. If in equilibrium $\pi > \pi_o$, then $\frac{\partial R}{\partial \pi} = \frac{\partial M}{\partial \pi} = 0$, and since (9) gives $\frac{\partial D}{\partial \pi} = 1$, it follows that $\frac{\partial U_1}{\partial D} = \frac{\partial U_1}{\partial \pi}$ when $\mu_1 = 0$. The ratio $\frac{\partial U_1/\partial R}{\partial U_1/\partial D}$ in (43) can be interpreted as the collective marginal rate of substitution between total sales revenue and net income, and (43) can be interpreted as the net income equivalent (in terms of collective utility) of a marginal change in sales revenue. Hence, the right-hand side of (40) can be interpreted as the adjusted marginal revenue product of capital, where the adjustment reflects the direct contribution of a change in total sales revenue to collective utility. According to (40) the firm modeled in (11) will expand its capital stock at each time t to the point at which the adjusted marginal revenue product of capital just equals the marginal cost of capital.[96]

Together (26) and (42) imply that a firm of the type modeled in (11) would tend to employ too much (in comparison to a short run profit maximizer) of both inputs. Moreover, it is easily seen from (42) that if the Baumol-type sales maximizer were placed in a multiperiod setting it too would tend to employ too much capital in the sense described above. Note that the marginal revenue product of capital could equal the marginal cost of capital for a firm of either the Baumol sales maximization type or the type modeled in (11) only if the firm were to become satiated with respect to total sales revenue beyond some point, i.e. only if $\frac{\partial U_1}{\partial R} = 0$ beyond some level of sales.

Putting together (24) and (40) enables the firm's expansion path to be characterized,[97]

$$\frac{\partial L}{\partial K} = \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{i/\left[\, p(1-\tau)+p\left(\frac{\partial U_1/\partial R}{\partial U_1/\partial D}\right)\right]}{w/\left[p+\left(\frac{1}{1-\tau}\right)p\left(\frac{\partial U_1/\partial R}{\partial U_1/\partial D}\right)\right]} = \frac{1}{1-\tau}\cdot\frac{i}{w}\ , \tag{44}$$

which agrees with Jorgenson's characterization of the traditional firm's expansion path when $\tau = 0$.[98] Moreover, it follows from (41) and the necessary condition $p\cdot\frac{\partial f}{\partial L} = w$ for the traditional firm's optimal employment of labor that (44) also characterizes the traditional firm's expansion path when there are proportional profit taxes. Hence, the firm modeled in (11) has the same expansion path as a traditional firm when its minimum profit constraint is not binding. Even though the firm modeled in (11) exhibits a preference for revenue and managerial emoluments, each at the expense of profits, it still selects the most profitable combination of input levels with which to produce each level of output, provided the profit constraint is not binding.

For each time t at which the minimum profit constraint is binding, the firm's expansion path will deviate from that of the traditional firm, unless the condition stated in the next theorem is satisfied.

Theorem III-3

For each time t, $0 \leq t \leq T$, at which the minimum profit constraint is not binding when the firm modeled in (11) is in equilibrium, the firm's expansion path will coincide with the expansion path of the traditional firm. For each time t, $0 \leq t \leq T$, at which the profit constraint is binding, the expansion path of the firm modeled in (11) will coincide with the expansion path of the traditional firm only if the rate of discount equals the percentage rate of increase of capital goods prices.

Proof

The first statement in the theorem was proved in developing (44). To prove the second part of the theorem it is necessary to show that $\frac{\partial L}{\partial K} = \frac{1}{1-\tau}\frac{i}{w}$ when $r = \dot{q}/q$, where r is the rate of discount and $\dot{q}/q$ is the percentage rate of increase of capital goods prices.

It follows from (18) that

$$\frac{\partial f}{\partial L} = \frac{(1-\tau)w(\partial U_1/\partial D + \mu_1 e^{rt})}{(1-\tau)p(\partial U_1/\partial D + \mu_1 e^{rt}) + p(\partial U_1/\partial R)} . \qquad (45)$$

Combining (38) and (39) yields

$$\frac{\partial f}{\partial K} = \frac{i(\ \partial U_1/\partial D) + (1-\tau)q\delta\mu_1 e^{rt}}{(1-\tau)p(\partial U_1/\partial D + \mu_1 e^{rt}) + p(\partial U_1/\partial R)} \ . \tag{46}$$

Dividing (46) by (45) gives

$$\frac{\partial L}{\partial K} = \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{i(\partial U_1/\partial D) + (1-\tau)q\delta\mu_1 e^{rt}}{(1-\tau)w(\partial U_1/\partial D + \mu_1 e^{rt})}$$

$$= \frac{1}{1-\tau} \cdot \frac{i}{w} \cdot \left[\frac{(\partial U_1/\partial D) + (1-\tau)(q\delta/i)\ \mu_1 e^{rt}}{(\partial U_1/\partial D) + \mu_1 e^{rt}} \right] \ , \tag{47}$$

which equals $\frac{1}{1-\tau}\ \frac{i}{w}$ only if

$$(1 - \tau)(q\delta/i) = 1 \ . \tag{48}$$

But (48) is equivalent to $(1 - \tau)q\delta = i \equiv rq + (1 - \tau)q\delta - \dot{q}$, so that (48) is equivalent to $rq = \dot{q}$, or $r = \dot{q}/q$, which was to be proved. Q.E.D.

Theorem III-3 may seem counterintuitive since along the traditional firm's expansion path long run total cost is minimized for each level of output. If the profit constraint were binding, then surely the firm could reduce costs, and thereby increase profits, by altering its input mix until it attained the mix that lay on the traditional firm's expansion path. However, as theorem III-3 states, this is not the case unless $r = \dot{q}/q$. The apparent contradiction lies in the crucial distinction made in section C of chapter one between profit in the economic sense - associated with which is the notion of economic cost that underlies the expansion path - and profit in the accounting sense of net income - which underlies the minimum (accounting) profit constraint in (11). Theorem III-3 implies that when the constraint on accounting profit is not binding, it is optimal for the firm to select the same input mixes as a maximizer of economic profit. However, when the constraint on accounting profit is binding, the firm's interest shifts from the overall cost of capital to the cost of depreciation, $(1-\tau)q\delta$, the only component of the cost of capital that has an immediate impact on net income, and this will cause the firm's input mix to differ from that of a profit (in the economic sense) maximizer - unless $r = \dot{q}/q$, in which case $i = (1-\tau)q\delta$ and the economic and accounting costs of capital are the same.[99, 100]

This subsection has described the formulation of the basic model under certainty and has examined its properties.[101] The next subsection presents several comparative dynamics results, which indicate the sensitivity of the optimal trajectories of the firm's operating policy variables and its capital stock to changes in each of several parameters. In the following section the basic model is extended to allow for uncertainty.

4. Comparative Dynamics and Comparative Statics Results

In the development of the model under certainty in the previous subsection, the following eight parameters were treated as exogenously determined constants: the tax rate (τ); the minimum net income level (π_o); the discount rate (r); the rate of depreciation (δ); and the price of capital goods (q), the rate of change of capital goods prices ($\dot{q}$), the wage rate (w) and the price of output (p) at each time t. Note that, by implication, the firm's cost of capital, $i \equiv q[r + (1-\tau)\delta] - \dot{q}$, was also treated as exogenously determined. The purpose of this subsection is to determine how changes in these parameters would affect the firm's behavior at each point in time (i.e. comparative statics) and the time paths of labor, investment, managerial emoluments, and the capital stock (i.e. comparative dynamics). Rather than let i vary directly, it will prove more instructive to vary τ, r, δ, q, and $\dot{q}$ separately.

a. The Tax Rate

Two different effects of a change in the tax rate are considered here. The first is its effect on the equilibrium time paths of dividends, total revenue, and managerial emoluments. The second is its effect on the firm's optimal input mix at each point in time, and by implication, the time paths of labor and capital.

From (18),

$$-\left(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\right)(1 - \tau)\left(p\cdot\frac{\partial f}{\partial L} - w\right) = \frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial f}{\partial L} . \qquad (49)$$

If the minimum profit constraint is not binding, then $\mu_1 = 0$. If the tax rate increases, the firm finds itself out of equilibrium and (49) becomes

$$-\frac{\partial U_1}{\partial D}(1 - \tau)\left(p\cdot\frac{\partial f}{\partial L} - w\right) < \frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial f}{\partial L} . \qquad (50)$$

To restore equilibrium the firm will expand output and hence total revenue and its use of labor, since this will, under the assumptions of diminishing returns and diminishing marginal utility, cause $\frac{\partial U_1}{\partial R}$ and $\frac{\partial f}{\partial L}$ to fall. But by (26), $p\cdot\frac{\partial f}{\partial L} - w < 0$, so that expanding output causes $-\left(p\cdot\frac{\partial f}{\partial L} - w\right)$ to increase and net income to fall. If dividends are cut, then under the assumption of diminishing marginal utility, $\frac{\partial U_1}{\partial D}$ increases. This continues - with total revenue rising relative to total dividends - until equilibrium, i.e. equality in (50), is restored. Similarly, it can be shown using (20) that when the profit constraint is not binding, a rise in the tax rate will tend to cause managerial emoluments to rise relative to dividends. Moreover, these effects are reversed when the tax rate decreases. Putting (18) and (20) together, a change in the tax rate will not affect the relative mix of revenue and managerial emoluments.

When the profit constraint is binding, however, the effect of a tax change in either direction may be reversed. If the profit constraint is binding, then $\mu_1 > 0$ in (49). But if the tax rate is increased, then the minimum profit constraint is no longer satisfied, and μ_1, which measures the value of a relaxation in the minimum profit constraint, will tend to increase. If $\frac{d\mu_1}{d\tau}$ is sufficiently large, then

$$- \left(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt} \right)(1 - \tau)\left(p \cdot \frac{\partial f}{\partial L} - w\right) > \frac{\partial U_1}{\partial R} \cdot p \cdot \frac{\partial f}{\partial L} , \qquad (51)$$

so that total revenue will fall (rather than rise) relative to dividends. A similar result holds for managerial emoluments.

The practical explanation for the foregoing results is that, when the profit constraint is not binding, an increase in the tax rate makes it relatively more expensive for the firm to pay dividends, since $\frac{\tau}{1 - \tau}$ dollars is paid in taxes for each dollar paid as dividends. It thus becomes relatively cheaper in terms of the impact of alternative operating policies on collective utility to increase revenue and managerial emoluments at the expense of dividends (although clearly the redistribution of utility does involve a loss of collective welfare), particularly if managers are large shareholders. However, when the profit constraint is binding, an increase in the tax rate causes the minimum profit constraint to become violated. To increase net income the firm will have to decrease either revenue or managerial emoluments (or both). Thus, while it is cheaper in terms of pure tax considerations to increase revenue and managerial emoluments, it is not cheaper when one adds in the cost implicit in violating the minimum profit constraint.

A second effect that a change in the tax rate may have on the firm is to cause a change in the firm's equilibrium capital-labor input mix. First, if the minimum profit constraint is not binding, then from (44) and the definition of i,

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial K}\right) = \frac{d}{d\tau}\left[\left(\frac{1}{1 - \tau}\right) \frac{i}{w} \right] = \frac{1}{w} \frac{(qr - \dot{q})}{(1 - \tau)^2} \; . \tag{52}$$

Then from (52),

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial K}\right)\left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \text{ depending on } r\left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} \frac{\dot{q}}{q} \; . \tag{53}$$

The implications of (53) are illustrated in figure III-1. A change in the tax rate affects both i and $(1 - \tau)w$ in (44). When the rate of discount exceeds the percentage rate of increase in the price of capital goods, a rise in the tax rate causes $(1 - \tau)w$, the wage rate net of tax, to fall more than i, the cost of capital net of tax. The firm reacts by substituting the relatively cheaper for the relatively more expensive input, i.e. labor for capital, and the firm's expansion path shifts upward in figure III-1. The reverse occurs when $r < \dot{q}/q$.

When $r = \dot{q}/q$ the effects on the wage rate and the cost of capital, both net of tax, are offsetting, and the expansion path does not shift. Since (44) also characterizes the expansion path of the traditional firm, a change in the tax rate would have the same impact on the traditional firm's expansion path.

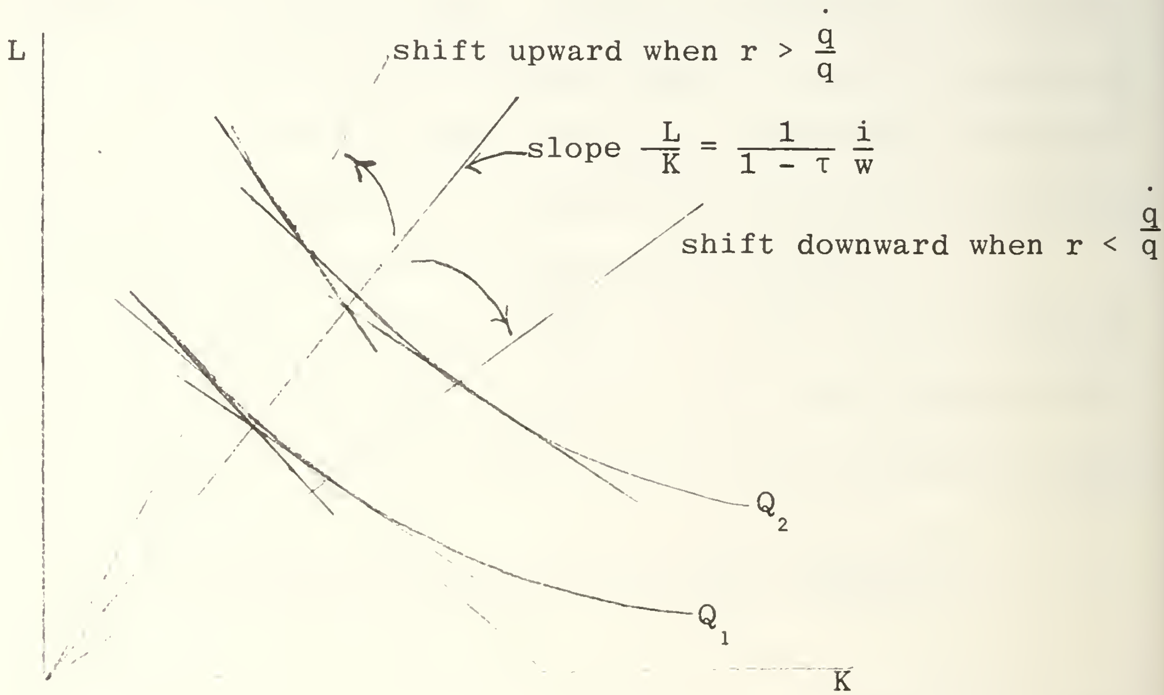


Figure III-1 Shift of the Firm's Expansion Path Due to a Change in the Tax Rate (τ)

When the minimum profit constraint is binding, $\mu_1 > 0$. Differentiating (47) with respect to τ gives

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial K}\right) = \frac{\frac{\partial U_1}{\partial D}(qr-\dot{q})\left[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt} - (1-\tau)e^{rt}\frac{d\mu_1}{d\tau}\right]}{w(1-\tau)^2\left(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\right)^2} . \qquad (54)$$

Since the denominator in (54) is always positive, the sign of $\frac{d}{d\tau}\left(\frac{\partial L}{\partial K}\right)$ depends on the sign of the numerator. If the term in brackets in the numerator of (54) is positive, then $\frac{d}{d\tau}(\frac{\partial L}{\partial K})$ has the same sign as $qr - \dot{q}$, which is identical to the unconstrained case summarized in (53). But note that when $\frac{d\mu_1}{d\tau}$ is sufficiently large, the effects are reversed. A rise in the tax rate would, in this case, cause the firm to use relatively less labor and relatively more capital (i.e. the shifts illustrated in figure III-1 are reversed, with the expansion path shifting downward when $r > \dot{q}/q$ and τ is increased). When $\frac{d\mu_1}{d\tau}$ is small, the expansion path shifts in the same direction as in the unconstrained case, though by a smaller amount. In this sense, the firm tends to use relatively too much capital due to the effects of the constraint on net income.

The effects of a change in the tax rate, then, depend on whether the profit constraint is binding.[102] When the constraint is not binding, a rise in the tax rate tends to cause revenue and managerial emoluments to rise relative to dividends and tends to cause labor to be substituted for capital when $r > \dot{q}/q$ or capital to be substituted for labor when $r < \dot{q}/q$. When the profit constraint is binding, these effects are less pronounced, and may in some cases be reversed.

b. The Minimum Net Income Level

By inspection of (11) it is clear that a rise in π_0 has no impact on the firm's operating policies unless the minimum profit constraint is binding or is caused to become binding. If the constraint is binding, then $\mu_1 > 0$, and intuitively one would expect the effect of an increase in π_0 to be the same as the effect of an increase in τ when the constraint is binding and μ_1 is large. From (18) and (20) it follows that the firm will tend to reduce total revenue and managerial emoluments in order to raise net income and restore equilibrium. In addition, from (47),

$$\frac{d}{d\pi_o}\left(\frac{\partial L}{\partial K}\right) = \left(\frac{1}{1-\tau}\right)\frac{i}{w} \frac{\frac{\partial U_1}{\partial D}\frac{d\mu_1}{d\pi_o} e^{rt}\left[(1-\tau)\frac{qs}{i} - 1 \right]}{\left(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\right)^2} . \tag{55}$$

From (55), the sign of $\frac{d}{d\pi_o}(\frac{\partial L}{\partial K})$ is the same as the sign of the term in brackets. But from the definition of i,

$$(1 - \tau)\frac{q\delta}{i}\{\gtrless\}1 \text{ depending on } r\{\lessgtr\}\frac{\dot{q}}{q} . \tag{56}$$

From (56), a rise in the minimum acceptable level of net income will cause the firm's expansion path to shift upward (downward) when the rate of discount is less than (greater than) the percentage rate of increase of the price of capital goods.

Comparing (56) with (53) it can be seen that an increase in π_o has the opposite effect on the input mix from the pure effect (profit constraint not binding) of an increase in the tax rate. This enables a clearer explanation to be provided for the effect of an increase in the tax rate when the profit constraint is binding. In that case, the effect of a tax increase is the composite of two effects. The first is a pure price-related effect, which causes revenue and managerial emoluments to be substituted for dividends and labor to be substituted for capital (when $r > \dot{q}/q$). The second is a constraint-related effect, which is due to the fact that an increase in τ when the constraint is binding is tantamount to an increase in π_o and which operates in the opposite direction to the pure price-related effect. Depending on the relative strengths of the two effects, total revenue and managerial emoluments could actually decrease relative to dividends and capital could actually be substituted for labor (when $r > \dot{q}/q$).

The effect of an increase in the minimum net income level, then, is to cause total revenue and managerial emoluments to decrease relative to dividends and capital to be substituted for labor when $r > \dot{q}/q$ and labor to be substituted for capital when $r < \dot{q}/q$.

c. The Discount Rate

A change in the discount rate has two effects that reinforce one another. An increase in the discount rate raises the firm's cost of capital, i, and also causes future utility levels to be discounted more heavily. Each of these effects induces the firm to pay higher dividends and managerial emoluments and to earn higher revenue in the present at the expense of investment (i.e. increments to the firm's capital stock that pay returns in future periods).

From (47)

$$\frac{d}{dr}\left(\frac{\partial L}{\partial K}\right) = \left(\frac{1}{1-\tau}\right)\frac{q}{w} \; \frac{\frac{\partial U_1}{\partial D}\left[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\{1-r(r-\dot{q}/q)\}\right]}{\left(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}\right)^2} \; . \tag{57}$$

If $\mu_1 = 0$, then clearly $\frac{d}{dr}(\frac{\partial L}{\partial K}) > 0$, and the expansion path shifts upward in figure III-1 as the firm substitutes labor for the now relatively more expensive capital. But $\frac{d}{dr}(\frac{\partial L}{\partial K}) > 0$ even when $\mu_1 > 0$. To see this, note that $\dot{q}/q \geq -1$ since prices can never fall by more than 100 percent. Since $0 < r < 1$ it follows that $1 - r(r - \dot{q}/q) > - 1$ and that the term in brackets in (57) is negative only if

$$r > \dot{q}/q \quad \text{and} \quad \partial U_1/\partial D < \mu_1 e^{rt} \ . \tag{58}$$

Since (58) holds only under exceptional circumstances - since it implies by (47) that the firm's expansion path is negatively sloped[103] - it follows that the term in brackets in (57) is normally positive, and hence that $\frac{d}{dr}(\frac{\partial L}{\partial K}) > 0$. Thus, except under exceptional circumstances,[104] the expansion path shifts upward due to an increase in r, whether or not the profit constraint is binding.

In addition, from (23),

$$\frac{d}{dr}\lambda(t) = \frac{\partial U_1}{\partial D} q(t)\cdot(-r)e^{-rt} = -r\cdot\lambda(t) < 0 \ . \tag{59}$$

Thus, an increase in the discount rate reduces the shadow price of capital, i.e. the value of an additional unit of capital in terms of collective utility. This causes the time path of the firm's capital stock to shift downward, as illustrated in figure III-2. A decrease in the discount rate would, of course, have the opposite effect. It may be observed that, ceteris paribus, the alternative time paths tend to diverge when the capital stock is increasing and tend to converge when it is decreasing.

A change in the discount rate, then, affects the time path of the firm's capital stock. If the discount rate increases, the firm tends to substitute labor for capital at each point in time and the time path of capital is lowered as a result.

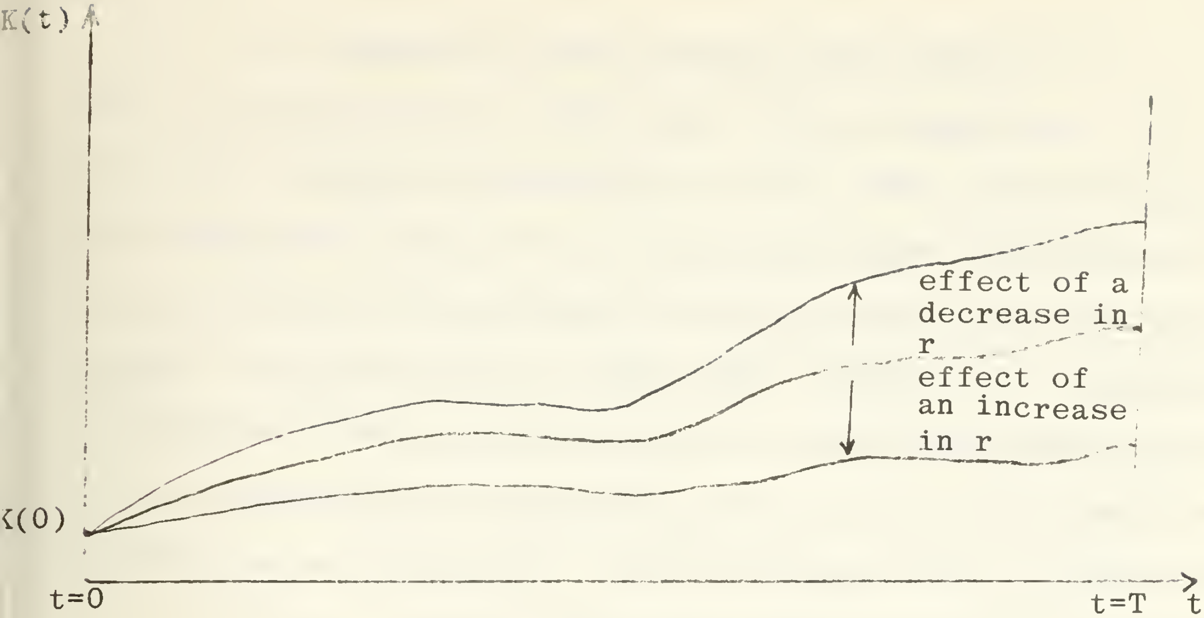


Figure III-2 Effect of a Change in the Discount Rate on the Time Path of the Capital Stock

d. The Rate of Depreciation, the Price of Capital Goods, and the Rate of Change of the Price of Capital Goods

An increase in the rate of depreciation (δ) or in the price of capital goods (q) causes the firm's cost of capital at any point in time to increase. An increase in the rate of increase of capital goods prices has the opposite effect. By differentiating (47) individually with respect to δ, q, and $\dot{q}$ it is easily shown that an increase in the rate of depreciation, an increase in the price of capital goods, and a decrease in the rate of increase of the price of capital goods will, by increasing the cost of capital, cause the firm to substitute labor for capital. Opposite changes in these three factors cause capital to be substituted for labor. Moreover, changes in these factors affect the firm modeled in (11) in the same manner as they affect the traditional firm.

e. The Wage Rate and the Price of Output

A change in the wage rate, i.e. the price of labor, has two effects. From (18), an increase in the wage rate will cause labor usage and output to fall, and this effect will be more pronounced when the minimum profit constraint is binding. In addition, from (47), an increase in the wage rate will cause the firm's expansion path to shift downward (see figure III-1) as the firm substitutes capital for the now relatively more expensive labor. Both effects are also observed in the case of the traditional firm.

A change in the price of output also has two effects. An increase in price has an immediate impact on total revenue, causing it to increase. From (18) and (20), the firm will also tend to increase dividends and managerial emoluments. When the profit constraint is not binding, there is no impact on the firm's expansion path. If, however, the profit constraint had been binding before the price increase, then, it can be shown by differentiating (47) that when $r \neq \dot{q}/q$ the firm will substitute labor for capital and shift back onto the traditional firm's expansion path. If the profit constraint is not binding in the new equilibrium for the firm, then its capital-labor ratio will satisfy the equation of the traditional firm's expansion path.

Thus, a change in the wage rate affects the firm modeled in (11) in the same manner as such a change would affect the traditional firm, but this is not true of the effect of a price change unless the profit constraint was not binding prior to the price increase.

5. Derivation of a Hicks-Slutsky-Type Equation

In the previous subsection it was argued that the effect of a change in the tax rate τ on the behavior of the firm depends on whether or not the net income constraint is binding. When the constraint is not binding there is a substitution effect. A rise in the tax rate leads to an increase in revenue and managerial emoluments relative to dividends. When the profit constraint is binding, this effect may be reversed. The purpose of this subsection is to explain the overall effect of a tax change as the resolution of two effects, one a 'substitution effect' and the other an 'income effect', which are analogous to the substitution and income effects of a price change in consumer theory.[105] This result is proved as the following theorem.

<u>Theorem III-4</u>

If the utility function U_1 and the production function f are both strictly concave, then the effect of a change in the tax rate τ on the equilibrium level of total sales revenue when the profit constraint is binding can be expressed as the resolution of two effects, one of which is positive (i.e. a 'substitution effect') and the other of which is negative (i.e. an 'income effect'). Moreover, when both total sales revenue and managerial emoluments are permitted to vary, the effect of a change in τ on total sales revenue is the resolution of three effects, the first two being the (pure) substitution and income effects already indicated, plus a cross substitution effect between total sales revenue and managerial emoluments that is negative.

Remark

The method of proof is to derive a Hicks-Slutsky-type equation for $\partial L/\partial\tau$, the instantaneous rate of change of the labor input with respect to the tax rate. This is accomplished for arbitrary time t with the capital stock K held fixed. The equation for $\partial L/\partial\tau$ will then imply the desired expression for $\partial R/\partial\tau$.

The Hicks-Slutsky-type equation is derived for the optimization problem:

$$\begin{aligned} &\underset{\{L,\ I,\ M\}}{\text{maximize}} \quad H(K,\ L,\ I,\ M,\ \lambda,\ t) \\ &\text{subject to} \quad \pi(t) = (1-\tau)\{p\cdot f(K,\ L) - wL - M \\ &\qquad\qquad\qquad - q\cdot[\ \delta\cdot K]= \pi_o \end{aligned} \tag{60}$$

which is necessary condition (14) for an optimal solution to (11) under the assumptions that the profit constraint is binding and the nonnegativity constraints are not. Problem (60) is the dynamic analogue of the static consumer budget allocation problem with π_o in the role of 'income'.[106] In the derivation of the Hicks-Slutsky-type equation it is assumed that the level of investment, I, in (60) is fixed. This is done for ease of exposition,[107] and does not affect the result qualitatively.

Proof of Theorem III-4

The necessary conditions for an optimal solution to (60) when I is held fixed are, in addition to the constraint in (60), (18) and (20) with μ_2 set equal to zero:

$$(1 - \tau)\{p\cdot f(K,\ L) - wL - M - q\cdot\delta\cdot K\} - \pi_o = 0 \tag{61}$$

$$\begin{aligned} &\{\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1 - \tau)(p\cdot\frac{\partial f}{\partial L} - w)\}e^{-rt} \\ &\qquad + \mu_1(1 - \tau)(p\cdot\frac{\partial f}{\partial L} - w) = 0 \end{aligned} \tag{18}$$

$$\left\{\frac{\partial U_1}{\partial D}(1-\tau)(-1)+\frac{\partial U_1}{\partial M}\right\}e^{-rt}-\mu_1(1-\tau)=0 \tag{20}$$

Define the bordered Hessian matrix H by

$$H \equiv \begin{bmatrix} 0 & \partial g/\partial L & \partial g/\partial M \\ \partial g/\partial L & \partial^2 H/\partial L^2 & \partial^2 H/\partial L\partial M \\ \partial g/\partial M & \partial^2 H/\partial M\partial L & \partial^2 H/\partial M^2 \end{bmatrix},$$

where $g(L, M) \equiv (1-\tau)\{p\cdot f(K, L) - wL - M - q\cdot\delta\cdot K\}$.

The elements of H are shown in table III-3. It is easily shown that

$$\det H = -(1-\tau)^2\left\{\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial^2 f}{\partial L^2}+\frac{\partial^2 U_1}{\partial R^2}\left(p\cdot\frac{f}{L}\right)^2\right.$$

$$+\frac{\partial U_1}{\partial D}(1-\tau)p\cdot\frac{\partial^2 f}{\partial L^2}+\mu_1 e^{rt}(1-\tau)p\cdot\frac{\partial^2 f}{\partial L^2} \tag{62}$$

$$\left.+\frac{\partial^2 U_1}{\partial M^2}\left(p\cdot\frac{\partial f}{\partial L}-w\right)^2\right\}e^{-rt} > 0 ,$$

since the strict concavity of U_1 and f imply that $\frac{\partial^2 U_1}{\partial R^2} < 0$, $\frac{\partial^2 U_1}{\partial M^2} < 0$, and $\frac{\partial^2 f}{\partial L^2} < 0$, and hence, that each term within the braces in (62) is strictly negative. But det H $\neq$ 0 implies by the implicit function theorem that equations (61), (18), and (20) can be used to obtain functions

$$\mu_1 = \mu_1(\tau, \pi_o) \quad L = L(\tau, \pi_o) \quad M = M(\tau, \pi_o)$$

that are continuously differentiable for all (τ, π_o) in some neighborhood of $(\hat{\tau}, \hat{\pi}_o)$ and that satisfy

$$\hat{\mu}_1 = \mu_1(\hat{\tau}, \hat{\pi}_o) \quad \hat{L} = L(\hat{\tau}, \hat{\pi}_o) \quad \hat{M} = M(\hat{\tau}, \hat{\pi}_o) ,$$

where the carat denotes the optimal values satisfying (61), (18), and (20). Treating μ_1, L, and M each as a function of τ and π_o and partially differentiating (61), (18), and (20) with respect to τ leads to the matrix equation

$$[H]\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \begin{pmatrix} p\cdot f(K,\ L) - wL - M - q\cdot\delta\cdot K \\ (\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1)(p\cdot\frac{\partial f}{\partial L} - w) \\ -(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1) \end{pmatrix} . \tag{63}$$

Note that $p\cdot f(K,\ L) - wL - M - q\cdot\delta\cdot K = \frac{\pi_o}{1-\tau}$. Denote $(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1)(p\cdot\frac{\partial f}{\partial L} - w)$ by α and $-(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1)$ by β and rewrite (63) as

$$[H]\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{1-\tau}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \tag{64}$$

where $\alpha < 0$ by lemma III-1 and $\beta < 0$ also. Since H is nonsingular in some neighborhood of $(\hat{\tau},\ \hat{\pi}_o)$, (64) can be solved for the partial derivatives of μ_1, L, and M with respect to τ:

$$\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{1-\tau} H^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha H^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta H^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} , \tag{65}$$

where H^{-1} denotes the inverse of the matrix H in table III-3. To facilitate the interpretation of (65), an alternative expression is obtained for $H^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. This is done by partially differentiati (61), (18), and (20) with respect to π_o. This gives the matrix equation

$$[H]\begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \iff \begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} = H^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad (66)$$

since H is nonsingular. Substituting (66) into (65) gives

$$\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{1-\tau}\begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} + \alpha H^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta H^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad .(67)$$

From (67),

$$\frac{\partial L}{\partial\tau} = \frac{\pi_o}{1-\tau}\frac{\partial L}{\partial\pi_o} + \alpha H^{-1}{}_{22} + \beta H^{-1}{}_{23} \, , \qquad (68)$$

where $H^{-1}{}_{ij}$ is the i-jth element of H^{-1} . From table III-3 ,

$$H^{-1}{}_{22} = \frac{-(1-\tau)^2}{\det H} \qquad\qquad H^{-1}{}_{23} = \frac{-(1-\tau)^2(p\cdot\frac{\partial f}{\partial L} - w)}{\det H} \, .$$

To prove the first part of the theorem, eliminate the effect of a change in managerial emoluments by setting $\partial M/\partial\tau = 0$. Then $\beta = 0$ by (63). Setting $\beta = 0$ and substituting for α and $H^{-1}{}_{22}$ in (68) gives

$$\frac{\partial L}{\partial\tau} = \frac{\pi_o}{1-\tau}\frac{\partial L}{\partial\pi_o} + (\frac{\partial U_1}{\partial D}e^{-rt} + \mu_1)(p\cdot\frac{\partial f}{\partial L} - w)[\frac{-(1-\tau)^2}{\det H}] \, . \qquad (69)$$

Table III-3   Elements of the Bordered Hessian Matrix H (under Certainty)

$$H \equiv \begin{bmatrix} 0 & (1-\tau)(p\cdot\frac{\partial f}{\partial L} - w) & -(1-\tau) \\ (1-\tau)(p\cdot\frac{\partial f}{\partial L} - w) & \begin{array}{l} [\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial^2 f}{\partial L^2} + \frac{\partial^2 U_1}{\partial R^2}(p\cdot\frac{\partial f}{\partial L})^2 \\ + \frac{\partial U_1}{\partial D}(1-\tau)\cdot p\cdot\frac{\partial^2 f}{\partial L^2} \\ + \frac{\partial^2 U_1}{\partial D^2}\{(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)\}^2]\, e^{-rt} \\ + \mu_1 (1-\tau) p\cdot\frac{\partial^2 f}{\partial L^2} \end{array} & -[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 (p\cdot\frac{\partial f}{\partial L} - w)]\, e^{-rt} \\ -(1-\tau) & -[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 (p\cdot\frac{\partial f}{\partial L} - w)]\, e^{-rt} & [\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 + \frac{\partial^2 U_1}{\partial M^2}]\, e^{-rt} \end{bmatrix}$$

It follows from (66) that $\partial L/\partial\pi_o = H^{-1}{}_{21} < 0$ [108] and hence that $\frac{\pi_o}{1-\tau}\frac{\partial L}{\partial \pi_o} < 0$. It follows from lemma III-1 and (62) that

$$\left(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1\right)\left(p\cdot\frac{\partial f}{\partial L} - w\right)\left[\frac{-(1-\tau)^2}{\det H}\right] > 0 .$$

To prove the second part of the theorem, substitute for α, β, $H^{-1}{}_{22}$ and $H^{-1}{}_{23}$ in (68) to obtain

$$\frac{\partial L}{\partial \tau} = \frac{\pi_o}{1-\tau}\frac{\partial L}{\partial \pi_o} + \left(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1\right)\left(p\cdot\frac{\partial f}{\partial L} - w\right)\left[\frac{-(1-\tau)^2}{\det H}\right] + \left(\frac{\partial U_1}{\partial D} + \mu_1\right)\left[\frac{(1-\tau)^2\left(p\frac{\partial f}{\partial L} - w\right)}{\det H}\right] \quad (70)$$

It follows from lemma III-1 and (62) that

$$\left(\frac{\partial U_1}{\partial D} + \mu_1\right)\left[\frac{(1-\tau)^2\left(p\cdot\frac{\partial f}{\partial L} - w\right)}{\det H}\right] < 0 .$$

Q.E.D.

The main results provided by theorem III-4 are equations (69) and (70). Equation (69) expresses the change in the amount of the labor input employed, and by implication the change in total sales revenue (in the same direction since $\partial f/\partial L > 0$ for a rational producer), as the sum of two effects. The first, $\frac{\pi_o}{1-\tau}\frac{\partial L}{\partial \pi_o}$, can be interpreted as an 'income effect' and measures the impact of the implicit increase in π_o (i.e. the 'tightening' of the constraint) that results from an increase in τ . From (61), the effect of such an increase is to cause L, and hence total sales revenue, to fall. This is analogous to the income effect of an increase in price on the quantity demanded of a normal good in consumer theory.[109]

The second term in (69) (and in (70) as well) can be interpreted as a substitution effect. In the previous subsection it was noted that if the profit constraint were not binding, then an increase in the tax rate would cause total sales revenue to increase relative to dividends as relatively cheaper sales revenue is 'substituted for' relatively more expensive (in terms of collective utility) dividends. But this is just what is implied by $(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1)(p \cdot \frac{\partial f}{\partial L} - w) \times [\frac{-(1-\tau)^2}{\det H}] > 0$ - an increase in τ tending to cause L, and hence total sales revenue, to increase. This is analogous to the substitution effect of a price change in consumer demand theory. Putting these two effects together, equation (69) can be expressed generically as:

$$\frac{\partial L}{\partial \tau} = \left(\frac{\partial L}{\partial \pi_o}\right)_{\tau \equiv \text{constant}} + \left(\frac{\partial L}{\partial \tau}\right)_{\pi_o \equiv \text{constant}} . \tag{71}$$

Equation (70) indicates that there is a third effect at work, in addition to the above two, when both total sales revenue and managerial emoluments are permitted to vary. The firm is able to substitute sales revenue for managerial emoluments and vice versa. This effect might be interpreted as a 'cross substitution effect' on total sales revenue to distinguish it from the 'price substitution' effect between dividends and total sales revenue. Since (61) implies that L and M must vary inversely for given π_o, one would expect this cross substitution effect to be negative. This is the case since, as shown in the proof of theorem III-4,

$(\frac{\partial U_1}{\partial D} + \mu_1)[\frac{(1-\tau)^2(p \cdot \frac{\partial f}{\partial L} - w)}{\det H}] < 0$. Putting this effect together with those embodied in (71), equation (70) can be expressed generically as:

$$\frac{\partial L}{\partial \tau} = \left(\frac{\partial L}{\partial \pi_o}\right)_{\substack{\tau \equiv \text{constant} \\ M \equiv \text{constant}}} + \left(\frac{\partial L}{\partial \tau}\right)_{\substack{\pi_o \equiv \text{constant} \\ M \equiv \text{constant}}} + \left(\frac{\partial L}{\partial M}\right)_{\substack{\tau \equiv \text{constant} \\ \pi_o \equiv \text{constant}}} . \qquad (72)$$

Results similar to those just obtained for the effect of a change in τ on total sales revenue can also be obtained for managerial emoluments. These are stated as the following corollary to theorem III-4.

Corollary III-4-1

If the utility function U_1 and the production function f are both strictly concave, then the effect of a change in the tax rate τ on the equilibrium level of managerial emoluments when the profit constraint is binding can be expressed as the resolution of two effects, one of which is positive (i.e. a 'substitution effect') and the other of which is negative (i.e. an 'income effect'). Moreover, when both managerial emoluments and total sales revenue are permitted to vary, the effect of a change in τ on managerial emoluments is the resolution of three effects, the first two being the (pure) substitution and income effects already indicated, plus a cross substitution effect between managerial emoluments and total sales revenue that is negative.

Proof

From (67),

$$\frac{\partial M}{\partial \tau} = \frac{\pi_o}{1-\tau}\frac{\partial M}{\partial \pi_o} + \beta H^{-1}{}_{33} + \alpha H^{-1}{}_{32} . \qquad (73)$$

Evaluate $H^{-1}{}_{33}$ and $H^{-1}{}_{32}$ and substitute the resulting expressions and the expressions for α and β into (73) to obtain equations analogous to (69) (when $\alpha = 0$) and (70). Q.E.D.

The interpretation of corollary III-4-1 is similar to the interpretation of theorem III-4, and equations analogous to (71) and (72) could be developed from the theorem.

This subsection has extended a basic result from consumer theory, the Hicks-Slutsky equation, to the theory of the firm. This result is important for at least four reasons. First, it demonstrates analytically the importance of whether or not the profit constraint is binding to the effect of a change in the tax rate on total sales revenue. An increase in the tax rate may increase or decrease sales revenue, depending on whether or not the profit constraint is binding, which has important implications for tax policy.[110] Second, the possibility that an increase in the tax rate can lead to an increase in sales revenue reverses one of the major predictions of Baumol's sales maximization model.[111] Third, the derivation of the Hicks-Slutsky-type equations (69) and (70) demonstrate results suggested and illustrated geometrically by Yarrow, and in addition, indicate the existence of cross substitution effects between alternative managerial objectives not considered in Yarrow's analysis.[112] Fourth, theorem III-4 suggests how to extend the Hicks-Slutsky equation from the analysis of single period consumer behavior to the analysis of the firm in a multiperiod setting. In particular, the derivation of the Hicks-Slutsky-type equation in the multiperiod setting should proceed from the maximization of the Hamiltonian.

Theorem III-4 was concerned with the effect on the behavior of the firm of a change in the tax rate τ. It should be noted that the same technique employed in the proof of theorem III-4 could be used to analyze the effects of changes in other parameters, such as the price of output, p, and the wage rate, w. For this reason the importance of theorem III-4 extends beyond the parametric analysis concerning the effect of changes in τ .

6. Section Summary

This section developed the model of the firm under certainty and derived the important properties and policy implications of the model. The firm was modeled in (11) as a maximizer of discounted collective utility over a finite planning horizon, where collective utility reflected managerial sources of satisfaction - sales revenue, managerial emoluments, and growth as well as a source of shareholder satisfaction - dividends - and where the maximization took place subject to a minimum net income constraint. It was shown that such a firm would tend to use more labor, and hence produce more output, than a short run profit maximizer (lemma III-1). In addition, it was shown that the equilibrium operating policies selected by the firm could be used to characterize an implicit ranking at the margin of the sales revenue, dividends, and managerial emoluments objectives (theorems III-1 and III-2 plus corollaries).

It was further shown that the extent of managerial discretion, as reflected in whether or not the profit constraint is binding, has important implications for the behavior of the firm. When the profit constraint is not binding, the expansion path of the firm modeled in (11) will coincide with the traditional firm's expansion path, but when the profit constraint is binding, the expansion paths will coincide only if the rate of discount is equal to the percentage rate of increase of the price of capital goods (theorem III-3). An increase in the tax rate may lead to either a decrease or an increase in output, sales revenue, and managerial emoluments, depending mainly on whether the profit constraint is or is not binding. It was shown that when the profit constraint is binding, the effect of a change in the tax rate on the behavior of the firm can be interpreted as the resolution of three effects, which are embodied in the Hicks-Slutsky-type equation developed for the dynamic optimization model (11) (theorem III-4)

The model (11) developed in this section assumes that the firm produces a single good under conditions of certainty and under conditions of perfect competition in product and factor markets. The extension of the model to permit uncertainty is carried out in the next section. The extension to permit imperfect competition in product or factor markets (or both) is straightforward and does not affect any of the qualitative results obtained in this section.[113] The extension of the model to the multiproduct firm is also straightforward.[114]

C. THE MODEL UNDER UNCERTAINTY

The purpose of this section is to extend the model of the collective utility maximizing firm (11) developed in the previous section to incorporate uncertainty. The uncertainty version of the model will be used in section E of this chapter to study the behavior of the firm over the business cycle.

1. The Model

In this section it is again assumed that the firm produces a single good that it sells in a perfectly competitive market. The firm is a quantity-setter of the Sandmo-Leland type,[115] setting the quantity of output to be produced prior to observing market price, which is assumed random and functionally dependent on the state of nature, θ_t, prevailing at time t. Assume that for every $t \varepsilon [0, T]$, θ_t has distribution function $F_t(\theta_t)$ for which a probability density function exists. Further assume that the probability distribution is time invariant so that the subscript t may be dropped and that the states of nature are defined in such a way that $dp/d\theta > 0$, i.e. a higher value for θ corresponds to a higher price p (and hence, to a more favorable state of the firm's operating environment).

Expressing price as a function of time, p(t), the family of random variables $\{p(t), 0 \leq t \leq T\}$ forms a continuous parameter stochastic process. Assume that this process has independent increments. This assumption, and possibly stronger assumptions, are needed to rule out pathological cases for which the model set out below would fail to have a feasible solution.

Since price is random, so is net income. Therefore, the minimum net income constraint (7) must be modified. Assume that the firm selects its operating policies each period such that the probability that net income will be greater than or equal to π_o is at least ε, where ε is a positive constant, $0 < \varepsilon < 1$. The constraint (7) becomes

$$P[\pi(t) = (1-\tau)\{p(\theta)\cdot f(K(t), L(t)) - w(t)\cdot L(t) - M(t) - q(t)\cdot[\delta\cdot K(t)]\} \geq \pi_o] \geq \varepsilon > 0 , \quad 0 \leq t \leq T , \tag{74}$$

where π_o is determined exogenously and where it is assumed that ε is also determined exogenously as of time t = 0. One possible interpretation of ε is that it reflects the board of directors' perception of the threat of an unfriendly takeover attempt, with the likelihood of such an attempt inversely related to the value of ε . Given management's aversion to unfriendly takeover bids, the attitudes of potential takeover raiders (i.e. how they would react to different net income levels), as perceived by top management, determine ε .

Rather than work directly with the probabilistic constraint it will prove more convenient to use the distribution of p, which is assumed known, to convert (74) into an equivalent deterministic constraint. To accomplish this, note that with the capital stock treated as fixed at time t, $\pi(t)$ in (74) is a function of L, M, and θ . Fix L and rewrite (74) as

$$P\{p(\theta)\cdot f(K,\ L) - M \geq \frac{1}{1-\tau}\pi_o + q\cdot[\ \delta\cdot K\ ] + wL\} \geq \varepsilon\ . \tag{75}$$

Since the left-hand side of the inequality inside the braces is a monotonically decreasing function of M, while the right-hand side is fixed (in terms of L and K) by assumption, the known distribution $p(\theta_t)$ implies a maximum value for managerial emoluments, $\hat{M}(L,\ K,\ \pi_o,\ \varepsilon)$, where the arguments L, K, π_o, and ε indicate the dependence of this maximum value on the amount of labor, the size of the capital stock, and also on π_o and ε .[116] It is easily seen from (75) that[117]

$$\partial\hat{M}/\partial\pi_o \leq 0 \qquad \text{and} \qquad \partial\hat{M}/\partial\varepsilon \leq 0\ . \tag{76}$$

It can be shown that for L sufficiently large,[118]

$$\partial\hat{M}/\partial L < 0\ . \tag{77}$$

The interpretation of (76) is that an increase in the ex post minimum net income level, π_o , or an increase in the fear of takeover leading to an increase in ε each leads to a decrease in the maximum permissible level of managerial emoluments. Similarly, the interpretation of (77) is that beyond some point further increases in labor usage lead to decreases in the maximum permissible level of managerial emoluments. The above procedure leads to a functional relationship that can be used to replace the probabilistic constraint (74) with the equivalent deterministic constraint

$$M(t) \leq \hat{M}(L(t),\ K(t),\ \pi_0,\ \varepsilon)\ ,\ 0 \leq t \leq T\ . \tag{78}$$

It also follows from the fact that price is a random variable that total revenue and total dividends paid are also random variables. Hence, the objective functional in (11) must be reformulated to take this into account. This was done in (3). The model of the firm (11) reformulated to take into account the existence of uncertainty is:

$$\begin{aligned} \underset{\{L(t),I(t),M(t)\}}{\text{maximize}} \quad & \int_0^T E\{U_1\,[p(\theta)\cdot f(K(t),L(t));(1-\tau)\{p(\theta)\cdot f(K(t),L(t)) \\ & \quad - w(t)\cdot L(t) - M(t)\} + \tau\cdot q(t)\cdot [\delta\cdot K(t)] \\ & \quad - q(t)\cdot I(t);\ M(t)]\ \}e^{-rt}dt + U_2(K(T))e^{-rT} \\ \text{subject to} \quad & \dot{K}(t) = I(t) - \delta\cdot K(t)\ ,\ 0\leq t\leq T\ ,\ K(0) \text{ given} \\ & M(t) \leq \hat{M}(L(t),K(t),\pi_0,\varepsilon)\ ,\ 0\leq t\leq T \\ & L(t),\ K(t),\ M(t) \geq 0\ ,\ 0\leq t\leq T \end{aligned} \tag{79}$$

It is assumed that the firm modeled in (79) exhibits risk aversion with respect to each of the arguments of the utility function U_1 , so that

$$\partial^2 U_1/\partial R^2 < 0 \qquad \partial^2 U_1/\partial D^2 < 0 \qquad \partial^2 U_1/\partial M^2 < 0\ .$$

As indicated in subsection 1 of section B above, the interpretation of U_1 in (79) is different from the interpretation of U_1 in (11), as it is assumed in (79) that U_1 satisfies the von Neumann-Morgenstern 'postulates of rational choice'.[119]

Note that since the probability distribution of $p(\theta)$ is known, E{ } at each time t is a function of the control variables L, I, and M and the state variable K at that time only. Hence, the deterministic maximum principle can be used to characterize the multiperiod equilibrium position of the expected collective utility maximizer modeled in (79).

2. The Firm's Optimal Operating Policies under Uncertainty

The Hamiltonian for the stochastic optimal control problem (79) is

$$H[K, L, I, M, \lambda, t] = E\{U_1[\quad]\}e^{-rt} + \lambda(t)[I(t) - \delta\cdot K(t)] , \quad (80)$$

where $E\{U_1[\quad]\}e^{-rt}$ denotes the integrand in (79). In (80) the costate variable $\lambda(t)$ measures the shadow price of capital in terms of the discounted value of expected collective utility. As in the deterministic case, define the Lagrangian L_μ by

$$L_\mu[K, L, I, M, \lambda, \mu_1, t] = H[K, L, I, M, \lambda, t] + \mu_1(t)\cdot[\hat{M}(L(t),K(t),\pi_o,\varepsilon) - M(t)], \quad (81)$$

where H is given by (80).

In order that the time paths L*(t), I*(t), and M*(t) provide an optimal solution to problem (79), it is necessary that they satisfy the following conditions:[120]

$$\{L^*(t), I^*(t), \text{ and } M^*(t)\} \text{ maximize } H(K,L,I,M,\lambda,t) \quad \text{subject to} \quad M(t) \leq \hat{M}(L(t),K(t),\pi_o,\varepsilon) , \; 0 \leq t \leq T \quad (82)$$

$$\dot{K}(t) = I^*(t) - \delta\cdot K(t) , \; 0 \leq t \leq T , \; K(0) \text{ given} \quad (83)$$

$$\dot{\lambda}^*(t) = -\frac{\partial L_\mu}{\partial K} , \; \lambda^*(T) = \frac{\partial U_2(K(T))}{\partial K(T)} e^{-rT} \quad (84)$$

where L_μ is given by (81). Since the second necessary condition (83) merely repeats the net investment constraint, it need not be considered further. The other two necessary conditions differ from (14) and (16) for the deterministic case and so are discussed below.

To begin, consider the necessary condition (82). The Kuhn-Tucker conditions necessary for an optimal solution to (82) are the following:

$$\frac{\partial L_\mu}{\partial L} = E\{\frac{\partial U_1}{\partial R}\cdot p(\theta)\cdot\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)\}e^{-rt} + \mu_1 \frac{\partial \hat{M}}{\partial L} = 0 \tag{85}$$

$$\frac{\partial L_\mu}{\partial I} = E\{\frac{\partial U_1}{\partial D}(-q(t))\}e^{-rt} + \lambda(t) = 0 \tag{86}$$

$$\frac{\partial L_\mu}{\partial M} = E\{\frac{\partial U_1}{\partial D}(1-\tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} - \mu_1 = 0 \tag{87}$$

$$M \leq \hat{M}(L,\ K,\ \pi_0,\ \varepsilon) \quad \mu_1 \geq 0 \quad \mu_1[\hat{M}(L,\ K,\ \pi_0,\ \varepsilon) - M] = 0 \tag{88}$$

To interpret necessary conditions (85) - (88) two cases need to be considered, depending on whether or not $\mu_1 = 0$. Since μ_1 does not appear in (86), that condition can be considered separately. Necessary condition (86) can be rewritten as:

$$\frac{\lambda(t)}{E(\partial U_1/\partial D)e^{-rt}} = q(t)\ , \tag{89}$$

which is analogous to (23) for the certainty case. The interpretation of (89) is similar to the interpretation of (23), with the obvious adjustments on account of the expected values in (89). The Lagrange multiplier $\lambda(t)$ is interpreted as in the certainty case, although it is expressed in terms of expected collective utility since those are the units in which the objective functional in (79) is expressed. In addition, the left-hand side of (89) differs from the left-hand side of (23) in that the former involves $E[\frac{\partial U_1}{\partial D}]$ in place of $\frac{\partial U_1}{\partial D}$. Analogous to the interpretation of (23), the ratio of the left-hand side of (89) can be interpreted as the marginal rate of substitution between physical capital and dividends, where numerator and denominator in (89) are expressed as discounted expected values. According to (89), in equilibrium the expected collective utility maximizer will have carried investment at each time t to the point at which the marginal rate of substitution between physical capital and dividends just equals the price of capital goods - at which point the internal and external trade offs between physical capital and dividends will be equated. This result is summarized as the following lemma:

Lemma III-2

Under certainty (uncertainty) the collective utility (expected collective utility) maximizing firm will carry investment at each time t up to the point at which its marginal rate of substitution between physical capital and dividends just equals the price of capital goods.

Consideration of necessary conditions (85) and (87) involves the following two cases:

case (i): $\mu_1 = 0$.

In this case the managerial emoluments constraint (78) is not necessarily binding at optimality. Setting $\mu_1 = 0$ in (85) leads to the following theorem:

Theorem III-5

When the risk averse expected collective utility maximizing firm modeled in (79) is in equilibrium, it will tend to produce more output than a similarly risk averse expected utility of total profit maximizer. The effect of uncertainty on both types of firms is to cause each to produce a smaller quantity of output than it would under certainty.

Proof

Since $\frac{\partial U_1}{\partial R}$, p, and $\frac{\partial f}{\partial L}$ are all strictly positive, it follows that $E[\frac{\partial U_1}{\partial R} p(\theta)\frac{\partial f}{\partial L}] > 0$ and hence, from (85) with $\mu_1 = 0$, that

$$E[\frac{\partial U_1}{\partial D}(1 - \tau)(p(\theta)\frac{\partial f}{\partial L} - w)] < 0 . \tag{90}$$

If I(t), K(t), and M(t) are fixed, it follows from (9) that a preference ordering over D also gives a preference ordering over π. Thus, (90) implies that

$$E[\frac{\partial U_1}{\partial \pi}(1 - \tau)(p(\theta)\frac{\partial f}{\partial L} - w)] < 0 . \tag{91}$$

For an expected utility of total profit maximizer with utility function $U(\pi)$ such that $\frac{dU}{d\pi} \equiv \frac{\partial U_1}{\partial \pi}$, (91) implies that the expected collective utility maximizer will produce greater output. This follows from (91) and Batra's and Ullah's statement of the first- and second-order conditions for expected utility of total profit maximization:[121]

$$\frac{\partial E[U(\pi)]}{\partial L} = 0 \qquad \frac{\partial^2 E[U(\pi)]}{\partial L^2} < 0 . \tag{92}$$

Hence, from (91) and (92), the expected collective utility maximizer uses more labor, and since $\partial f/\partial L > 0$, it will, as a consequence, produce greater output.

The proof of the second statement utilizes a technique introduced by Sandmo and since utilized by others.[122] The first-order condition (85) with $\mu_1 = 0$ can be written as

$$E\{\frac{\partial U_1}{\partial R}\, p(\theta)\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)p(\theta)\frac{\partial f}{\partial L}\} = E\{\frac{\partial U_1}{\partial D}(1-\tau)w\} \quad . \tag{93}$$

Subtract $E\{\frac{\partial U_1}{\partial R}\cdot\overline{p}\cdot\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)\overline{p}\cdot\frac{\partial f}{\partial L}\}$ from each side of (93) to obtain

$$E\{(\frac{\partial U_1}{\partial R}\,\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)\frac{\partial f}{\partial L})(p(\theta) - \overline{p})\}$$

$$= E\{\frac{\partial U_1}{\partial D}(1-\tau)w\} - E\{\frac{\partial U_1}{\partial R}\cdot\overline{p}\cdot\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)\overline{p}\cdot\frac{\partial f}{\partial L}\} \; , \tag{94}$$

where $\overline{p} = E[\,p(\theta)\,]$ is the expected price of output.

By definition, $R = p(\theta)\cdot f(K, L)$, and taking expectations, $E[\,R\,] = \overline{p}\cdot f(K, L)$. Therefore, R can be expressed as $R = E[\,R\,] + (p - \overline{p})\cdot f(K, L)$. If $p \geq \overline{p}$, then $E[\,R\,] \leq R$, and by the assumed risk aversion on the part of the firm,

$$\frac{\partial U_1}{\partial R}(R) \leq \frac{\partial U_1}{\partial R}\,[E(R)] \quad , \text{ if } p \geq \overline{p} \; , \tag{95}$$

where $\frac{\partial U_1}{\partial R}(R)$ denotes $\frac{\partial U_1}{\partial R}$ evaluated at arbitrary $R(\geq E(R))$ and where $\frac{\partial U_1}{\partial R}\,[E(R)]$ denotes $\frac{\partial U_1}{\partial R}$ evaluated at E(R).

It is clear from (95) that

$$\frac{\partial U_1}{\partial R}(R)\cdot(p - \overline{p}) \leq \frac{\partial U_1}{\partial R}[E(R)]\cdot(p - \overline{p}) . \qquad (96)$$

But (96) must hold for all p, since when $p \leq \overline{p}$ the inequality in (95) is reversed, while multiplication by $(p - \overline{p})$ reverses the direction of the inequality and gives (96). Taking expectations in (96) and noting that $\frac{\partial U_1}{\partial R}[E(R)]$ is a constant yields

$$E[\frac{\partial U_1}{\partial R}\cdot(p - \overline{p})] \leq \frac{\partial U_1}{\partial R}[E(R)]\cdot E(p - \overline{p}) = 0 , \qquad (97)$$

since $\overline{p} = E(p)$. Similarly it can be shown that

$$E[\frac{\partial U_1}{\partial D}\cdot(p - \overline{p})] \leq \frac{\partial U_1}{\partial D}[E(D)]\cdot E(p - \overline{p}) = 0 . \qquad (98)$$

Since $\frac{\partial f}{\partial L}$ and $(1 - \tau)$ are positive constants, and hence will not alter the inequalities in (97) and (98) when $\frac{\partial U_1}{\partial R}$ is multiplied by $\frac{\partial f}{\partial L}$ and $\frac{\partial U_1}{\partial D}$ is multiplied by $(1 - \tau)\frac{\partial f}{\partial L}$, (97) and (98) imply that the expression on the right-hand side of (94) is nonpositive. Rearranging terms this implies

$$E[\frac{\partial U_1}{\partial R}]\overline{p}\cdot\frac{\partial f}{\partial L} + E[\frac{\partial U_1}{\partial D}](1 - \tau)(\overline{p}\cdot\frac{\partial f}{\partial L} - w) \geq 0 , \qquad (99)$$

which is similar to (24) for the certainty case with mathematical expectations in place of $\frac{\partial U_1}{\partial R}$, $\frac{\partial U_1}{\partial D}$, and p. If the firm knew that the price p would take on its mathematical expectation $\overline{p}$ with certainty, then (99) would hold as an equality. Since $\partial f/\partial L > 0$, (99) implies that the existence of uncertainty causes the firm modeled in (79) to produce less than it would under certainty, a result that Sandmo and others have demonstrated for the single period expected utility of total profit maximizer.[123]

Q.E.D.

It should be noted that when the expected collective utility maximizer is in equilibrium, (99) implies that $\bar{p}\cdot\frac{\partial f}{\partial L} - w < 0$ does not necessarily hold. That is, in the certainty case the firm will, according to lemma III-1, hire labor beyond the point at which its marginal revenue product equals the wage. Under uncertainty, the firm may, depending on the shape of U_1 and the probability distribution of $p(\theta)$, hire labor beyond the point at which its expected marginal revenue product equals the wage. The complication introduced under uncertainty is the firm's attitude toward risk, as embodied in the shape of the firm's utility function U_1. This is made clear by the following corollary to theorem III-5.

Corollary III-5-1

If the firm modeled in (79) exhibits risk neutrality with respect to revenue and total dividends paid, then it will in equilibrium hire labor beyond the point at which the expected marginal revenue product of labor equals the wage rate.

Proof

Risk neutrality implies $\frac{\partial U_1}{\partial R} = k_1 > 0$ and $\frac{\partial U_1}{\partial D} = k_2 > 0$, where k_1 and k_2 are constants that are not necessarily equal. Then with $\mu_1 = 0$ (85) becomes

$$k_1 \cdot \bar{p}\cdot\frac{\partial f}{\partial L} + k_2(1 - \tau)(\bar{p}\cdot\frac{\partial f}{\partial L} - w) = 0 \ , \qquad (100)$$

which implies that $\bar{p}\cdot\frac{\partial f}{\partial L} - w < 0$. Q.E.D.

The interpretation of corollary III-5-1 is that risk neutrality on the part of the firm makes it, in effect, insensitive to the existence of uncertainty. Note that (100) is similar in form to (24), which holds in the certainty case. Since the risk neutral short run expected utility of total profit maximizer would hire labor up to the point at which $\bar{p} \cdot \frac{\partial f}{\partial L} = w$, under risk neutrality it is much easier to see that the expected collective utility maximizer will hire more labor and produce more output than the traditional firm. Under risk aversion (or under less plausible risk seeking behavior) the marginal utilities $\frac{\partial U_1}{\partial R}$ and $\frac{\partial U_1}{\partial D}$ are not constant, and it may be inferred on the basis of theorem III-5, and in particular (99), that if the expected collective utility maximizer is sufficiently risk averse, then it is possible that $\bar{p} \cdot \frac{\partial f}{\partial L} > w$. Theorem III-5 and corollary III-5-1 suggest, then, that under uncertainty a highly risk averse expected collective utility maximizer and a risk neutral or barely risk averse traditional firm could reach identical output decisions. Under uncertainty, then, the firm's attitude toward risk, as well as the nature of the sources of utility, can have a strong impact on the behavior of the firm. If managerial firms are highly security conscious, as Rothschild, Galbraith, Marris, and others have argued,[124] and if as a consequence they are strongly risk averse, then to the extent that traditional firms are less risk averse, it would become very difficult empirically to distinguish managerial behavior from traditional behavior and from expected collective utility maximizing behavior.

Turning next to necessary condition (87), when $\mu_1 = 0$,

$$\frac{E[\partial U_1/\partial M]}{E[\partial U_1/\partial D]} = (1 - \tau), \qquad (101)$$

which is analogous to (28) for the certainty case. Unfortunately, the ratio of two expected values is not in general equal to the expected value of the ratio, so that the left-hand side of (101) cannot be interpreted as the expected value of the marginal rate of substitution between managerial emoluments and dividends. However, given the analogy between (101) and (28), it is, in the opinion of this writer, still meaningful to interpret the left-hand side of (101) as the marginal rate of substitution between managerial emoluments and dividends, with the understanding that expected utility indifference curves replace the utility indifference curves of the certainty version, so that correspondingly, the ratio of expected marginal utilities replaces the ratio of marginal utilities of the certainty case in the calculation of the marginal rate of substitution (i.e. the negative of the slope of the [expected] utility indifference curve at a point). Then (101) can be interpreted as the requirement that the marginal rate of substituti[on] between managerial emoluments and dividends equal one minus the tax rate when the expected collective utility maximizer modeled in (79) is in equilibrium.

case (ii): $\mu_1 \neq 0$.

In this case the managerial emoluments constraint (78) is binding at optimality, so that $M = \hat{M}$. By analogy with the certainty case, (77) must hold, for otherwise an increase in output would relax the constraint. By (88), $\mu_1 > 0$.

Hence, (85) and (87) imply that

$$\frac{\partial H}{\partial L} = E\{\frac{\partial U_1}{\partial R} p(\theta)\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)\}e^{-rt} > 0 \qquad (102)$$

$$\frac{\partial H}{\partial M} = E\{\frac{\partial U_1}{\partial D}(1 - \tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} > 0 , \qquad (103)$$

which are analogous to (32) and (33), respectively, for the certainty case. According to (102) and (103), the Hamiltonian is a constrained maximum with respect to L and M. Were it not for the constraint (78), the firm could increase both revenue and managerial emoluments and thereby reach a higher level of discounted expected collective utility. But the firm cannot do this because it would increase the 'risk' (i.e. the probability) that actual net income would fall short of π_o.

The foregoing was concerned with necessary condition (82) and the optimal time paths of labor, investment, and managerial emoluments. Next, the implications of necessary condition (84) for the optimal time path of the firm's capital stock are considered. Together (84) and (86) imply that

$$\begin{aligned} E[(\frac{\partial U_1}{\partial D})\cdot(-rq + \dot{q})] = - [& E(p(\theta)\frac{\partial U_1}{\partial R})\frac{\partial f}{\partial K} \\ & + E(p(\theta)\frac{\partial U_1}{\partial D})\cdot(1-\tau)\cdot\frac{\partial f}{\partial K} \qquad (104) \\ & + E(\frac{\partial U_1}{\partial D})\cdot\tau q\delta - \delta q\cdot E(\frac{\partial U_1}{\partial D})] , \end{aligned}$$

where, for convenience, only the case $\mu_1 = 0$ is considered. Equation (104) can be used to establish the following theorem and corollary.

Theorem III-6

When the risk averse expected collective utility maximizing firm modeled in (79) is in equilibrium, the effect of uncertainty is to cause it to use less capital and to produce less output than it would under certainty.

Proof

Rewrite (104) as

$$E\{\frac{\partial U_1}{\partial R}p(\theta)\frac{\partial f}{\partial K} + \frac{\partial U_1}{\partial D}(1-\tau)p(\theta)\frac{\partial f}{\partial K}\}=E\{\frac{\partial U_1}{\partial D}(q[r+(1-\tau)\delta]-\dot{q})\} \ , \tag{105}$$

which is just (93) with $\frac{\partial f}{\partial K}$ in place of $\frac{\partial f}{\partial L}$ and $q[r+(1-\tau)\delta]-\dot{q}$, the cost of capital net of tax, in place of $(1-\tau)w$, the cost of labor net of tax. Applying the technique utilized in the proof of theorem III-5 gives

$$E\left[\frac{\partial U_1}{\partial R}\right]\bar{p}\ \frac{\partial f}{\partial K} + E\left[\frac{\partial U_1}{\partial D}\right]\cdot\{(1-\tau)\bar{p}\ \frac{\partial f}{\partial K} - (q[r+(1-\tau)\delta]-\dot{q})\} \geq 0 \ , \tag{106}$$

which is analogous to (99). But if the firm knew with certainty that output price would be $\bar{p}$, then it follows from (40) that equality would hold in (106). Since $\frac{\partial f}{\partial K} > 0$, (106) implies that uncertainty has the effect of reducing the firm's output and its use of capital. Q.E.D.

Collectively, theorems III-5 and III-6 imply that the existence of uncertainty causes the risk averse firm to use less of both inputs, and therefore, to produce less output than it would under certainty. As in the case of theorem III-5, risk neutrality on the part of the firm leads to a simpler result.

Corollary III-6-1

If the firm modeled in (79) exhibits risk neutrality with respect to revenue and total dividends paid, then it will hire capital beyond the point at which the expected marginal revenue product of capital equals the cost of capital.

Proof

Same as corollary III-5-1. Q.E.D.

The existence of uncertainty causes the risk averse firm to use less of each input. The question remains as to whether there is any built-in bias against one of the inputs. Put differently, will the existence of uncertainty force the firm off the expansion path characterized by (44) for the certainty case? The answer is 'no'. To demonstrate this result, which is contained in theorem III-7, it is assumed that $\mu_1 = 0$ so that the effects of uncertainty only are reflected in the results (recall from section B above that when the net income constraint is binding the firm's expansion path may change).

Theorem III-7

Regardless of the firm's attitude toward risk, the expected collective utility maximizer modeled in (79) will have the same expansion path under uncertainty as it would under certainty, provided the profit constraint is not binding.

Proof

To prove the theorem it is sufficient to show that (44) holds. From (85),

$$\frac{\partial f}{\partial L} = \frac{E[\partial U_1/\partial D](1-\tau)w}{E[(\partial U_1/\partial R)p(\theta)+(\partial U_1/\partial D)(1-\tau)\cdot p(\theta)]}, \qquad (107)$$

and from (105),

$$\frac{\partial f}{\partial K} = \frac{E[\partial U_1/\partial D]\cdot i}{E[(\partial U_1/\partial R)p(\theta) + (\partial U_1/\partial D)(1-\tau)\cdot p(\theta)]}\ , \tag{108}$$

where $i \equiv q(r + (1 - \tau)\delta) - \dot{q}$ (as in the certainty case). Combining (107) and (108) gives

$$\frac{\partial L}{\partial K} = \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{E[\partial U_1/\partial D]i}{E[\partial U_1/\partial D](1-\tau)w} = \frac{1}{1-\tau}\cdot\frac{i}{w}\ . \qquad \text{Q.E.D.}$$

Theorem III-7 may appear surprising. One might suspect that due to uncertainty the firm would try to hold less capital, and instead employ relatively more labor in order to obtain greater flexibility in its use of inputs. However, when the market for capital goods is perfect and investment is reversible,[125] as it is for the firm modeled in (79), the firm has the ability to adjust both capital and labor instantaneously. Also the firm is assumed to reach its output decision prior to observing market price. Hence, the existence of uncertainty does not affect the input mix, given this output decision.

This subsection has been concerned with the characterization of the equilibrium position of the firm modeled in (79) and with the effect of uncertainty on that equilibrium position. Nothing has yet been said concerning the sensitivity of this multiperiod equilibrium to changes in the parameters ε, τ, and π_0. The next subsection presents this sensitivity analysis.

3. Comparative Dynamics and Comparative Statics Results

This subsection explores the sensitivity of the behavior of the expected collective utility maximizer modeled in (79) to changes in several key parameters. The main results, which are summarized in theorems III-8 and III-9, are shown to be analogous to results obtained earlier for the certainty case.

Theorem III-8

An increase in ε , the probability that net income will exceed π_o , will cause the firm to reduce its level of output at each time t at which the profit constraint is (or is caused to become) binding.

Proof

Given that $\partial f/\partial L > 0$, to prove the theorem it is sufficient to show that $\partial L/\partial \varepsilon < 0$. To accomplish this, use (85) to define the function

$$G(L,\varepsilon) = E\{\frac{\partial U_1}{\partial R} p(\theta)\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)\} e^{-rt} + \mu_1 \frac{\partial \hat{M}}{\partial L} .$$

Note that when the profit constraint is binding, $\mu_1 > 0$ and $\frac{\partial \mu_1}{\partial \varepsilon} \geq 0$. It can also be shown that $\frac{\partial^2 \hat{M}}{\partial L^2} < 0$ and $\frac{\partial^2 \hat{M}}{\partial L \partial \varepsilon} < 0$.[126] Then since

$$\frac{\partial G}{\partial L} = E\{\frac{\partial^2 U_1}{\partial R^2}[p(\theta)\frac{\partial f}{\partial L}]^2 + \frac{\partial U_1}{\partial R} p(\theta)\frac{\partial^2 f}{\partial L^2}$$

$$+ \frac{\partial^2 U_1}{\partial D^2}[(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)]^2 \frac{\partial U_1}{\partial D}(1-\tau)p(\theta)\frac{\partial^2 f}{\partial L^2}\} e^{-rt}$$

$$+\mu_1 \frac{\partial^2 \hat{M}}{\partial L^2} < 0 ,$$

by the assumed concavity of U_1 and f, and since

$$\frac{\partial G}{\partial \varepsilon} = \frac{\partial \mu_1}{\partial \varepsilon}\,\frac{\partial \hat{M}}{\partial L} + \mu_1 \cdot \frac{\partial^2 \hat{M}}{\partial L \partial \varepsilon} < 0 \ ,$$

it follows by the implicit function theorem that

$$\frac{\partial L}{\partial \varepsilon} = -\frac{\partial G/\partial \varepsilon}{\partial G/\partial L} < 0 \ . \qquad \text{Q.E.D.}$$

The practical interpretation of theorem III-8 is that if the firm should become more risk averse in the sense that it raises ε (thereby reducing the probability that it will fail to generate net income at least as great as π_0), it will find it necessary to reduce output accordingly.

The second main result developed in this subsection concerns the derivation of a Hicks-Slutsky-type equation similar to (69) and (70), which were developed in subsection 5 of section B for the certainty case. While the form of the equation becomes somewhat more complicated under uncertainty, the interpretation of the equation is similar. This second main result is proved as theorem III-9 and the special case of risk neutrality is presented as a corollary.

Theorem III-9

If the utility function U_1 and the production function f are both strictly concave, then the effect of a change in the tax rate τ on the equilibrium level of total sales revenue when both total sales revenue and managerial emoluments are permitted to vary and when the profit constraint is binding can be expressed as the resolution of three effects, one of which can be interpreted as a substitution effect, one of which can be interpreted as an income effect, and the third of which can be interpreted as a cross substitution effect, with the first and third being negative and the second one being, in general, positive.

Proof

As in the certainty case, write the necessary conditions for the optimization problem:

$$\begin{aligned} &\underset{\{L,\ M\}}{\text{maximize}} \quad H(K, L, I, M, \lambda, t) \\ &\text{subject to} \quad \hat{M}(L, K, \pi_o, \varepsilon) - M(t) = 0 \ , \end{aligned} \tag{109}$$

where the level of investment, I, is treated as fixed. The necessary conditions for an optimal solution to problem (109) are (85) and (87) together with the constraint in (109):

$$\hat{M}(L, K, \pi_o, \varepsilon) - M(t) = 0 \tag{110}$$

$$E\{\frac{\partial U_1}{\partial R}\, p(\theta)\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)\}e^{-rt} + \mu_1 \cdot \frac{\partial \hat{M}}{\partial L} = 0 \tag{85}$$

$$E\{\frac{\partial U_1}{\partial D}(1 - \tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} - \mu_1 = 0 \ . \tag{87}$$

Also as in the certainty case, define the bordered Hessian H by

$$H \equiv \begin{bmatrix} 0 & \partial g/\partial L & \partial g/\partial M \\ \partial g/\partial L & \partial^2 H/\partial L^2 & \partial^2 H/\partial L \partial M \\ \partial g/\partial M & \partial^2 H/\partial M \partial L & \partial^2 H/\partial M^2 \end{bmatrix} ,$$

where $g(L, M) \equiv \hat{M}(L, K, \pi_o, \varepsilon) - M(t)$. The elements of H are shown in table III-4. It is easily verified that

$$\det H = -2\,\frac{\partial\hat{M}}{\partial L}\Big(-E\,\Big\{\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2\Big(p\cdot\frac{\partial f}{\partial L} - w\Big)\Big\}e^{-rt}\Big)$$

$$-E\Big[\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial^2 f}{\partial L^2} + \frac{\partial^2 U_1}{\partial R^2}\Big(p\cdot\frac{\partial f}{\partial L}\Big)^2 + \frac{\partial U_1}{\partial D}(1-\tau)p\cdot\frac{\partial^2 f}{\partial L^2}$$

$$+\frac{\partial^2 U_1}{\partial D^2}\Big\{(1-\tau)\Big(p\cdot\frac{\partial f}{\partial L} - w\Big)\Big\}^2\Big]e^{-rt} - \mu_1\cdot\frac{\partial^2\hat{M}}{\partial L^2} \tag{111}$$

$$-\Big(\frac{\partial\hat{M}}{\partial L}\Big)^2 E\Big[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 + \frac{\partial^2 U_1}{\partial M^2}\Big]\,e^{-rt} > 0\ ,$$

provided $E\{\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2(p\cdot\frac{\partial f}{\partial L} - w)\} < 0$.[127] Thus, by the implicit function theorem, equations (110), (85), and (87) can be used to obtain functions

$$\mu_1 = \mu_1(\tau,\ \pi_o) \qquad L = L(\tau,\ \pi_o) \qquad M = M(\tau,\ \pi_o)$$

that are continuously differentiable for all $(\tau,\ \pi_o)$ in some neighborhood of $(\hat{\tau},\ \hat{\pi}_o)$ at which $\det H > 0$ and that satisfy

$$\hat{\mu}_1 = \mu_1(\hat{\tau},\ \hat{\pi}_o) \qquad \hat{L} = L(\hat{\tau},\ \hat{\pi}_o) \qquad \hat{M} = M(\hat{\tau},\ \hat{\pi}_o)\ ,$$

where the carat denotes the optimal values satisfying (110), (85), and (87).[128] Treating μ_1, L, and M each as a function of τ and π_o and partially differentiating (110), (85), and (87) with respect to τ leads to the matrix equation

$$[H]\begin{pmatrix}\partial\mu_1/\partial\tau\\ \partial L/\partial\tau\\ \partial M/\partial\tau\end{pmatrix} = \begin{pmatrix}\pi_o/(1-\tau)^2\\ E[(\partial U_1/\partial D)(p(\theta)\frac{\partial f}{\partial L} - w)]e^{-rt}\\ -E[\partial U_1/\partial D]e^{-rt}\end{pmatrix} . \tag{112}$$

Denote $E[(\partial U_1/\partial D)(p(\theta)\frac{\partial f}{\partial L} - w)]e^{-rt}$ by α and $-E[\partial U_1/\partial D]e^{-rt}$ by β . Note that $\beta < 0$, but that, unlike the certainty case, the sign of α is indeterminate. However, by analogy with the certainty case, it is reasonable to expect that in general $\alpha < 0$ as well.[129] Rewriting (112) as

$$[H]\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{(1-\tau)^2}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{113}$$

and solving for the vector of partial derivatives with respect to τ in (113) yields

$$\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{(1-\tau)^2} H^{-1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha H^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta H^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{114}$$

To facilitate the interpretation of (114), next partially differentiate (110), (85), and (87) with respect to π_o to obtain

$$[H]\begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} = \begin{pmatrix} 1/(1-\tau) \\ 0 \\ 0 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} = H^{-1}\begin{pmatrix} 1/(1-\tau) \\ 0 \\ 0 \end{pmatrix}. \tag{115}$$

Substituting (115) into (114) gives

$$\begin{pmatrix} \partial\mu_1/\partial\tau \\ \partial L/\partial\tau \\ \partial M/\partial\tau \end{pmatrix} = \frac{\pi_o}{1-\tau}\begin{pmatrix} \partial\mu_1/\partial\pi_o \\ \partial L/\partial\pi_o \\ \partial M/\partial\pi_o \end{pmatrix} + \alpha H^{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \beta H^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{116}$$

which is analogous to (67) for the certainty case. From (116),

$$\frac{\partial L}{\partial\tau} = \frac{\pi_o}{1-\tau}\frac{\partial L}{\partial\pi_o} + \alpha H^{-1}{}_{22} + \beta H^{-1}{}_{23}, \tag{117}$$

which is analogous to (68) for the certainty case.

Table III-4 Elements of the Bordered Hessian Matrix H (under Uncertainty)

$$
H \equiv \begin{bmatrix}
0 & \partial\hat{M}/\partial L & -1 \\
\partial\hat{M}/\partial L & \begin{array}{l} E[\frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial^2 f}{\partial L^2} + \frac{\partial^2 U_1}{\partial R^2}(p\cdot\frac{\partial f}{\partial L})^2 + \frac{\partial U_1}{\partial D}(1-\tau)p\cdot\frac{\partial^2 f}{\partial L^2} \\ + \frac{\partial^2 U_1}{\partial D^2}\{(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)\}^2]e^{-rt} \\ + \mu_1\cdot\frac{\partial^2\hat{M}}{\partial L^2} \end{array} & \begin{array}{l} - E[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 \times \\ (p\cdot\frac{\partial f}{\partial L} - w)]e^{-rt} \end{array} \\
-1 & - E[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2(p\cdot\frac{\partial f}{\partial L} - w)]e^{-rt} & \begin{array}{l} E[\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 \\ + \frac{\partial^2 U_1}{\partial M^2}]e^{-rt} \end{array}
\end{bmatrix}
$$

From table III-4,

$$H^{-1}{}_{22} = \frac{-1}{\det H} \qquad\qquad H^{-1}{}_{23} = \frac{-\partial\hat{M}/\partial L}{\det H} .$$

With these expressions for $H^{-1}{}_{22}$ and $H^{-1}{}_{23}$ (117) becomes

$$\frac{\partial L}{\partial \tau} = \frac{\pi_o}{1-\tau}\frac{\partial L}{\partial \pi_o} - \frac{E[(\partial U_1/\partial D)(p(\theta)\frac{\partial f}{\partial L} - w)]e^{-rt}}{\det H} + \frac{E[\partial U_1/\partial D]e^{-rt}(\partial\hat{M}/\partial L)}{\det H} . \tag{118}$$

It follows from (110) that $\partial L/\partial\pi_o < 0$.[130] Hence, the first term in (118) is negative. It follows from the fact that, in general, $E[(\partial U_1/\partial D)(p(\theta)\frac{\partial f}{\partial L} - w)] < 0$ that the second term in (118) is in general positive. Since $E[\partial U_1/\partial D] > 0$ and $\partial\hat{M}/\partial L < 0$, it follows that the third term in (118) is negative.

Q.E.D.

<u>Corollary III-9-1</u>

Under risk neutrality the qualification 'in general' can be dropped from the interpretation of the second term in (118).

<u>Proof</u>

From corollary III-5-1, $\bar{p}\,\frac{\partial f}{\partial L} - w < 0$. Hence, for some $k > 0$, $E[(\partial U_1/\partial D)(p(\theta)\cdot\frac{\partial f}{\partial L} - w)] = k\cdot(\bar{p}\cdot\frac{\partial f}{\partial L} - w) < 0$. Q.E.D.

In interpreting theorem III-9 and corollary III-9-1 it should be noted that the Hicks-Slutsky-type equation (118) is expressed generically by equation (72), which furthers the analogy between theorems III-4 and III-9 for the certainty and uncertainty cases, respectively. The importance of theorem III-9 and the accompanying corollary III-9-1 is the demonstration that under uncertainty the expected collective utility maximizer will respond to a change in either π_o or τ in a manner qualitatively the same as its response to each such change under certainty.

4. Section Summary

This section has extended the model of the firm developed in the previous section to incorporate uncertainty. It was shown that the existence of uncertainty causes the risk averse firm to use less of both inputs and to produce less output (theorems III-5 and III-6), but does not induce a change in the expansion path (theorem III-7). It was also shown that an increase in ε, the probability that net income will exceed π_o, will tend to cause the firm to reduce output (theorem III-8) and that a change in either π_o or τ will elicit the same qualitative response under uncertainty as it did under certainty (theorem III-9)

D. RECONCILING PROFIT MAXIMIZATION AND MANAGERIAL UTILITY MAXIMIZATION: THE TWO-STATE CASE

The main purpose of this chapter is to model the behavior of the firm over the business cycle and to demonstrate that, depending on the state of the firm's immediate operating environment (or loosely, the state of the business cycle), the firm may alternate between modes of traditional and managerial behavior. This section presents a simple two-state Markov model of the behavior of the firm over the business cycle. In the next section the basic theoretical model developed in sections B and C of this chapter is adapted to incorporate the effects of the business cycle and the number of states of the firm's operating environment is permitted to be arbitrary.

O.E. Williamson has developed a Markov model in which two states of the environment were hypothesized and in which managers could select either of two modes of behavior: managerial utility maximizing behavior or profit maximizing behavior.[131] The firm's transition from one state of the environment to another was modeled as a nonstationary Markov process in which the transition probabilities were dependent on the mode of behavior selected, as well as on the current state of the firm's environment. Williamson showed by numerical example that systematic shifts between modes of behavior can occur, depending on the state of the environment.

Albin and Alcaly have described a model in which corporate managers choose between growth maximizing policies and profit maximizing policies and in which the state of the environment is indicated by the macroeconomic rate of growth.[132] Their model is mainly concerned with the interactions between the policy choices of individual firms and how these policy choices determine the general equilibrium macroeconomic growth rate (and how expectations concerning the macroeconomic growth rate affect these policy choices). Though their model shows that it is "reasonable and appropriate for the same firm to be apparently management motivated at one time and owner-interest motivated at another,"[133] the level of abstraction at which the individual firm is treated obscures the workings of the mechanism by which systematic shifts in objectives take place.

It is this writer's view that Williamson and Albin and Alcaly are correct in asserting that the typical large firm's mode of behavior adjusts to the state of the firm's operating environment. During the downswing in the business cycle sales and profits tend to fall, firms' financial positions tend to worsen, and firms try to divest themselves of unprofitable or marginally profitable operations.[134] In terms of the model of the firm developed in sections B and C of this chapter, under severe business conditions the profit constraint is not satisfied, and firms are forced to behave as profit maximizers (i.e. the traditional mode of behavior). During the upswing in the business cycle the state of the firm's operating environment is much more favorable. Sales and profits are increasing, firms' financial positions tend to improve, and firms become more sales- and growth-oriented.[135] In terms of the model developed in sections B and C of this chapter, under favorable business conditions the profit constraint is easily satisfied, and firms have greater discretion to pursue managerial objectives.

1. The Two-State Markov Model

To model the cyclical pattern of behavior just described, assume there are just two possible states of the firm's operating envirnoment, one being prosperity (or the 'upswing' denoted by H) and the other being adversity (or the 'downswing' denoted by L). Let time be measured in discrete periods denoted by t. During each period t the state of the firm's operating environment must be either H or L. Also during each period, there is some maximum level of total profit π^*_t. The actual level of total profit, π_t, depends on the firm's operating posture. Define $x_t = \pi_t/\pi^*_t$. Then $x_t \leq 1$, where $x_t = 1$ indicates the choice of a traditional operating posture. That is $x_t = 1$ implies that total profit is being maximized, $\pi_t = \pi^*_t$, and the firm is employing those operating policies that would be adopted by a short run profit maximizer. During each period there is also some minimum required level of total profit, $\bar{\pi}_t$, which is assumed known with certainty. Define $\bar{x}_t = \bar{\pi}_t/\pi^*_t$. When $x_t = \bar{x}_t$ the firm is said to have chosen a managerial operating posture. That is, $x_t = \bar{x}_t$ implies that total profit is not being maximized (unless $\bar{\pi}_t \equiv \pi^*_t$), and in view of the sacrifice of total profit inherent in the pursuit of managerial objectives (as discussed throughout this chapter), the firm's behavior when it selects $x_t = \bar{x}_t$ may be characterized as managerial. It follows from the foregoing that the firm's choice of operating posture, x_t, must satisfy the double constraint

$$\bar{x}_t \leq x_t \leq 1 , \tag{119}$$

where x_t is a continuous variable and where values of x_t such that $\bar{x}_t < x_t < 1$ indicate varying degrees of traditional and managerial behavior between the two extremes ($x_t = 1 \equiv$ 'pure' traditional and $x_t = \bar{x}_t \equiv$ 'pure' managerial).

It is assumed that the firm's transitions from one state of the environment to another follow a Markov process. That is, a transition from either state at time t to either state at time t + 1 is (in each case) a function of the state and the firm's choice of operating posture at time t only and is independent of the firm's prior history. The transition probabilities, which are written as explicit functions of the firm's current choice of operating posture, are arrayed in the transition probability matrix shown in table III-5.

Table III-5 Transition Probability Matrix for a Two-State Markov Decision Process

$$P = \begin{bmatrix} p_{LL}(x_t) & p_{LH}(x_t) \\ p_{HL}(x_t) & p_{HH}(x_t) \end{bmatrix}$$

It is assumed that all four transition probabilities are strictly positive. The transition probability matrix is stochastic; that is,

$$\left.\begin{aligned} p_{LL}(x_t) + p_{LH}(x_t) &= 1 \\ p_{HL}(x_t) + p_{HH}(x_t) &= 1 \end{aligned}\right\} \qquad (120)$$

It is assumed that

$$\frac{d}{dx_t}\, p_{LL}(x_t) < 0 \text{ and } \frac{d}{dx_t}\, p_{HL}(x_t) < 0 . \qquad (121)$$

According to (121) the more traditional the firm's current choice of operating posture, the less is the likelihood of a transition to state L, and correspondingly, by (120), the greater is the likelihood of a transition to state H. The rationale underlying (121) is that adopting the managerial posture, and in so doing increasing sales and managerial emoluments beyond the respective profit-maximizing levels, tends to shrink future choice sets, and thereby increase the likelihood of adversity setting in.[136]

The firm's policy choices, as embodied in x_t , determine not only the transition probabilities, but also the returns, which are arrayed in the return matrix shown in table III-6.

Table III-6 Return Matrix for a Two-State Markov Decision Process

$$R = \begin{bmatrix} U_{LL}(x_t) & U_{LH}(x_t) \\ U_{HL}(x_t) & U_{HH}(x_t) \end{bmatrix}$$

The return $U_{ij}(x_t)$ expresses the collective utility resulting from the choice of current operating posture x_t when the firm transitions from state i to state j. It is assumed that all returns are positive and that for each choice of posture, x_t ,

$$U_{LL}(x_t) < U_{LH}(x_t) \text{ and } U_{HL}(x_t) < U_{HH}(x_t) \ . \tag{122}$$

The rationale underlying (122) is that, regardless of the posture chosen, the firm is better off in the sense of having achieved a higher immediate level of collective utility if it transitions to H than if it transitions to L. In addition, it is assumed that for all x_t ,

$$\frac{d}{dx_t}U_{ij}(x_t) < 0 \qquad i = L, H \ ; \ j = L, H \ . \tag{123}$$

The rationale underlying (123) is that the level of collective utility in the current period would always be enhanced if the firm were to adopt a more managerial (and less traditional) posture. This treatment of the relative immediate returns from the alternative modes of behavior is consistent with Williamson's assumptions concerning the relative returns from 'pure' traditional and 'pure' managerial behavior.[137] As in the Williamson model, it will be shown below that consideration of the future implications of the current policy choice by the firm will prevent the firm from always assuming a managerial operating posture, and may in fact cause the firm's behavior to alternate systematically between the two 'pure' modes of behavior. But unlike the Williamson model, the model developed below allows for intermediate modes of behavior that the firm may exhibit, depending on the nature of the matrices P and R.

2. The Behavior of the Firm Over Time

Given the matrices P and R, the firm's choices of operating postures over time are modeled as a Markov decision process.[138] Let r denote the firm's rate of discount, which is assumed to remain constant over time, and let U_τ^i denote the expected present value of returns resulting from all remaining transitions over the remainder of the planning period $(\tau, \ldots, T)$ given that state i is occupied at the end of period $\tau - 1$. The expected present value of returns U_τ^i can be expressed in the following recursive form:

$$U_\tau^i = p_{iL}(x_\tau)\cdot[U_{iL}(x_\tau) + (\tfrac{1}{1+r})U_{\tau+1}^L] + p_{iH}(x_\tau)\cdot[U_{iH}(x_\tau) + (\tfrac{1}{1+r})U_{\tau+1}^H] \quad , \tag{124}$$

where $i = L, H$ and $\tau = 1, \ldots, T$ and where $U_{\tau+1}^L$ and $U_{\tau+1}^H$ are independent of x_τ by the Markov property. The firm's decision problem for each time τ, $\tau=1, \ldots, T$, is modeled as:

$$\underset{\{x_\tau,\ldots,x_T\}}{\text{maximize}} \quad U_\tau^i \qquad \text{subject to} \quad \bar{x}_t \le x_t \le 1 \; , \; t = \tau, \ldots, T \tag{125}$$

The solution to problem (125) can be obtained by decomposing problem (125) into the sequential optimization problem:

$$\underset{\{x_t,\ldots,x_T\}}{\text{maximize}} \quad U_t^i = \underset{\{x_\tau\}}{\text{maximize}} \sum_j p_{ij}(x_\tau)[\, U_{ij}(x_\tau) + (\tfrac{1}{1+r}) \underset{\{x_{\tau+1},\ldots,x_T\}}{\max} U_{\tau+1}^j] \tag{126}$$

$$\text{subject to} \quad \bar{x}_t \le x_t \le 1 \; , \; t = \tau, \ldots, T$$

The use of decomposition is justified by the following lemma.

Lemma III-3

The optimal solution to problem (126) is also the optimal solution to problem (125).

Proof

To prove the lemma it must be shown that both the separability condition and the monotonicity condition (i.e. Mitten's condition) are satisfied.[139] Since $U^j_{\tau+1}$ is a function of $x_{\tau+1}, \ldots, x_T$ only (by the Markov property), separability is assured. Since the transition probabilities $p_{ij}(x_t)$ are strictly positive by assumption, U^i_t is a monotonically nondecreasing function of $U^j_{\tau+1}$ for every x_τ, and thus Mitten's condition is satisfied also. Q.E.D.

Denote by $\tilde{U}^j_{\tau+1}$ the solution to the problem

$$\begin{aligned} &\underset{\{x_{\tau+1},\ldots,x_T\}}{\text{maximize}} \quad U^j_{\tau+1} \\ &\text{subject to} \quad \bar{x}_t \le x_t \le 1 , \quad t = \tau+1, \ldots , T \end{aligned} \tag{127}$$

Note that $\tilde{U}^H_t > \tilde{U}^L_t$ for all t since, by definition, H represents the more favorable state of the firm's operating environment, and hence, permits the firm the greater degree of discretion in the selection of its operating posture. Substituting (127) into (126) the latter becomes

$$\begin{aligned} &\underset{\{x_\tau\}}{\text{maximize}} \quad \sum_j p_{ij}(x_\tau)[U_{ij}(x_\tau) + (\frac{1}{1+r})\tilde{U}^j_{\tau+1}] \\ &\text{subject to} \quad \bar{x}_\tau \le x_\tau \le 1 , \end{aligned} \tag{128}$$

which is solved for each time τ, τ=1, ... , T. In obtaining (128), decomposition was used to express the expected present value of total returns over the planning period as a function of immediate returns $U_{ij}(x_\tau)$ and the optimal discounted value of future returns, $(\frac{1}{1+r})\tilde{U}^j_{\tau+1}$. In particular, note that the current policy decision, x_τ , affects immediate returns through $U_{ij}(x_\tau)$ as well as future returns through $p_{ij}(x_\tau)(\frac{1}{1+r})\tilde{U}^j_{\tau+1}$.

To characterize the solution to (128), form the Lagrangian

$$L_\lambda = p_{iL}(x_\tau)[U_{iL}(x_\tau) + (\frac{1}{1+r})\tilde{U}^L_{\tau+1}]$$

$$+p_{iH}(x_\tau)[U_{iH}(x_\tau) + (\frac{1}{1+r})\tilde{U}^H_{\tau+1}] \tag{129}$$

$$+ \lambda_1(1-x_\tau) + \lambda_2(\bar{x}_\tau - x_\tau) .$$

The Kuhn-Tucker conditions for an optimal solution to (128), which are obtained by taking the appropriate derivatives of (129), are:

$$\frac{\partial L_\lambda}{\partial x_\tau} = p_{iL}\frac{dU_{iL}}{dx_\tau} + \frac{dp_{iL}}{dx_\tau}[\,U_{iL} + (\frac{1}{1+r})\,\tilde{U}^L_{\tau+1}] + p_{iH}\frac{dU_{iH}}{dx_\tau}$$

$$+ \frac{dp_{iH}}{dx_\tau}[\,U_{iH} + (\frac{1}{1+r})\,\tilde{U}^H_{\tau+1}\,] - \lambda_1 - \lambda_2 = 0 \tag{130}$$

$$\lambda_1(1 - x) = 0 \qquad x \leq 1 \qquad \lambda_1 \geq 0 \tag{131}$$

$$\lambda_2(\bar{x} - x) = 0 \qquad \bar{x} \leq x \qquad \lambda_2 \leq 0 \tag{132}$$

To interpret necessary conditions (130) - (132) three cases are distinguished depending on whether or not $\lambda_1 = 0$ and $\lambda_2 = 0$ (note that either or both must be zero).

case (i): $\lambda_2 < 0$ (managerial behavior).

From (132), $x = \bar{x}$, and then from (131), $\lambda_1 = 0$. From (130),

$$p_{iL}\frac{dU_{iL}}{dx_\tau} + \frac{dp_{iL}}{dx_\tau}[U_{iL} + (\frac{1}{1+r})\,\tilde{U}^L_{\tau+1}] + p_{iH}\frac{dU_{iH}}{dx_\tau}$$

$$+ \frac{dp_{iH}}{dx_\tau}[U_{iH} + (\frac{1}{1+r})\,\tilde{U}^H_{\tau+1}] < 0 . \tag{133}$$

The interpretation of (133) is that the expected value of discounted returns could be increased were it not for the profit constraint preventing the firm from adopting a more managerial posture.

<u>case (ii): $\lambda_1 > 0$ (traditional behavior).</u>

From (131), x = 1, and then from (132), $\lambda_2 = 0$. From (130), the inequality in (133) is reversed, which is interpreted to mean that, not only is the firm maximizing current period total profit, but the expected value of discounted returns could be increased if somehow sales revenue and/or managerial emoluments could be traded for additional profit.

<u>case (iii): $\lambda_1 = \lambda_2 = 0$ (the intermediate case).</u>

In this case equality holds in (133) and U^i_τ is an unconstrained maximum. Greater insight is achieved by rearranging terms in (133) - reexpressed as an equality - to obtain:

$$\frac{dp_{iL}}{dx_\tau}[\, U_{iL} + (\tfrac{1}{1+r})\, \tilde{U}^L_{\tau+1} \,] + \frac{dp_{iH}}{dx_\tau}[\, U_{iH} + (\tfrac{1}{1+r})\, \tilde{U}^H_{\tau+1} \,]$$

$$= -\{\, p_{iL} \frac{dU_{iL}}{dx_\tau} + p_{iH} \frac{dU_{iH}}{dx_\tau} \} \,. \tag{134}$$

It is shown below in theorem III-10 that the expression on the left-hand side of (134) must be positive whether i = L or i = H, and it is further shown in theorem III-10 that the expression in braces on the right-hand side of (134) must be negative whether i = L or i = H. The left-hand side of (134) can be interpreted as the marginal value of adopting a more traditional operating posture (i.e. increasing x_τ), which is expressed in terms of the impact on the present value of discounted total returns of an increased likelihood of transitioning to H. The right-hand side of (134) can be interpreted as the negative of the marginal cost of adopting a more traditional operating posture, which is expressed in terms of the expected decrease in immediate returns resulting from such a change in posture. Thus, (134) is the familiar marginal value equals (in absolute value) marginal cost necessary condition for optimality, this time applied to the choice of optimal operating posture. Moreover, the nature of the 'marginal value' and 'marginal cost' make clear the trade off that must be made between the improved likelihood of transitionin to the more favorable state of the environment on the one hand and increased immediate returns on the other.

Cases (i) and (ii) suggest that under certain conditions the firm's optimal operating posture would alternate between the managerial mode of behavior and the traditional mode of behavior. This is stated formally as the following theorem:

Theorem III-10

Define $\alpha_i \equiv p_{iL} \dfrac{dU_{iL}}{dx_\tau} + p_{iH} \dfrac{dU_{iH}}{dx_\tau}$ and define

$$\beta_i \equiv \frac{dp_{iL}}{dx_\tau} [U_{iL} + (\frac{1}{1+r}) \tilde{U}^L_{\tau+1}] + \frac{dp_{iH}}{dx_\tau} [U_{iH} + (\frac{1}{1+r}) \tilde{U}^H_{\tau+1}] .$$ For

$i = L, H$, $\alpha_i < 0$ and $\beta_i > 0$. If $\alpha_L + \beta_L > 0$ and $\alpha_H + \beta_H < 0$, then the firm's behavior will alternate systematically between the traditional and managerial modes, being managerial in H and being traditional in L.

Proof

From (123), $dU_{ij}/dx_\tau < 0$. Since the transition probabilities are strictly positive by assumption, it follows that $\alpha_i < 0$, $i = L, H$. From (120) and (121), $dp_{iL}/dx_\tau = - dp_{iH}/dx_\tau < 0$.

Hence, $\beta_i = \frac{dp_{iH}}{dx_\tau}[(U_{iH} - U_{iL}) + (\frac{1}{1+r})(\tilde{U}^H_{\tau+1} - \tilde{U}^L_{\tau+1})]$. From (122) and the fact that $\tilde{U}^H_{\tau+1} > \tilde{U}^L_{\tau+1}$ it follows that $\beta_i > 0$, $i = L, H$.

If $\alpha_L + \beta_L > 0$, then the inequality in (133) is reversed and case (ii) applies. The firm adopts the traditional operating posture. If $\alpha_H + \beta_H < 0$, then (133) holds and case (i) applies. The firm adopts the managerial operating posture. Q.E.D.

The importance of theorem III-10 is that it shows first, that the marginal value of adopting a more traditional operating posture, β_i, is positive whether i = H or i = L and that the marginal cost of adopting a more traditional operating posture, α_i, is negative whether i = H or i = L; and second, that the firm's behavior may alternate systematically between the traditional and managerial modes. If H is interpreted as the upswing portion of the business cycle and if L is interpreted as the downswing portion of the business cycle, then theorem III-10 is interpreted to mean that the firm will act in accordance with the managerial theories during the upswing and will act in accordance with the traditional theories during the downswing.

3. Section Summary

This section has generalized the model of O.E. Williamson and has shown how the behavior of the collective expected utility maximizer may vary systematically over the business cycle. It was shown (in (133)) that when the firm's behavior does not oscillate between these two extremes, the firm is balancing the marginal value of a change in operating posture against the (absolute value of the) marginal cost of a change in operating posture. It was also shown (theorem III-10) that oscillation between these two extremes results when marginal value is greater than (less than) marginal cost in absolute value for all feasible operating postures in state L (state H). The next section demonstrates similar results for the basic theoretical model developed in sections B and C of this chapter.

E. RECONCILING PROFIT MAXIMIZATION AND MANAGERIAL UTILITY MAXIMIZATION: THE BEHAVIOR OF THE FIRM OVER THE BUSINESS CYCLE

In this section the model of the firm developed in sections B and C of this chapter is used to characterize the behavior of the firm over the business cycle. The two main results are stated as theorems III-11 and III-12, the former characterizing the firm's behavior when future changes in the state of the firm's operating environment are known with certainty and the latter characterizing the firm's behavior under uncertainty.

Let the parameter θ denote the state of the firm's operating environment. It is again assumed that the firm acts as a price taker in all possible states of the environment. It is also assumed that changes in the state of the environment are instantaneously reflected in changes in the price of output p, so that $p = p(\theta_t)$, where θ_t denotes the state of the firm's operating environment at time t. If it is assumed that more favorable states of the environment are associated with higher product price, then $dp/d\theta > 0$ for all θ.

The business cycle is interpreted in terms of variations in θ as a sequence of periods in which θ is alternatively rising and falling. During the upswing the firm's operating environment becomes more favorable, and θ and p increase. During the downswing the firm's operating environment becomes less favorable, and θ and p decrease. Theorem III-11 characterizes the behavior of output (as reflected in changes in labor usage, L), investment, and managerial emoluments over the business cycle.

Theorem III-11

For the collective utility maximizer modeled in (11), both investment and managerial emoluments vary systematically over the business cycle: increasing during the upswing and decreasing during the downswing. Total output follows the same pattern provided $-\frac{R}{\partial U_1/\partial R} \cdot \frac{\partial(\partial U_1/\partial R)}{\partial R} < 1$ at the margin.

Proof

To prove the first statement it is sufficient to show that $dI/d\theta > 0$ and $dM/d\theta > 0$. Necessary condition (19) implicitly defines I as a function of θ. By the implicit function theorem,

$$\frac{dI}{d\theta} = -\frac{(\partial^2 U_1/\partial D^2)(1-\tau)p'(\theta)\, f(K, L)(-q(t))e^{-rt}}{(\partial^2 U_1/\partial D^2)[q(t)]^2 e^{-rt}} > 0 \,. \tag{135}$$

By the same argument applied to (20),

$$\frac{dM}{d\theta} = -\frac{(\partial^2 U_1/\partial D^2)(1-\tau)^2 p'(\theta) f(K, L)(-1)e^{-rt}}{[(\partial^2 U_1/\partial D^2)(1-\tau)^2 + (\partial^2 U_1/\partial M^2)]e^{-rt}} > 0 \,. \tag{136}$$

To prove the second statement it is sufficient to show that $dL/d\theta > 0$ when $-\frac{R}{\partial U_1/\partial R}\cdot\frac{\partial(\partial U_1/\partial R)}{\partial R} < 1$. It follows from (18) and the implicit function theorem that

$$\frac{dL}{d\theta} = -\frac{\partial^2 L_{\mu,t}/\partial L\partial\theta}{\partial^2 L_{\mu,t}/\partial L^2} \,, \tag{137}$$

where

$$\frac{\partial^2 L_{\mu,t}}{\partial L \partial \theta} = \left[\frac{\partial^2 U_1}{\partial R^2} \cdot p \cdot f(K, L) + \frac{\partial U_1}{\partial R} \right] p'(\theta) \frac{\partial f}{\partial L} e^{-rt}$$

$$+ \left(\frac{\partial U_1}{\partial D} e^{-rt} + \mu_1 \right)(1-\tau) p'(\theta) \frac{\partial f}{\partial L} \tag{138}$$

$$+ \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 p'(\theta) f(K, L)\left(p \cdot \frac{\partial f}{\partial L} - w\right) e^{-rt}$$

$$> 0 \ ,$$

provided

$$\frac{\partial^2 U_1}{\partial R^2} \cdot p \cdot f(K, L) + \frac{\partial U_1}{\partial R} \equiv \frac{\partial^2 U_1}{\partial R^2} R + \frac{\partial U_1}{\partial R} > 0 \ ,$$

and where

$$\frac{\partial^2 L_{\mu,t}}{\partial L^2} = \left[\frac{\partial^2 U_1}{\partial R^2}\left(p \cdot \frac{\partial f}{\partial L}\right)^2 + \frac{\partial U_1}{\partial R} \cdot p \cdot \frac{\partial^2 f}{\partial L^2} + \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 \left(p \cdot \frac{\partial f}{\partial L} - w\right)^2 \right.$$

$$\left. + \frac{\partial U_1}{\partial D}(1-\tau) p \cdot \frac{\partial^2 f}{\partial L^2} \right] e^{-rt} + \mu_1 (1-\tau) p \cdot \frac{\partial^2 f}{\partial L^2} \tag{139}$$

$$< 0 \ .$$

Combining (138) and (139) gives the desired result. Q.E.D.

The significance of theorem III-11 is that the behavior of the collective utility maximizer varies systematically over the business cycle. During the upswing, with price increasing, its opportunities for exercising managerial discretion increase. In particular, managerial emoluments increase. At the same time, both total revenue and investment increase, and with π_0 fixed, the firm can increase revenue and managerial emoluments while still satisfying the profit constraint. Moreover, the increased investment signifies growth, which is an important source of managerial utility. It is also possible - and indeed likely - that the firm will also increase dividend payments to shareholders so that they benefit from the improved business environment as well.

During the downswing, however, price falls and managers' discretion to pursue their own objectives decreases as the need to generate sufficient earnings to meet the profit constraint becomes increasingly important. In particular, managerial emoluments fall.[140] This notion of a shrinking opportunity set for managers is best illustrated by reexpressing the minimum net income constraint (7) as a constraint on managerial emoluments, as was done in section C. This procedure leads to the constraint

$$M \leq p(\theta)\cdot f(K, L) - wL - q\delta K - \pi_o/(1-\tau) \equiv \hat{M}(K, L, \pi_o, \theta) \; . \tag{140}$$

From (140) it is easily seen that $\partial\hat{M}/\partial\theta = p'(\theta)f(K, L) > 0$. During the upswing $\hat{M}$ increases with θ and so does the degree of managerial discretion, while during the downswing $\hat{M}$ decreases with θ and so does the degree of managerial discretion.

Before proving the second main result of this section, the requirement that

$$-\frac{R}{\partial U_1/\partial R}\cdot\frac{\partial(\partial U_1/\partial R)}{\partial R} < 1 \tag{141}$$

at the margin in theorem III-11 deserves comment. The left-hand side of (141) can be interpreted as the elasticity of the marginal utility of total revenue with respect to a change in total revenue. According to theorem III-11, output will vary cyclically if the elasticity of the marginal utility of total revenue with respect to a change in total revenue is less than one at the margin. If the inequality in (141) is reversed, however, output may fall as the firm's operating environment improves - i.e. output may behave countercyclically. This happens when $-\frac{\partial^2 U_1}{\partial R^2}$ is 'large', which means that the marginal utility of revenue is falling 'rapidly'. In effect, then, rapidly falling marginal utility of revenue causes the firm to restrain the rate of growth of revenue so that dividends and managerial emoluments (which are relatively more important at the margin due to the rapid decrease in $\partial U_1/\partial R$) can be increased. The direction of the inequality in (141) can be viewed as an indicator of whether this shift in objectives has occurred.

The second main result of this section is stated as the following theorem:

Theorem III-12

For the expected collective utility maximizer modeled in (79), both investment and managerial emoluments vary systematically over the business cycle: increasing during the upswing and decreasing during the downswing. Total output follows the same pattern provided

$$E\{[\frac{\partial^2 U_1}{\partial R^2} R + \frac{\partial U_1}{\partial R}]\frac{\partial f}{\partial L} + \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 f(K,L)[(p(\theta) + \gamma)\frac{\partial f}{\partial L} - w]\}e^{-rt} > 0 ,$$

where γ is a shift parameter.

Remark

In section C the parameter θ was used to denote the distribution of p at each time t. To indicate the change in the firm's operating environment under uncertainty the additive shift parameter γ is added to $p(\theta)$. This has the effect of shifting the entire probability distribution of p without altering its shape.[141]

After adding the shift parameter, price is given by $p = p(\theta) + \gamma$.

Proof of Theorem III-12

The method of proof is similar to that used in proving theorem III-11. It follows from (86) and the implicit function theorem that

$$\frac{dI}{d\gamma} = -\frac{E\{(\partial^2 U_1/\partial D^2)(1-\tau)f(K,L)(-q(t))\}e^{-rt}}{E\{(\partial^2 U_1/\partial D^2)[q(t)]^2\}e^{-rt}} > 0 ,$$

and it follows from (87) and the implicit function theorem that

$$\frac{dM}{d\gamma} = -\frac{E\{(\partial^2 U_1/\partial D^2)(1-\tau)^2 f(K,L)(-1)\}e^{-rt}}{E\{(\partial^2 U_1/\partial D^2)(1-\tau)^2 + (\partial^2 U_1/\partial M^2)\}e^{-rt}} > 0 .$$

It follows from (85) and the implicit function theorem that

$$\frac{dL}{d\gamma} = -\frac{\partial^2 L_\mu/\partial L\partial\gamma}{\partial^2 L_\mu/\partial L^2} , \tag{142}$$

where[142]

$$\frac{\partial^2 L_\mu}{\partial L\partial\gamma} = E\{[\frac{\partial^2 U_1}{\partial R^2}[p(\theta) + \gamma]f(K,L) + \frac{\partial U_1}{\partial R}]\frac{\partial f}{\partial L}\}e^{-rt}$$

$$+ E\{\frac{\partial U_1}{\partial D}(1-\tau)\frac{\partial f}{\partial L} + \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 f(K,L)\times \tag{143}$$

$$[(p(\theta) + \gamma)\frac{\partial f}{\partial L} - w]\}e^{-rt} + \mu_1(\partial^2\hat{M}/\partial L\partial\gamma)$$

$$> 0$$

provided

$$E\{[\frac{\partial^2 U_1}{\partial R^2}[p(\theta) + \gamma]f(K,L) + \frac{\partial U_1}{\partial R}]\frac{\partial f}{\partial L}$$

$$+ \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 f(K,L)[(p(\theta) + \gamma)\frac{\partial f}{\partial L} - w]\}e^{-rt}$$

$$> 0, \tag{144}$$

and where

$$\frac{\partial^2 L_\mu}{\partial L^2} = E\{\frac{\partial^2 U_1}{\partial R^2}(p\cdot\frac{\partial f}{\partial L})^2 + \frac{\partial U_1}{\partial R}\cdot p\cdot\frac{\partial^2 f}{\partial L^2} + \frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2(p\cdot\frac{\partial f}{\partial L} - w)^2$$
$$+ \frac{\partial U_1}{\partial D}(1-\tau)p\cdot\frac{\partial^2 f}{\partial L^2}\}e^{-rt} + \mu_1(\partial^2\hat{M}/\partial L^2) < 0 \ . \tag{145}$$

Combining (143) and (145) gives $dL/d\gamma > 0$. Q.E.D.

It should be noted that (144) is analogous to (141), though the latter is more complicated in form due to the presence of the expectations operator and due also to the fact that $(p(\theta) + \gamma)\frac{\partial f}{\partial L} - w < 0$ cannot be assured under uncertainty.

This section has modeled the behavior of the firm over the business cycle.[143] As in the two-state model developed in section D, it was shown that the behavior of the firm varies systematically over the business cycle. The importance of these results (theorems III-11 and III-12) is that they suggest a possible reconciliation of the traditional and managerial theories of the firm. During the upswing managers have relatively more discretion than they do during the downswing, since the profit constraint is more easily satisfied, and managers are freer to pursue their own objectives. During the downswing their discretion is restricted due to the need to generate sufficient earnings with which to satisfy the minimum profit constraint. Hence, it is this writer's opinion that the 'pure' managerial models are best interpreted as models of the behavior of the firm during the upswing, and that the traditional models are best interpreted as models of the behavior of the firm during the downswing.

F. CHAPTER SUMMARY

In this chapter the modern corporate enterprise was modeled, first under certainty and then under uncertainty, as a constrained optimal control problem. The firm was modeled as a discounted collective utility maximizer, and under uncertainty, as a discounted expected collective utility maximizer (in each case over a finite planning horizon), where collective utility was expressed as a function of total sales revenue, total dividends paid, and managerial emoluments, (and also indirectly, growth), and where the trade offs embodied in the collective utility function were assumed to be established by the corporate board of directors. Collective utility maximization was modeled as taking place subject to an exogenously determined minimum net income constraint, and subject to an appropriately formulated probabilistic minimum net income constraint in the uncertainty case.

The model of the firm under certainty, the optimal operating policies implied by the model, and the sensitivity of these policies to changes in key parameters were presented in section B. It was shown that such a firm would behave like a Baumol sales maximizer and would tend to produce more output than a short run profit maximizer. It was also shown that the equilibrium operating policies selected by the firm could be used to characterize an implicit ranking at the margin of the sales revenue, dividends, and managerial emoluments objectives. It was further shown that the extent of managerial discretion, as indicated by whether or not the profit constraint is binding, had a significant impact on the behavior of the firm, and that when the profit constraint is binding, the effect of a change in the tax rate on the behavior of the firm can be expressed as a Hicks-Slutsky-type equation.

The model of the firm under certainty was extended to incorporate uncertainty in section C. It was shown that the presence of uncertainty causes the risk averse firm to produce less output than it would under certainty, but that for the quantity-setting firm at least, the presence of uncertainty does not alter the firm's expansion path. It was also shown that an increase in the required probability of meeting the profit constraint would cause the firm to reduce output, and further, that changes in either the minimum net income level or the tax rate would elicit the same qualitative responses under uncertainty as they did in the certainty case.

Two models of the behavior of the firm that can be interpreted in terms of the business cycle were developed in sections D and E. The first, a two-state Markov model, generalized a model previously developed by O.E. Williamson. The second, a multi-state model, was based more directly on the models developed in sections B and C of this chapter. Each model was used to show that the behavior of the (expected) collective utility maximizer tends to vary systematically over the business cycle, with the firm's behavior during the upswing more closely fitting that which would be predicted on the basis of the managerial models and with its behavior during the downswing more closely fitting that which would be predicted on the basis of the traditional models of the firm.

This chapter was mainly concerned with modeling the behavior of the firm over the business cycle - or at least with modeling the firm's behavior subject to a pattern of variation in the state of its operating environment that could be interpreted in terms of the business cycle. To focus on the impact of these external factors in real terms, the analysis abstracted from both financial factors and the role of resource allocation within the firm. The purpose of the next two chapters is to explore the implications of each of these for the behavior of the firm of the type modeled in this chapter.

CHAPTER THREE FOOTNOTES

1. This introductory section is intended to serve as both an executive summary of chapter two - in order that chapter three be essentially self-contained - and an introduction to this chapter's discussion of the author's research results concerning the behavior of the firm over the business cycle.

2. Hereafter in this paper the 'traditional' models and the 'modern traditional' models of the firm will be referred to collectively as the 'traditional theory of the firm'.

3. Such models involve a single time period and typically assume certainty. See sections B through E of chapter two. See also subsection 2 of section I of chapter two for a discussion of a profit maximization model due to Vickers that permits uncertainty, but subsumes its impact within the functional form of the average interest rate on debt funtion.

4. Such models typically involve a single time period and normally assume certainty. See sections F and L of chapter two. See also subsection 2 of section I of chapter two for a discussion of a single period stock market value maximization model.

5. See the discussion of the Lintner model in section J of chapter two.

6. See the discussion of the Meyer model in subsection 3 of section K of chapter two.

7. See the discussion of Leland's models of the quantity-setting firm and the price-setting firm in subsection 2 of section K of chapter two. Under the assumption that owners are risk neutral, this objective function can be simplified to one of maximizing expected profit.

8. See the discussion of Leland's model of the firm in the context of stock market equilibrium in subsection 4 of section K of chapter two.

9. See footnotes 4, 6, and 7 of chapter one for references to empirical studies.

10. See the discussion of the Baumol sales maximization model in subsection 1a of section G of chapter two.

11. See the discussions of the Baumol growth maximization model in subsection 1b and the Marris growth maximization model in subsection 2a of section G and the discussion of the Herendeen model in subsection 3 of section I of chapter two.

12. See the discussion of the Marris utility maximization model in subsection 2b of section G of chapter two.

13. See the discussion of the O.E. Williamson model in subsection 3 of section G of chapter two.

14. See the discussion of Leland's managerial model in subsection 3b of section L of chapter two.

15. See section G, and in particular subsections 2c and 4, of chapter two for a discussion of some of this empirical evidence.

16. As, for example, the study by Kuehn that attempts to determine whether firms maximize profits or growth. Kuehn, <u>Takeovers and the Theory of the Firm: An Empirical Analysis for the United Kingdom, 1957-1969</u>, <u>op. cit.</u>

17. O.E. Williamson, <u>A Dynamic Stochastic Theory of Managerial Behavior</u>, <u>op. cit.</u>

18. Solow, <u>op. cit.</u>

19. Leland, <u>Why Profit Maximization May Be A Better Assumption Than You Think</u>, <u>op. cit.</u> The managerial model developed by Leland is discussed in subsection 3b of section L of chapter two.

20. Wong, <u>op. cit.</u> Wong's model is discussed in subsection 3a of section L of chapter two.

21. See section H of chapter two.

22. See also Machlup, <u>op. cit.</u>, on this point. Indeed, if the behavior of firms under objectives other than profit (or value) maximization is consistent with profit- (or value-) maximizing behavior, the assumption of profit (or value) maximization might have the advantage of greater mathematical tractability and might therefore permit a wider range of phenomena to be modeled.

23. During the long upswing of the 1960s, economists became complacent regarding the business cycle. There was a general feeling that the careful application of Keynesian demand management policies would strongly dampen the amplitude of cyclical fluctuations. But as a recent conference of economists concluded - and as the recent recession would seem to bear out - the business cycle is not (yet at least) obsolete. See M. Bronfenbrenner, ed., <u>Is the Business Cycle Obsolete?</u> (Wiley-Interscience;

New York; 1969). See also "The business cycle is alive and well," Business Week (April 19, 1976).

24. One of the classic works relating to the business cycle is J. Duesenberry, Business Cycles and Economic Growth (McGraw-Hill; New York; 1958). A discussion of various models of the business cycle can be found in S. Bober, The Economics of Cycles and Growth (Wiley; New York; 1968). The history of U.S. business cycles, including the 'long cycle' of 1961-70, can be found in M.W. Lee, Macroeconomics: Fluctuations, Growth, and Stability, 5th ed. (Irwin; Homewood, Ill; 1971). The business cycle, particularly as it affects manufacturing firms, is discussed in T. Hultgren, Cost, Prices, and Profits: Their Cyclical Relations (Columbia University Press; New York; 1965).

25. The recent post-recession upsurge in profits has been heralded widely in the business literature. See, for example, "First Quarter's Spurt In Corporate Earnings Is Steepest in 17 Years," Wall Street Journal (April 29, 1976); "Profits: Better than expected," Business Week (May 17, 1976); and "Profits are growing across the board," Business Week (August 2, 1976). The upswing has also improved the sales outlook for most industries. See, for example, "Semiconductor Makers See the Recovery From '75 Slump Gaining Strength in '77," Wall Street Journal (March 16, 1977), and "Ford Motor Chairman Says Neither Profit Nor Market Share Is Strong as Possible," Wall Street Journal (May 14, 1976).

26. See, for example, "Mergers, Acquisitions Come Back Into Style - But the Style Is New," Wall Street Journal (April 28, 1976), and "A seller's market in executive talent," Business Week (July 5, 1976).

27. See, for example, "Westinghouse's Ongoing Business Posts Profit Drop," Wall Street Journal (January 30, 1975); "GE: Not Recession Proof, But Recession Resistant," Forbes (March 15, 1975); and "The recession balks Genesco's turnaround," Business Week (July 7, 1975).

28. See, for example, "Capital Budgets Are Cut by Firms Due to Recession," Wall Street Journal (March 10, 1975), and "Firms Spend Warily In Planning Spending On Plant, Equipment," Wall Street Journal (November 10, 1975).

29. See, for example, "A & P to Close Third Of Its Stores, Set $195 Million Reserve," Wall Street Journal (March 14, 1975); "Westinghouse Moves To Halt Old Drains And Avoid New Ones," Wall Street Journal (March 7, 1975); "Firms Drop Operations To Lower Their Costs And Preserve Capital," Wall Street Journal (March 17, 1975); and "How High-Flying Firm Came Crashing Down And Survived the Fall," Wall Street Journal (August 19, 1975).

30. See, for example, Mergers, Acquisitions Come Back Into Style - But the Style Is New, op. cit., and A seller's market in executive talent, op. cit.

31. See, for example, "Hercules: A plan that failed," Business Week (December 1, 1975), and "More Firms Slow Drive For Growth, Bid to Lift Return on Investment," Wall Street Journal (December 16, 1975). This tendency is more pronounced among smaller, less diversified companies. See "Small Business: The maddening struggle to survive," Business Week (June 30, 1975).

32. See, for example, "How American Standard cured its conglomeritis," Business Week (September 28, 1974); "Bludhorn the raider as elder statesman," Business Week (January 20, 1975); Westinghouse Moves To Halt Old Drains And Avoid New Ones, op. cit.; "How Rucker cured its conglomerate fever," Business Week (April 7, 1975); "Why the profits vanished at Singer," Business Week (June 30, 1975); "Bringing Order to a Billion-Dollar Empire," Business Week (September 8, 1975); "With A Little Bit Of Luck," Forbes (December 15, 1975); and "The Hard Road of the Food Processors," Business Week (March 8, 1976). In each case the emphasis shifted from growth to profits.

33. R.M. Cyert, "Oligopoly Price Behavior and the Business Cycle," Journal of Political Economy (vol. 63; no. 1; February 1955), pp. 41-51; W.A.H. Godley and W.D. Nordhaus, "Pricing in the Trade Cycle," Economic Journal (vol. 82; no. 327; September 1972), pp. 853-882; R.L. Nelson, "Business Cycle Factors in the Choice between Internal and External Growth," in W. Alberts and J. Segall, eds., The Corporate Merger (University of Chicago Press; Chicago; 1966), pp. 52-66; and D.J. Aigner and C.M. Sprenkle, "On Optimal Financing of Cyclical Cash Needs," Journal of Finance (vol. 28; no. 5; December 1973), pp. 1249-1254.

34. O.E. Williamson, A Dynamic Stochastic Theory of Managerial Behavior, op. cit.; K.J. Arrow, Optimal Capital Policy with Irreversible Investment, op. cit. (see subsection 2 of section L of chapter two); and E.M. Birch and C.D. Siebert, "Uncertainty, Permanent Demand, and Investment Behavior," American Economic Review (vol. 66; no. 1; March 1976), pp. 15-27.

35. R. Stone, "A Dynamic Model of Demand," in H. Townsend, ed., Price Theory (Penguin; Middlesex, England; 1971), pp. 155-169, and A.A. Alchian, "Costs and Outputs," in Townsend, Price Theory, pp. 228-249.

36. L.J. Maccini, "An Aggregative Dynamic Model of Short-Run Price and Output Behavior," Quarterly Journal of Economics (vol. 90; no. 2; May 1976), pp. 177-196, and E.A. Kervinen, "An Adaptive Production Planning Model for Seasonal Goods," Management Sciences Research Report No. 355 (Graduate School of Industrial Administration, Carnegie-Mellon University; Pittsburgh; December 1974).

37. See sections J and K of chapter two and the references provided therein. See also A.J. Douglas, "Stochastic Returns and the Theory of the Firm," American Economic Review (vol. 63; no. 2; May 1973), pp. 129-133; Birch and Siebert, op. cit.; M.C. Gupta, "Optimal Financing Policy for a Firm with Uncertain Fund Requirements," Journal of Financial and Quantitative Analysis (vol. 8; no. 5; December 1973), pp. 731-747; and S. Wu, R. Rozek, and D. Dutton, "The Effects of Uncertainty on the Firm's Demand for Money," Working Paper Series No. 75-13 (College of Business Administration, The University of Iowa; Iowa City; June 1975).

38. There has been some debate in recent years as to how powerful boards of directors really are. See the discussion of this point in chapter one. See also Brown, op. cit., and R. Malik, And Tomorrow ... The World? Inside IBM (Millington; London; 1975). The latter points out that the real decision-making power at IBM in recent years has resided in the Management Review Committee, rather than the Board of Directors. However, for many years this committee included a member of the Watson family, who were the major shareholders. Hence, even where the board of directors is relatively weak, this does not imply that shareholder objectives are ignored. The form of the collective utility function presented below is general enough to permit different implicit weightings to be attached to shareholder and managerial objectives, so that different degrees of 'management domination' of the board of directors, management review committee, or whatever, and even the extreme case of a board of directors dominated by an autocratic chairman, can be accommodated. It is assumed, however, that regardless of the composition of the primary decision-making body, shareholder objectives are considered explicitly when then firm's policies are formulated.

39. As in the Baumol sales maximization model discussed in subsection 1a of section G of chapter two.

40. See the introduction to section G of chapter two.

41. As in the O.E. Williamson model discussed in subsection 3 of section G of chapter two.

42. O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., p. 1035. See also subsection 3 of section G of chapter two.

43. As in the Baumol sales growth maximization model discussed in subsection 1b of section G of chapter two and in the Marris growth maximization and managerial utility maximization models discussed in subsection 2 of section G of chapter two. Unlike these models, however, the model developed below will not require steady state growth.

44. See the introduction to section G of chapter two.

45. As in the modern traditional models that assume stock market value maximization. See sections F and L of chapter two.

46. The reason for including this second source, rather than merely conjecturing present and future dividends (suitably discounted) over an infinite time horizon, will be made clear below.

47. However, dividend payments could be a direct source of satisfaction if top managers and directors own shares of the company and if the dividends they receive form a significant portion of their incomes. In this case, of course, the top managers and directors who have significant share holdings might be expected to make personal trade offs of the sort embodied in the collective utility function U_1 given below in (1).

48. See the introduction to subsection 2 of section G of chapter two. Marris incorporated this factor directly into the managerial utility function in the form of the firm's valuation ratio. See subsection 2b of section G of chapter two.

49. See, for example, "Battle of Titans," Newsweek (June 10, 1974); "Ruthlessness By The Rules," Forbes (February 1, 1976); and "Posner Prowls Again," Newsweek (August 9, 1976).

50. Under certainty and perfect capital markets, the share price will, in stock market equilibrium, equal the discounted stream of dividends that would be paid to the holder of that share. See subsection 1 of section I of chapter two.

51. A common ploy used to thwart a takeover attempt is to increase the annual dividend rate (per share), as Foremost-McKesson Inc. recently did (in spite of lower quarterly net income). See "Foremost-McKesson Meeting Dominated By Absent V. Posner," Wall Street Journal (July 30, 1976). Microdot Inc. employed the same tactic. See "Microdot struggles to stay single," Business Week (February 2, 1976).

52. A similar approach (a 'managerial-owner preference function') has been suggested by Albin and Alcaly. See P.S. Albin and R.E. Alcaly, "Corporate Objectives and the Economy: Systematic Shifts between Growth and Profit Goals," Journal of Economic Issues (vol. 10; no. 2; June 1976), p. 277.

53. Arrow, Social Choice and Individual Values, op. cit. See also Heal, op. cit., ch. 2.

54. The transitivity property requires that if society prefers alternative A to alternative B and if it also prefers alternative B to alternative C, then it must also prefer alternative A to alternative C. That is, preferences must be consistent. However, it is easily shown that majority voting - the most obvious candidate for a social choice rule - fails to satisfy this property. Ibid., pp. 35-36.

55. Ibid., pp. 29-34.

56. Ibid., p. 59.

57. It should be noted that in the certainty case the decisions made at time zero for any time t, $0 < t \leq T$, are the same as those that would be made at time t. That is, open loop control (all decisions made at $t = 0$) and closed loop control (decisions made sequentially) yield identical results under certainty. See Intriligator, op. cit., pp. 299-302, on this point.

58. If these functions change in a real time sense, then open loop control and closed loop control do not yield identical results in terms of actual behavior. See preceding footnote. In that case interpreting the models developed below as planning models becomes important.

59. Sandmo, op. cit., pp. 65-66.

60. How effective boards of directors are in promoting the interests of shareholders (i.e. to what extent they are selected by and dominated by top management) is a moot point. See "Outside Directors Get More Careful, Tougher After Payoff Scandals," Wall Street Journal (March 24, 1976); "Annual Meeting Time," Forbes (April 15, 1976); and Brown, op. cit. Differences in effectiveness would, of course, affect the specific functional forms of U_1 and U_2, and in particular, the trade offs between dividends and other objectives.

61. The derivation of (2) follows. From the definition of g,

$$K(T) = K(0)\cdot(1 + g)^T . \qquad (*)$$

Dividing each side of (*) by K(0), taking logarithms, and solving for g yields (2).

62. Hirshleifer, Investment, Interest, and Capital, op. cit., p. 226, and Sandmo, op. cit., p. 65.

63. von Neumann and Morgenstern, op. cit., pp. 15-31, and Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 216-224.

64. Ibid., pp. 224-226.

65. It should be repeated that the functional forms U_1 and U_2 are assumed to be stable during the planning cycle.

They may change in a real time sense - in a manner suggested by the behavioralists - when the planning cycle repeats. See footnotes 57 and 58.

66. In particular, the assumption that f has a full set of continuous second partial derivatives implies that f is continuous with respect to K and with respect to L and that $\frac{\partial^2 f}{\partial L \partial K} = \frac{\partial^2 f}{\partial K \partial L}$. In addition, it is normally assumed that the production function f is strictly concave (or at least locally strictly concave) in order to satisfy the sufficiency conditions for an optimum. Strict concavity requires that

$$\frac{\partial^2 f}{\partial L^2} < 0 \qquad \frac{\partial^2 f}{\partial K^2} < 0 \qquad \frac{\partial^2 f}{\partial L^2}\frac{\partial^2 f}{\partial K^2} > \left(\frac{\partial^2 f}{\partial L \partial K}\right)^2 .$$

67. The direct contribution of managers to production is allowed for in the composite labor index L(t) and their opportunity cost, the market-determined 'wage rate' for managers, is allowed for in the composite wage index w(t). The definition of managerial emoluments in the text is consistent with O.E. Williamson's. O.E. Williamson, Managerial Discretion and Business Behavior, op. cit., p. 1035.

68. It is assumed here that depreciation for tax purposes is figured on this basis only, so that the amount of depreciation expense is equal to the product of the amount of depreciation in physical units, $\delta \cdot K(t)$, and the unit value of capital goods, q(t). It should be noted that replacement cost accounting is not generally practiced in the United States, although it is used in The Netherlands and the United Kingdom. See A.J.H. Enthoven, "Replacement-Value Accounting: Wave of the Future?," Harvard Business Review (January-February 1976), and "Inflation Accounting: Report of the Inflation Accounting Committee," Cmnd Document 6225 (H.M. Stationery Office; London; 1975). Moreover, a recent decision by the Securities and Exchange Commission requiring companies to provide replacement cost data in the notes to their financial statements, as well as the strong support that exists for replacement cost accounting in the business and accounting communities, suggests that it may become general accounting practice in the United States in the not-too-distant future. See Securities and Exchange Commission Accounting Series Release No. 190 (Washington, D.C.; March 1976) and "SEC Adopts Disclosure Rules for Replacement Costs," Journal of Accountancy (May 1976), pp. 11-12. See also J.C. Burton, "Financial Reporting in an Age of Inflation," Journal of Accountancy (February 1975); F.T. Weston, "Adjust Your Accounting for Inflation," Harvard Business Review (January-February 1975); R.F. Vancil, "Inflation Accounting - The Great Controversy," Harvard Business Review (March-April 1976); and "How Recently Issued Accounting Rules Are Affecting Corporate Annual Reports," Wall Street Journal (March 30, 1977). Therefore, the assumption of replacement cost accounting is not, in the opinion of this writer, unreasonable.

69. It is assumed in this chapter that the firm owns all its capital and that it has no bonds outstanding, so that no rental expense and no interest expense show up in the firm's income statement.

70. The importance of (7) will become clearer in chapter four where the firm will also be required to generate sufficient pretax income to meet bond interest obligations.

71. See subsection G of chapter two.

72. See R.W. Clower, "A Reconsideration of the Microfoundations of Monetary Theory," Western Economic Journal (vol. 6; no. 1; December 1967), pp. 1-8. Under uncertainty, however, the firm may have several motives for holding cash. Traditionally, these motives have been categorized as a transactions demand for cash (i.e. for use in normal transactions such as purchasing inputs), a precautionary demand for cash (e.g. to pay bond interest when pretax income is insufficient, and to thereby prevent bankruptcy), and a speculative demand for cash (e.g. to hold cash in anticipation of price changes favorable to the firm and its planned purchase of some capital asset). D. Patinkin, Money, Interest, and Prices, 2nd ed. (Harper & Row; New York; 1965). The role of cash is explored in chapter four of this thesis.

73. That is, $\partial U_1/\partial\pi \neq 0$ and $\frac{\partial U_1/\partial R}{\partial U_1/\partial\pi}$ and $\frac{\partial U_1/\partial M}{\partial U_1/\partial\pi}$ both exist for all $R \geq 0$ and for all $M \geq 0$.

74. As shown below, the firm modeled in (11), like the Baumol sales maximizer, will tend to produce more output than a short run profit maximizer. Such a firm could increase profits by reducing output, thereby having additional funds with which to purchase new plant and equipment. Therein lies the trade off between revenue and capital embodied in the Hamiltonian (12).

75. The nonnegativity constraint on the firm's capital stock at each time t might be considered explicitly if, for example, the firm were permitted to go bankrupt (i.e. to jail), with $K(t) = 0$ used as the indicator of bankruptcy.

76. Arrow, Applications of Control Theory to Economic Growth, op. cit., and Takayama, op. cit., pp. 646-651.

77. Ibid., pp. 648-649. As in the case of static optimization, conditions (13)-(15) are necessary provided the "constraint qualification" holds. For conditions that provide the constraint qualification see the lemma preceding theorem 8.C.1 in ibid., p. 648.

78. This is perfectly analogous to the interpretation given the maximization of the Hamiltonians in section L of chapter two.

79. Note that even though the Lagrangian (17) applies to a single point in time, whereas the Lagrangian (13) applies to the period $0 \leq t \leq T$, the value of μ_1 must be the same for each time t in both (13) and (17) when the firm is in multiperiod equilibrium. Hence, there are no conceptual difficulties involved in using the same symbol in (13) and (17).

80. Note that this follows if it is assumed (as is normally done) that the production function is strictly concave. It also follows under weaker assumptions (e.g. local strict concavity). See footnote 66.

81. That is, one dollar less in managerial emoluments enables the firm to pay out an additional $1 - \tau$ dollars in dividends (the other τ dollars must be paid in taxes).

82. See subsection 1a of section G in chapter two of this thesis.

83. Note what happens when π_o approaches the maximum short run level of profits. Since μ_1 in (31) can be expressed as

$$\mu_1 = \frac{(\partial U_1/\partial R)p\cdot\frac{\partial f}{\partial L}\, e^{-rt}}{(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)} - \frac{\partial U_1}{\partial D}\, e^{-rt}, \qquad (*)$$

it follows that $\mu_1 \to \infty$ as $p\cdot\frac{\partial f}{\partial L} - w \to 0$. Of course, (*) is undefined when $p\cdot\frac{\partial f}{\partial L} - w = 0$ (the necessary condition for selecting the short run profit maximizing level of labor usage). The interpretation of (*) is that the implicit price of meeting the constraint increases as π_o increases, becoming infinite as π_o approaches the short run maximum level of total profit.

84. In the intermediate case, $\frac{\mu_2}{\mu_1} = (1 - \tau)$ and $\frac{\partial H}{\partial M} = 0$. Both constraints are binding, and by coincidence, the Hamiltonian is simultaneously a maximum with respect to M.

85. Recall that μ_1 and μ_2 have discount factors built into them. That is, μ_1 and μ_2 are shadow prices expressed in terms of present value. The factor e^{rt} "undoes the discounting" in order to convert the expression into one in terms of current value units.

86. Note that the inequalities that result in cases (i)-(iv) are insensitive to the particular utility scale adopted for U_1. That is, since preference orderings are invariant under monotonically increasing transformations of the utility scale, the results summarized in theorem III-1 are independent of the utility scale. This also means, however, that no quantitative significance can be attached to differences of marginal utilities, as these differences are scale sensitive.

87. Note that this procedure has the effect of converting from an ordinal scale, e.g. $\partial U/\partial D$, to a ratio scale, e.g. $(\partial U/\partial D)/(\partial U/\partial M)$. This compensates for the arbitrariness of the utility scale (see footnote 86) since any monotonically increasing transformation of U will affect numerator and denominator proportionately, and therefore not affect the value of the ratio.

88. It deserves to be emphasized that the orderings of objectives given below hold at the margin when the firm is in equilibrium. That is, the orderings describe the firm's relative preferences with regard to marginal increments in the values of the arguments of its utility function when the firm is in equilibrium. They do not imply, for example, that under some circumstances (i.e. $-(p\cdot\frac{\partial f}{\partial L} - w) < p\cdot\frac{\partial f}{\partial L}$) the firm would always most prefer an increment in dividends to equal increments in total revenue and managerial emoluments regardless of the values of these other variables.

89. Recall that $0 < p\cdot\frac{\partial f}{\partial L} < w$, so that $p\cdot\frac{\partial f}{\partial L} - w < 0$, in each of the four cases considered above.

90. The reason for introducing this concept is that it will be needed below in interpreting the possible preference relations between R and D stated in theorem III-2.

91. Or equivalently, $\frac{1}{2}w < p\cdot\frac{\partial f}{\partial L}$, i.e. the marginal revenue product of labor exceeds one half the wage rate. This, together with lemma III-1, implies that when $-(p\cdot\frac{\partial f}{\partial L} - w) < p\cdot\frac{\partial f}{\partial L}$ the marginal revenue product of labor is less than the wage rate but greater than one half the wage rate. Similarly,

$$-(p\cdot\frac{\partial f}{\partial L} - w) = p\,\frac{\partial f}{\partial L} \Longleftrightarrow \tfrac{1}{2}w = p\cdot\frac{\partial f}{\partial L}$$

$$p\,\frac{\partial f}{\partial L} < -(p\cdot\frac{\partial f}{\partial L} - w) \Longleftrightarrow \tfrac{1}{2}w > p\cdot\frac{\partial f}{\partial L}$$

In the latter case, the marginal revenue product of labor is less than one half the wage rate.

92. That is, the analysis proceeds in terms of the inequality $-(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)\{\lesseqgtr\}p\cdot\frac{\partial f}{\partial L}$, which is equivalent to $(\frac{1}{1-\tau})p\cdot\frac{\partial f}{\partial L}\{\lesseqgtr\} -(p\cdot\frac{\partial f}{\partial L} - w)$ in (iii)-(v) in theorem III-2. Making this conversion simplifies the exposition.

93. Arrow, Applications of Control Theory to Economic Growth, op. cit., pp. 88-89, and Takayama, op. cit., pp. 621-622.

94. See equation (281) in chapter two. Here the term 'cost of capital' is used in the sense of the economist's

'price' of capital, rather than in the sense of the financial 'cost of raising money capital'.

95. This implies that the firm is rational in the sense that it employs capital only in the region within which the marginal physical product of capital is positive - a result that also has been shown to hold for the short run profit maximizer. See section B of chapter two.

96. Similarly, note that for case (i) considered above, (24) can be rewritten as

$$w = p\cdot\frac{\partial f}{\partial L} + \left[\left(\frac{1}{1-\tau}\right)p\cdot\frac{\partial f}{\partial L}\left(\frac{\partial U_1/\partial R}{\partial U_1/\partial D}\right) \right], \qquad (*)$$

which is analogous to (40). The expression in brackets on the right-hand side of (*) can be interpreted as the pretax income equivalent (in terms of collective utility) of a marginal change in sales revenue. The right-hand side of (*) can be interpreted as the adjusted marginal revenue product of labor (which is figured pretax) and condition (*) is interpreted as the requirement that in equilibrium the firm modeled in (11) will hire labor up to the point at which its adjusted marginal revenue product just equals its marginal cost (under perfect competition in factor markets, the wage rate). Cases (ii)-(iv) yield the same condition, but with $\partial U_1/\partial D + \mu_1 e^{rt}$ in place of $\partial U_1/\partial D$ in (*) when the minimum profit constraint is binding. Thus (*) above and (40) in the text are the logical extensions of the neoclassical equilibrium criteria to the case of the firm modeled in (11).

97. For ease of exposition only case (i) developed above is considered here. The analysis for the other cases would proceed analogously.

98. See equation (284) in chapter two.

99. Recall that it has been assumed that the firm owns its capital, rather than rents it, so that there is no interest cost shown on the firm's income statement. See table III-1. If the firm rented some portion of its capital, then the firm would be concerned about this cost in addition to $(1 - \tau)q\delta$ when the constraint $\pi \geq \pi_o$ was binding.

100. Another interpretation of theorem III-3 is that in long run equilibrium, in which prices, interest rates, input levels, output levels, etc., would be determined such that each firm's input mix would lead to minimum cost in both the accounting sense and the economic sense, the condition $r = \dot{q}/q$ implies that equilibrium interest rates are determined by real factors and not by monetary factors (i.e. the equilibrium price of capital goods, q, expressed as an interest rate, $\dot{q}/q$, determines the interest rate r in general economic equilibrium).

101. For completeness, it should be noted that sufficiency conditions could be established from a theorem proved by Kamien and Schwartz. See M.I. Kamien and N.L. Schwartz, "Sufficient Conditions in Optimal Control Theory," Journal of Economic Theory (vol. 3; no. 2; June 1971), pp. 207-214. This was not done in the text since the list of conditions is lengthy and is not for the most part amenable to economic interpretation. It should also be noted that Kamien's and Schwartz's result is more general than Mangasarian's, which was referred to earlier in this thesis. Mangasarian, op. cit.

102. A similar result has been obtained by Yarrow via a geometric approach using Baumol's sales maximization model. See G.K. Yarrow, "On the Predictions of Managerial Theories of the Firm," Journal of Industrial Economics (vol. 24; no. 4; June 1976), pp. 267-279.

103. Note that

$$r > \dot{q}/q \Leftrightarrow r\dot{q} - q > 0 \Longleftrightarrow \frac{(1-\tau)q\delta}{i} < 1 .$$

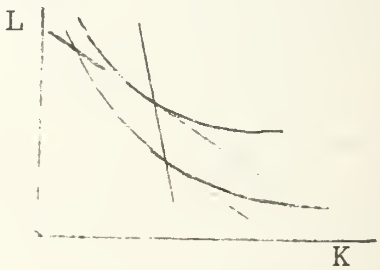
If also

$$\partial U_1/\partial D < \mu_1 e^{rt} ,$$

then it follows from (47) that

$$\partial L/\partial K < 0 ,$$

and the firm's expansion path is negatively sloped, as in the figure to the right.



Figure

104. Note that conditions (58) are necessary but not sufficient for $\frac{d}{dr}(\partial L/\partial K) < 0$. Thus, even when the firm's expansion path is negatively sloped, as in the figure in the previous footnote, $\frac{d}{dr}(\partial L/\partial K)$ may still be positive. In order that

$$\frac{d}{dr}\left(\frac{\partial L}{\partial K}\right) < 0 \qquad (*)$$

it is necessary that not only (58) hold, but that

$$1 - r(r - \dot{q}/q) \simeq -1 ,$$

which requires that r be large (at least $(\sqrt{5} - 1)/2 \simeq 0.618$) and that the price of capital goods be falling rapidly (i.e. $\dot{q}/q \simeq -1$). Thus, (*) will hold only under (these) exceptional circumstances.

105. Hicks, Value and Capital, op. cit., ch. 2 and pp. 307-311, and Samuelson, Foundations of Economic Analysis, op. cit., pp. 100-105. See also Takayama, op. cit., pp. 156-160. The Hicks-Slutsky equation can be expressed as

$$\frac{\partial q_i}{\partial p_i} = \left(\frac{\partial q_i}{\partial p_i}\right)_{I \equiv \text{constant}} - q_i\left(\frac{\partial q_i}{\partial I}\right)_{\text{prices} \equiv \text{constant}} , \qquad (*)$$

where q_i and p_i are the quantity demanded and the price, respectively, of the i-th good and where I is the consumer's income. The first term in (*) corresponds to the substitution effect and the second term corresponds to the income effect.

106. Ibid., p. 156.

107. As demonstrated in the proof of theorem III-4, the derivation of the Hicks-Slutsky-type equation when L and M vary involves the inverse of a 3 by 3 matrix. If I were also permitted to vary, the derivation would involve the inverse of a 4 by 4 matrix, thus making it more difficult to obtain and to interpret the basic result.

108.

$$H^{-1}{}_{21} = \frac{-(1-\tau)(p\cdot\frac{\partial f}{\partial L} - w)\frac{\partial^2 U_1}{\partial M^2} e^{-rt}}{\det H} < 0$$

since $p\cdot\frac{\partial f}{\partial L} - w < 0$ and $\frac{\partial^2 U_1}{\partial M^2} < 0$, while by (62), $\det H > 0$.

109. That is, the price increase causes real income to fall. This fall in real income is tantamount to the budget constraint becoming 'tighter'. In consumer theory a 'normal' (or 'superior') good is one the quantity demanded of which varies directly with (real) income. Under the assumptions stated in the theorem, total sales revenue is a 'normal good' with respect to the collective utility of the firm.

110. In particular, a tax rate increase intended as a deflationary policy action may lead to increases, rather than decreases, in output (and hence increases in demand for inputs such as labor) in certain sectors of the economy, thereby increasing inflationary pressures in those sectors. Note also that when the profit constraint is not binding, an increase in the tax rate will tend to increase managerial utility at the expense of shareholder utility by causing sales revenue and managerial emoluments to increase and dividends to decrease (or at least, it will tend to cause managerial utility to rise relative to shareholder utility as sales revenue and managerial emoluments increase relative to dividends). The reverse tends to happen when the constraint is binding.

111. Baumol, Business Behavior, Value and Growth, op. cit., ch. 8. This has also been recognized by Yarrow. See Yarrow, On the Predictions of Managerial Theories of the Firm, op. cit., p. 272.

112. Ibid., pp. 277-278.

113. The chief modifications to quantitative results are handled in the following manner. When the product market is characterized by imperfect competition, quantity

demanded is a function of price and the expression for total revenue becomes $R = p\cdot Q = p\cdot Q(p) = p(Q)\cdot Q$, where $p = Q^{-1}$ is obtained by appealing to the inverse function theorem. Under imperfect competition $\frac{\partial R}{\partial L} = p\cdot\frac{\partial f}{\partial L} + \frac{dp}{dQ}\cdot\frac{\partial f}{\partial L}\cdot Q = (p + \frac{dp}{dQ}\cdot Q)\frac{\partial f}{\partial L}$, whereas under perfect competition $\frac{\partial R}{\partial L} = p\cdot\frac{\partial f}{\partial L}$ since $\frac{dp}{dQ} \equiv 0$. Thus $p\cdot\frac{\partial f}{\partial L}$ would have to be replaced by

$$(p + \frac{dp}{dQ}\cdot Q)\frac{\partial f}{\partial L}$$

in condition (18). If there is imperfect competition in the market for labor, then $w = w(L)$ and $\frac{d}{dL}(w(L)\cdot L) = \frac{dw}{dL}\cdot L + w$, and w would have to be replaced by

$$\frac{dw}{dL}\cdot L + w$$

in condition (18).

114. This extension as well as the extension to more than two inputs can be handled easily in the model by introducing the implicit production function

$$F(\overline{q}, \overline{x}) = 0 \ ,$$

which was discussed in chapters one and two, and by expressing total revenue as the sum of the amounts of revenue earned on sales of the n goods and expressing variable cost as the sum of the amounts spent on the m variable inputs:

$$R(\overline{q}) = \sum_{i=1}^{n} p_i q_i \qquad C(\overline{x}) = \sum_{j=1}^{m} w_j L_j \ .$$

If different classes of capital goods were introduced, then additional differential equations for net investment of the form $\dot{K} = I - \delta\cdot K$ would have to be introduced (again assuming depreciation at a constant percentage rate).

115. Sandmo, <u>op. cit.</u>, and Leland, <u>Theory of the Firm Facing Uncertain Demand (1972)</u>, <u>op. cit.</u>

116. Specifically, from (75),

$$P\{p(\theta) \geq \frac{1}{f(K,L)}[\frac{1}{1-\tau}\pi_o + q\cdot[\delta\cdot K] + wL + M]\} \geq \varepsilon. \qquad (*)$$

Since p is a monotonically increasing function of θ, the probability distribution over θ defines a probability distribution over p, i.e.

$$G(p(\theta_1)) \equiv P[p(\theta) \leq p(\theta_1)] = P[\theta \leq \theta_1] = F(\theta_1) \ .$$

Since by assumption F is a continuously differentiable function of θ and p is a differentiable function of θ with $p'(\theta) > 0$ for all θ, it follows that G is a differentiable function of p. Using the continuity of G, (*) requires that

$$1 - G\{\frac{1}{f(K,L)}[\frac{1}{1-\tau}\pi_o + q\cdot[\delta\cdot K] + wL + M]\} \geq \varepsilon,$$

or

$$G\{\frac{1}{f(K,L)}[\frac{1}{1-\tau}\pi_o + q\cdot[\delta\cdot K] + wL + M]\} \leq 1 - \varepsilon$$

or

$$\frac{1}{f(K,L)}[\frac{1}{1-\tau}\pi_o + q\cdot[\delta\cdot K] + wL + M] \leq G^{-1}(1-\varepsilon)$$

or

$$M \leq f(K, L)\cdot G^{-1}(1 - \varepsilon) - \frac{1}{1-\tau}\pi_o - q\cdot[\delta\cdot K] - wL ,$$

where G^{-1} denotes the inverse of the cumulative distribution function of p. Hence,

$$\hat{M}(L,K,\pi_o,\varepsilon) = f(K,L)\cdot G^{-1}(1-\varepsilon) - \frac{1}{1-\tau}\pi_o - q\cdot[\delta\cdot K] - wL. \quad (**)$$

117. From (**) in footnote 116,

$$\frac{\partial\hat{M}}{\partial\pi_o} = -\frac{1}{1-\tau} < 0$$

and

$$\frac{\partial\hat{M}}{\partial\varepsilon} = f(K,L)\cdot\frac{d}{d\varepsilon}G^{-1}(1-\varepsilon) = -f(K,L)\cdot\frac{1}{g(G^{-1}(1-\varepsilon))} < 0 ,$$

provided $g(G^{-1}(1-\varepsilon)) \neq 0$, where $g(x) = dG/dx$.

118. From (**) in footnote 116,

$$\frac{\partial\hat{M}}{\partial L} = \frac{\partial f}{\partial L}G^{-1}(1 - \varepsilon) - w < 0 \qquad (*)$$

for L sufficiently large since $G^{-1}(1-\varepsilon)$ and w are fixed while, by assumption, $\partial^2 f/\partial L^2 < 0$.

119. For references see footnote 63.

120. Assuming, as in the case of problem (11), that L(t) and K(t) are both strictly positive at each point along their respective optimal trajectories. It is also assumed that M(t) > 0 at each point along its optimal trajectory. This simplifies the analysis of the model by reducing by one the number of cases that need to be considered. Permitting M(t) = 0 can be handled in the same manner as in the certainty case.

121. Batra and Ullah, _op. cit._, p. 539.

122. Sandmo, _op. cit._, and Batra and Ullah, _op. cit._

123. Sandmo, _op. cit._

124. K.W. Rothschild, op. cit.; Galbraith, The New Industrial State, op. cit.; and Marris, Managerial Capitalism, op. cit.

125. See subsection 2 in section L of chapter two of this thesis for a discussion of the meaning and significance of 'reversible' and 'irreversible' investment.

126. From footnote 118, $\frac{\partial^2 \hat{M}}{\partial L^2} = \frac{\partial^2 f}{\partial L^2} G^{-1}(1 - \varepsilon) < 0$

since $\partial^2 f/\partial L^2 < 0$ and $G^{-1}(1 - \varepsilon)$ represents the $(1 - \varepsilon) \times 100$ percentile of the distribution of output price, which must be strictly positive. From footnote 117,

$$\frac{\partial^2 \hat{M}}{\partial L \partial \varepsilon} = \frac{\partial}{\partial L}\left[-f(K,L)\frac{1}{g(G^{-1}(1-\varepsilon))}\right] = \frac{-\partial f/\partial L}{g(G^{-1}(1-\varepsilon))} < 0$$

since $\partial f/\partial L > 0$ and $g(G^{-1}(1 - \varepsilon)) > 0$ (recall that g is the probability density function for output price).

127. Actually, weaker conditions would suffice to ensure this since the assumed strict concavity of U_1 and f implies that each of the last three terms in (111) is strictly positive.

128. Assume det $H > 0$ at this point.

129. Note that if the distribution of p is symmetric and that if L is chosen so that $\bar{p} \cdot \frac{\partial f}{\partial L} - w = 0$, then the strict concavity of U_1 implies that $E[(\partial U_1/\partial D)(p(\theta) \times \frac{\partial f}{\partial L} - w)] < 0$. If $\bar{p} \cdot (\partial f/\partial L) - w < 0$, this effect is more pronounced. Even if $\bar{p} \cdot (\partial f/\partial L) - w > 0$, strict concavity will ensure that $E[(\partial U_1/\partial D)(p(\theta)\frac{\partial f}{\partial L} - w)] < 0$ as long as $\bar{p}(\partial f/\partial L) - w$ remains relatively small. When p is nonsymmetric the above arguments must be modified according to the direction of skewness. It is conceivable, however, that examples could be constructed in which $\alpha > 0$.

130. From footnotes 117 and 118,

$$\frac{\partial L}{\partial \pi_o} = -\frac{\partial \hat{M}/\partial \pi_o}{\partial \hat{M}/\partial L} < 0$$

since $\partial \hat{M}/\partial \pi_o < 0$ and $\partial \hat{M}/\partial L < 0$.

131. O.E. Williamson, A Dynamic Stochastic Theory of Managerial Behavior, op. cit., pp. 11-31.

132. Albin and Alcaly, op. cit., pp. 260-297.

133. Ibid., p. 285.

134. See "Getting Out," Wall Street Journal (March 21, 1975); "An urge to purge misfit operations," Business Week (April 21, 1975); and "Black Future for White?" Time (May 31, 1976).

135. See, for example, "Cash-Laden Firms Are In a Marrying Mood, Spurring Merger Wave," Wall Street Journal (February 18, 1977).

136. O.E. Williamson, A Dynamic Stochastic Theory of Managerial Behavior, op. cit., pp. 21-22. Specifically, "A decline in the condition of the environment is therefore more probable and an improvement less probable when the organization is operated along managerial lines than when a profit-maximizing posture is adopted. Opportunities that the profit-maximizing organization will recognize or develop will simply go unrecognized or undeveloped if the managerial syndrome prevails." Ibid., p. 22.

137. Ibid., p. 24.

138. R.A. Howard, Dynamic Programming and Markov Processes (Wiley; New York; 1960), and G.L. Nemhauser, Introduction to Dynamic Programming (Wiley; New York; 1966).

139. Ibid. pp. 34-35.

140. One practical example of such variation is the recent set of sharp increases in the levels of compensation of the chief executives of the U.S. automobile manufacturers. Total annual compensation of the chairman of Ford Motor Co. increased from $334,000 in 1975 to $970,000 in 1976. Total annual compensation of the chairman of General Motors increased from $575,000 in 1975 to $950,000 in 1976. Total annual compensation of the chairman of Chrysler Corp. increased from $216,000 in 1975 to $620,000 in 1976. "GM and Ford Chiefs Are Each Awarded Nearly $1 Million," Wall Street Journal (April 18, 1977).

141. It also holds the risk level fixed, which is important because, under the assumption of risk aversion on the part of the firm, increases in price that involve an increase in the level of risk can have offsetting effects on output when the state of the firm's operating environment improves.

142. Note that by (*) in footnote 118, $\partial^2\hat{M}/\partial L\partial\gamma = \partial f/\partial L > 0$ (since $G^{-1}(1 - \varepsilon)$ is the $(1 - \varepsilon) \times 100$ percentile of the distribution of p, which increases with γ).

143. Strictly speaking, the cycle in question could be that of a single industry if θ is so interpreted. That is, the model could apply with equal validity to a counter-cyclical industry, with θ and $p(\theta)$ varying counter-cyclically. Second, it should be noted that, as long

as the assumption of perfect competition (i.e. price-taking behavior on the part of the firm) is retained, firms will continue to respond in a passive manner to these cyclical influences; admitting the possibility of imperfect competition would permit behavior of an anticipatory nature to be considered.

IV. THE FIRM'S FINANCIAL DECISIONS UNDER UNCERTAINTY

A. INTRODUCTION

In a world of certainty where capital markets are perfect,[1] the firm's production and investment decisions (or collectively, its operating decisions) are separable from its financial decisions.[2] Even when there is uncertainty, if there exists a set of complete markets for contingent claims,[3] the firm's operating decisions and its financial decisions remain separable.[4] However, when capital markets are imperfect or when the markets for contingent claims are incomplete, separability is no longer assured.[5]

In actual capital markets there exist imperfections in the form of transactions costs, bankruptcy penalties, and taxes.[6] Information is generally not costless, and interest rates generally vary according to the financial strength of the borrower and the amount borrowed.[7] Nor are actual financial markets complete. Indeed, where there are positive costs of establishing and maintaining markets for contingent claims, it is virtually impossible for complete markets to exist.[8] For actual firms, then, operating decisions are not, in general, separable from financial decisions and the interests of holders of the firm's securities (and in particular the firm's stockholders).[9]

The model of the firm developed in chapter three abstracted from the firm's financial decisions. The purpose of this chapter is to extend the model to incorporate financial considerations and to use the model to study the firm's optimal financial decisions under uncertainty. Of particular interest is the impact of the incompleteness of markets on these decisions, and this is explored in section E.

Firms generally make three major classes of financial policy decisions.[10] First, the firm must decide how large a stock of cash to maintain. The firm's motives for holding cash and the other factors affecting its *cash management policy* are discussed in section B. Since cash is an asset, this first decision affects the composition of the assets side of the balance sheet, as illustrated in table IV-1. Second, the firm must determine the relative proportions of debt and equity (i.e. leverage) to be used in financing its activities. The factors affecting the firm's *leverage policy* are discussed in section C. Third, the firm must choose the proportion of net income to be distributed to its shareholders as dividends (and therefore what proportion to add to retained earnings). The factors affecting its *dividend policy* are discussed in section D. The latter two policies affect the composition of the total liabilities and stockholders' equity side of the firm's balance sheet, as illustrated in table IV-1. The balance sheet provided in table IV-1 is a simplified version of table II-1 and will be referred to throughout the chapter.

Table IV-1 Typical Firm's Balance Sheet

Assets		Liabilities		
Cash	C	Debt		B
Inventories	V			
Fixed Assets	qK	Contributed Capital	xxx	
		Retained Earnings	xxx	
		Equity		E
Total Assets	C + V + qK	Total Liabilities and Stockholders' Equity		B + E

In table IV-1 there are five entries that will be of interest in the analysis presented in this chapter. On the assets side of the balance sheet, C denotes the stock of cash, which is assumed to be non-interest bearing. Inventories, V, are assumed to be directly related to the firm's level of output. That is, in the model developed below inventories, measured in dollars, will be expressed as a function of the firm's output level. Fixed assets, K, and the unit price of capital goods, q, are interpreted as before. On the liabilities side of the balance sheet, B denotes debt, which consists of fixed interest obligations, chiefly bonds and bank borrowings.[11] Equity, E, consists of contributed capital and retained earnings. New share issues add to contributed capital and the portion of net income not paid out as dividends is added to retained earnings. In what follows, however, these two components of equity are treated collectively.

The firm's financial decisions are important for several reasons, which are explored at length throughout the chapter. Briefly, financial capital is important because production is a time-consuming process. The firm must hire inputs and carry out production before it can sell its output. The firm needs financial capital in order to be able to purchase capital goods that will provide a stream of productive services over many future time periods. In general, the firm must also maintain stocks of raw materials and finished goods. To finance its investment in plant, equipment, and inventories - i.e. real capital - the firm must obtain money capital either through retained earnings or through the issuance of debt or equity instruments. In addition, the firm must hire labor and pay for raw materials, and due to the time lag between receipts and expenditures of cash, it must maintain a stock of cash with which to make these transactions.[12] If the firm is unable to raise the needed financial capital, then it may be forced to eschew profitable investment opportunities. If it suffers a cash shortage, it may have to reduce its hiring of inputs or sell off a portion of its inventories, or worse yet, it may fail to meet its bond interest obligations and be forced to liquidate.[13]

The main purpose of this chapter is to explore the relationship between the firm's operating decisions and its financial policy (i.e. cash management policy, leverage policy, and dividend policy) decisions. In contrast to the recent studies that have explored the firm's production decisions in the context of single period stock market equilibrium, usually in attempts to derive shareholder unanimity theorems or to prove Pareto optimality (or lack of it) of stock market allocation,[14] this chapter is concerned with the firm's integrated financial planning within a multiperiod framework.[15] The first three sections below discuss the firm's financial policies individually. In section E the three classes of financial policies are integrated into a modified version of the model developed in the previous chapter and then the optimal operating and financial policies implied by the model are derived.

B. CASH MANAGEMENT POLICY

The purpose of this section is to explore the first of the firm's three major classes of financial policy decisions, cash management policy. First, the firm's motives for holding cash are discussed. Then a simple model of the firm's transactions demand for cash is presented.

1. The Firm's Demand for Cash

Keynesian monetary theory identifies three motives for holding money: a transactions motive, a speculative motive, and a precautionary motive.[16] The transactions motive arises out of the imperfect synchronization of cash receipts and cash expenditures.[17] The speculative motive arises out of the possibility for profit by holding cash and arranging securities purchases to take advantage of interest rate changes.[18] The precautionary motive arises out of the need to maintain cash balances for protection against unforeseen circumstances, such as a severe decrease in net income that leaves insufficient funds with which to pay bond interest.[19]

In a world of certainty the firm's only motive for holding cash is the transactions motive. Baumol and others have modeled the firm's transactions demand for money as an inventory management problem,[20] with the stock of cash playing the role of the inventory of 'goods'.[21] Such models are based on the fact that cash inflows and outflows cannot be synchronized perfectly and that there exist costs (e.g. transactions costs) associated with converting securities (or inventories of goods or fixed assets) into cash. These models assumed (at least implicitly) that money serving as a medium of exchange is a nonproductive asset.

In contrast, Saving has argued that money itself is productive in that it reduces transactions costs.[22] While Saving carefully distinguishes the productiveness of money from the productivity of real resources like capital and labor - in particular, Saving does not treat money as a factor of production[23] - Friedman has argued that the firm's money balances yield real productive services and that the value product of money depends on the technical conditions of production embodied in the firm's production function.[24] Fischer and Wu, Rozek, and Dutton have gone a step further and treated the firm's real money balances[25] as a factor of production by incorporating them in the firm's production function:[26]

$$Q = f(K, L, m) \; , \tag{1}$$

where m denotes the firm's real money balances. Though Fischer took great pains to justify his derivation of (1), in the opinion of this writer the theoretical issues that have been raised concerning the direct inclusion of real money balances in the consumer's utility function can also be raised concerning the direct inclusion of real money balances in the firm's production function.[27] In both cases the demand for real money balances is a derived demand. Under certainty, where there is neither a precautionary demand nor a speculative demand for real money balances, real money balances are valued for the role they play in making transactions possible.[28] Money is useful to the firm - or 'productive' in the sense described by Saving - not because it makes a direct contribution to production, but because its use frees up real resources that would otherwise have had to have been used in the execution of transactions. Therefore, before one can employ the production function (1) in a model of the firm, it is necessary to specify the economic role of m and to indicate the derivation of (1) from the traditional production function

$$Q = g(K, L) \ , \tag{2}$$

where the symbol g is used in (2) to distinguish that function from f in (1). The next subsection is concerned with the derivation of (1) from (2).

2. Money and the Production Function

The derivation of (1) from (2) is demonstrated in this subsection by showing how a function of the form of (1) could be constructed from (2) when certain additional summary information concerning the firm's cash inflows and outflows is provided. Given any output level, Q, the production function (2) gives the locus of input combinations (K, L) with which Q can be produced with maximum technical efficiency. Assume that for each output level, Q, and for each input combination (K, L) satisfying (2), the time pattern of cash outflows for the purchase of inputs and cash inflows from the sale of output can be used to determine the minimum level of real cash balances, $m > 0$, that will facilitate all such transactions.[29] The different values of m, together with the corresponding triplet (Q, K, L) for each, define the minimum balance function,

$$m = m(Q, K, L) \; . \tag{3}$$

That is, underlying (3) there is a particular, possibly stochastic, time pattern of cash inflows and outflows for each triplet (Q, K, L) that determines the functional relationship between m and (Q, K, L).[30]

If r denotes the market rate of interest, then the minimum cost (including allowance for the opportunity cost, $r \cdot m$, of holding real money balances) combination of inputs for producing any particular level of output, $\overline{Q}$, can be obtained by solving the following mathematical programming problem:[31]

$$\begin{aligned} &\underset{\{K,\, L\}}{\text{minimize}} \quad wL + iK + r \cdot m(K, L) \\ &\text{subject to } \overline{Q} = g(K, L) \end{aligned} \tag{4}$$

where w and i are the unit costs of labor and capital, respectively, and where m given by (3) has been written as a function of K and L only since Q is fixed. The necessary conditions for an optimal solution to (4) are:

$$\left.\begin{aligned} w + r\frac{\partial m}{\partial L} - \lambda\frac{\partial g}{\partial L} &= 0 \\ i + r\frac{\partial m}{\partial K} - \lambda\frac{\partial g}{\partial K} &= 0 \end{aligned}\right\} \tag{5}$$

where the Lagrangian is $F_\lambda = wL + iK + r \cdot m(K, L) + \lambda(\bar{Q} - g(K,L))$. The necessary conditions (5) lead to the following characterization of the optimal input mix:

$$-\frac{\partial K}{\partial L} = \frac{w + r(\partial m/\partial L)}{i + r(\partial m/\partial K)} ,$$

which is analogous to the optimality conditions arising out of the two versions of the Vickers model discussed in section I of chapter two.[32]

Returning to (3), note that if $\partial m/\partial Q \neq 0$ in some region, then the implicit function theorem may be invoked to use (3) to define a function

$$Q = h(K, L, m) \tag{6}$$

that holds locally (within the region where $\partial m/\partial Q \neq 0$). Note that (6) is really just another way of characterizing the relationship between minimum real money balances, output, and input levels that is embodied in (3). To allow in addition for the technological relationships that also underlie (1), define the function

$$f(K, L, m) \equiv g(K, L) + h(K, L, m) , \tag{7}$$

where g is given by (2) and where h is given by (6). The following two lemmas demonstrate that the *derived production function* (7) can be used in models of the firm without introducing distortions, provided of course that they are correctly formulated (and interpreted).

Lemma IV-1

If the derived production function (7) were used in place of the production function (2) in determining the firm's least cost combination of inputs for any given output level Q, the choice of input mix would not be affected, i.e. the optimal input mix would again satisfy (5).

Proof

Using (7), the least cost combination of inputs can be obtained by solving the following mathematical programming problem:

$$\begin{aligned} &\underset{\{K,L,m\}}{\text{minimize}} \quad wL + iK + rm \\ &\text{subject to} \quad \overline{Q} = f(K, L, m) \end{aligned} \tag{8}$$

where $\overline{Q}$ is given. The necessary conditions for an optimal solution to (8) are:

$$\left.\begin{aligned} w - \lambda\left(\frac{\partial g}{\partial L} + \frac{\partial h}{\partial L}\right) &= 0 \\ i - \lambda\left(\frac{\partial g}{\partial K} + \frac{\partial h}{\partial K}\right) &= 0 \\ r - \lambda\left(\frac{\partial h}{\partial m}\right) &= 0 \ , \end{aligned}\right\} \tag{9}$$

where the Lagrangian is $F_\lambda = wL + iK + rm + \lambda(\overline{Q} - f(K, L, m))$. The necessary conditions (9) are easily rearranged to obtain (5). Q.E.D.

The next lemma demonstrates that the derived production function (7) exhibits the same mathematical properties as Fischer's production function incorporating real money balances.[33]

Lemma IV-2

If $\partial m/\partial Q > 0$, $\partial m/\partial K < 0$, $\partial m/\partial L < 0$; and if g given by (2) and h given by (6) are concave; then $\partial f/\partial K > 0$,

$\partial f/\partial L > 0$, and $\partial f/\partial m > 0$ for f given by (7), and moreover, f is concave.

Remark

The assumed signs of the first partial derivatives of the function m given by (3) can be given the following interpretation. An increase in the 'average' rate of output Q with the 'average' input levels held fixed requires greater (minimum) real money balances in order to fund the consequent increase in the number of transactions (per unit time period). On the other hand, increasing the 'average' input levels while holding the 'average' rate of output constant has the opposite effect, i.e. the number of transactions (per unit time period) falls and so do the required (minimum) real money balances.

Proof of Lemma IV-2

From (7), $\frac{\partial f}{\partial K} = \frac{\partial g}{\partial K} + \frac{\partial h}{\partial K}$. By the implicit function theorem, $\frac{\partial h}{\partial K} = -\frac{\partial m/\partial K}{\partial m/\partial Q}$. From the stated assumptions it follows that $\frac{\partial h}{\partial K} > 0$ for all K, and hence, that $\frac{\partial f}{\partial K} > 0$ whenever $\frac{\partial g}{\partial K} > 0$. The proof that $\frac{\partial f}{\partial L} > 0$ is similar. To show that $\frac{\partial f}{\partial m} > 0$, note that from (7), $\frac{\partial f}{\partial m} = \frac{\partial h}{\partial m}$, and from the implicit function theorem, that $\frac{\partial h}{\partial m} = -\frac{1}{-\partial m/\partial Q}$. But by assumption $\frac{\partial m}{\partial Q} > 0$, so that $\frac{\partial h}{\partial m} > 0$ also.

The second statement follows from the well-known result that the sum of two concave functions is concave.

Q.E.D.

The significance of lemmas IV-1 and IV-2 is that the use of a production function of the form of (1) in the model of a firm will not lead to distortions in the policy implications derived from the model, provided of course that the function is defined correctly. In particular, it follows from lemma IV-1 that, for any output level Q, the correct application of (7) leads to the true minimum cost combination of inputs for producing Q. It follows that the use of (7) does not affect the shape or position of the firm's total cost curve. Since the use of (7) clearly would not alter the treatment of demand or revenue conditions in the model, it follows that production functions of the form of (1), provided they are correctly formulated and correctly interpreted, can be incorporated into models of the firm in which the firm's transactions demand for real money balances is to be given a role to play.

In subsection 4 the model of the firm developed in chapter three will be modified for the purpose of studying the firm's transactions demand for real money balances. In that analysis the derived production function (7) will be used in place of the neoclassical production function (2).

3. Other Motives for Holding Money Balances

The transactions motive for holding money exists whether or not there is uncertainty.[34] In contrast, the speculative and precautionary motives for holding cash are due to uncertainty. Precautionary balances, in particular, provide a buffer to protect the firm's viability when economic conditions worsen unexpectedly and when, as a result, its operations fail to generate sufficient cash to meet the firm's cash obligations. The firm can employ the precautionary balances, and then replenish its precautionary stock of cash when conditions improve.[35]

In the model of the firm under uncertainty developed later in this chapter the precautionary motive (as well as the transactions motive) is treated explicitly. The speculative motive, which in some circumstances may be incidental to the precautionary motive, is not treated explicitly. In the model developed below the firm is motivated to hold precautionary cash balances when markets are incomplete - though not when markets are complete - to compensate for risks (such as the risk of bankruptcy) that cannot be fully diversified away due to the incompleteness of markets.

4. A Simple Model of the Firm's Transactions Demand for Cash

The purpose of this subsection is to modify the model of the firm under certainty developed in section B of chapter three to incorporate the role of cash management. Only the deterministic case is considered here.[36] The firm's demand for cash under uncertainty is explored in sections E and F later in this chapter. The main results of this subsection are first, that the collective utility maximizer modeled in chapter three tends to maintain greater real cash balances than a short run profit maximizer, and second, that its transactions demand for real cash balances is inversely related to the interest rate.

To modify the model (11) of the firm under certainty formulated in section B of chapter three, the production function (1) needs to be substituted for f(K, L). Assume that the production function (1) is concave and non-decreasing in each of its arguments. In addition, denote the change in the firm's nominal (i.e. unadjusted for inflation) stock of cash by N and denote by J the price index (e.g.

the GNP deflator) used to deflate the nominal addition to the stock of real money balances. Then

$$\dot{m}(t) = N(t)/J(t) \ , \tag{10}$$

where the dot again denotes differentiation with respect to time. Assume that there are no borrowings and no new issues, so that $N(t)$ represents the difference between total cash generated from operations less total cash applied to dividends and the purchase of capital goods, as illustrated in table IV-2. Then total dividends paid satisfies the following accounting identity:

$$D(t) = \pi(t) - q(t)[I(t) - \delta \cdot K(t)] - N(t) \ , \tag{11}$$

which is just equation (9) of chapter three with a $-N(t)$ term on the right-hand side. The only modifications that need to be made to the model (11) in chapter three, then, are the addition of the constraint (10); the substitution of (1) for the production function and (11) for the expression for dividends; and the addition of a term $U_3(m(T))e^{-rT}$ to the objective function to reflect the value the firm attaches to having nonzero terminal real money balances (i.e. $m(T) > 0$). The introduction of U_3, which plays a role analogous to that of U_2, serves the important purpose of preventing terminal real money balances from falling to zero. For the convenience of the reader the modified model is set out below:

$$\begin{aligned}\underset{\{L(t),I(t),\,M(t),N(t)\}}{\text{maximize}} \int_o^T U_1[&p(t)\cdot f(K(t),L(t),m(t));\\ &(1-\tau)\{p(t)\cdot f(K(t),L(t),m(t))-w(t)\cdot L(t)-M(t)\}\\ &+\tau\cdot q(t)\cdot[\delta\cdot K(t)]-q(t)\cdot I(t)-N(t);M(t)]e^{-rt}dt\\ &+U_2(K(T))e^{-rT}+U_3(m(T))e^{-rT}\end{aligned} \tag{12}$$

subject to $\dot{K}(t) = I(t) - \delta \cdot K(t),\ 0 \leq t \leq T,\ K(0)$ given

$\dot{m}(t) = N(t)/J(t),\ 0 \leq t \leq T,\ m(0)$ given

$(1-\tau)\{p(t) \cdot f(K(t),L(t),m(t)) - w(t) \cdot L(t) - M(t)$

$- q(t) \cdot [\delta \cdot K(t)]\} \geq \pi_o,\ 0 \leq t \leq T$

$L(t),\ K(t),\ M(t),\ m(t) \geq 0,\ 0 \leq t \leq T$

The Lagrangian for problem (4) is [37]

$$L_\mu[K,m,L,I,M,N,\lambda_1,\lambda_2,t] = \tag{13}$$

$$U_1[\quad]e^{-rt} + \lambda_1(t) \cdot [I(t) - \delta \cdot K(t)] + \lambda_2(t) \cdot [N(t)/J(t)]$$

$$+ \mu_1(t)[(1-\tau)\{p(t) \cdot f(K(t),L(t),m(t)) - w(t) \cdot L(t) - M(t)$$

$$- q(t) \cdot [\delta \cdot K(t)]\} - \pi_o] .$$

The necessary conditions for an optimal solution to (12) are conditions (14), (15), and (17)-(21) of chapter three, and in addition,

$$\dot{m}(t) = N(t)/J(t),\ 0 \leq t \leq T,\ m(0) \text{ given} \tag{14}$$

$$\dot{\lambda}_2^*(t) = -\frac{\partial L_\mu}{\partial m},\quad \lambda_2^*(T) = \frac{\partial U_3}{\partial m} e^{-rT} \tag{15}$$

$$\frac{\partial L_\mu}{\partial N} = \frac{\partial U_1}{\partial D}(-1)e^{-rT} + \lambda_2(t)/J(t) = 0 \tag{16}$$

where $\frac{\partial U_3}{\partial m} e^{-rT}$ measures the (discounted) marginal utility of the firm's terminal stock of cash. Since (14) merely repeats one of the constraints, it need not be considered further.

Of the two remaining conditions, first consider (16), which can be reexpressed as:

$$\frac{\lambda_2(t)}{(\partial U_1/\partial D)e^{-rt}} = J(t) . \tag{17}$$

Table IV-2 Typical Firm's Sources and Uses of Cash under Certainty Subject to Transactions Demand for Cash

Sources of cash:		
Sales revenue		$p(t) \cdot Q(t)$
Total Expenses and Taxes	$(1 - \tau)\{w(t) \cdot L(t) + M(t) + q(t) \cdot [\delta \cdot K(t)]\} + \tau \cdot p(t) \cdot Q(t)$	
Adjustment for noncash outlay	$q(t) \cdot [\delta \cdot K(t)]$	
Cash outflow for expenses		$(1 - \tau)\{w(t) \cdot L(t) + M(t)\} + \tau\{p(t) \cdot Q(t) - q(t) \cdot [\delta \cdot K(t)]\}$
Total cash generated		$(1 - \tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t)\} + \tau \cdot q(t) \cdot [\delta \cdot K(t)]$ $= \pi(t) + q(t) \cdot [\delta \cdot K(t)]$
Uses of cash:		
To pay dividends	$D(t)$	
To purchase capital goods	$q(t) \cdot I(t)$	
Total cash applied	$D(t) + q(t) \cdot I(t)$	
Increase (decrease) in stock of cash	$N(t)$	

Note that equation (17), which relates to the accumulation of cash, is analogous to equation (22) of chapter three, which relates to the accumulation of physical capital. In (17) $\lambda_2(t)$ is interpretable as the shadow price of cash, i.e. the value in terms of collective utility of an additional dollar added to the stock of cash. The ratio $\lambda_2(t)/(\partial U_1/\partial D)e^{-rt}$ expresses the internal (to the firm) trade off between real cash balances and dividends, where numerator and denominator are each expressed in terms of discounted marginal collective utility. The right-hand side of (17), the price index $J(t)$, measures the externally imposed rate at which real cash balances and dividends can be traded off. According to (17), then, in equilibrium the firm modeled in (12) will have equated its internal rate of trade off between real cash balances and dividends to the externally imposed rate of trade off between these quantities, $J(t)$. This is, of course, perfectly analogous to the interpretation of equation (22) of chapter three.

Turning next to (15), evaluate $-\partial L_\mu/\partial m$, use (17) to obtain a second expression for $\dot{\lambda}_2(t)$, equate the two expressions, and rearrange terms to obtain

$$r - \dot{J}/J = \bar{p}\cdot\frac{\partial f}{\partial m}(1-\tau) + \bar{p}\cdot\frac{\partial f}{\partial m}\left(\frac{\partial U_1/\partial R + (1-\tau)\mu_1 e^{rt}}{\partial U_1/\partial D}\right), \qquad (18)$$

where $\bar{p} \equiv p/J$ is the price of output in constant dollars. Note that equation (18) is analogous to equation (39) of chapter three. In particular, the left-hand side of (18) can be interpreted as the firm's opportunity cost of maintaining a stock of cash, expressed as the nominal rate of interest, r, adjusted for inflation. The difference $r - \dot{J}/J$ is called the real rate of interest. The right-hand side of (18) can be interpreted as the marginal revenue product of real money balances adjusted upward by the marginal revenue-equivalent of the increased dividends that the increase in real money balances makes possible.[38] According to (18), in equilibrium the firm modeled in (12) will increase real balances up to the point at which the marginal revenue product of real balances just equals the marginal cost ($\equiv$ average cost in this case) of real balances.

Note that, since $\bar{p}$, $\partial f/\partial m$, and the terms in parentheses on the right-hand side of (18) are all positive and since f is assumed to be strictly concave, the firm modeled in (12) will tend to maintain larger real cash balances than a short run profit maximizer. This follows from the fact that the traditional firm's criterion for optimal real balances is $r - \dot{J}/J = \bar{p}\, \frac{\partial f}{\partial m}(1-\tau)$, which is perfectly analogous to equation (40) of chapter three.[39] This important result is stated as the following theorem.

Theorem IV-1

When the firm modeled in (12) is in equilibrium, it will maintain greater real cash balances than a short run profit maximizer would under identical cash inflow and outflow conditions (i.e. the same functions $m = m(Q, K, L)$ and $Q = f(K, L, m)$).

The practical interpretation of the theorem is that the firm modeled in (12) produces greater output, using more of both inputs, than a short run profit maximizer. Ceteris paribus, the firm modeled in (12) will make more transactions, and thus have to maintain greater real balances than a short run profit maximizer.

This section concludes with a theorem that establishes the inverse relationship between the firm's transactions demand for real balances and the market rate of interest r when the firm's profit constraint is not binding.

Theorem IV-2

If the production function f and the utility function U_1 are strictly concave, then the collective utility maximizer modeled in (12) exhibits a transactions demand for cash that is inversely related to the interest rate.

Proof

To prove the theorem it is sufficient to show that $\frac{\partial m}{\partial r} < 0$. First define $\bar{\mu}_1 = \mu_1 e^{rt}$ and substitute into (18).[40] Then use (18) to define the function,

$$G(r,m)=\bar{p}\ \frac{\partial f}{\partial m}[(1-\tau)\frac{\partial U_1}{\partial D} + \frac{\partial U_1}{\partial R} +(1-\tau)\bar{\mu}_1]-r\ \frac{\partial U_1}{\partial D} +(\dot{J}/J)\frac{\partial U_1}{\partial D}\ .$$

Then, by the implicit function theorem,

$$\frac{\partial m}{\partial r} = -\ \frac{\partial G/\partial r}{\partial G/\partial m} \tag{19}$$

$$= -\ \frac{-\partial U_1/\partial D}{\bar{p}\ \frac{\partial^2 f}{\partial m^2}[(1-\tau)\frac{\partial U_1}{\partial D} + \frac{\partial U_1}{\partial R} +(1-\tau)\bar{\mu}_1]+\bar{p}\cdot p(\frac{\partial f}{\partial m})^2[(1-\tau)^2\frac{\partial^2 U_1}{\partial D^2} + \frac{\partial^2 U_1}{\partial R^2}]}$$

It follows by the strict concavity of f and U_1 that the denominator in (19) is negative. Hence $\frac{\partial m}{\partial r} < 0$. Q.E.D

The practical interpretation of the theorem, which also holds for the short run profit maximizer,[41] is that the firm's opportunity cost of holding real cash balances increases with the interest rate, inducing the firm to reduce its stock of cash. Alternatively, the higher interest rate makes the firm more willing and better able to bear the transactions costs associated with converting other assets into cash when the (now smaller) stock of cash runs out.

This section discussed the firm's cash management policy. The next two sections discuss the firm's leverage and dividend policies, and then all three classes of financial policy decisions are integrated into the model of the collective utility maximizer under uncertainty.[42]

C. LEVERAGE POLICY

The firm's choice of capital structure was discussed in sections F and K of chapter two. The purpose of this section is to review briefly the salient points of that discussion and to explore the implications of introducing debt into the model of the firm under certainty developed in chapter three.

1. The Meaning and Importance of Leverage

The firm's leverage policy, or choice of capital structure, involves the choice of the portion of the firm's total investment to be financed through the issuance of debt instruments (i.e. bonds and bank borrowings[43]) and the portion to be financed through either the issuance of equity instruments (i.e. shares of common stock) or an increase in the pool of retained earnings. In terms of the balance sheet illustrated in table IV-1, given the firm's total assets C + V + K, its leverage policy involves determining the relative sizes of B and E in the identity,

$$C + V + K = B + E \ .$$

One indicator of the firm's leverage policy is the value of its *leverage ratio* (or *debt-equity ratio*), defined here as

$$\text{leverage ratio} \equiv B/E \ .$$

According to traditional financial theory,[44] an increase in leverage, as indicated by an increase in the value of the firm's leverage ratio, can have two important effects. Since bondholders are paid a rate of interest that is fixed at the time the bond is sold, if the firm's rate of return on investment exceeds the interest rate on debt, the increase in net income resulting from an increase in the leverage ratio (i.e. net of bond interest as well as operating costs) causes the total market value of the firm (i.e. the sum of the market values of its debt and equity) to increase. This continues as the leverage ratio increases. The increased leverage, however, also has a second effect. Since the firm's total fixed interest obligations must be met when they are due, or else the firm can be forced to liquidate, the increased leverage ratio carries with it increased risk. Eventually, the risk effect predominates, and the total market value of the firm falls as the leverage ratio increases further. This is illustrated in figure IV-1. The firm's (unique) optimal capital structure occurs at the point A where the total market value of the firm is maximized.

According to the Modigliani-Miller view,[45] there is no leverage effect and the total market

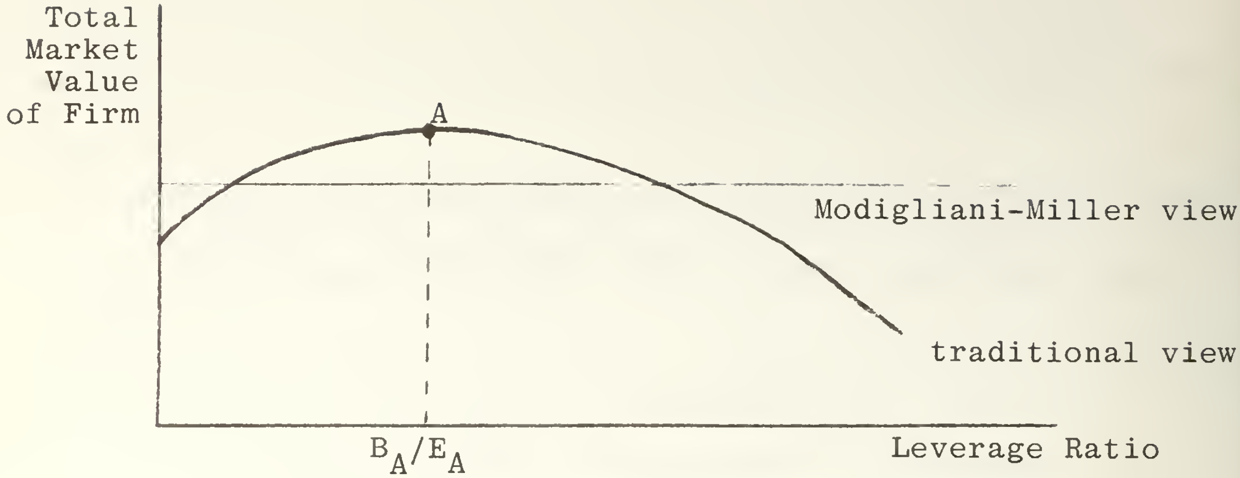


Figure IV-1 Total Market Value Versus Leverage Ratio

value of the firm is independent of the firm's leverage ratio, provided capital markets are perfect and there are no corporate income taxes.[46] The assumptions of perfect capital markets and the absence of corporate income taxes - or at least the absence of a tax that treats dividends and interest paid by the firm differently[47] - are both crucial.[48] Under uncertainty, the Modigliani-Miller proposition on the irrelevance of the firm's debt-equity ratio can be shown to hold, provided financial markets are complete[49] and also provided that either bankruptcy is impossible or that, if it is possible, there do not exist any bankruptcy penalties.[50]

One would expect on the basis of imperfections in actual capital markets and a tax system that gives special treatment to interest payments that the firm's leverage ratio and changes in that ratio would be of material interest to the managers, the creditors, and the shareholders of the firm.[51]

2. A Simple Model of the Impact of Leverage[52]

The purpose of this subsection is to modify the model of the firm under certainty developed in section B of chapter three. In this subsection debt is introduced into the model. Let B(t) denote the firm's amount of outstanding debt at time t. By assuming that each bond sells for one dollar, B(t) can simultaneously represent both the number of bonds and the amount of debt outstanding at time t. Let i(t) denote the average interest rate on debt at time t, so that the product $i(t) \cdot B(t)$ represents the firm's interest expense at time t. Denote the amount of funds raised at time t through new debt issues by Y(t). Then, by definition,

$$\dot{B}(t) = Y(t) . \tag{20}$$

Assume that due to bond market imperfections the interest rate the firm pays on new issues is a monotonically increasing function of the amount borrowed and that the average interest rate on debt at time t satisfies the functional relation

$$i(t) = i[B(t), Y(t)] , \tag{21}$$

where $\partial i/\partial B > 0$ and $\partial i/\partial Y > 0$ reflect the impact on i of increased borrowings at time t.[53] Further assume that $\partial^2 i/\partial Y^2 > 0$, i.e. an increase in new debt issues causes the average rate of interest to increase at an increasing rate; that $\partial^2 i/\partial B^2 > 0$, i.e. an increase in the level of debt outstanding causes the average interest rate to rise at an increasing rate; and that $\partial^2 i/\partial B \partial Y > 0$. Taking into account the interest expense on debt, the typical firm's income statement is illustrated in table IV-3, which is a modified version of table III-1. The typical firm's

Table IV-3 Typical Firm's Income Statement With Interest Expense

Sales revenue		$p(t) \cdot Q(t)$
Expenses:		
Labor	$w(t) \cdot L(t)$	
Emoluments	$M(t)$	
Depreciation	$q(t) \cdot [\delta \cdot K(t)]$	
Interest	$i(t) \cdot B(t)$	
Total Expenses		$w(t) \cdot L(t) + M(t) + q(t) \cdot [\delta \cdot K(t)] + i(t) \cdot B(t)$
Pretax Income		$p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - q(t) \cdot [\delta \cdot K(t)] - i(t) \cdot B(t)$
Income tax		$\tau\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - q(t) \cdot [\delta \cdot K(t)] - i(t) \cdot B(t)\}$
Net Income		$\pi(t) = (1-\tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - q(t) \cdot [\delta \cdot K(t)] - i(t) \cdot B(t)\}$

statement of sources and uses of cash that takes into account bond interest expense is illustrated in table IV-4. Note that the firm is assumed not to hold cash balances. This is done in order to focus the analysis on the impact of debt issuance.

It follows from table IV-4 that total dividends paid at time t must satisfy the following accounting identity:

$$D(t) = \pi(t) - q(t)[I(t) - \delta \cdot K(t)] + Y(t) , \qquad (22)$$

which differs from equation (9) of chapter three in that $\pi(t)$ in (22) allows for interest expense.

For the convenience of the reader the modified version of the model of the collective utility maximizing firm is set out below:

$$\underset{\{L(t),I(t),M(t),Y(t)\}}{\text{maximize:}} \int_0^T U_1[p(t)\cdot f(K(t),L(t));(1-\tau)\{p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - M(t) - i(B(t),Y(t))\cdot B(t)\} + \tau\cdot q(t)\cdot[\delta\cdot K(t)] - q(t)\cdot I(t) + Y(t);M(t)]e^{-rt}dt + U_2(K(T))e^{-rT} + U_4(B(T))e^{-rT} \qquad (23)$$

subject to:

$$\dot{K}(t) = I(t) - \delta\cdot K(t),\ 0 \le t \le T,\ K(0) \text{ given}$$

$$\dot{B}(t) = Y(t),\ 0 \le t \le T,\ B(0) \text{ given}$$

$$(1-\tau)\{p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - M(t) - i(B(t),Y(t))\cdot B(t) - q(t)\cdot[\delta\cdot K(t)]\} \ge \pi_0,\quad 0 \le t \le T$$

$$L(t),\ K(t),\ M(t),\ B(t) \ge 0,\ 0 \le t \le T$$

The modified model (23) differs from (11) in chapter three due to the addition of the constraint (20); the allowance

Table IV-4 Typical Firm's Sources and Uses of Cash under Certainty When Uses Include Bond Interest Payments

Sources of cash:	
From Operations:	
Sales revenue	$p(t) \cdot Q(t)$
Total expenses and taxes	$(1-\tau)\{w(t) \cdot L(t) + M(t) + q(t)[\delta \cdot K(t)] + i(t) \cdot B(t)\} + \tau \cdot p(t) \cdot Q(t)$
Adjustment for noncash outlay	$q(t) \cdot [\delta \cdot K(t)]$
Cash outflow for expenses	$(1-\tau)\{w(t) \cdot L(t) + M(t) + i(t) \cdot B(t)\} + \tau\{p(t) \cdot Q(t) - q(t) \cdot [\delta \cdot K(t)]\}$
Total cash generated by operations	$(1-\tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - i(t) \cdot B(t)\} + \tau \cdot q(t) \cdot [\delta \cdot K(t)]$
From other sources:	
New debt issues	$Y(t)$
Total cash generated	$(1-\tau)\{p(t) \cdot Q(t) - w(t) \cdot L(t) - M(t) - i(t) \cdot B(t)\} + \tau \cdot q(t) \cdot [\delta \cdot K(t)] + Y(t)$ $= \pi(t) + q(t) \cdot [\delta \cdot K(t)] + Y(t)$
Uses of cash:	
To pay dividends:	$D(t)$
To purchase capital goods:	$q(t) \cdot I(t)$
Total cash applied:	$D(t) + q(t) \cdot I(t) \equiv \pi(t) + q(t) \cdot [\delta \cdot K(t)] + Y(t)$
Increase in stock of cash	0

for debt interest expense in the expression for total dividends paid and also in the expression for net income in the profit constraint; and the introduction of the utility function U_4, which is analogous to U_3 in (12).[55] The Lagrangian for problem (15) is[54]

$$L_\mu[K,B,L,I,M,Y,\lambda_1,\lambda_2,t] =$$

$$U_1[\quad]e^{-rt} + \lambda_1(t)\cdot[I(t) - \delta\cdot K(t)] + \lambda_2(t)\cdot[Y(t)] \quad (24)$$

$$+ \mu_1(t)[(1-\tau)\{p(t)\cdot f(K(t),L(t)) - w(t)\cdot L(t) - M(t)$$
$$- i(B(t),Y(t))\cdot B(t) - q(t)[\delta\cdot K(t)]-\pi_o].$$

The necessary conditions for an optimal solution to (24) are conditions (14), (15), and (17)-(21) of chapter three, and in addition,

$$\dot{B}(t) = Y(t),\ 0 \leq t \leq T,\ B(0) \text{ given} \quad (25)$$

$$\dot{\lambda}^*_2(t) = -\frac{\partial L_\mu}{\partial B},\ \lambda^*_2(T) = \frac{\partial U_4}{\partial B}e^{-rT} \quad (26)$$

$$\frac{\partial L_\mu}{\partial Y} = \frac{\partial U_1}{\partial D}\{(1-\tau)(-\frac{\partial i}{\partial Y}B) + 1\}e^{-rt} + \mu_1(1-\tau)(-\frac{\partial i}{\partial Y}B) + \lambda_2(t) = 0 \quad (27)$$

where $\frac{\partial U_4}{\partial B}e^{-rT}$ measures the (discounted) marginal utility attached to having a nonzero terminal stock of outstanding debt, $B(T) > 0$. Note that since (25) merely repeats one of the constraints, it need not be considered further.

Of the two remaining conditions, first consider (27), which can be reexpressed as:

$$-\lambda_2(t) = \frac{\partial U_1}{\partial D}\{1-(1-\tau)\frac{\partial i}{\partial Y}B\}e^{-rt} - \mu_1(1-\tau)\frac{\partial i}{\partial Y}B\ . \quad (28)$$

Note that equation (28) is analogous to equation (22) of chapter three. In (28) $\lambda_2(t)$ is interpretable as the implicit cost of debt, i.e. the cost in terms of discounted collective utility of having an additional dollar of debt outstanding at time t. Note that λ_2 is negative since the additional dollar of debt requires future interest payments (and hence smaller dividends) and, if the debt is retired, a dollar decrease in future dividends in order to purchase the dollar of debt (and thereby 'retire' the dollar's worth of bonds). Issuing an additional dollar of debt at time t enables the firm to increase dividends at time t by an amount equal to one dollar less the marginal cost of debt, $1 - (1 - \tau)\frac{\partial i}{\partial Y} B$, where the factor $(1 - \tau)$ allows for the deductibility of interest expense for tax purposes. The term $-\mu_1(1-\tau)\frac{\partial i}{\partial Y} B$, which is positive when the profit constraint is binding since $\mu_1 > 0$ and $\partial i/\partial Y > 0$, measures the negative impact on collective utility implied by an additional dollar of new debt issues, and the resulting increase in interest expense, $(1 - \tau)\frac{\partial i}{\partial Y} B$. The right-hand side of (28), then, is interpretable as the discounted marginal collective utility of an additional dollar of new debt issues at time t. According to (28), then, the firm should continue to increase new debt issues at each time t up to the point at which the discounted marginal cost of the new debt issues just equals the marginal value of the new issues, where marginal cost and marginal value are measured in terms of discounted collective utility.

Turning next to (26), evaluate $-\partial L_\mu/\partial B$, use (28) to obtain a second expression for $\dot{\lambda}_2(t)$, equate the two expressions, and rearrange terms to obtain[56]

$$(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt})(1-\tau)[\frac{\partial i}{\partial Y}(Y-rB)-i- \frac{\partial i}{\partial B} B]-(1-r)\frac{\partial U_1}{\partial D} = 0 \ . \qquad (29)$$

Equation (29) is used to demonstrate the main result of this section, which is stated as the following theorem:

Theorem IV-3

When the profit constraint is not binding, the firm modeled in (23) will substitute debt for equity in its capital structure in response to an increase in the tax rate τ. When the profit constraint is binding, the overall effect is indeterminate, being the sum of an income effect and a substitution effect, which act in opposing directions.

Proof

When the profit constraint is not binding, $\mu_1 = 0$. Set $\mu_1 = 0$ in (29) and define the function

$$G(Y,\tau) = (1-\tau)[\frac{\partial i}{\partial Y}(Y-rB)-i- \frac{\partial i}{\partial B} B]-(1-r)\frac{\partial U_1}{\partial D} . \qquad (30)$$

Note that since $r < 1$ it follows from (29) that the expression in brackets in (30) is positive. Since $i > 0$, $B > 0$, $\partial i/\partial Y > 0$, and $\partial i/\partial B > 0$ it also follows that $Y-rB > 0$.

By the implicit function theorem,

$$\frac{\partial Y}{\partial \tau} = - \frac{\partial G/\partial \tau}{\partial G/\partial Y} = - \frac{-[\frac{\partial i}{\partial Y}(Y-rB)-i- \frac{\partial i}{\partial B} B]}{(1-\tau)[\frac{\partial^2 i}{\partial Y^2}(Y-rB)+ \frac{\partial i}{\partial Y}]} . \qquad (31)$$

By the foregoing the numerator in (31) is negative. By assumption $\partial i/\partial Y > 0$ and $\partial^2 i/\partial Y^2 > 0$, so that the denominator is positive. Hence $\partial Y/\partial \tau > 0$, and an increase in the tax rate causes the amount of new debt issues to be increased. Since everything else was held fixed, the first statement has been proved.

To prove the second statement, define the function $G(Y, \tau)$ by the expression on the left-hand side of (29).

By the implicit function theorem,

$$\frac{\partial Y}{\partial \tau} = -\frac{\partial G/\partial \tau}{\partial G/\partial Y} = -\{\frac{-(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt})[\frac{\partial i}{\partial Y}(Y-rB)-i-\frac{\partial i}{\partial B}B]}{\partial G/\partial Y}$$

$$+\frac{(\frac{\partial \mu_1}{\partial \tau} e^{rt})(1-\tau)[\frac{\partial i}{\partial Y}(Y-rB)-i-\frac{\partial i}{\partial B}B]}{\partial G/\partial Y}\}$$

$$= \frac{(\frac{\partial U_1}{\partial D} + \mu_1 e^{rt})[\frac{\partial i}{\partial Y}(Y-rB)-i-\frac{\partial i}{\partial B}B]}{\partial G/\partial Y} \tag{32}$$

$$-\frac{(\frac{\partial \mu_1}{\partial \tau} e^{rt})(1-\tau)[\frac{\partial i}{\partial Y}(Y-rB)-i-\frac{\partial i}{\partial B}B]}{\partial G/\partial Y}$$

where
$$\frac{\partial G}{\partial Y} = (\frac{\partial U_1}{\partial D} + \mu_1 e^{rt})(1-\tau)[\frac{\partial^2 i}{\partial Y^2}(Y-rB) + \frac{\partial i}{\partial Y}]$$

$$+\{(1-\tau)[\frac{\partial i}{\partial Y}(Y-rB)-i-\frac{\partial i}{\partial B}B]-(1-r)\}\frac{\partial^2 U_1}{\partial D^2} \tag{33}$$

is positive since the expression in brackets in (29) must be positive, which implies that the expression in braces in (33) must be negative. Hence, the first term in (32) is positive, and, by analogy with (31), corresponds to a substitution effect. The second term in (32) is negative, and corresponds to an income effect. Q.E.D.

Theorem IV-3 leads to the Hicks-Slutsky-type equation (32), which can be expressed generically as:

$$\frac{\partial Y}{\partial \tau} = (\frac{\partial Y}{\partial \tau})_{\pi_o \equiv \text{constant}} + (\frac{\partial Y}{\partial \pi_o})_{\tau \equiv \text{constant}}, \tag{34}$$

where the first term in (34) corresponds to a substitution effect and where the second term corresponds to an income effect. The interpretation of theorem IV-3 is that an increase in the tax rate τ increases the attractiveness of debt financing since interest expense is tax deductible. For the firm in equilibrium with the profit constraint not binding, the increase in τ would induce the firm to substitute debt for equity in its capital structure, i.e. to increase dividends (thereby reducing retained earnings) and to increase debt in order to maintain the pool of money capital available for investment purposes. When the profit constraint is binding there is a second effect. Since increasing new debt issues increases the firm's debt interest obligations, the profit constraint becomes violated unless other adjustments are made. In the extreme case, the increase in the tax rate causes equity to be substituted for debt and that happens when the second term of the Hicks-Slutsky-type equation (32) is greater in value than the first. As in chapter three, the profit constraint, if it is binding, can cause the firm to exhibit the opposite form of behavior to that which it exhibits when the constraint is not binding.

D. DIVIDEND POLICY

The preceding section studied the firm's choice of leverage policy - its choice of the (optimal) debt-equity mix within its capital structure. This section discusses the firm's choice of dividend policy; it takes the firm's choice of leverage policy as given and considers the firm's choice involving the size of the cash dividend. The firm's dividend policy determines what portion of net income will be distributed as dividends to shareholders, and given its investment policy and its leverage policy, also determines the required amount of new equity issues and the composition of equity, E, in table IV-1 between contributed capital and retained earnings.[57]

The relevance of the firm's dividend policy for the stock market value of its equity shares was discussed in section I of chapter two. That discussion will not be repeated here. Rather, the salient points will be reviewed and the role of the firm's dividend policy in the model to be developed in the next section will be suggested.

The traditional view of the relevance of the firm's dividend policy is that a marginal increase in dividends (at the expense of retentions) will raise the share price.[58] Several empirical studies have suggested that a higher dividend payout ratio leads to a higher price-earnings ratio,[59] i.e. to a higher share price for a given level of earnings.[60] It is argued that shareholders prefer immediate dividends to future, less certain capital gains.[61]

An alternative view has been put forward by Miller and Modigliani, who argue that, given the firm's investment policy, its dividend policy is irrelevant to the stock market value of the firm, provided certain assumptions are satisfied.[62] As shown in section I and in the discussion of the Krouse model in section K of chapter two, these assumptions require that capital markets be perfect. In addition, Rubinstein has shown that for irrelevancy to hold under uncertainty, stronger assumptions, such as the assumption of complete markets, are required.[63]

The Krouse model, which was discussed in subsection 4 in section K of chapter two, illustrates the interdependence of the firm's investment and dividend policies when capital markets are imperfect. Krouse's model concerned a value maximizing firm under certainty. In the extension of the author's basic model in the next section of this chapter, the question of separability is studied for an expected collective utility maximizing firm under uncertainty.

E. THE EXPECTED COLLECTIVE UTILITY MAXIMIZER'S OPTIMAL OPERATING AND FINANCIAL POLICIES

The purpose of this section is to characterize the financial policy decisions of the firm under uncertainty. Sections B, C, and D of this chapter discussed the firm's financial policies individually, and each of the illustrative models discussed in those sections abstracted from uncertainty. In this section the three major classes of financial policy decisions - cash management policy, leverage policy, and dividend policy - are considered within the same model under uncertainty, and the relationship between the firm's financial policy decisions and its operating policy decisions is explored.

1. The Model

The purpose of this subsection is to develop the model of the expected discounted collective utility maximizing firm. The modeling technique employed below in formulating the model is the time-state-preference framework.[64]

As before, it is assumed that the firm seeks to maximize expected discounted collective utility over a finite planning horizon extending T periods into the future. In chapter three collective utility during each period t was expressed as a function U_1 of total revenue, total dividends paid, and managerial emoluments, and in addition, as a function U_2 of the terminal capital stock. But in chapter three the firm's choice of dividend policy and the impact of this policy choice on the firm's share valuation were not treated explicitly. In this chapter the firm's choice of dividend policy as it affects share valuation and collective utility is a matter of interest. In the remainder of this chapter it is assumed that dividends are valued by shareholders because of their contribution to the value of the shares they hold. In the remainder of this chapter it is the share value, v, rather than total dividends paid, D, that is the direct source of shareholder satisfaction; and v replaces D in U_1. As described below, v is a function of the firm's choice of dividend policy. But v is a function of the riskiness attached to the shares as well as a function of the dividend per share. Thus, a framework for handling uncertainty (or risk[65]) must be developed before an expression for v can be obtained.

To deal explicitly with uncertainty it is assumed that at each time t there are S distinguishable states of nature. These are designated s = 1, ..., S. While it is assumed for convenience that the number of possible states is the same for each time t, it is not required that the set of possible states be the same for each t, nor is it required that the states be numbered in any particular order. Time is measured in discrete units. The time periods are designated t = 0, ..., T, where t = 0 denotes the present, at which the state of nature is assumed known with certainty. Henceforth, the double subscript t,s, $1 \leq t \leq T$, $s = 1, \ldots, S$, will designate the state of nature at time t.

The possible states of nature s at each time t are meant to reflect the different possible states of demand for the firm's product, the different possible conditions prevailing in factor markets (implying different factor prices) and in financial markets (implying different interest rates and different stock market values), the different possible acts of nature that might affect the firm, and other conditions of the firm's operating environment that might have a nonnegligible impact on the firm. It is assumed that the possible states of nature at each time t lie beyond the firm's control, and that the firm can attach a (possibly subjective) probability of occurrence, $\phi_{t,s}$, to each.

In this section it is not required that financial markets be complete.[66] It is also not required that a set of complete markets for contingent output claims exist.[67] Let $Q_{t,s}$ denote the quantity of output sold during period t in state of nature s. It is assumed that the alternative output levels, $Q_{t,s}$, for each time t represent contingent output levels. Inputs are purchased during period t and output for period t is produced only after state s has been realized. The set of output levels, $Q_{t,s}$, $t = 1, \ldots, T$, $s = 1, \ldots, S$, that arise out of the model developed below are interpreted, then, as planned output levels contingent on the state s that actually obtains at each time t, rather than as output levels to which the firm is contractually committed.[68]

In the remainder of this chapter a modified version of the Sharpe-Lintner-Mossin share valuation formula[69] is used to express the value of a share, v_t, at each time t. It is assumed that the share value at each time t, v_t, satisfies

$$v_t = \frac{E[d] - \beta \cdot Var(d)}{\rho}, \tag{35}$$

where E[d] denotes the expected dividend at time t, Var(d) denotes the variance of the dividend at time t, β denotes an exogenously determined positive constant that measures the stock market's collective aversion to risk,[70] and ρ denotes the exogenously determined riskless rate at which the stock market capitalizes the firm's risk-adjusted expected return, $E[d] - \beta \cdot Var(d)$. Thus, in (35) it is assumed that the variance of the dividend, Var(d), serves as the measure of the riskiness attached to the shares by the stock market. It is assumed that the parameter ρ takes into account the market's assessment of the firm's long run growth prospects.[71]

Denoting by $d_{t,s}$ the dividend per share at time t in state of nature s,

$$E[d] = \sum_{s=1}^{S} \phi_{t,s} \cdot d_{t,s}$$

$$Var(d) = E[d^2]-(E[d])^2 = \sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 - \{ \sum_{s=1}^{S} \phi_{t,s} d_{t,s} \}^2 .$$

Substituting for E[d] and Var(d) in (35) yields

$$v_t = \frac{\sum_{s=1}^{S} \phi_{t,s} d_{t,s} - \beta[\sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 - \{ \sum_{s=1}^{S} \phi_{t,s} d_{t,s} \}^2]}{\rho} . \tag{36}$$

v_t given by (36) is one of the three arguments of the collective utility function U_1, and the firm can influence v_t through its choice of dividend policy $\{d_{t,s}\}_{s=1}^{S}$ for each time t.

Denoting total revenue at time t in state of nature s by $R_{t,s}$, and similarly interpreting $M_{t,s}$ and $K_{T,s}$, the objective functional is expressed as

$$\text{maximize } \sum_{t=1}^{T} \sum_{s=1}^{S} \phi_{t,s} \cdot U_1(R_{t,s}; v_t; M_{t,s}) \left(\frac{1}{1+r}\right)^t + \sum_{s=1}^{S} \phi_{T,s} U_2(K_{T,s}) \left(\frac{1}{1+r}\right)^T , \tag{37}$$

where, as discussed above, the stock market value of a share, v_t, given by (36) replaces total dividends paid, $D_{t,s}$, as the argument of U_1 reflecting the main source of shareholder utility. For each date t and state s denote the price of output by $p_{t,s}$ and denote the wage rate, the price of capital goods, and the average interest rate on debt by $w_{t,s}$, $q_{t,s}$, and $i_{t,s}$, respectively. If it is again assumed that the product, labor, and capital goods markets are perfectly competitive and that the average interest rate on debt for each date and state satisfies (21), then total revenue for each period t and state of nature s is given by

$$R_{t,s} = p_{t,s} \cdot Q_{t,s} , \tag{38}$$

where $Q_{t,s}$ denotes the quantity of output at date t in state s. Total profit at date t in state s is given by

$$\pi_{t,s} = (1-\tau)\{p_{t,s} \cdot Q_{t,s} - w_{t,s} L_{t,s} - M_{t,s} - q_{t,s} \cdot \delta \cdot K_{t,s} - i_{t,s}(B_{t,s}, Y_{t,s}) \cdot B_{t,s}\} , \tag{39}$$

where τ and δ are the exogenously determined tax rate and rate of depreciation, respectively, and where depreciation is again figured on a replacement cost basis.

Previously it has been assumed that net income given by (39) had to satisfy some minimum required level determined by financial factors, such as the need to generate sufficient cash from operations with which to pay bond interest and the need to maintain sufficient profits to satisfy the firm's shareholders that the firm's assets were being well-managed and to reduce thereby the threat of takeover. In the remainder of this chapter these financial factors are treated explicitly, so that a minimum net income constraint would be redundant. Specifically, the threat of takeover is linked directly to the share value.[72] By incorporating v_t as an argument of U_1 in (37) this factor is taken into account. A higher share value increases managerial utility, in addition to shareholder utility, because it reduces the likelihood of a takeover. The second aspect of the previously employed net income constraint that is incorporated directly into the model in this section is the explicit treatment of cash, part of which must be used to meet bond interest obligations, and the introduction of precautionary cash balances, which are held as a reserve to enable the firm to meet its bond interest obligations when poor market demand conditions cause the firm's net income to be inadequate.

Turning to production, it is assumed as before that the firm has a production function that is strictly concave and continuously differentiable with strictly positive first partial derivatives. In addition, it is assumed that the same technological relationship between inputs and outputs holds for each date and state, so that

$$Q_{t,s} = f(K_{t,s}, L_{t,s}) \tag{40}$$

for all t and for all s, with

$$\left.\begin{array}{ll} \partial Q_{t,s}/\partial K_{t,s} > 0 & \partial Q_{t,s}/\partial L_{t,s} > 0 \\ \\ \partial^2 Q_{t,s}/\partial K_{t,s}^2 < 0 & \partial^2 Q_{t,s}/\partial L_{t,s}^2 < 0 \;. \end{array}\right\} \tag{41}$$

The capital stock, $K_{t,s}$, and investment, $I_{t,s}$, for each date and state must satisfy the constraint

$$\Delta K_{t,s} \equiv K_{t,s} - K_{t-1,s'} = I_{t,s} - \delta \cdot K_{t,s} \;, \tag{42}$$

where the change in the capital stock in (42) is dependent on the state s' obtaining during period t - 1, and where it is assumed that investment is made at the beginning of the period and that depreciation is reckoned on the basis of the current period's capital stock. Note that (42) is just the discrete analogue of the net investment constraint employed in chapter three and earlier in this chapter. Solving (42) for $K_{t,s}$ yields

$$K_{t,s} = \left(\frac{1}{1+\delta}\right)K_{t-1,s'} + \left(\frac{1}{1+\delta}\right)I_{t,s} \;. \tag{43}$$

To complete the model it is necessary to formulate the financial identities. First, there is a balance sheet identity that needs to be satisfied. With reference to the typical firm's balance sheet in table IV-1, the following accounting identity must hold for each date t and state s:

$$C_{t,s} + V_{t,s} + q_{t,s} \cdot K_{t,s} = B_{t,s} + E_{t,s} \;. \tag{44}$$

Cash on hand, $C_{t,s}$, is assumed to equal some minimum level, $\overline{C}_{t,s}$, which is needed to fund transactions and which is a function of the level of output, plus some additional amount (possibly zero), $\hat{C}_{t,s}$, held as precautionary balances, or in equation form,

$$C_{t,s} = \bar{C}_{t,s}(Q_{t,s}) + \hat{C}_{t,s} , \tag{45}$$

where

$$d\bar{C}/dQ > 0 \qquad\qquad d^2\bar{C}/dQ^2 < 0 . \tag{46}$$

It is also assumed that there is some minimum value of inventories held at each time t and state s that is a function of the level of output,

$$V_{t,s} = V_{t,s}(Q_{t,s}) , \tag{47}$$

where

$$dV/dQ > 0 \qquad\qquad d^2V/dQ^2 < 0 . \tag{48}$$

Substituting (45) and (47) into (44) yields the balance sheet identity,

$$\bar{C}_{t,s}(Q_{t,s}) + \hat{C}_{t,s} + V_{t,s}(Q_{t,s}) + q_{t,s} \cdot K_{t,s} = B_{t,s} + E_{t,s} . \tag{49}$$

Turning to the liabilities side of the balance sheet in table IV-1, there are several identities that must be satisfied. Letting $Y_{t,s}$ denote the amount of bonds issued/redeemed at time t in state of nature s, where for convenience each bond is assumed to be worth one dollar, and letting $Z_{t,s}$ denote the number of shares of stock issued/repurchased at time t in state of nature s, (20) becomes

$$B_{t,s} = B_{t-1,s'} + Y_{t,s} , \tag{50}$$

and the analogous identity for shares is

$$n_{t,s} = n_{t-1,s'} + Z_{t,s} , \tag{51}$$

where $n_{t,s}$ denotes the number of shares outstanding at time t in state of nature s.

Next, consider the change in equity. As indicated by table IV-1, total equity consists of contributed capital plus accumulated retained earnings. Let $E_{t,s}$ denote the amount of equity at time t in state of nature s, and let ${}_K E_{t,s}$ and ${}_R E_{t,s}$ denote the portions that represent contributed capital and retained earnings, respectively, or in equation form,

$$E_{t,s} = {}_K E_{t,s} + {}_R E_{t,s} \ . \tag{52}$$

It follows from (52) that

$$\Delta E_{t,s} = \Delta_K E_{t,s} + \Delta_R E_{t,s} \ . \tag{53}$$

But $\Delta_K E_{t,s}$ is the change in contributed capital, which is just the share price times the number of shares issued/ repurchased. With v_t given by (36), the following identity must hold

$$\Delta_K E_{t,s} = Z_{t,s} \cdot v_t \ . \tag{54}$$

As indicated by table IV-5,

$$\Delta_R E_{t,s} = \pi_{t,s} - D_{t,s} \ . \tag{55}$$

Substituting (54) and (55) into (53) and rearranging terms gives

$$E_{t,s} = E_{t-1,s'} + Z_{t,s} \cdot v_t + \pi_{t,s} - D_{t,s} \ . \tag{56}$$

Unlike the version of the expected discounted collective utility maximizer presented in chapter three, in this section the dividend per share for each date and state, $d_{t,s}$, is a decision variable. It is assumed that new shares are issued with dividends (i.e. they are issued at the beginning of the period), so that total dividends paid amount to

$$D_{t,s} = n_{t,s} \cdot d_{t,s} \ . \tag{57}$$

Table IV-5 Statement of Retained Earnings

Beginning balance	$R^E{}_{t-1,s'}$
Add net income for the year	$\pi_{t,s}$
Total	$R^E{}_{t-1,s'} + \pi_{t,s}$
Less total dividends paid during the year	$D_{t,s}$
Ending balance, retained earnings	$R^E{}_{t,s} = \underline{\underline{R^E{}_{t-1,s'} + \pi_{t,s} - D_{t,s}}}$

If it is assumed that all dividends are paid in cash, rather than through the issuance of new shares, then it follows from table IV-6 and the identity between sources of cash on the one hand and uses of cash plus the increase (or decrease) in the stock of cash on the other that total dividends paid at time t in state of nature s must satisfy the identity

$$\begin{aligned} D_{t,s} = n_{t,s} \cdot d_{t,s} = {} & (1-\tau)\{p_{t,s} \cdot Q_{t,s} - w_{t,s} \cdot L_{t,s} - M_{t,s} \\ & - i_{t,s} \cdot B_{t,s}\} + \tau \cdot q_{t,s} \cdot \delta \cdot K_{t,s} + Y_{t,s} \\ & + Z_{t,s} \cdot v_t - q_{t,s} \cdot I_{t,s} - (C_{t,s} - C_{t-1,s'}) \\ & - (V_{t,s} - V_{t-1,s'}) \,. \end{aligned} \tag{58}$$

Substituting (36) for v_t, (38) for $R_{t,s}$, (39) for $\pi_{t,s}$, (40) for $Q_{t,s}$, (45) for $C_{t,s}$, and (57) for $D_{t,s}$, and collecting the constraints (43), (49), (50), (51), (56), and (58), the model of the expected discounted collective utility maximizer generalized to incorporate the firm's financial policy variables can be formulated as the following stochastic optimal control problem:

$$\underset{\substack{\{L_{t,s}, I_{t,s},\\ M_{t,s}, Y_{t,s},\\ Z_{t,s}, d_{t,s},\\ \hat{C}_{t,s}\}}}{\text{maximize}} \quad \sum_{t=1}^{T} \sum_{s=1}^{S} \phi_{t,s} \cdot U_1\Big[p_{t,s} \cdot f(K_{t,s}, L_{t,s}); \frac{1}{\rho}\Big(\sum_{s=1}^{S} \phi_{t,s} d_{t,s} - \beta\Big[\sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 - \Big\{\sum_{s=1}^{S} \phi_{t,s} d_{t,s}\Big\}^2\Big]\Big); M_{t,s}\Big]\Big(\frac{1}{1+r}\Big)^t + \sum_{s=1}^{S} \phi_{T,s} \cdot U_2(K_{T,s})\Big(\frac{1}{1+r}\Big)^T \tag{59}$$

subject to

$$K_{t,s} = \Big(\frac{1}{1+\delta}\Big)K_{t-1,s'} + \Big(\frac{1}{1+\delta}\Big)I_{t,s}, \quad K_{o,s_o} \text{ given} \tag{59a}$$

$$B_{t,s} = B_{t-1,s'} + Y_{t,s}, \quad B_{o,s_o} \text{ given} \tag{59b}$$

$$n_{t,s} = n_{t-1,s'} + Z_{t,s}, \quad n_{o,s_o} \text{ given} \tag{59c}$$

$$\begin{aligned} E_{t,s} = {} & E_{t-1,s'} + Z_{t,s} \cdot \frac{1}{\rho}\Big(\sum_{s=1}^{S} \phi_{t,s} d_{t,s} \\ & - \beta\Big[\sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 - \Big\{\sum_{s=1}^{S} \phi_{t,s} d_{t,s}\Big\}^2\Big]\Big) \\ & + (1-\tau)\{p_{t,s} \cdot f(K_{t,s}, L_{t,s}) - w_{t,s} \cdot L_{t,s} \\ & - M_{t,s} - q_{t,s} \cdot \delta \cdot K_{t,s} - i_{t,s}(B_{t,s}, Y_{t,s}) \cdot B_{t,s}\} \\ & - d_{t,s} \cdot n_{t,s}, \quad E_{o,s_o} \text{ given} \end{aligned} \tag{59d}$$

$$\begin{aligned} \bar{C}_{t,s}[f(K_{t,s}, L_{t,s})] + \hat{C}_{t,s} + V_{t,s}[f(K_{t,s}, L_{t,s})] & \\ + q_{t,s} \cdot K_{t,s} = B_{t,s} + E_{t,s} & \end{aligned} \tag{59e}$$

$$\begin{aligned} n_{t,s} \cdot d_{t,s} = {} & (1-\tau)\{p_{t,s} \cdot f(K_{t,s}, L_{t,s}) - w_{t,s} \cdot L_{t,s} \\ & - M_{t,s} - i_{t,s}(B_{t,s}, Y_{t,s}) \cdot B_{t,s}\} \\ & + \tau \cdot q_{t,s} \cdot \delta \cdot K_{t,s} + Y_{t,s} + Z_{t,s} \times \\ & \frac{1}{\rho}\Big(\sum_{s=1}^{S} \phi_{t,s} d_{t,s} - \beta\Big[\sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 \end{aligned} \tag{59f}$$

$$-\{\sum_{s=1}^{S}\phi_{t,s}d_{t,s}\}^2])-q_{t,s}\cdot I_{t,s}$$

$$-(\overline{C}_{t,s}[f(K_{t,s},L_{t,s})]+\hat{C}_{t,s}$$

$$-\overline{C}_{t-1,s'}[f(K_{t-1,s'},L_{t-1,s'})]-\hat{C}_{t-1,s'})$$

$$-(V_{t,s}[f(K_{t,s},L_{t,s})]$$

$$-V_{t-1,s'}[f(K_{t-1,s'},L_{t-1,s'})]),$$

$$\overline{C}_{o,s_o}\ ,\ \hat{C}_{o,s_o}\ ,\ V_{o,s_o}\ \text{given}$$

$$K_{t,s},L_{t,s},M_{t,s},B_{t,s},n_{t,s},E_{t,s},\hat{C}_{t,s},d_{t,s} \geq 0$$

where s_o denotes the (certain) state at time t = 0.

According to (59), the objective of the firm is to maximize expected discounted collective utility subject to a net investment constraint (59a), a bond issue/ redemption constraint (59b), a share issue/repurchase constraint (59c), a change in total equity identity (59d), the basic balance sheet identity (59e), and the sources of, uses of, and change in the stock of cash identity (59f), plus, of course, nonnegativity constraints. The decision variables are shown in table IV-7, according to the type of policy involved. The next two subsections are concerned with the optimal operating and financial policies implied by the model (59) and with the relationship between these two sets of policies.

Table IV-6 Typical Firm's Statement of Sources and Uses of Cash

Sources of cash:	
From operations:	
Sales revenue	$p_{t,s} \cdot Q_{t,s}$
Total expenses and taxes	$(1-\tau)\{w_{t,s} \cdot L_{t,s} + M_{t,s} + q_{t,s} \cdot \delta \cdot K_{t,s} + i_{t,s} \cdot B_{t,s}\} + \tau \cdot p_{t,s} \cdot Q_{t,s}$
Adjustment for noncash outlay	$q_{t,s} \cdot \delta \cdot K_{t,s}$
Cash outflow for expenses	$(1-\tau)\{w_{t,s} \cdot L_{t,s} + M_{t,s} + i_{t,s} \cdot B_{t,s}\} + \tau\{p_{t,s} \cdot Q_{t,s} - q_{t,s} \cdot \delta \cdot K_{t,s}\}$
Total cash generated by operations	$(1-\tau)\{p_{t,s} \cdot Q_{t,s} - w_{t,s} \cdot L_{t,s} - M_{t,s} - i_{t,s} \cdot B_{t,s}\} + \tau \cdot q_{t,s} \cdot \delta \cdot K_{t,s}$
From other sources:	
New debt issues	$Y_{t,s}$
New equity issues	$Z_{t,s} \cdot v_t$
Total cash generated	$(1-\tau)\{p_{t,s} \cdot Q_{t,s} - w_{t,s} \cdot L_{t,s} - M_{t,s} - i_{t,s} \cdot B_{t,s}\} + \tau \cdot q_{t,s} \cdot \delta \cdot K_{t,s} + Y_{t,s} + Z_{t,s} \cdot v_t$
Uses of cash:	
To pay dividends	$D_{t,s}$
To purchase capital goods	$q_{t,s} \cdot I_{t,s}$
To increase inventories	$V_{t,s} - V_{t-1,s'}$
Total cash applied	$D_{t,s} + q_{t,s} \cdot I_{t,s} + V_{t,s} - V_{t-1,s'}$

Increase (decrease) in stock of cash $C_{t,s} - C_{t-1,s'} \equiv (1-\tau)\{p_{t,s} \cdot Q_{t,s} - w_{t,s} \cdot L_{t,s} - M_{t,s} - i_{t,s} \cdot B_{t,s}\}$

$$+\tau \cdot q_{t,s} \cdot \delta \cdot K_{t,s} + Y_{t,s} + Z_{t,s} \cdot v_t - D_{t,s} - q_{t,s} \cdot I_{t,s}$$

$$-(V_{t,s} - V_{t-1,s'})$$

Table IV-7 The Firm's Policy Variables

Operating Policy		Financial Policy	
labor input	$L_{t,s}$	bond issue/redempt.	$Y_{t,s}$
investment	$I_{t,s}$	share issue/repurch.	$Z_{t,s}$
mgr. emol.	$M_{t,s}$	dividend	$d_{t,s}$
		cash	$\hat{C}_{t,s}$

2. Optimal Operating Policies

Define the following generalized Lagrangian for (59):

$$
\begin{aligned}
L_\lambda = & \sum_{t=1}^{T} \sum_{s=1}^{S} \phi_{t,s} \cdot U_1 \Big[p_{t,s} \cdot f(K_{t,s}, L_{t,s}); \frac{1}{\rho} \Big(\sum_{s=1}^{S} \phi_{t,s} d_{t,s} - \beta \Big[\sum_{s=1}^{S} \phi_{t,s} d_{t,s}^2 \\
& - \Big\{ \sum_{s=1}^{S} \phi_{t,s} \cdot d_{t,s} \Big\}^2 \Big] \Big); M_{t,s} \Big] \Big(\frac{1}{1+r}\Big)^t + \sum_{s=1}^{S} \phi_{T,s} \cdot U_2(K_{T,s}) \Big(\frac{1}{1+r}\Big)^T \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{1,t,s,s'} \Big[\Big(\frac{1}{1+\delta}\Big) K_{t-1,s'} + \Big(\frac{1}{1+\delta}\Big) I_{t,s} - K_{t,s} \Big] \qquad (60) \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{2,t,s,s'} [B_{t-1,s'} + Y_{t,s} - B_{t,s}] \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{3,t,s,s'} [n_{t,s} - Z_{t,s} - n_{t-1,s'}] \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{4,t,s,s'} \Big[E_{t-1,s'} + Z_{t,s} \cdot \frac{1}{\rho} \Big(\sum_{s=1}^{S} \phi_{t,s} \cdot d_{t,s} \\
& \qquad - \beta \Big[\sum_{s=1}^{S} \phi_{t,s} \, d_{t,s}^2 - \Big\{ \sum_{s=1}^{S} \phi_{t,s} d_{t,s} \Big\}^2 \Big] \Big) \\
& \qquad + (1-\tau) \{ p_{t,s} \cdot f(K_{t,s}, L_{t,s}) - w_{t,s} \cdot L_{t,s} - M_{t,s} - q_{t,s} \cdot \delta \cdot K_{t,s} \\
& \qquad - i_{t,s}(B_{t,s}, Y_{t,s}) \cdot B_{t,s} \} - d_{t,s} \cdot n_{t,s} - E_{t,s} \Big] \\
& + \sum_{t=1}^{T} \sum_{s=1}^{S} \lambda_{5,t,s} \Big[B_{t,s} + E_{t,s} - \bar{C}_{t,s}[f(K_{t,s}, L_{t,s})] - \hat{C}_{t,s} \\
& \qquad - V_{t,s}[f(K_{t,s}, L_{t,s})] - q_{t,s} \cdot K_{t,s} \Big]
\end{aligned}
$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{6,t,s,s'}[(1-\tau)\{p_{t,s}\cdot f(K_{t,s},L_{t,s})-w_{t,s}\cdot L_{t,s}$$

$$-M_{t,s}-i_{t,s}(B_{t,s},Y_{t,s})\cdot B_{t,s}\}+\tau\cdot q_{t,s}\cdot\delta\cdot K_{t,s}$$

$$+Y_{t,s}+Z_{t,s}\cdot\frac{1}{\rho}(\sum_{s=1}^{S}\phi_{t,s}d_{t,s}-\beta[\sum_{s=1}^{S}\phi_{t,s}\cdot d_{t,s}^2$$

$$-\{\sum_{s=1}^{S}\phi_{t,s}\cdot d_{t,s}\}^2])-q_{t,s}\cdot I_{t,s}-\overline{C}_{t,s}[f(K_{t,s},L_{t,s})]$$

$$-\hat{C}_{t,s}+\overline{C}_{t-1,s'}[f(K_{t-1,s'},L_{t-1,s'})]+\hat{C}_{t-1,s'}$$

$$-V_{t,s}[f(K_{t,s},L_{t,s})]+V_{t-1,s'}[f(K_{t-1,s'},L_{t-1,s'})]$$

$$-n_{t,s}\cdot d_{t,s}]$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{7,t,s}\hat{C}_{t,s}\ ,$$

where the last term in (60) corresponds to the nonnegativity constraint $\hat{C}_{t,s} \geq 0$ in (59).[73] In the next subsection the firm's decision regarding precautionary cash balances will be considered and conditions under which $\hat{C}_{t,s} = 0$ at optimality will be distinguished from those under which $\hat{C}_{t,s} > 0$ at optimality.

The necessary conditions for an optimal solution to (59) are the following, where the constraints in (59) have not been repeated and where all decision variables but $\hat{C}_{t,s}$ have been assumed to be strictly positive all along their respective optimal time paths:

$$\frac{\partial L_\lambda}{\partial L_{t,s}} = \phi_{t,s}\frac{\partial U_1}{\partial R}\, p_{t,s}\frac{\partial f}{\partial L_{t,s}}(\frac{1}{1+r})^t + \sum_{s'=1}^{S}\lambda_{4,t,s,s'}(1-\tau)\{p_{t,s}\frac{\partial f}{\partial L_{t,s}}$$

$$-w_{t,s}\} + \lambda_{5,t,s}[-\frac{\partial\overline{C}_{t,s}}{\partial Q_{t,s}}\frac{\partial f}{\partial L_{t,s}} - \frac{\partial V_{t,s}}{\partial Q_{t,s}}\frac{\partial f}{\partial L_{t,s}}] \tag{61}$$

$$+ \sum_{s'=1}^{S} \lambda_{6,t,s,s'}[(1-\tau)\{p_{t,s} \frac{\partial f}{\partial L_{t,s}} - w_{t,s}\} - \frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial L_{t,s}}$$

$$- \frac{\partial V_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial L_{t,s}}]$$

$$+ \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} [\frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial L_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial L_{t,s}}] = 0$$

$$\frac{\partial L_{\lambda}}{\partial I_{t,s}} = \sum_{s'=1}^{S} \lambda_{1,t,s,s'} (\frac{1}{1+\delta}) + \sum_{s'=1}^{S} \lambda_{6,t,s,s'}[-q_{t,s}] = 0 \quad (62)$$

$$\frac{\partial L_{\lambda}}{\partial M_{t,s}} = \phi_{t,s} \frac{\partial U_1}{\partial M_{t,s}} (\frac{1}{1+r})^t + \sum_{s'=1}^{S} \lambda_{4,t,s,s'}[-(1-\tau)]$$

$$+ \sum_{s'=1}^{S} \lambda_{6,t,s,s'}[-(1-\tau)] = 0 \quad (63)$$

$$\frac{\partial L_{\lambda}}{\partial K_{t,s}} = \phi_{t,s} \frac{\partial U_1}{\partial R} p_{t,s} \frac{\partial f}{\partial K_{t,s}} (\frac{1}{1+r})^t + \sum_{s'=1}^{S} \lambda_{1,t,s,s'}(-1)$$

$$+ \sum_{s'=1}^{S} \lambda_{1,t+1,s',s} (\frac{1}{1+\delta}) + \sum_{s'=1}^{S} \lambda_{4,t,s,s'}(1-\tau)\{p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$

$$-q_{t,s} \cdot \delta\} + \lambda_{5,t,s}[- \frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} - \frac{\partial V}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} - q_{t,s}] \quad (64)$$

$$+ \sum_{s'=1}^{S} \lambda_{6,t,s,s'}[(1-\tau)\{p_{t,s} \frac{\partial f}{\partial K_{t,s}}\} + \tau \cdot q_{t,s} \cdot \delta$$

$$- \frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} - \frac{\partial V}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}}]$$

$$+ \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} [\frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} + \frac{\partial V}{\partial Q_{t,s}} \cdot \frac{\partial f}{\partial K_{t,s}}] = 0 ,$$

$$0 < t < T$$

$$\frac{\partial L_\lambda}{\partial K_{T,s}} = \phi_{T,s}\frac{\partial U_1}{\partial R}p_{T,s}\frac{\partial f}{\partial K_{T,s}}\left(\frac{1}{1+r}\right)^T+\phi_{T,s}\frac{\partial U_2}{\partial K_{T,s}}\left(\frac{1}{1+r}\right)^T$$

$$+\sum_{s'=1}^{S}\lambda_{1,T,s,s'}(-1)+\sum_{s'=1}^{S}\lambda_{4,T,s,s'}(1-\tau)\{p_{T,s}\frac{\partial f}{\partial K_{T,s}}$$

$$-q_{T,s}\cdot\delta\}+\lambda_{5,T,s}[-\frac{\partial \bar{C}_{T,s}}{\partial Q_{T,s}}\frac{\partial f}{\partial K_{T,s}}-\frac{\partial V_{T,s}}{\partial Q_{T,s}}\frac{\partial f}{\partial K_{T,s}}-q_{T,s}] \tag{65}$$

$$+\sum_{s'=1}^{S}\lambda_{6,T,s,s'}[(1-\tau)\{p_{T,s}\frac{\partial f}{\partial K_{T,s}}\}+\tau\cdot q_{T,s}\cdot\delta$$

$$-\frac{\partial \bar{C}_{T,s}}{\partial Q_{T,s}}\frac{\partial f}{\partial K_{T,s}}-\frac{\partial V_{T,s}}{\partial Q_{T,s}}\frac{\partial f}{\partial K_{T,s}}] = 0$$

$$\frac{\partial L_\lambda}{\partial Y_{t,s}} = \sum_{s'=1}^{S}\lambda_{2,t,s,s'}+\sum_{s'=1}^{S}\lambda_{4,t,s,s'}(1-\tau)(-\frac{\partial i_{t,s}}{\partial Y_{t,s}}B_{t,s})$$

$$+\sum_{s'=1}^{S}\lambda_{6,t,s,s'}[(1-\tau)(-\frac{\partial i_{t,s}}{\partial Y_{t,s}}B_{t,s})+1] = 0 \tag{66}$$

$$\frac{\partial L_\lambda}{\partial B_{t,s}} = \sum_{s'=1}^{S}\lambda_{2,t,s,s'}(-1)+\sum_{s'=1}^{S}\lambda_{2,t+1,s',s}$$

$$+\sum_{s'=1}^{S}\lambda_{4,t,s,s'}(1-\tau)(-i_{t,s}-\frac{\partial i_{t,s}}{\partial B_{t,s}}B_{t,s})+\lambda_{5,t,s} \tag{67}$$

$$+\sum_{s'=1}^{S}\lambda_{6,t,s,s'}[(1-\tau)(-i_{t,s}-\frac{\partial i_{t,s}}{\partial B_{t,s}}B_{t,s}] = 0$$

$$\frac{\partial L_\lambda}{\partial Z_{t,s}} = \sum_{s'=1}^{S}\lambda_{3,t,s,s'}(-1)+\sum_{s'=1}^{S}\lambda_{4,t,s,s'}v_{t,s}$$

$$+\sum_{s'=1}^{S}\lambda_{6,t,s,s'}v_{t,s} = 0 \tag{68}$$

$$\frac{\partial L_\lambda}{\partial n_{t,s}} = \sum_{s'=1}^{S}\lambda_{3,t,s,s'}+\sum_{s'=1}^{S}\lambda_{3,t+1,s',s}(-1)+\sum_{s'=1}^{S}\lambda_{4,t,s,s'}(-d_{t,s})$$

$$+\sum_{s'=1}^{S}\lambda_{6,t,s,s'}(-d_{t,s}) = 0 \tag{69}$$

$$\frac{\partial L_\lambda}{\partial E_{t,s}} = \sum_{s'=1}^{S} \lambda_{4,t,s,s'}(-1) + \sum_{s'=1}^{S} \lambda_{4,t+1,s',s} + \lambda_{5,t,s} = 0 \tag{70}$$

$$\frac{\partial L_\lambda}{\partial d_{t,s}} = \frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \phi_{t,s} \frac{\partial U_1}{\partial v_t}\left(\frac{1}{1+r}\right)^t + \frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{4,t,s,s'} \times$$

$$Z_{t,s} + \sum_{s'=1}^{S} \lambda_{4,t,s,s'}(-n_{t,s}) + \frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{6,t,s,s'} \times \tag{71}$$

$$Z_{t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'}(-n_{t,s}) = 0$$

$$\frac{\partial L_\lambda}{\partial \hat{C}_{t,s}} = \lambda_{5,t,s}(-1) + \sum_{s'=1}^{S} \lambda_{6,t,s,s'}(-1) + \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}$$

$$+\lambda_{7,t,s} = 0 \tag{72}$$

$$\hat{C}_{t,s} \geq 0 \qquad \lambda_{7,t,s} \geq 0 \qquad \lambda_{7,t,s} \cdot \hat{C}_{t,s} = 0 \tag{73}$$

where it follows from (36) that

$$\frac{\partial v_t}{\partial d_{t,s}} = \frac{1}{\rho}(\phi_{t,s} - \beta[2\phi_{t,s} d_{t,s} - 2\phi_{t,s} \sum_{s'=1}^{S} \phi_{t,s'} d_{t,s'}]) \tag{74}$$

in (71). Note that (74) implies that

$$\left.\begin{aligned} \frac{\partial v_t}{\partial d_{t,s}} > 0 \Leftrightarrow \frac{\phi_{t,s}}{\rho} &> \frac{\beta}{\rho}[2\phi_{t,s} d_{t,s} - 2\phi_{t,s} \sum_{s'=1}^{S} \phi_{t,s'} d_{t,s'}] \\ \Leftrightarrow \frac{1}{2\beta} &> d_{t,s} - E[d] \ . \end{aligned}\right\} \tag{75}$$

(75) implies that an increase in the dividend per share planned for time t and state of nature s may decrease the share value at t if its marginal impact on the expected dividend per share is less than the marginal impact on the riskiness per share, but that if the distribution of the planned dividend per share across states of nature is clustered tightly around the mean[74] (i.e. $d_{t,s} - E[d] < \frac{1}{2\beta}$ for all $d_{t,s}$), then an increase in any $d_{t,s}$ will lead to an increase

in v_t. In particular, if $d_{t,s}$ is independent of s, then $d_{t,s} \equiv E[d]$ for all s, and since $\beta > 0$ by assumption, $\frac{\partial v_t}{\partial d_{t,s}} > 0$ as one would expect. Note that (74) also implies that

$$\frac{\partial^2 v_t}{\partial d_{t,s}^2} = - \frac{2\beta\phi_{t,s}}{\rho}(1-\phi_{t,s}) < 0 \tag{76}$$

since $\beta > 0$, $\rho > 0$, and $0 < \phi_{t,s} < 1$. According to (76), even if increases in the planned dividend lead to increases in v_t, they do so at a decreasing rate.

The remainder of this subsection characterizes the firm's optimal operating policies, which are implied by (61)-(65), and the next subsection characterizes its optimal financial policies, which are implied by (66)-(73), and explores the relationship between the firm's financial policy decisions and its operating policy decisions. In each case the optimal policies under certainty are determined by fixing the states of nature s at t and s' at t + 1, and then the optimal policies under uncertainty are determined by permitting multiple possible states at t and at t + 1.

First, consider the policy implications of (61) under certainty. Fix s at t and s' at t + 1. That is, $\phi_{t,s} = 1$ for some s at t (with $\phi_{t,s} \equiv 0$ for all other s at t) and $\phi_{t+1,s'} = 1$ for some s' at t + 1 (with $\phi_{t+1,s'} \equiv 0$ for all other s' at t + 1). But then the subscripts s and s' in (61) (as well as in (62)-(73)) are redundant and $\lambda_{6,t}$ replaces $\sum_{s'=1}^{S} \lambda_{6,t,s,s'}$ and $\lambda_{6,t+1}$ replaces $\sum_{s'=1}^{S} \lambda_{6,t+1,s',s}$ in (61). Simplifying (61) in this manner and rearranging terms yields

$$\{\frac{\partial U_1}{\partial R} p_t(\frac{1}{1+r})^t - [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\}\frac{\partial f}{\partial L_t} + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})(p_t \frac{\partial f}{\partial L_t} - w_t) = 0 \quad (77)$$

The first term on the left-hand side of (77) can be interpreted as the total revenue-related impact on marginal collective utility of an incremental change in labor usage, which is adjusted for the implicit cost of the marginal cash balance and working capital (i.e. inventory) requirements associated with the change in output. It is shown below that $\lambda_{5,t}$ can be interpreted as the firm's implicit marginal cost of money capital and that, in general, $\lambda_{5,t} > 0$. The difference $\lambda_{6,t} - \lambda_{6,t+1}$ can be interpreted on the basis of (60) as the implicit cost in terms of dividend payments this period and next of an increase in cash balance and working capital requirements, where this difference reflects the fact that an increase in cash balances and working capital totaling one dollar this period will, ceteris paribus, reduce the amount of cash available for paying dividends this period by one dollar but will also, ceteris paribus, increase the amount of cash available next period by one dollar (by reducing the required increase or increasing the permitted decrease in the levels of cash and value of inventories). Thus, the implicit cost of the marginal cash balance and working capital requirements,

$$- [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}) , \quad (78)$$

is comprised of an opportunity cost term $-\lambda_{5,t}(\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})$ and a dividends-related cash flow impact term $-[\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})$.

Before interpreting the second term on the left-hand side of (77), it will prove helpful to prove the following lemma.

Lemma IV-3

Under certainty $\lambda_{4,t} + \lambda_{6,t} > 0$ when the firm modeled in (59) is in equilibrium.

Proof

It follows from (63) that

$$\frac{\partial U_1}{\partial M}\left(\frac{1}{1+r}\right)^t = (1-\tau)(\lambda_{4,t}+\lambda_{6,t}) \ .$$

But $\partial U_1/\partial M > 0$ and $0 < \tau < 1$ by assumption, so that $\lambda_{4,t} + \lambda_{6,t} > 0$ at optimality. Q.E.D.

The sum $\lambda_{4,t} + \lambda_{6,t}$ can be interpreted as the implicit cost of a change in net income, measured in terms of the change's impact on total equity, $\lambda_{4,t}$, plus its impact on current period cash flow, $\lambda_{6,t}$. According to lemma IV-3, this unit cost is strictly positive at optimality.

The second term on the left-hand side of (77) can be interpreted as the net income-related impact on collective utility of a change in labor usage, where this marginal impact is measured as the product of the change in net income $(1-\tau)(p \frac{\partial f}{\partial L} -w)$, which is negative when $p \frac{\partial f}{\partial L} <w$, and the implicit marginal cost of a change in net income, $\lambda_{4,t}+\lambda_{6,t}$, which is positive by lemma IV-3. According to (77), then, under certainty a firm of the type modeled in (59) should continue to expand its labor usage until the point at which the marginal increase in collective utility resulting from the increase in total revenue adjusted for the implicit cost of the increased

cash balance and working capital requirements plus the implicit cost of the increased labor usage in terms of its impact on net income is zero. More importantly, (77) leads to the following result.

Theorem IV-4

Under certainty for a firm of the type modeled in (59), if a unit increase in output contributes more to collective utility than the required increases in cash balance and working capital requirements subtract, i.e. if $\frac{\partial U}{\partial R} p_t(\frac{1}{1+r})^t - [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}) > 0$, then the firm will produce more output than a short run profit maximizer would when it is in equilibrium.

Proof

It follows from (41) and the stated assumption that the first term on the left-hand side of (77) is strictly positive at optimality. By lemma IV-3, $\lambda_{4,t}+\lambda_{6,t} > 0$ at optimality. Hence $p_t \frac{\partial f}{\partial L_t} - w_t < 0$, and the conclusion follows by the assumed concavity of f. Q.E.D.

A somewhat different interpretation of (77) is possible when the necessary condition is rewritten as

$$\frac{\partial U_1}{\partial R} p_t(\frac{1}{1+r})^t \frac{\partial f}{\partial L_t} +(1-\tau)(\lambda_{4,t}+\lambda_{6,t})(p_t \frac{\partial f}{\partial L_t} -w_t) = [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\frac{\partial f}{\partial L_t} . \tag{79}$$

(79) leads to the following corollary.

Corollary IV-4-1

Under certainty a firm of the type modeled in (59) will increase its usage of labor up to the point at which the marginal increase in collective utility due to the increase in total revenue, $\frac{\partial U_1}{\partial R} p_t(\frac{1}{1+r})^t \frac{\partial f}{\partial L_t}$, plus the marginal impact on collective utility due to the change in net income, $(1-\tau)(\lambda_{4,t}+\lambda_{6,t})(p_t \frac{\partial f}{\partial L_t} - w_t)$, just equals the implicit marginal cost of the increased cash balance and working capital requirements, $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\frac{\partial f}{\partial L_t}$.

Also of interest is how the firm modeled in (59) would be expected to react to changes in the implicit cost of marginal cash balance and working capital requirements. To explore this, three cases are considered, according to whether $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}$ is positive, zero, or negative. Since $\lambda_{5,t} > 0$ and since one would expect $\lambda_{6,t}$ and $-\lambda_{6,t+1}$ to be roughly offsetting under most circumstances, the case in which $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} > 0$ is the most interesting economically.

case (i): $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} > 0$ (marginal cost is positive).

Corollary IV-4-2

If the implicit marginal cost of the increased cash balance and working capital requirements decreases less rapidly with an increase in output than the sum of the marginal impacts on collective utility of the changes in total revenue and net income, i.e. if

$$\frac{\partial}{\partial L_t}\{\frac{\partial U_1}{\partial R} p_t(\frac{1}{1+r})^t \frac{\partial f}{\partial L_t} + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})(p_t \frac{\partial f}{\partial L_t} - w_t) < \frac{\partial}{\partial L_t}\{[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\frac{\partial f}{\partial L_t}\} \quad (80)$$

then an increase in the firm's cost of financial capital, $\lambda_{5,t}$, or more generally, an increase in the implicit cost per unit of increased cash balance and working capital requirements, $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$, leads to a fall in the firm's optimal level of output. If the implicit marginal cost of the increased cash balance and working capital requirements decreases more rapidly, i.e. if the sign in (80) is reversed, then the increase in $\lambda_{5,t}$ (or $\lambda_{5,t}+\lambda_{6,t}$ $-\lambda_{6,t+1}$) leads to a rise in the firm's optimal level of output.

Proof

It follows from (41), (46), and (48) that

$$\frac{\partial L}{\partial[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]} = -\frac{-(\frac{\partial \overline{C}}{\partial Q}+\frac{\partial V}{\partial Q})\frac{\partial f}{\partial L}}{B}, \tag{81}$$

where

$$B = \frac{\partial^2 U_1}{\partial R^2}(p\frac{\partial f}{\partial L})^2(\frac{1}{1+r})^t + \frac{\partial U_1}{\partial R}p(\frac{1}{1+r})^t\frac{\partial^2 f}{\partial L^2} + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})p\frac{\partial^2 f}{\partial L^2}$$
$$- [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]\{(\frac{\partial^2\overline{C}}{\partial Q^2}+\frac{\partial V^2}{\partial Q^2})(\frac{\partial f}{\partial L})^2 + (\frac{\partial \overline{C}}{\partial Q}+\frac{\partial V}{\partial Q})\frac{\partial^2 f}{\partial L^2}\}, \tag{82}$$

has the same sign as B given by (82). But (80) implies that $B < 0$. Hence $\partial L/\partial[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}] < 0$ by (80), and thus, by (41), Q falls when $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}$ increases. Moreover, $B > 0$ when the inequality in (80) is reversed, so that in that case, by (81) and (41), Q rises when $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}$ increases. Q.E.D.

Corollary IV-4-3

If (80) holds, then an increase in the marginal cash balance and working capital requirements, $\frac{\partial \overline{C}}{\partial Q}+\frac{\partial V}{\partial Q}$, for each output level causes the firm's optimal output level to fall. But when the inequality in (80) is reversed, the increase in $\frac{\partial \overline{C}}{\partial Q}+\frac{\partial V}{\partial Q}$ for each output level causes the firm's optimal output level to increase.

Proof

Follows directly from

$$\frac{\partial L}{\partial\left(\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}\right)} = - \frac{-[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]\frac{\partial f}{\partial L}}{B} , \tag{83}$$

where B is given by (82). Q.E.D.

The results presented in corollaries IV-4-2 and IV-4-3 are summarized graphically in figure IV-2. In each part of the overall figure curve A represents the graph of $\frac{\partial U_1}{\partial R} p(\frac{1}{1+r})^t \frac{\partial f}{\partial L} + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})(p \frac{\partial f}{\partial L} -w)$ and curve B represents the graph of $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\frac{\partial f}{\partial L}$. Note that under the stated assumptions (of this case), each curve must be downward sloping. Note also that an increase in either $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$ or $(\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})$ shifts B upward while leaving the position of A unchanged. In figure IV-2(a)

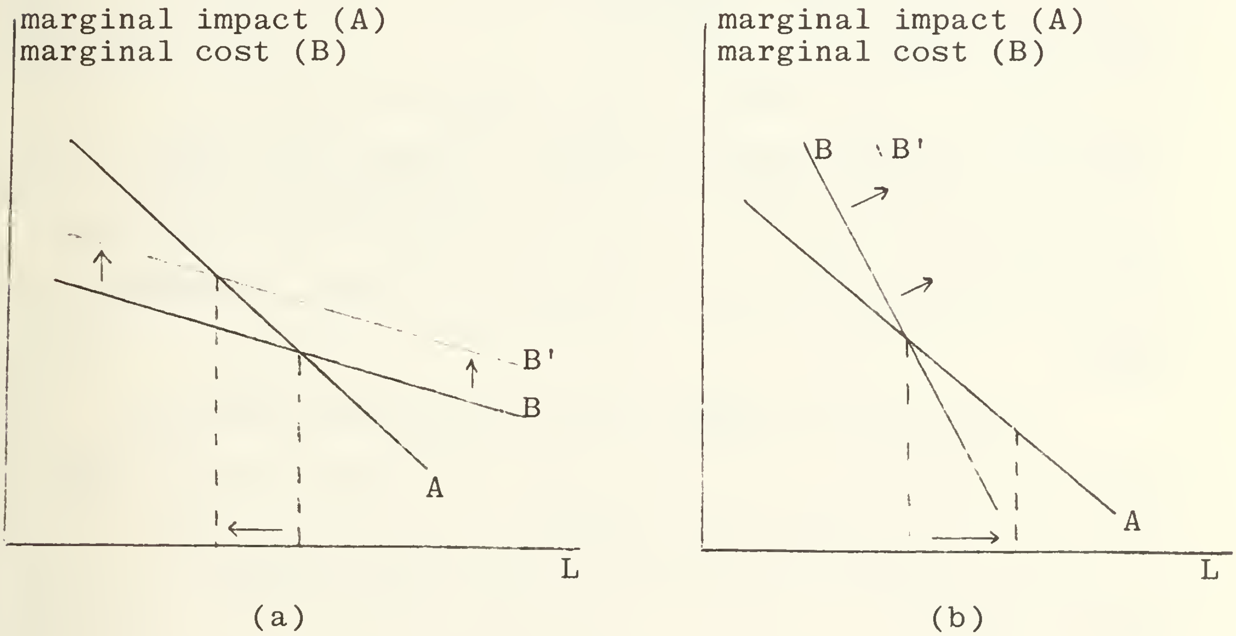


Figure IV-2 Effect on Labor Usage of a Change in the Implicit Cost of and Level of Cash Balance and Working Capital Requirements (positive marginal cost) curve B has a smaller slope than curve A, i.e. (80) holds, so that labor usage (and hence output) falls when B shifts upward to B' due to the increase in the implicit marginal cost of cash balance and working capital requirements. The reverse happens in figure IV-2(b) because there B is steeper than A, i.e. the inequality in (80) is reversed.

It should be noted that if $\bar{C}$ and V were convex, rather than concave, then the curve B in figure IV-2 would be upward sloping. In that case an increase in either $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$ or $\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}$ would cause labor usage, and hence output, to fall. However, it seems more reasonable to this writer, in view of the EOQ formulas from inventory theory, to treat $\bar{C}$ and V as concave functions of Q, as was done in (46) and (48).[75]

<u>case (ii): $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} = 0$ (marginal cost is zero).</u>

<u>Corollary IV-4-4</u>

An increase in the implicit cost $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$ causes both labor usage and output to fall, while an increase in cash balance and working capital requirements, $\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}$, has no effect on either labor usage or output.

<u>Proof</u>

The first statement follows from (81) and (82), where $B < 0$ when $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} = 0$ in (82). The second statement follows from (83) with $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}$ set equal to zero. Q.E.D.

<u>case (iii): $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} < 0$.(marginal cost is negative)</u>

<u>Corollary IV-4-5</u>

An increase in the implicit cost $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$ causes both labor usage and output to fall, while an increase in cash balance and working capital requirements, $\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}$, causes labor usage and output to increase.

Proof

The first statement follows from (46), (48), (81), and (82) since B < 0 according to (82) when $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} < 0$. The second statement follows from (83) since B < 0 and $-[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}] \frac{\partial f}{\partial L} > 0$. Q.E.D.

The results presented in corollary IV-4-5 are summarized in figure IV-3. Since $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1} < 0$, increasing $\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}$ causes curve B, which is now positively sloped, to shift upward, while increasing $\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}$ causes B to shift downward (unlike figure IV-2 in which an increase in either factor caused curve B to shift upward). In figure IV-3(a) curve B shifts upward

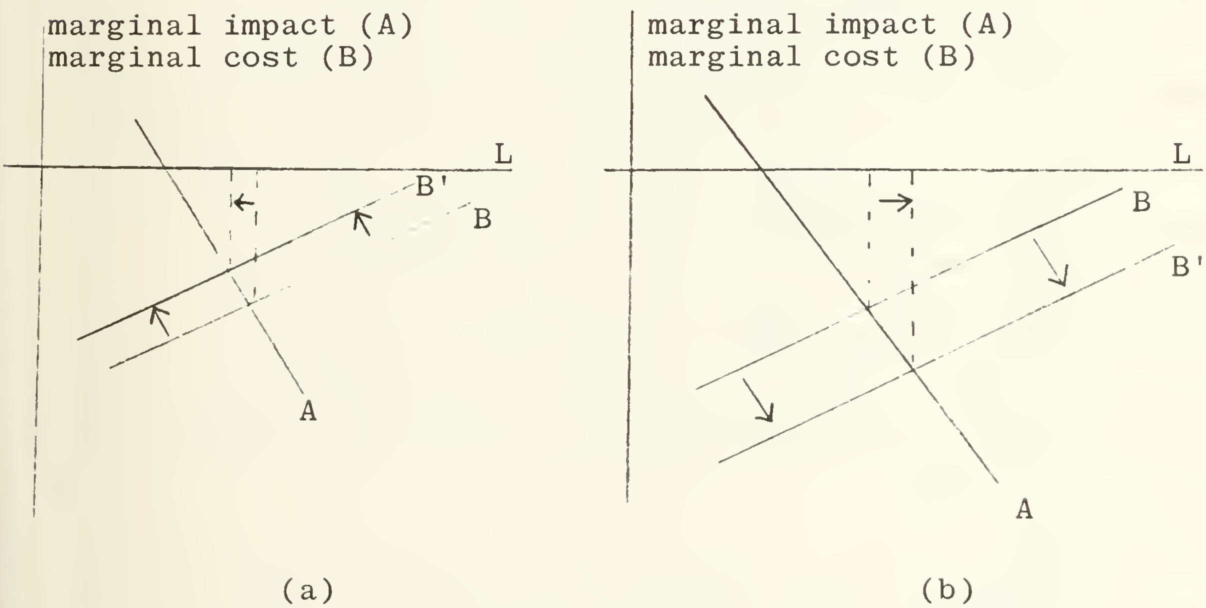


Figure IV-3 Effect on Labor Usage of a Change in the Implicit Cost of and Level of Cash Balance and Working Capital Requirements (negative marginal cost) to B' and labor usage (and hence output) falls due to the increase in $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$. In figure IV-3(b) curve B shifts downward and labor usage increases.

The importance of theorem IV-4 and its corollaries is that it demonstrates clearly the relationship between the firm's output decisions and the cost of obtaining the needed financial resources to support the required transactions cash balance and working capital requirements. Where such requirements exist, the firm's choice of output level is not, in general, independent of financial cost considerations. The results suggest, for example, that when the implicit cost $[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]$ is positive it is not unlikely that the firm's choice of output level is constrained in the sense that a greater output level would be chosen were the implicit cost lower. In the extreme case, the firm might even find itself facing a liquidity crisis, in which the implicit cost (and possibly even the explicit interest cost as well) of the marginal cash balance and working capital requirements was very large indeed, and in which, as a consequence, financial restrictions were severely affecting operating decisions.

Turning to (62) and (63), the following theorem is easily proved.

<u>Theorem IV-5</u>

Under certainty a firm of the type modeled in (59) will pay a level of managerial emoluments at each time t such that the collective marginal rate of substitution between managerial emoluments and net income just equals one minus the tax rate. The firm will select the level of investment at each time t such that the collective marginal rate of substitution between physical capital and dividends just equals the price of capital goods at t.

<u>Proof</u>

It follows from (63) that

$$\frac{[\partial U_1/\partial M](\frac{1}{1+r})^t}{\lambda_{4,t}+\lambda_{6,t}} = (1-\tau) \tag{84}$$

and it follows from (62) that

$$\frac{(\lambda_{1,t})(\frac{1}{1+\delta})}{\lambda_{6,t}} = q_t \ , \tag{85}$$

where in each case the stated necessary condition was reexpressed as the appropriate necessary condition under certainty. Q.E.D.

Theorem IV-5 merely verifies results proved in chapter three. Note in particular that (84) is analogous to (28) in chapter three and that (85) is analogous to (23) in chapter three, where $\lambda_{4,t}+\lambda_{6,t}$ in (84) measures the impact of a change in managerial emoluments on net income, and indirectly, on total equity ($\lambda_{4,t}$) and cash flow ($\lambda_{6,t}$), and where $(\lambda_{1,t})(\frac{1}{1+\delta})$ in (85) measures the value of a unit of investment in terms of the undepreciated balance remaining at the end of the period. Thus, as in chapter three, the firm selects the optimal levels of managerial emoluments and investment at each time t by equating the appropriate internal rate of trade off, or marginal rate of substitution, on the left-hand side of (84) and (85), respectively, with the relevant externally imposed rate of trade off on the right-hand side of (84) and (85), respectively.

Turning next to the selection of the optimal capital stock at each time t, K_t, consider (64). The analysis for the terminal capital stock, K_T, proceeds analogously using (65), and so is not considered here explicitly. For the certainty case (64) can be rewritten

$$\begin{aligned}
&\{\frac{\partial U_1}{\partial R}\left(\frac{1}{1+r}\right)^t + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})\}p_t \frac{\partial f}{\partial K_t} - \lambda_{1,t} + \left(\frac{1}{1+\delta}\right)\lambda_{1,t+1} \\
&\qquad - \lambda_{5,t}q_t - (1-\tau)\,\lambda_{4,t}\cdot q_t\cdot\delta + \lambda_{6,t}\cdot\tau\cdot q_t\cdot\delta \qquad (86) \\
&\qquad - (\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1})\left(\frac{\partial\bar{C}}{\partial Q}\frac{\partial f}{\partial K_t} + \frac{\partial V}{\partial Q}\frac{\partial f}{\partial K_t}\right) = 0 \ .
\end{aligned}$$

It follows from (62) that

$$\left.\begin{aligned}
\lambda_{1,t} &= \lambda_{6,t}\cdot q_t\cdot(1+\delta) \\
\lambda_{1,t+1} &= \lambda_{6,t+1}\cdot q_{t+1}\cdot(1+\delta)\ .
\end{aligned}\right\} \qquad (87)$$

Substituting (87) into (86) and rearranging terms yields

$$\begin{aligned}
&(1-\tau)(\lambda_{4,t}+\lambda_{6,t})p_t \frac{\partial f}{\partial K_t} + \frac{\partial U_1}{\partial R}\, p_t \frac{\partial f}{\partial K_t}\left(\frac{1}{1+r}\right)^t \\
&\qquad - (\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1})\left(\frac{\partial\bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}\right)\frac{\partial f}{\partial K_t} \qquad (88) \\
&= \lambda_{5,t}q_t + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})q_t\cdot\delta - [\lambda_{6,t+1}q_{t+1}-\lambda_{6,t}\cdot q_t]\ ,
\end{aligned}$$

or equivalently, that [76]

$$\begin{aligned}
&(1-\tau)p\,\frac{\partial f}{\partial K} + p\,\frac{\partial f}{\partial K}\left[\frac{(\partial U_1/\partial R)/(1+r)^t}{\lambda_{4,t}+\lambda_{6,t}} - \frac{[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]}{\lambda_{4,t}+\lambda_{6,t}}\ \times \right. \\
&\qquad \left.\left(\frac{\partial\bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}\right)\right] \qquad (89) \\
&\qquad = \frac{\lambda_{5,t}}{\lambda_{4,t}+\lambda_{6,t}}\, q_t + (1-\tau)q_t\cdot\delta - \left[\frac{\lambda_{6,t+1}}{\lambda_{4,t}+\lambda_{6,t}}\, q_{t+1}\right. \\
&\qquad\qquad \left. - \frac{\lambda_{6,t}}{\lambda_{4,t}+\lambda_{6,t}}\, q_t\right]\ .
\end{aligned}$$

The equilibrium condition (89) is analogous to the equilibrium condition (40) in chapter three, though (89) is somewhat more complex due to the explicit treatment given financial factors in the model of the firm (59). In particular, the left-hand side of (89) is analogous to the right-hand side of (40) in chapter three and can be interpreted as the marginal value of an additional unit of physical capital adjusted (as were the corresponding expressions for labor, (75) and (77)) for the implicit cost of the marginal cash balance and working capital requirements associated with the increase in output resulting from the incremental unit of physical capital applied in production, where the cost adjustment term is $-\frac{[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}]}{\lambda_{4,t}+\lambda_{6,t}}(\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})\frac{\partial f}{\partial K}$. The right-hand side of (89) is analogous to the left-hand side of (40) in chapter three and can be interpreted as the marginal cost of physical capital expressed as the sum of the opportunity cost of the unit of capital $\frac{\lambda_{5,t}}{\lambda_{4,t}+\lambda_{6,t}} \cdot q_t$, and the implicit depreciation cost (net of tax), $(1-\tau)q_t \cdot \delta$, less the implicit value (in terms of cash flow) of the increase in capital goods prices between t and t + 1, $[\frac{\lambda_{6,t+1}}{\lambda_{4,t}+\lambda_{6,t}} q_{t+1} - \frac{\lambda_{6,t}}{\lambda_{4,t}+\lambda_{6,t}} q_t]$. Hence, the marginal cost of physical capital has the same three components as before. The main difference between the marginal cost of physical capital implied by (89) and the one implied by (40) in chapter three is that the values attached to the various cost elements in (89) are determined implicitly.

In view of the analogy between (88) and (77), results analogous to those presented in theorem IV-4 and its corollaries could be proved for physical capital. Rather than essentially reproduce those results, (88) will be used together with (77) to prove the following result.

Theorem IV-6

Under certainty the expansion path for a firm of the type modeled in (59) is given by $\frac{\partial L}{\partial K} = \frac{1}{1-\tau}\,\frac{i}{w}$, where

$$i \equiv \frac{\lambda_{5,t}}{\lambda_{4,t}+\lambda_{6,t}}\,q_t + (1-\tau)q_t\cdot\delta - \left[\frac{\lambda_{6,t+1}}{\lambda_{4,t}+\lambda_{6,t}}q_{t+1} - \frac{\lambda_{6,t}}{\lambda_{4,t}+\lambda_{6,t}}\,q_t\right]. \quad (90)$$

Proof

It follows from (77) that

$$\frac{\partial f}{\partial L} = \frac{(1-\tau)w(\lambda_{4,t}+\lambda_{6,t})}{\frac{\partial U_1}{\partial R}\,p(\frac{1}{1+r})^t - [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q}) + (1-\tau)(\lambda_{4,t}+\lambda_{6,t})p} \quad (91)$$

and it follows from (88) that

$$\frac{\partial f}{\partial K} = \frac{i(\lambda_{4,t}+\lambda_{6,t})}{\frac{\partial U_1}{\partial R}\,p(\frac{1}{1+r})^t - [\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})+(1-\tau)(\lambda_{4,t}+\lambda_{6,t})p}\,. \quad (92)$$

Dividing (92) by (91) yields

$$\frac{\partial L}{\partial K} = \frac{\partial f/\partial K}{\partial f/\partial L} = \frac{1}{1-\tau}\,\frac{i}{w}\,, \quad (93)$$

where i in (92) and (93) is given by (90). Q.E.D.

The significance of theorem IV-6 is that whatever restraints on the firm's choice of output level are implied by the cost adjustment term $-[\lambda_{5,t}+\lambda_{6,t}-\lambda_{6,t+1}](\frac{\partial \bar{C}}{\partial Q} + \frac{\partial V}{\partial Q})$ are neutral with regard to the firm's selection of its optimal input mix since the cost adjustment term does not appear in (90), and hence, does not affect (93). However, it cannot be concluded that the expansion path of the firm modeled in (59) is independent of financial considerations, even under certainty. Even though (93) above and (44) in chapter three are identical in form, the marginal cost of physical capital, i, is not necessarily the same in both cases.

Corollary IV-6-1

Under certainty the expansion path for a firm of the type modeled in (59) will coincide with the expansion path of a short run profit maximizer only if i given by (90) is identically equal to $rq_t + (1-\tau)q_t\delta + q_{t+1} - q_t$ for all output levels at each time t.

Proof

Under the stated assumption (93) above and (44) in chapter three (written in discrete time form) agree.

Q.E.D.

Conditions when the two formulas would agree and conditions when they would not agree were discussed in chapter three. It follows from (90) that the two would agree if there were effectively no constraint on total equity, i.e. $\lambda_{4,t} = 0$; if the relative value of an additional unit of cash this period and next were identically one, i.e. $\lambda_{6,t} \equiv \lambda_{6,t+1}$; and if the normalized marginal cost of financial capital were equal to the interest rate, $\frac{\lambda_{5,t}}{\lambda_{4,t}+\lambda_{6,t}} = r$; all of which would require at the very least the sort of frictionless world hypothesized in neoclassical models of the firm.[77]

Next considering the firm's optimal operating policies under uncertainty, begin with (61), which can be rewritten

$$\phi_{t,s} \frac{\partial U_1}{\partial R} p_{t,s} \left(\frac{1}{1+r}\right)^t \frac{\partial f}{\partial L_{t,s}} + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) \times$$

$$\left(p_{t,s} \frac{\partial f}{\partial L_{t,s}} - w_{t,s}\right)$$

$$= \left[\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}\right] \times \tag{94}$$

$$\left(\frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}}\right) \frac{\partial f}{\partial L_{t,s}} .$$

(94) is perfectly analogous to (79), with the main differences being that

- For each time t there is a necessary condition (94) for each state s that might obtain at t.
- Each term in (94) is probability weighted, with the probability $\phi_{t,s}$ appearing explicitly in the first term on the left-hand side of (94) and being incorporated within the Lagrange multipliers in the other terms in (94).
- Where intertemporal effects are to be taken into account in (94) there are Lagrange multiplier sums, e.g. $\sum_{s'=1}^{S} \lambda_{4,t,s,s'}$, that can be interpreted as expected values contingent upon state s at t, where the expectation is taken with respect to the states s' that might obtain at t + 1.

Before interpreting (94) it will prove helpful to prove the following lemma, which is analogous to lemma IV-3.

Lemma IV-4

Under uncertainty $\sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) > 0$ when the firm modeled in (59) is in equilibrium.

Proof

It follows from (63) that

$$\phi_{t,s} \frac{\partial U_1}{\partial M_{t,s}} \left(\frac{1}{1+r}\right)^t = (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) .$$

But $\frac{\partial U_1}{\partial M_{t,s}} > 0$ and $\phi_{t,s} > 0$ for all s and $0 < \tau < 1$ by assumption, so that the conclusion follows. Q.E.D.

According to lemma IV-4, the expected implicit cost of a change in state s net income, $\sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'})$, where the expectation is taken over the possible states s' that might obtain at t + 1, is strictly positive at optimality.

The implications of (94) are summarized in the following theorem and corollaries.

Theorem IV-7

If a unit increase in output at time t in state of nature s would contribute more to expected collective utility than the required increases in cash balance and working capital requirements would be expected to subtract, i.e. if

$$\phi_{t,s} \frac{\partial U_1}{\partial R} p_{t,s} \left(\frac{1}{1+r}\right)^t - [\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}] \times$$

$\left(\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}}\right) > 0$, then the firm modeled in (59) will produce greater output in that state of nature when it is in equilibrium than a short run profit maximizer.

Proof

It follows from (41), lemma IV-4, and the stated assumption that $p_{t,s} \frac{\partial f}{\partial L_{t,s}} - w_{t,s} < 0$, which leads to the desired result by the assumed concavity of f. Q.E.D.

Corollary IV-7-1

Under uncertainty the firm modeled in (59) will expand its usage of labor up to the point at which the marginal increase in expected collective utility due to the increase in total revenue in state s, $\phi_{t,s} \frac{\partial U_1}{\partial R} p_{t,s}(\frac{1}{1+r})^t \frac{\partial f}{\partial L_{t,s}}$, plus the expected marginal impact on expected collective utility due to the change in net income in state s, $(1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})(p_{t,s} \frac{\partial f}{\partial L_{t,s}} - w_{t,s})$, just equals the expected implicit marginal cost in state s of the increased cash balance and working capital requirements.

Corollary IV-7-2

If the expected implicit marginal cost in state s of each unit of additional cash balance and working capital requirements is strictly positive, i.e. if

$$\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} > 0 ,$$

and if this implicit marginal cost in state s decreases less rapidly with an increase in output than the sum of the marginal (expected) impacts on expected collective utility of changes in total revenue and net income in state s, i.e. if

$$\frac{\partial}{\partial L_{t,s}}\{\phi_{t,s} \frac{\partial U_1}{\partial R} p_{t,s}(\frac{1}{1+r})^t \frac{\partial f}{\partial L_{t,s}} + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'} +\lambda_{6,t,s,s'})(p_t \frac{\partial f}{\partial L_{t,s}} - w_{t,s}) \tag{95}$$

$$< \frac{\partial}{\partial L_{t,s}}\{[\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}]\times (\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}})\frac{\partial f}{\partial L_{t,s}}\} ,$$

then an increase in the firm's marginal cost of financial capital in state s, $\lambda_{5,t,s}$, or more generally, an increase in the expected implicit cost per unit of increased cash balance and working capital requirements, $[\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}]$, leads to a fall in the firm's optimal level of labor usage, and hence output, in state s. The opposite is true if the expected implicit marginal cost decreases more rapidly, i.e. if the sign is reversed in (95).

Proof

Follow the steps in the proof of corollary IV-4-2 using (94) and (95). Q.E.D.

Corollary IV-7-2 for the uncertainty case is analogous to corollary IV-4-2 for the certainty case. Similarly, results analogous to corollaries IV-4-3, IV-4-4, and IV-4-5 could be proved for the uncertainty case. But since the steps in the proof under uncertainty are the same as the steps in the corresponding proof under certainty, as suggested by the proofs of corollaries IV-4-2 and IV-7-2, the details need not be provided here.

Turning to (62) and (63), the following theorem, which is analogous to theorem IV-5, is easily proved.

Theorem IV-8

Under uncertainty the firm modeled in (59) will set a level of managerial emoluments for each time t and state of nature s such that the collective marginal rate of substitution between managerial emoluments and net income in s at t just equals one minus the tax rate. The firm will also set a level of investment in s at t such that the collective marginal rate of substitution between physical capital and dividends in s at t just equals the price of capital goods in s at t.

Proof

It follows from (63) that

$$\frac{\phi_{t,s}[\partial U_1/\partial M_{t,s}](\frac{1}{1+r})^t}{\sum_{s'=1}^{S}(\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})} = (1-\tau) \tag{96}$$

and it follows from (62) that

$$\frac{\sum_{s'=1}^{S}\lambda_{1,t,s,s'}(\frac{1}{1+\delta})}{\sum_{s'=1}^{S}\lambda_{6,t,s,s'}} = q_{t,s} . \tag{97}$$

Q.E.D.

Note that (96) and (97) are the logical extensions of (84) and (85), respectively, to the uncertainty case, where it should be repeated that the Lagrange multiplier sums, such as those in (96) and (97), represent measures of value that are associated with intertemporal phenomena and that are computed as expectations taken over the possible states s' in the next period.

Next consider the firm's selection of the optimal stock of capital for each period t and state of nature s, $K_{t,s}$, and accordingly, from (43), its optimal investment policy for each period and state, $I_{t,s}$. As in the certainty case, only the case $0 < t < T$ is considered explicitly since the case $t = T$ proceeds analogously using (65) in place of (64).

It follows from (64) that the optimal capital stock for each date and state, $K_{t,s}$, must satisfy

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R}(\frac{1}{1+r})^t + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})\}p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$

$$- \sum_{s'=1}^{S} \lambda_{1,t,s,s'} + (\frac{1}{1+\delta}) \sum_{s'=1}^{S} \lambda_{1,t+1,s',s} - \lambda_{5,t,s} \cdot q_{t,s}$$

$$- (1-\tau) \sum_{s'=1}^{S} \lambda_{4,t,s,s'} q_{t,s} \cdot \delta + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} \tau \cdot q_{t,s} \cdot \delta \tag{98}$$

$$- (\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}) \times$$

$$(\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} + \frac{\partial V}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}}) = 0 ,$$

which is perfectly analogous to (86) for the certainty case. The optimal investment levels, $I_{t,s}$, must satisfy (62), which can be rewritten as

$$\sum_{s'=1}^{S} \lambda_{1,t,s,s'} = (1+\delta) \sum_{s'=1}^{S} \lambda_{6,t,s,s'} \cdot q_{t,s} . \tag{99}$$

Substituting (99) into (98) and rearranging terms yields

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R}(\frac{1}{1+r})^t + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})\}p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$

$$- (\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}) \times$$

$$(\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}} \frac{\partial f}{\partial K_{t,s}}) \tag{100}$$

$$= \lambda_{5,t,s} q_{t,s} + (1-\tau)(\sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'}))q_{t,s} \cdot \delta$$

$$- (\frac{1}{1+\delta}) \sum_{s'=1}^{S} \lambda_{1,t+1,s',s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} q_{t,s} ,$$

which is analogous to (88) for the certainty case. (100) differs from (88) in that the marginal value of an additional unit of physical capital on the left-hand side of (100) is probability weighted as well as discounted because it pertains to a particular state s at period t. Similarly, the marginal cost of capital on the right-hand side of (100) is also state-specific. But the last two terms on the right-hand side of (100), which together play the role of the term in brackets on the right-hand side of (88) is different in form since $\sum_{s'=1}^{S} \lambda_{1,t+1,s',s}$ appears in place of $\lambda_{6,t+1,s',s}$. Obtaining an expression for the impact of changing capital goods prices over time is more difficult under uncertainty,[78] because the impact must be averaged over all possible states of nature during both time periods (the 'present', or t, and the 'future', or t + 1). To allow for this, sum both sides of (99) over s, and, since (99) must hold for each time t when the firm is in equilibrium, substitute t + 1 for t. After reindexing this yields

$$\sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{1,t+1,s',s} = (1+\delta) \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}\, q_{t+1,s'} \quad (101)$$

Summing each side of (100) over the states s at t and substituting using (101) gives

$$\sum_{s=1}^{S} \{\phi_{t,s} \frac{\partial U_1}{\partial R}(\frac{1}{1+r})^t + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})\} p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$
$$- \sum_{s=1}^{S} (\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}) \times$$
$$(\frac{\partial \overline{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}}) \frac{\partial f}{\partial K_{t,s}} \quad (102)$$
$$= \sum_{s=1}^{S} \lambda_{5,t,s} q_{t,s} + (1-\tau)(\sum_{s=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})) q_{t,s} \cdot \delta$$
$$- [\sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} q_{t+1,s'} - \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{6,t,s,s'} q_{t,s}] .$$

Comparing (102) with (88) leads to the following theorem.

Theorem IV-9

Under certainty (uncertainty) when the firm modeled in (59) is in equilibrium it will have expanded investment at each time t to the point at which the (expected) marginal value of an additional unit of physical capital just equals the (expected) marginal cost of the required financial capital, plus the (expected) marginal cost of depreciation net of tax, less the (expected) cash flow impact of changing capital goods prices.

(102) also suggests the following investment criterion for each period t and state of nature s.

Corollary IV-9-1

A sufficient condition for meeting the investment criterion (102) for optimal investment in capital at each time t in state of nature s is the following: the firm should expand investment at each date t for each state s to the point at which the marginal value of an additional unit of capital at t in s,

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R}\left(\frac{1}{1+r}\right)^t + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'})\} p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$

$$- (\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}) \times$$

$$\left(\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}}\right)\frac{\partial f}{\partial K_{t,s}} ,$$

just equals the marginal cost of the required financial capital, $\lambda_{5,t,s} \cdot q_{t,s}$, plus the implied marginal cost (net of tax) of depreciation, $(1-\tau)(\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) q_{t,s} \cdot \delta$, less the expected impact of changing capital goods prices, $\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} q_{t+1,s'} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} q_{t,s}$, or in equation form,

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R}(\frac{1}{1+r})^t + (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'})\} p_{t,s} \frac{\partial f}{\partial K_{t,s}}$$

$$- (\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}) \times$$

$$(\frac{\partial \bar{C}_{t,s}}{\partial Q_{t,s}} + \frac{\partial V_{t,s}}{\partial Q_{t,s}}) \frac{\partial f}{\partial K_{t,s}} \tag{103}$$

$$= \lambda_{5,t,s} \cdot q_{t,s} + (1-\tau)(\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) q_{t,s} \cdot \delta$$

$$- [\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} q_{t+1,s'} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} q_{t,s}] .$$

Proof

For given period t, if (103) holds for each possible state of nature s at t, then (102) follows by summing each side of (103) over s. Q.E.D.

The significance of theorems IV-7, IV-8, and IV-9 is that they demonstrate that the optimality criteria for the firm's operating policies under certainty can be extended in a straightforward manner to each specified state of nature under uncertainty. Moreover, by dealing directly with the probability distribution over the possible states of nature, rather than with the parameters of the distribution as in the mean-variance approach to uncertainty that was adopted in the previous chapter, more definitive results concerning output levels, input levels, and investment levels can be obtained.[79]

3. Optimal Financial Policies

The previous subsection was concerned with the policy implications of necessary conditions (61)-(65). This subsection is concerned with the policy implications of necessary conditions (66)-(73). Proceeding as in the previous subsection, first the policy implications of (66)-(73) under certainty are explored, and then the policy implications under uncertainty are developed by extension of the certainty results.

Beginning with the bond issue/redemption policy, the amount of issues/redemptions during each period t, Y_t, must satisfy the following necessary condition:

$$\lambda_{2,t} + (\lambda_{4,t}+\lambda_{6,t})(1-\tau)(- \frac{\partial i_t}{\partial Y_t} B_t) + \lambda_{6,t} = 0 \ , \tag{104}$$

which was obtained from (66). Similarly, (67) implies that under certainty the amount of bonds outstanding at each date, B_t, must satisfy the following necessary condition:

$$-\lambda_{2,t} + \lambda_{2,t+1} + \lambda_{5,t} + (\lambda_{4,t}+\lambda_{6,t})(1-\tau)(-i_t-\frac{\partial i_t}{\partial B_t} B_t) = 0 \ . \tag{105}$$

Solving (105) for $\lambda_{2,t}$, substituting into (104), and solving for $\lambda_{5,t}$ gives

$$\lambda_{5,t} = (1-\tau)(\lambda_{4,t}+\lambda_{6,t})(i_t+ \frac{\partial i_t}{\partial B_t} B_t + \frac{\partial i_t}{\partial Y_t} B_t) - \lambda_{6,t} - \lambda_{2,t+1} \ . \tag{106}$$

To interpret (106) note that the first term on the right-hand side can be interpreted as the immediate impact on net income, and indirectly on collective utility, of a change in the level of debt and a corresponding change in the rate of new issues/redemptions, where the two effects are related through (50) and are transmitted through the induced changes in the interest rate given by (21). Moreover, it follows from lemma IV-3 and the assumptions that $\partial i/\partial B > 0$ and $\partial i/\partial Y > 0$ that the first term on the right-hand side of (106) is strictly positive at optimality. The second term on the right-hand side of (106) can be interpreted as the direct cash flow impact in the current period of an additional dollar of new debt. The third term on the right-hand side of (106) can be interpreted as the value of an additional dollar of debt in terms of reduced future needs, as can be seen from (50). As such, one would normally expect this term to be nonnegative. Collectively, the three terms on the right-hand side of (106), and hence $\lambda_{5,t,s}$ itself, can be interpreted as the equilibrium marginal cost of debt capital at time t, where this marginal cost is adjusted for the direct impact of a change in issues/redemptions on current period cash flow and is also adjusted for the impact of increasing the current stock of debt on future debt requirements.[80]

Turning to the firm's share issue/redemption policy, the number of shares outstanding during each period t, n_t, must be such that the following necessary condition is satisfied:

$$\lambda_{3,t} - \lambda_{3,t+1} - d_t(\lambda_{4,t} + \lambda_{6,t}) = 0 \ , \tag{107}$$

which was obtained from (69). The issues/redemptions at each date, Z_t, must be such that the following necessary condition is satisfied:

$$- \lambda_{3,t} + v_t(\lambda_{4,t} + \lambda_{6,t}) = 0 \ , \tag{108}$$

which was obtained from (68). In addition, total stockholders' equity at each time t, E_t, must be such that the following necessary condition is satisfied:

$$-\lambda_{4,t} + \lambda_{4,t+1} + \lambda_{5,t} = 0 \ , \tag{109}$$

which was obtained from (70). Solving (108) for $\lambda_{3,t}$, substituting into (107), and rearranging terms gives

$$-\lambda_{3,t+1} + (v_t - d_t)(\lambda_{4,t} + \lambda_{6,t}) = 0 \ . \tag{110}$$

Solving (109) and (110) for $\lambda_{4,t}$, equating the resulting expressions, and solving for $\lambda_{5,t}$ yields

$$\lambda_{5,t} = \frac{\lambda_{3,t+1}}{v_t - d_t} - \lambda_{6,t} - \lambda_{4,t+1} \ . \tag{111}$$

To interpret (111) note that the first term on the right-hand side can be interpreted as the value of an additional dollar's worth of newly issued shares. To see this recall that each new share is issued at the beginning of the period, so that net receipts for the period are equal to the share price less the dividend, $v_t - d_t$. Also, since $\lambda_{3,t+1}$ applies to the new issues/redemptions constraint in (59), the effect of dividing $\lambda_{3,t+1}$ by $v_t - d_t$ is to convert the value measure in terms of shares into one in terms of a dollar's worth of newly issued shares. The second and third terms on the right-hand side of (111) represent adjustments for the current cash flow impact of a change in share issues/redemptions and the impact of increasing the current amount of total stockholders' equity on future equity requirements. Collectively, the three terms on the right-hand side of (111) can be interpreted as the equilibrium marginal cost of external equity capital adjusted for the current cash flow and future equity requirements impacts.

The equilibrium conditions (106) and (111) lead to the following theorem.

Theorem IV-10

In equilibrium under certainty the firm modeled in (59) will have adjusted its capital structure so that the marginal cost of debt capital equals the marginal cost of external equity capital.

Proof

Equate (106) and (111). Q.E.D.

The importance of theorem IV-10 is that $\lambda_{5,t}$ can be interpreted as the firm's marginal cost of financial capital (from either external source). (106) also leads to the following corollary.

Corollary IV-10-1

Under certainty, an increase in the tax rate τ leads to a reduction in the firm's cost of debt capital.

Proof

It follows from (106) and lemma IV-3 that

$$\frac{d}{d\tau}\lambda_{5,t} = -(\lambda_{4,t}+\lambda_{6,t})(i_t + \frac{\partial i_t}{\partial B_t} B_t + \frac{\partial i_t}{\partial Y_t} B_t) < 0 .$$

Q.E.D.

The significance of corollary IV-10-1 is that the marginal cost of debt capital falls, while according to (111), the marginal cost of equity capital is unaffected. This would, of course, induce the firm to increase its leverage, i.e. to substitute debt for equity in its optimal capital structure.

A more important result arising out of (106) and (111) is the following theorem, which states that, in general, the firm's choice of capital structure is not irrelevant to its investment decision.

Theorem IV-11

Under certainty, the choice of capital structure for the firm modeled in (59) is not, in general, irrelevant to its investment decision, and therefore, the separability theorems do not, in general, apply.[81]

Proof

Consider (106) and (111). If the firm's choice of capital structure were irrelevant, then a marginal increase in the stock of equity would have to have the same value as a marginal increase in the stock of debt, i.e. $\lambda_{2,t} \equiv \lambda_{4,t}$ in (106) and (111). In that case the two expressions for $\lambda_{5,t}$ are equal only if

$$(1-\tau)(\lambda_{4,t}+\lambda_{6,t})(i_t + \frac{\partial i_t}{\partial B_t} B_t + \frac{\partial i_t}{\partial Y_t} B_t) = \frac{\lambda_{3,t+1}}{v_t - d_t} . \qquad (112)$$

But it follows from (110) that

$$\frac{\lambda_{3,t+1}}{v_t - d_t} = \lambda_{4,t} + \lambda_{6,t} . \qquad (113)$$

Substituting (113) into (112) and simplifying yields

$$(1-\tau)(i_t + \frac{\partial i_t}{\partial B_t} B_t + \frac{\partial i_t}{\partial Y_t} B_t) = 1 . \qquad (114)$$

But the left-hand side of (114) is a monotonically increasing function of B_t,[82] so that (114) is satisfied (if at all) by a unique value of B_t, and given the sum $B_t + E_t$, by a unique value of the leverage ratio, B_t/E_t. Hence, the firm's cost of financial capital is not, in general, independent of the firm's choice of capital structure. Q.E.D.

The significance of theorem IV-11 is that the firm's choice of capital structure, at least in the certainty case, will affect its cost of financial capital, and by

(88) and (89), its investment decision. Hence, its choice of leverage is not, in general, separable from its investment decision.

Turning to the firm's dividend policy under certainty, necessary condition (71) leads to the following condition that must be satisfied under certainty:

$$\frac{\partial U_1}{\partial v_t} \cdot \frac{dv_t}{dd_t} \cdot \left(\frac{1}{1+r}\right)^t + Z_t \cdot \frac{dv_t}{dd_t}(\lambda_{4,t}+\lambda_{6,t}) = n_t(\lambda_{4,t}+\lambda_{6,t}). \qquad (115)$$

The first term on the left-hand side of (115) can be interpreted as the direct impact on discounted collective utility of a change in the dividend per share. The second term on the left-hand side can be interpreted as the indirect impact on discounted collective utility of a change in the dividend per share that results from the fact that a change in the dividend induces a change in the share price, which alters the number of shares, Z_t, that need to be issued (or redeemed) to meet any particular total equity requirement. The term on the right in (115) can be interpreted as the impact on discounted collective utility of a change in the dividend that, given the number of shares outstanding, n_t, increases the total required dividend payout, thereby reducing retained earnings (and total equity) and the firm's stock of cash. This interpretation of (115) leads to the following theorem.

<u>Theorem IV-12</u>

Under certainty the firm modeled in (59) will set its dividend policy, d_t, per share, such that the direct impact on discounted collective utility due to a marginal increase in d_t plus the indirect impact resulting from the increased contributed capital and cash that can be raised through any predetermined level of new issues just equals the cash flow and retained earnings impact of the marginal increase in d_t.

(115) also implies the following interesting result.

Corollary IV-12-1

Under certainty the firm modeled in (59) will set its dividend policy for each time t so that the marginal rate of substitution between the share value v_t and retained earnings at t, $\frac{[\partial U_1/\partial v_t]/(1+r)^t}{\lambda_{4,t}+\lambda_{6,t}}$, just equals the net share equivalent, $\rho \cdot n_t - Z_t$, of the dividend/share issue/redemptions policies.

Proof

It follows from (35) that under certainty $v_t = \frac{d_t}{\rho}$, so that

$$\frac{dv_t}{dd_t} = \frac{1}{\rho} . \tag{116}$$

Substituting (116) into (115) and rearranging terms gives

$$\frac{[\partial U_1/\partial v_t]/(1+r)^t}{\lambda_{4,t}+\lambda_{6,t}} = \rho \cdot n_t - Z_t , \tag{117}$$

which was to be proved. Q.E.D.

Remark

Note that in (117) the left-hand side represents a marginal rate of substitution - that between the share value v_t and the total equity and cash flow impacts of a change in retained earnings due to a change in dividend policy. The right-hand side of (117) contains the stock market parameter ρ and the policy variables n_t and Z_t. But $v_t = \frac{d_t}{\rho}$, so that $\rho \cdot v_t = d_t$, and $n_t \cdot \rho \cdot v_t = n_t \cdot d_t \equiv D_t$, and hence, $\rho \cdot n_t = D_t/v_t$, where D_t represents total dividends paid. Therefore, $\rho \cdot n_t$ can be interpreted as the share equivalent of the total dividend payout, whereupon $\rho \cdot n_t - Z_t$ can be given the interpretation stated in corollary IV-12-1. The point is that n_t and Z_t are related by (51) by definition, and in addition, are related by (117) due to the preferences of the firm on the one hand and the stock market on the other.

Corollary IV-12-2

The net share equivalent, $\rho \cdot n_t - Z_t$, of the firm's dividend/share issues/redemptions policies is strictly positive for each period t when the firm modeled in (59) is in equilibrium.

Proof

By assumption, $\partial U_1/\partial v_t > 0$ for all values of v_t. By lemma IV-3, $\lambda_{4,t} + \lambda_{6,t} > 0$ at optimality. Therefore, the left-hand side of (117) is strictly positive at optimality. Therefore, $\rho \cdot n_t - Z_t > 0$ when the firm is in equilibrium.

Q.E.D.

The significance of corollary IV-12-2 lies in the fact that, if new share issues are viewed as negative dividends,[83] then the corollary states that the net flow of dividends must always be strictly positive. That is, the firm modeled in (59) will never find itself with such large financing needs that the flow of new equity into the firm exceeds (or is even equal to) the flow of dividend payments. The reason for this result is that the firm adjusts its equity issues/redemptions each period in line with present, as well as future, requirements. In terms of (111), future equity needs affect $\lambda_{4,t+1}$; ceteris paribus, an increase in future equity requirements would be expected to increase $\lambda_{4,t+1}$ and to reduce thereby the effective marginal cost of external equity for the current period. As a result, new share issues would be expected to increase (or redemptions to decrease) during the current period. Thus, the fact that future equity requirements are implicitly taken into account in (59) means that the stream of issues/redemptions over time undergoes a sort of smoothing due to the implicit consideration of intertemporal financial needs.

Turning next to the explicit consideration of the firm's financial policy choices under uncertainty, consider (72) and (73), which govern the firm's choice of precautionary cash balances. At each time t for each state of nature s the optimal level of precautionary cash balances, $\hat{C}_{t,s}$, must be chosen so that (72) and (73) are satisfied. This leads to the following two cases:

case (i): $\lambda_{7,t,s} > 0$. (zero precautionary cash balances)

It follows from (73) that $\hat{C}_{t,s} = 0$, so that the optimal level of precautionary cash balances is zero. It follows from (72) that

$$\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} < \lambda_{5,t,s} . \qquad (118)$$

The left-hand side of (118) can be interpreted as the net marginal value of an additional dollar of precautionary cash balances at t in s, which is expressed as the difference between the expected value of additional precautionary cash balances at t in s contingent upon s' at t + 1 on the one hand and the expected value of additional precautionary cash balances at t in s contingent upon s' at t - 1 on the other. The right-hand side of (118) is the familiar marginal cost of financial capital. According to (118), optimal precautionary cash balances are zero (at t in s) when the opportunity cost of precautionary cash balances, $\lambda_{5,t,s}$, exceeds their marginal value (at t in s).

case (ii): $\hat{C}_{t,s} > 0$. (positive precautionary cash balances)

It follows from (73) that $\lambda_{7,t,s} = 0$ and from (72) that equality must hold in (118). In this case the marginal value of precautionary cash balances at t in s just equals the marginal cost of financial capital (i.e. the opportunity cost of precautionary cash balances) at t in s.

The above cases are summarized in the following theorem.

Theorem IV-13

For any period t and state of nature s, the equilibrium net marginal value of an additional dollar of precautionary cash balances, $\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'}$, will never exceed the firm's equilibrium marginal cost of financial capital, $\lambda_{5,t,s}$. If the equilibrium net marginal value of precautionary cash balances is strictly less than the equilibrium marginal cost of financial capital, then the optimal level of precautionary cash balances is zero. If the optimal level of precautionary cash balances is strictly positive at optimality, then the equilibrium net marginal value of precautionary cash balances just equals the firm's equilibrium marginal cost of financial capital.

Proof

The first statement follows directly from (72) and (73) since $\lambda_{7,t,s} \geq 0$ and thus $\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} \leq \lambda_{5,t,s}$. The second statement characterizes case (i) and the third statement characterizes case (ii).

Q.E.D.

Theorem IV-13 leads to the following corollaries.

Corollary IV-13-1

The implicit marginal cost of an increase in cash balance and working capital requirements associated with an increase in output, $\lambda_{5,t} + \lambda_{6,t} - \lambda_{6,t+1}$ under certainty and $\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s}$ under uncertainty, is nonnegative. Under uncertainty, during any period t and state of nature s in which the equilibrium net marginal value of precautionary cash balances is strictly less than the firm's equilibrium marginal cost of financial capital, then the optimal level of precautionary cash balances is zero and the implicit marginal cost of increased cash balance and working capital requirements is strictly positive.

Proof

The first statement follows from (72) and (73) since under certainty $\lambda_{7,t} \geq 0$ and $\lambda_{5,t} + \lambda_{6,t} - \lambda_{6,t+1} \geq 0$, while under uncertainty $\lambda_{7,t,s} \geq 0$ and $\lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} \geq 0$. The second statement follows from theorem IV-13 and the fact that

$$\sum_{s'=1}^{S} \lambda_{6,t+1,s',s} - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} < \lambda_{5,t,s} \Leftrightarrow \lambda_{5,t,s} + \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} > 0 .$$

Q.E.D.

Corollary IV-13-2

Under uncertainty during time periods t and states of nature s when the equilibrium net marginal value of precautionary cash balances is strictly less than the firm's equilibrium marginal cost of financial capital, and as a result, the optimal level of precautionary cash balances is zero, the firm's output decision is constrained by the impact a change in the level of output would have on the firm's cash balance and working capital requirements.

Proof

It follows from the stated assumptions that the expression on the left-hand side of (94) is strictly positive at optimality. In the absence of cash balance or working capital requirements (or if their cost were zero), then this expression would be zero. The conclusion then follows by the assumed strict concavity of U_1 and f. Q.E.D.

Corollary IV-13-3

Under uncertainty the cash balance and working capital requirements associated with an increase in output tend to restrict the firm's output decision.

Proof

Since $\lambda_{7,t} \geq 0$, it follows that $\lambda_{5,t} + \lambda_{6,t} - \lambda_{6,t+1} \geq 0$. Hence, the left-hand side of (79) is nonnegative at optimality. The conclusion then follows by the assumed strict concavity of U_1 and f. Q.E.D.

Reconsidering the bond issue/redemption policy, this time under uncertainty, the amount of new issues at t in s, $Y_{t,s}$, must satisfy (66), and the amount of debt outstanding at t in s, $B_{t,s}$, must satisfy (67).

Solving (66) for $\sum_{s'=1}^{S} \lambda_{2,t,s,s'}$, substituting into (67), and solving the resulting expression for $\lambda_{5,t,s}$ yields

$$\lambda_{5,t,s} = (1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})(i_{t,s}+ \frac{\partial i_{t,s}}{\partial B_{t,s}} B_{t,s} + \frac{\partial i_{t,s}}{\partial Y_{t,s}} B_{t,s}) - \sum_{s'=1}^{S} \lambda_{6,t,s,s'} - \sum_{s'=1}^{S} \lambda_{2,t+1,s',s} \tag{119}$$

which is analogous to (106). (119) expresses the marginal cost of debt capital at time t in state of nature s, and because it applies under uncertainty, the intertemporal net income-related total equity and cash flow impacts, as measured by $\sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})$, the intertemporal direct cash flow impact, as measured by $\sum_{s'=1}^{S} \lambda_{6,t,s,s'}$, and the intertemporal impact on future debt requirements, as measured by $\sum_{s'=1}^{S} \lambda_{2,t+1,s',s}$, each must be computed as an expected value in order to take into account the S possible transitions from some state s' at t - 1 to state s at t, in the case of the net income-related total equity and cash flow impacts and the direct cash flow impact, and the S possible transitions from state s at t to some state s' at t + 1, in the case of the impact on future debt requirements. The exact interpretation of (119), as it relates to the interpretation of (106), is given below in theorem IV-14.

Next, reconsidering the share issue/redemption policy, the number of shares issued at each time t in each state of nature s, $Z_{t,s}$, must satisfy (68); the number of shares outstanding at each date and state, $n_{t,s}$, must satisfy (69); and total equity for each date and state, $E_{t,s}$, must satisfy (70). Solving (68) for $\sum_{s'=1}^{S} \lambda_{3,t,s,s'}$, substituting into (69), and rearranging terms gives

$$-\sum_{s'=1}^{S}\lambda_{3,t+1,s',s} + (v_t - d_{t,s})\sum_{s'=1}^{S}(\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'}) = 0. \quad (120)$$

Solving (70) and (120) for $\sum_{s'=1}^{S}\lambda_{4,t,s,s'}$, equating the resulting expressions, and solving for $\lambda_{5,t,s}$ gives

$$\lambda_{5,t,s} = \frac{1}{v_t-d_{t,s}}\sum_{s'=1}^{S}\lambda_{3,t+1,s',s} - \sum_{s'=1}^{S}\lambda_{6,t,s,s'} - \sum_{s'=1}^{S}\lambda_{4,t+1,s',s} \quad (121)$$

which is analogous to (111). The exact interpretation of (121), as it relates to the interpretation of (111), is given in the next theorem.

<u>Theorem IV-14</u>

Under certainty (uncertainty) the marginal cost of debt capital at each time t (at each time t in state of nature s) for the firm modeled in (59) is equal to the (expected) implicit cost of the change in net income resulting from a change in the amount of debt and rate of new issues/redemptions, less the (expected) implicit value of the direct cash flow impact of the new issues/redemptions, and also less the (expected) implicit value of the impact of increased debt in the current period on future debt requirements. Under certainty (uncertainty) the marginal cost of external equity capital at each time t (at each time t in state of nature s) for the firm modeled in (59) is equal to the (expected) implicit value of an additional dollar's worth of newly issued shares, less the (expected) implicit value of the direct cash flow impact of the new share issues/redemptions, and also less the (expected) implicit value of the impact of increased total equity in the current period on future equity requirements. In equilibrium under certainty (uncertainty) the firm modeled in (59) will have adjusted its capital structure for each period t (for each period t and state of nature s) so that the marginal cost of debt equals the marginal cost of external equity.

Proof

The first statement follows from (106) and (119). The second statement follows from (111) and (121). The third statement repeats theorem IV-10 for the certainty case and follows for the uncertainty case by equating (119) and (121). Q.E.D.

Similarly, corollary IV-14-1 is perfectly analogous to corollary IV-10-1.

Corollary IV-14-1

Under certainty (uncertainty) an increase in the tax rate τ leads to a reduction in the firm's cost of debt capital.

Proof

The result for the certainty case was proved as corollary IV-10-1. The result for the uncertainty case is proved in the same manner, but proceeding from (119) and using lemma IV-4. Q.E.D.

A more important result arising out of (119) and (121) is the following theorem, which extends theorem IV-11 and which states that, in general, the choice of capital structure for the firm modeled in (59) is not irrelevant to its investment decision.

Theorem IV-15

Under certainty (uncertainty), the choice of capital structure for the firm modeled in (59) is not, in general, irrelevant to its investment decision, and therefore, the separability theorems do not, in general, apply.

Proof

The result for the certainty case was proved as theorem IV-11. The proof for the uncertainty case proceeds analogously. Consider (119) and (121). If the firm's choice of capital structure were irrelevant, then $\lambda_{2,t+1,s',s} = \lambda_{4,t+1,s',s}$ for all t, s, and s'. But then $\sum_{s'=1}^{S} \lambda_{2,t+1,s',s} = \sum_{s'=1}^{S} \lambda_{4,t+1,s',s}$ and the two expressions (119) and (121) yield the same value for $\lambda_{5,t,s}$ only if

$$(1-\tau) \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})(i_{t,s}+ \frac{\partial i_{t,s}}{\partial B_{t,s}} B_{t,s} + \frac{\partial i_{t,s}}{\partial Y_{t,s}} B_{t,s}) = \frac{1}{v_t-d_{t,s}} \sum_{s'=1}^{S} \lambda_{3,t+1,s',s} \; . \tag{122}$$

But solving (120) for $\frac{1}{v_t-d_{t,s}} \sum_{s'=1}^{S} \lambda_{3,t+1,s',s}$ and substituting into (122) yields

$$(1-\tau)(i_{t,s}+ \frac{\partial i_{t,s}}{\partial B_{t,s}} B_{t,s}+ \frac{\partial i_{t,s}}{\partial Y_{t,s}} B_{t,s}) = 1 \; , \tag{123}$$

which implies that (114) must hold for each date and state. As argued in the proof of theorem IV-11, (114), and hence (123), implies a unique leverage ratio. Hence, it is also true under uncertainty that the firm's cost of financial capital is not, in general, independent of its choice of capital structure. Q.E.D.

The significance of theorem IV-15 is that the firm's investment decision and its choice of capital structure are not, in general, separable. That is, given the level of investment, it is not true, in general, that the cost of capital for the firm modeled in (59) is independent of its choice of capital structure. It follows then, that investment and financing decisions are, in general, related inextricably and must therefore be made concurrently.

The last type of financial policy to be explored under uncertainty in this subsection is the firm's dividend policy. Necessary condition (71) must be satisfied at optimality. (71) can be rewritten as[84]

$$\frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \phi_{t,s} \frac{\partial U_1}{\partial v_t}\left(\frac{1}{1+r}\right)^t + \frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \sum_{s'=1}^{S} Z_{t,s}(\lambda_{4,t,s,s'} + \lambda_{6,t,s,s'}) = n_{t,s} \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'}) , \tag{124}$$

which is analogous to (115). The analogy between (124) and (115) leads to the following theorem.

Theorem IV-16

Under uncertainty the firm modeled in (59) will set its dividend policy, $d_{t,s}$ per share at time t in state of nature s, such that the expected direct impact on discounted expected collective utility for period t due to a marginal increase in $d_{t,s}$, $\frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \phi_{t,s} \frac{\partial U_1}{\partial v_t}(\frac{1}{1+r})^t$, plus the expected indirect impact resulting from the increased contributed capital and cash that can be raised through any predetermined level of new issues, $\frac{\partial v_t}{\partial d_{t,s}} \sum_{s=1}^{S} \sum_{s'=1}^{S} Z_{t,s}(\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})$, just equals the cash flow and retained earnings impact of the marginal increase in $d_{t,s}$, $n_{t,s} \sum_{s'=1}^{S} (\lambda_{4,t,s,s'}+\lambda_{6,t,s,s'})$.

The significance of theorem IV-16 is that it extends theorem IV-12 for the certainty case to the uncertainty case. As in the certainty case, the firm modeled in (59) must balance the direct effect on collective utility and the indirect effect on contributed capital and cash flow associated with new issues/redemptions against the effect on cash flow and retained earnings associated with the total dividend payout when it sets its dividend policy. In both cases the potential cash flow impact of a change in the firm's dividend policy weighs heavily in the selection of the optimal dividend policy. Theorem IV-16 is also significant because it demonstrates, along with theorems IV-14 and IV-15, the straightforward extension of financial policy results obtained under the assumption of certainty to the case of uncertainty.

4. Summary

In this section an integrated production-finance model of the firm was formulated and the model's implications for the optimal operating policies and the optimal financial policies of the firm were derived. The model was developed by incorporating financial factors, and specifically, those factors relevant to the firm's cash management policy, leverage policy, and dividend policy, all of which were discussed earlier in this chapter, into the model of the discounted (expected) collective utility maximizing firm discussed in chapter three of this thesis.

The model developed in this section lends itself to either of two interpretations depending on the wider economic milieu within which the firm is assumed to operate. At one extreme, if the alternative output levels $Q_{t,s}$ are interpreted as contingent output claims, and if the set of markets for such claims is assumed to be complete,[85] then with regard to its output decision the firm behaves as if it were functioning within an Arrow-Debreu world,[86] and the plan characterized by (61)-(73) will, when all such markets are in equilibrium, constitute a multiperiod equilibrium for the firm such that its production plans are consistent with those of all other transactors in the firm's product market. In such a world the trading in contingent output claims gives the firm perfect insurance coverage in the sense that there is no economic incentive for the firm to alter its production plans prior to the planning horizon - although depending on the assumed state of input markets and financial markets, its choice of production techniques and its financial policy decisions may require alteration.

At the opposite extreme is the interpretation suggested at the beginning of this section. If markets for contingent output claims are nonexistent, then the plan constitutes a temporary equilibrium in the sense that the firm's production plan may not be consistent with the future period plans of the other transactors in the firm's product market.[87] In such a world the firm may have to revise its production plans each period as the product market (and the economy) shifts to a new temporary equilibrium. According to this interpretation of the model developed in this section, the firm's plan, as characterized by (61)-(73), is of a contingency nature only. The difference between this interpretation and the previous one lies in the different assumptions made concerning the character of the product markets in the wider institutional environment.

The main results obtained in this section include the following. The firm modeled in (59) tends to produce more output in each period (theorem IV-4) and in each state of nature (theorem IV-7) than a short run profit maximizer. The firm modeled in (59) pays managerial emoluments at a level that equates its marginal rate of substitution between managerial emoluments and dividends to one minus the tax rate (theorem IV-5 for the certainty case and theorem IV-8 for the uncertainty case). Cash balance and working capital requirements do not affect the expansion path of the firm modeled in (59) (theorem IV-6), although they do affect the firm's output decision (theorem IV-4, theorem IV-7, and corollary IV-13-2). It was found that at optimality, the marginal cost of debt capital for the firm modeled in (59) equals its marginal cost of equity capital (theorem IV-10 for the certainty case and theorem IV-14 for the uncertainty case); that the firm's decision regarding precautionary cash balances depends on whether or not the marginal value of such balances is as great as their marginal cost (theorem IV-13); and that the firm's choice of capital structure is not, in general, irrelevant to its cost of capital (theorem IV-11 for the certainty case and theorem IV-15 for the uncertainty case). In addition, optimality rules were derived for the firm's investment decision (theorem IV-9) and for the firm's dividend policy (theorem IV-12 for the certainty case and theorem IV-16 for the uncertainty case).

It deserves to be emphasized that several of the results obtained in this section demonstrate the relationship that exists between the firm's operating policy decisions and its financial policy decisions. In general, the firm's output decision is not independent of the firm's cash management policy (theorems IV-4 and IV-7 and corollary IV-13-2), and, also in general, the firm's cost of capital, and hence its investment decision, is not independent of its leverage policy (theorems IV-11 and IV-15).

F. CHAPTER SUMMARY

This chapter has extended the model of the firm developed in the previous chapter to incorporate those decision variables and constraints relevant to the firm's financial policy decisions. Three classes of financial policies were distinguished: cash management policy, leverage policy, and dividend policy. An integrated production-finance model of the firm was formulated and the model's implications for the optimal operating policies and the optimal financial policies of the firm were derived. In addition, the relationship between these two sets of policy decisions was explored.

The main results obtained in this chapter include the following. The collective utility maximizer tends to maintain greater transactions cash balances than a short run profit maximizer (theorem IV-1) and, as in the case of the neoclassical firm, its demand for transactions cash balances is inversely related to the interest rate (theorem IV-2). It was also found that an increase in the tax rate τ will tend to cause the (expected) collective utility maximizer to substitute debt for equity in its capital structure (theorem IV-3 and corollaries IV-10-1 and IV-14-1).

It was also found that the discounted expected collective utility maximizing firm modeled in section E tends to produce more output in each period (theorem IV-4) and in each state of nature (theorem IV-7) than a short run profit maximizer. It was found that cash balance and working capital requirements do not affect the expansion path of the firm (theorem IV-6), although they do affect its output decision (theorem IV-4, theorem IV-7, and corollary IV-13-2). At optimality the firm's marginal cost of debt capital equals its marginal cost of equity capital (theorem IV-10 for the certainty case and theorem IV-14 for the uncertainty case); that the firm's decision as to whether to maintain precautionary cash balances depends on whether the marginal value of such balances is as great as their marginal cost (theorem IV-13); and that the firm's choice of capital structure is not, in general, irrelevant to its cost of capital (theorem IV-11 for the certainty case and theorem IV-15 for the uncertainty case). In addition, optimality rules were derived for the firm's investment decision (theorem IV-9) and for the firm's dividend policy (theorem IV-12 for the certainty case and theorem IV-16 for the uncertainty case).

It deserves to be emphasized that several of the results obtained in this chapter demonstrate the relationship that exists between the firm's operating policy decisions and its financial policy decisions. In general, the firm's output decision is not independent of the firm's cash management policy (theorems IV-4 and IV-7 and corollary IV-13-2), and, also in general, the firm's cost of capital, and hence its investment decision, is not independent of its leverage policy (theorems IV-11 and IV-15).

CHAPTER FOUR FOOTNOTES

1. See footnote 126 in chapter two of this thesis for a definition of a perfect market.

2. Hirshleifer, Investment, Interest, and Capital, op. cit., ch. 3. See also subsection 1 in section I of chapter two of this thesis.

3. See footnote 529 in chapter two of this thesis for a definition of complete markets.

4. Ibid., pp. 261-264, and Rubinstein, The Irrelevancy of Dividend Policy in an Arrow-Debreu Economy, op. cit. See also subsection 1 in section K of chapter two of this thesis.

5. The effects of market imperfections are explored in Hirshleifer, Investment, Interest, and Capital, op. cit., ch. 7. The effects of the incompleteness of markets are analyzed in ibid., pp. 264-272.

6. G.J. Stigler, "Imperfections in the Capital Market," Journal of Political Economy (vol. 75; no. 3; June 1967), pp. 287-292, reprinted in Stigler, The Organization of Industry, op. cit., ch. 10. See also Fama and Miller, op. cit., pp. 76-77, 170-175.

7. At times small firms may find it impossible to obtain financial capital from external sources regardless of the rate of interest they are willing to pay. See "The Capital Crisis," Business Week (September 22, 1975).

8. See Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 264-265, on this point.

9. See, for example, Stiglitz, On the Optimality of the Stock Market Allocation of Investment, op. cit.; Jensen and Long, op. cit.; Fama, Perfect Competition and Optimal Production Decisions under Uncertainty, op. cit.; Diamond, op. cit.; Wilson, op. cit.; Ekern, op. cit.; and Leland, Production Theory and the Stock Market, op. cit. Leland's model is discussed in subsection 4 in section K of chapter two of this thesis. Even if the firm's operating decisions and its financial decisions are not, strictly speaking, separable, there remains the question as to how severe are the distortions introduced by assuming that they are separable. See, for example, Fama and Miller, op. cit., p. 77, footnote 8, for a discussion of this point as it relates to imperfect markets.

10. A good general discussion of these three types of financial policy decisions is provided in Bierman, op. cit.

11. Later in this chapter the cost of debt, i.e. 'the interest rate on debt', will be defined as the weighted average cost of outstanding bonds and current bank borrowings.

12. This transactions demand for cash exists even under certainty. Under uncertainty there are additional motives, which are discussed below in section B.

13. It should also be noted that Scitovsky has argued that the availability of financial capital is what limits the size of the firm. Scitovsky, op. cit., pp. 189-200.

14. See the references listed in footnote 9.

15. The model developed below is analytic. Simulation models have been developed by Carleton, et. al., and by Stone. See W.T. Carleton, C.L. Dick, Jr., and D.H. Downes, "Financial Policy Models: Theory and Practice," Journal of Financial and Quantitative Analysis (vol. 8; no. 5; December 1973), pp. 691-709, and B.K. Stone, "Cash Planning and Credit-Line Determination with a Financial Statement Simulator: A Case Report on Short-Term Financial Planning," Journal of Financial and Quantitative Analysis (vol. 8; no. 5; December 1973), pp. 711-729.

16. J.M. Keynes, The General Theory of Employment, Interest and Money (Harcourt, Brace and Co.; New York; 1935). See also Patinkin, op. cit., chs. 5-6. A model incorporating all three motives for the firm to hold cash is developed in Wu, Rozek, and Dutton, op. cit.

17. W.J. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics (vol. 66; no. 4; November 1952), pp. 545-556; J. Tobin, "The Interest-Elasticity of Transactions Demand for Cash," Review of Economics and Statistics (vol. 38; no. 3; August 1956), pp. 241-247; M.H. Miller and D. Orr, "A Model of the Demand for Money by Firms," Quarterly Journal of Economics (vol. 80; no. 3; August 1966), pp. 413-435; M. Weitzman, "A Model of the Demand for Money by Firms: Comment," Quarterly Journal of Economics (vol. 82; no. 1; February 1968), pp. 161-164; and Aigner and Sprenkle, op. cit., pp. 1249-1254. See also "The recovery produces a flood of liquidity," Business Week (July 26, 1976).

18. Wu, Rozek, and Dutton, op. cit.

19. N. Cunningham, "Business Investment and the Marginal Cost of Funds," _Metroeconomica_ (part II; vol. 10; no. 3; December 1958), pp. 155-181; Tinsley, _op. cit._; and Gupta, _op. cit._ Gupta's introduction of precautionary balances is an application of Roy's 'safety-first' principle. _Ibid._, p. 737, and A.D. Roy, "Safety First and the Holding of Assets," _Econometrica_ (vol. 20; no. 3; July 1952), pp. 431-449.

20. See T.M. Whitin, _The Theory of Inventory Management_, 2nd. ed. (Princeton University Press; Princeton, N.J.; 1957) or G. Hadley and T.M. Whitin, _Analysis of Inventory Systems_ (Prentice-Hall; Englewood Cliffs, N.J.; 1963).

21. Baumol, _The Transactions Demand for Cash: An Inventory Theoretic Approach_, _op. cit._, and Tobin, _op. cit._ are the classic works in this area. For an excellent discussion of the Baumol and Tobin models see K. Brunner and A.H. Meltzer, "Economies of Scale in Cash Balances Reconsidered," _Quarterly Journal of Economics_ (vol. 81; no. 3; August 1967), pp. 422-436. See also Miller and Orr, _op. cit._, and Weitzman, _op. cit._, which extend the basic Baumol model; Aigner and Sprenkle, _op. cit._; Bierman, _op. cit._, ch. 1; and J. Mensching, S. Garstka, and T. Morton, "Protective Planning Horizon Procedures for a Deterministic Cash Balance Problem," Management Sciences Research Report No. 361 (Graduate School of Industrial Administration, Carnegie-Mellon University; Pittsburgh; April 1975).

22. T.R. Saving, "Transactions Costs and the Firm's Demand for Money," _Journal of Money, Credit and Banking_ (vol. 4; no. 2; May 1972), pp. 245-259.

23. _Ibid._, p. 246.

24. M. Friedman, _The Optimum Quantity of Money and Other Essays_ (Aldine; Chicago; 1969), pp. 58-59.

25. A firm's real money balances are equal to its stock of cash adjusted for inflation, i.e. calculated as the dollar amount of cash carried on the firm's balance sheet divided by a suitable price index to convert the amount into 'base year' dollars (or dollars of 'constant purchasing power').

26. S. Fischer, "Money and the Production Function," _Economic Inquiry_ (vol. 12; no. 4; December 1974), pp. 517-533, and Wu, Rozek, and Dutton, _op. cit._

27. Patinkin, _op. cit._, chs. 5-7; C. Lloyd, "The Real-Balance Effect and the Slutsky Equation," _Journal of Political Economy_ (vol. 72; no. 3 ; June 1964), pp. 295-299; C. Lloyd, "Preferences, Separability, and the Patinkin Model," _Journal of Political Economy_ (vol. 79; no. 3; May-June 1971), pp. 642-651; R. Dusansky and P.J. Kalman, "The Real Balance Effect and the Traditional Theory of Consumer Behavior: A Reconciliation," _Journal of Economic Theory_ (vol. 5;

no. 3; December 1972),pp.336-347; and J.A. Alexander and C. Lloyd, "The Real Balance Effect and the Traditional Theory of Consumer Behavior: An Annulment," Journal of Economic Theory (vol. 11; no. 2; October 1975), pp. 289-291. To summarize briefly the arguments contained in the foregoing, placing real money balances in the utility function requires that the utility function satisfy certain somewhat restrictive conditions (what Lloyd calls 'partial separability') in order that the function continue to exhibit the set of properties generally considered desirable in view of traditional consumer theory.

28. Strictly speaking, one must be careful in pursuing this line of argument, for if there is complete certainty with regard to the future, then the economic system becomes formally equivalent to a barter economy, and even the transactions demand for money vanishes. See R.W. Clower, "A Reconsideration of the Microfoundations of Monetary Theory," Western Economic Journal (vol. 6; no. 1; December 1967), pp. 1-8. As discussed below, to justify the existence of a transactions demand for money on the part of the firm, one must assume at least some minimal degree of uncertainty on the part of the firm concerning the timing of cash inflows and outflows. As long as no other uncertainty exists (e.g. future interest rates are known with certainty), the only source of demand for real money balances is the transactions motive. In the remainder of this subsection this situation of 'minimal' uncertainty will be referred to loosely as one of 'certainty' to distinguish it from the more general situation of uncertainty in which there exist precautionary and speculative demands for real money balances.

29. The optimization required to obtain m for any particular triplet (Q, K, L) might be carried out in the manner suggested by Patinkin, Fischer, or Saving. See Patinkin op. cit., ch. 5 (plus special appendix); Fischer, op. cit.; and Saving, op. cit.

30. The exact nature of this process is left unstated, as it is incidental to the main purpose of this section. All that is important is the fact that the essential (from the standpoint of the problem at hand) features of the process can be summarized mathematically in (3). Note that in a similar manner, the actual production technologies underlying (2) are usually not stated explicitly either.

31. Since, as demonstrated in chapter three, the (expected) collective utility maximizer will (except in special circumstances) select the minimum cost combination of inputs with which to produce each level of output Q, the optimization embodied in (4) is consistent with the assumptions made and the results obtained in chapter three of this thesis.

32. See equations (148) and (158) of chapter two.

33. Fischer, op. cit., p. 524.

34. See footnote 28 of this chapter.

35. See, for example, "Many Firms Manage To Pare Role of Debt, Build Cash Reserves," Wall Street Journal (June 1, 1976), and "Firms Spend Carefully, Pay Off Much Debt And Build Liquidity," Wall Street Journal (January 13, 1977).

36. Stochastic cash balance problems are considered in G.D. Eppen and E.F. Fama, "Cash Balance and Simple Dynamic Portfolio Problems with Proportional Costs," International Economic Review (vol. 10; no. 2; June 1969), pp. 119-133; G.D. Eppen and E.F. Fama, "Solutions for Cash-Balance and Simple Dynamic-Portfolio Problems," Journal of Business (vol. 41; no. 1; January 1968), pp. 94-112; N.M. Girgis, "Optimal Cash Balance Levels," Management Science (vol. 15A; no. 3; November 1968), pp. 130-140; E.H. Neave, "The Stochastic Cash Balance Problem with Fixed Costs for Increases and Decreases," Management Science (vol. 16A; no. 7; March 1970), pp. 472-490; Miller and Orr, op. cit.; Tinsley, op. cit.; and Gupta, op. cit.

37. This assumes, for simplicity, that all decision variables and all state variables are strictly positive along their optimal trajectories.

38. The interpretation of the adjustment term in (18),

$$\bar{p}\,\frac{\partial f}{\partial m}\left(\frac{\partial U_1/\partial R + (1-\tau)\mu_1 e^{rt}}{\partial U_1/\partial D}\right)$$

is the same as the interpretation of (42) in chapter three (for which μ_1 had previously been set equal to zero).

39. Wu, Rozek, and Dutton, op. cit., p. 5.

40. Note that since μ_1 is expressed in present value units it is of the form $\mu_1 = \bar{\mu}_1 e^{-rt}$. Hence, $\mu_1 e^{rt} = \bar{\mu}_1 e^{-rt} e^{rt} = \bar{\mu}_1$. Thus, $\bar{\mu}_1$ is expressed in current value units and is independent of r.

41. Ibid., p. 5. The lemma is consistent with a property that Keynesian monetary theory holds to be generally valid for the demand for money balances. Patinkin, op. cit., ch. 9.

42. One question that is of interest in this regard is the extent to which the firm's working capital requirements contain what Weston and Brigham call a 'permanent working capital component' that can be financed more cheaply by long term capital than by repeated short term borrowing. See J.F. Weston and E.F. Brigham, Managerial Finance (Holt, Rinehart

and Winston; New York; 1972), pp. 526-544. See also Gupta, op. cit., which treats this problem.

43. For completeness it should be noted that, according to the definition of debt provided in section A of chapter two, debt also includes preferred stock. For convenience, preferred stock is not treated separately in this chapter, but rather, is subsumed within the composite measure of debt, B, defined in section A of this chapter.

44. See Philippatos, op. cit., ch. 10.

45. Modigliani and Miller, The Cost of Capital, Corporation Finance and the Theory of Investment, op. cit.; Modigliani and Miller, 'The Cost of Capital, Corporation Finance, and the Theory of Investment': Reply, op. cit.; and Modigliani and Miller, Corporate Income Taxes and the Cost of Capital: A Correction, op. cit.

46. As noted in chapter two, Modigliani's and Miller's 'irrelevance proposition' on the firm's leverage ratio has been extended by Stiglitz, A Re-Examination of the Modigliani-Miller Theorem, op. cit.; Stiglitz, On the Irrelevance of Corporate Financial Policy, op. cit.; and Mossin, Security Pricing and Investment Criteria in Competitive Markets, op. cit.

47. Currently, interest expense is deductible for tax purposes while dividends are not.

48. Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 263-264; J.E. Stiglitz, "Taxation, Risk Taking, and the Allocation of Investment in a Competitive Economy," in M. Jensen, ed., Studies in the Theory of Capital Markets (Praeger; New York; 1972), pp.294-374; and Stapleton, op. cit.

49. This result was demonstrated with the aid of time-state-preference theory in subsection 1 in section K of chapter two of this thesis.

50. Stiglitz, A Re-Examination of the Modigliani-Miller Theorem, op. cit.; Smith, Corporate Financial Theory under Uncertainty, op. cit.; Smith, Default Risk, Scale, and the Homemade Leverage Theorem, op. cit.; and Hirshleifer, Investment, Interest, and Capital, op. cit., pp. 263-264.

51. See, for example, "The Big Squeeze on U.S. Companies," Business Week (September 22, 1975); "Stock-Market Surge Beckons Firms Back To Equity Financings After Long Hiatus," Wall Street Journal (February 23, 1976); "AT & T to Sell 12 Million Shares About June 16," Wall Street Journal (May 19, 1976); and "Braniff to Announce Buying $70 Million Of Planes, Establishing New Credit Line," Wall Street Journal (March 30, 1977).

52. Inselbag explores the same problem considered below, but for a value maximizing firm in the absence of corporate taxes. Inselbag, op. cit.

53. Inselbag employs a relation similar to (21), although he works directly with total interest expense, rather than just the interest rate, and incorporates the capital stock at time t as one of the arguments of the function in order to allow for the impact of uncertainty (as transmitted through changes in the leverage ratio). Ibid., pp. 765-768. In order for (21) to be meaningful, it is necessary that debt be rolled over instantaneously so that the actual borrowing histories (amounts with corresponding interest rates up to time t) are irrelevant. Without such a restriction the Markov property embodied in (21) cannot be satisfied.

54. Once again under the simplifying assumption that all control variables and all state variables are strictly positive along their optimal trajectories.

55. Mathematically, U_4 is needed to prevent the terminal stock of debt from falling to zero. The economic role of U_4 is easily justified since a firm's planners would rarely want to reduce the terminal stock of debt to zero (and if they did, U_4 could be defined as the zero function).

56. The derivation of equation (29) follows.
From (26),

$$\dot{\lambda}_2 = -\partial L_\mu/\partial B = -[\frac{\partial U_1}{\partial D}(1-\tau)(-i-\frac{\partial i}{\partial B}B)e^{-rt}+\mu_1(1-\tau)(-i-\frac{\partial i}{\partial B}B)] .$$

From (28),

$$\dot{\lambda}_2 = -[\frac{\partial U_1}{\partial D}\{1-(1-\tau)\frac{\partial i}{\partial Y}B\}(-r)e^{-rt}+\frac{\partial U_1}{\partial D}\{1-(1-\tau)\frac{\partial i}{\partial Y}\dot{B}\}e^{-rt}$$

$$-\bar{\mu}_1(-r)e^{-rt}(1-\tau)\frac{\partial i}{\partial Y}B-\bar{\mu}_1e^{rt}(1-\tau)\frac{\partial i}{\partial Y}\dot{B} ,$$

where $\bar{\mu}_1 \equiv \mu_1 e^{rt}$. Equating these two expressions and simplifying yields

$$(\frac{\partial U_1}{\partial D}+\bar{\mu}_1)(1-\tau)(-i-\frac{\partial i}{\partial B}B) =$$

$$(1-r)\frac{\partial U_1}{\partial D}+r(\frac{\partial U_1}{\partial D}+\bar{\mu}_1)(1-\tau)\frac{\partial i}{\partial Y}B-(\frac{\partial U_1}{\partial D}+\bar{\mu}_1)(1-\tau)\frac{\partial i}{\partial Y}Y ,$$

which, after terms have been rearranged and $\mu_1 e^{rt}$ has been substituted for $\bar{\mu}_1$, gives equation (29).

57. There is a second aspect of the firm's dividend policy that has been discussed in the finance literature - the actual form the distribution of dividends is to take (i.e. cash vs. non-cash forms of distribution). See Bierman, op. cit., chs. 8-10.

58. Graham, Dodd, and Cottle, op. cit.

59. The price-earnings ratio for a particular stock is equal to the stock market value of a share divided by the company's earnings (i.e. net income) per share (of common stock).

60. See, for example, Gordon, Dividends, Earnings, and Stock Prices, op. cit. See also Friend and Puckett, op. cit., for a discussion of the biases in favor of retained earnings built into the Gordon study and other similar studies.

61. This has been christened the 'bird-in-the-hand' argument after the proverb that states that 'a bird in the hand is worth two in the bush'. See Fama and Miller, op. cit., pp. 84-85. See also "Dividend-hungry investors cry for more," Business Week (August 2, 1976).

62. Miller and Modigliani, op. cit.

63. Rubinstein, The Irrelevancy of Dividend Policy in an Arrow-Debreu Economy, op. cit.

64. See section K in chapter two of this thesis.

65. The terms 'risk' and 'uncertainty' are used synonymously in this thesis. See section C in chapter one of this thesis.

66. See footnotes 529 and 530 of chapter two of this thesis for a discussion of the meaning of 'complete' markets.

67. However, as discussed later in this section, this interpretation could be given to the model developed below if the appropriate additional assumptions are made concerning the firm's institutional milieu.

68. If it were assumed that there exists a complete set of markets for contingent output claims, then such contractual commitments could be the result of the firm's trading within these markets.

69. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, op. cit.; Lintner, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, op. cit.; and Mossin, Equilibrium in a Capital Asset Market, op. cit.

70. $\beta \equiv 0$ would correspond to risk neutrality and $\beta < 0$ would correspond to risk preference on the part of the stock market as a whole.

71. In the simplest case, $\rho = r - g$, where r is the interest rate and g is the firm's long term average rate of growth (say, of total assets). That is, under certainty with perfect markets and steady state growth, v_t in (35) becomes simply

$$v_t = \frac{d}{r-g} ,$$

where d is the dividend per share, as discussed in chapter two of this thesis.

72. As discussed in section G of chapter two of this thesis.

73. All other decision variables are assumed to be strictly positive for each date and state along their respective optimal trajectories.

74. (75) would also hold if β were sufficiently small, i.e. if the stock market's degree of risk aversion were sufficiently mild. See footnote 70.

75. Hadley and Whitin, op. cit., and Bierman, op. cit.

76. The main advantage of (89) is that the Lagrange multipliers in (88), which are expressed as marginal utilities (since the objective functional in (59) is expressed in terms of utility units), are replaced by ratios of marginal utilities in (89). Thus, the marginal cost of financial capital, $\lambda_{5,t}$, which is meaningful in an ordinal sense, but not in a cardinal sense, is replaced by the (normalized) marginal cost of financial capital $\frac{\lambda_{5,t}}{\lambda_{4,t}+\lambda_{6,t}}$, which is meaningful in a cardinal sense. Thus, in what follows it will be important to work with the right-hand side of (89) as 'the' marginal cost of physical capital when, as in theorem IV-6, the absolute size of this marginal cost matters. Similarly, $\lambda_{5,t}$ can be spoken of as the marginal cost of financial capital when only relative size matters. But when absolute size matters, the Lagrange multiplier terms, including $\lambda_{5,t}$, will have to be normalized by dividing through by $(\lambda_{4,t}+\lambda_{6,t})$.

77. The requirement that $\lambda_{4,t} = 0$ provides the connection between the model developed in chapter three and the model developed in this chapter. In chapter three it was found that, in general, the expansion path of the collective utility maximizer coincides with the expansion path of the short run profit maximizer only if the net income constraint is not

binding. But this corresponds to $\lambda_{4,t} = 0$ in (90) since the value of additional net income, which is measured by $\lambda_{4,t}$ in (60) in terms of net income's contribution to total equity, is zero in (11) in chapter three when the profit constraint is not binding.

78. Note that (99) does *not* imply that

$$\sum_{s'=1}^{S} \lambda_{1,t+1,s',s} = (1+\delta) \sum_{s'=1}^{S} \lambda_{6,t+1,s',s} .$$

79. The time-state-preference approach is also consistent with the detailing of scenarios (or states of nature) that accompanies actual corporate planning. See chapter six of this thesis for a further discussion of this point as it applies to the U.S. airframe industry.

80. That is, a change in the debt level has an immediate impact on cash flow since the firm's bond interest obligations are altered. This affects net income and collective utility through the impact of a change in net income on total equity ($\lambda_{4,t}$) and cash generated from operations ($\lambda_{6,t}$). A change in the rate of issues/redemptions also has an immediate cash impact as new debt issues are a source of cash while redemptions use up cash. In addition, a change in the level of debt outstanding in the current period will, given the firm's debt requirements next period, affect next period's required issues/redemptions decision, which in turn will affect next period's average interest rate $i = i(B, Y)$. This impact is measured by $\lambda_{2,t+1}$ in (106).

81. The separability theorems are discussed in sections I and K in chapter two of this thesis.

82. $\frac{\partial}{\partial B}(1-\tau)(i + \frac{\partial i}{\partial B} B + \frac{\partial i}{\partial Y} B) = (1-\tau)\{2 \frac{\partial i}{\partial B} + \frac{\partial i}{\partial Y} + B(\frac{\partial^2 i}{\partial B^2} + \frac{\partial^2 i}{\partial B \partial Y})\} > 0$ by the assumed properties of the function i.

83. As was done in the Marris models that were discussed in section G of chapter two of this thesis and as was also done in the Herendeen model that was discussed in section I of chapter two of this thesis.

84. Note that the double sum on the left-hand side of (124) reflects the fact that a change in any dividend per share, $d_{t,s}$, planned for any particular period and state of nature that period affects the share value, v_t, that period, according to (36), and hence, affects the equity and cash flow impacts of any planned share issue/redemption in each possible state s that might obtain that period.

85. See footnotes 529 and 530 of chapter two of this thesis for a discussion of the meaning of 'complete' markets.

86. Arrow, The Role of Securities in the Optimal Allocation of Risk-Bearing, op. cit., and Debreu, op. cit.

87. A survey of the temporary equilibrium literature can be found in J.M. Grandmont, "Temporary General Equilibrium Theory," Econometrica (vol. 45; no. 3; April 1977), pp. 535-572.

A DYNAMIC STOCHASTIC MODEL OF CORPORATE BEHAVIOR OVER THE BUSINESS CYCLE WITH A SPECIAL APPLICATION TO THE MAJOR U.S. MILITARY AIRFRAME BUILDERS

John Dudley Finnerty

NAVAL POSTGRADUATE SCHOOL

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis contains a formulation of a dynamic stochastic model of corporate behavior over the business cycle and applies the basic model to firms in the U.S. airframe industry. The literature dealing with the theory of the firm is surveyed and a taxonomy is developed within which the major contributions to the literature are appraised.

The basic model is formulated as an optimal control problem.

block 19 continued

Organizational Slack; Planning Algorithm; Decentralized Firm; U.S. Aerospace Industry; U.S. Airframe Industry; Defense Contracting; Corporate Planning; Defense Contractor Risk; Airframe Builder Model; Progress Payments; Profit '76.

block 20 continued

The model is used to study the behavior of the firm over the business cycle and to suggest a possible reconciliation of the traditional and managerial theories of the firm. Financial considerations are incorporated into the model and the relationship between the firm's optimal operating decisions and its optimal financial decisions is examined. Organizational factors are introduced and some of the consequences of decentralized decision-making for the loss of control and X-efficiency are studied.

The basic model is extended to the major airframe builders by incorporating factors specific to that industry's institutional milieu. A model of a representative airframe builder is formulated as a stochastic optimal control problem and is used to study the impact of the government's progress payments policy and the likely impact of making interest expense an allowable cost under government contracts.

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A Dynamic Stochastic Model of Corporate Behavior Over the Business Cycle With A Special Application To The Major U.S. Military Airframe Builders

by

John Dudley Finnerty
A.B., Williams College, 1971
B.A., Cambridge University, 1973
M.A., Cambridge University, 1977

Submitted in partial fulfillment of the requirements for the degree of

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V. CORPORATE PLANNING AND THE INTERNAL ALLOCATION OF THE FIRM'S RESOURCES

A. INTRODUCTION

With the exception of the behavioral theories of the firm, the models discussed in chapter two of this thesis were formulated for the purpose of studying phenomena external to the firm, e.g. how actual price and output were related to (external) product demand. More generally, this is true of microeconomics, which traditionally has been concerned with allocative efficiency - i.e. the efficiency with which the market system allocates resources and goods between firms (and also between firms and other economic units) - and much less concerned with what Leibenstein calls 'X-efficiency' - the efficiency with which resources are allocated and utilized within firms.[1] The main purpose of this chapter is to extend the model developed in chapters three and four of this thesis to incorporate phenomena internal to the firm, such as the nature of its organizational structure, and to explore the relationship between internal phenomena and the behavior exhibited by the firm.

In the traditional theory of the firm, the firm's organizational structure was given only an indirect role to play. It was argued that within the entrepreneurial firm the limited decision-making capacity of the entrepreneur was one of the major factors limiting the size of the firm.[2] The larger the organization, it was argued, the weaker the degree of control exercised by the decision maker.[3] As a result, internal diseconomies of size would set in beyond some point, causing the firm's long run average total cost curve to become upward-sloping beyond that point. These organizational factors were not, however, integrated directly into the model of the firm, but rather, were invoked as one justification for the U-shaped long run average cost curve, and hence, the optimum firm size, hypothesized in the neoclassical theory of the firm.[4]

In recent years economists have come to appreciate better the multidivision form of corporate organization pioneered by General Motors in the 1920s and 1930s[5] and have begun to explore its implications for the coordination and control of the corporate enterprise.[6] Several economists have explored the economics of the divisionalized firm, noting in particular that the separate divisions may be viewed as quasi-firms linked to corporate headquarters and to each other by a set of internal markets for both human and nonhuman resources.[7] It has also been noted that different organizational forms exist within the basic multidivision form,[8] although the implications of these different structures for coordination and control are not entirely clear as yet.

One of the consequences of the multidivision firm and decentralized decision-making is the need for planning procedures to guide the internal allocation of resources.[9] Capital, labor (particularly managers and scientists), and other resources may be transferred from one division to another. In addition, the output of one division may be used as an input by another division. Since there are seldom market-determined prices to guide these allocations - as there normally are for transactions between firms - the firm must rely on some other means of determining the (optimal) allocations. For this purpose the firm might adopt price- or nonprice- guided planning procedures, such as those employed in planned economies.[10] In some cases, the planning process supplants the market, as for example, when vertical integration takes place and resources that were previously transferred between firms via markets are thereafter transferred between divisions of the firm,[11] while in other cases the planning process is intended to serve a function for which markets are poorly designed, as for example, the allocation of managers on the basis of differences in quality and performance.[12]

The question regarding the efficiency with which resources are allocated and utilized within the firm is an important one. Leibenstein argues that the potential gains from improved X-efficiency - through the improved allocation of managers and workers, better motivation of workers, better organization of production, and quicker adoption of innovations - exceed the potential gains from improved allocative efficiency.[13] Organizational structure is important to the extent that it affects the coordination and control of the enterprise.[14] Where controls are weak, the firm may operate at a low level of X-efficiency, becoming more efficient only when adverse business conditions force cost cutting, productivity improvements, and other changes.[15] The multidivision form of organizational structure has evolved due to the need for effective coordination and control in large, diversified firms, although as Williamson's analysis suggests, managerial preferences for staff and emoluments make the multidivision organizational form a necessary but not a sufficient condition for X-efficiency in large manager-controlled firms.[16]

Most studies that have been concerned with the allocation of resources within firms have emphasized the informational aspects of the problem.[17] These studies have suggested that "the firm, in large part, consists of nonmarket institutions whose function is to deal with resource allocation in the presence of informational constraints that markets handle poorly or do not handle at all."[18] Several studies have explored how these informational constraints affect the structure of internal labor markets and the allocation of human resources within the firm,[19] while others have analyzed the functioning of internal capital markets, how they differ from external capital markets, and the relationship between the quality of information (e.g. quality of forecasts) and the internal investment decision.[20] While these studies have contributed significantly to the understanding of what goes on inside the firm, the relationship between what goes on inside the firm and the state of the firm's external environment remains largely unexplored.[21]

To reiterate a point made earlier, the main purpose of this chapter is to extend the model developed in chapters three and four of this thesis to incorporate phenomena internal to the firm and to explore the relationship between internal phenomena and the behavior exhibited by the firm. Section B distinguishes different classes of labor and develops a model of organizational slack that relates the degree of organizational slack to the state of the firm's operating environment. Section C presents the model of the multidivision firm. Under the assumption that decision-making is centralized, rules are derived for the optimal allocation of human and physical capital among the firm's divisions. These optimality rules are utilized in section D to construct an algorithm to solve the decentralized multidivision firm's multiperiod finite horizon planning problem. Also in section D, the problem of achieving compliance with corporate objectives in the decentralized multidivision firm when division managers attempt to maximize their own utility is studied and the implications of imperfect compliance for the loss of X-efficiency are discussed.

B. A MODEL OF ORGANIZATIONAL SLACK

1. Introduction

The model developed in chapters three and four assumed all labor to be homogeneous and assumed that inputs were employed with maximum technical efficiency. In this chapter two classes of labor are distinguished: manufacturing labor and administrative labor. In addition, it is no longer required that inputs be combined with maximum technical efficiency. In this section the concept of organizational slack is introduced, and the model developed in chapter three is modified so that variation in the degree of organizational slack over the business cycle can be studied.

2. The Model

It is assumed that the firm hires labor of two types: manufacturing labor, which is productive, and administrative labor, some of which is productive and some of which is not. Specifically, it is assumed that the units of administrative labor hired by the firm are homogenous, but that a portion of these units are allocated to staff jobs in which their productivity is zero.[22] Let the amount of manufacturing labor hired by the firm at time t be denoted by L(t) and let the amounts of productive and nonproductive administrative labor be denoted by $A_p(t)$ and $A_n(t)$, respectively. It is assumed that the markets for both types of labor are perfectly competitive, with the constant unit costs being denoted by w(t) for manufacturing labor and by s(t) for administrative labor. Since administrative labor is assumed to be homogeneous, both productive and nonproductive units are paid the same salary s(t) (per period).

Productive administrative labor is valued by the firm for its contribution to production. Its contribution is recognized by introducing $A_p(t)$ as an argument of the neoclassical production function,

$$Q = f(K(t), L(t), A_p(t)) \ , \tag{1}$$

where it is assumed, as in chapter three, that the firm produces a single output and that the production function (1) has a full set of continuous second partial derivatives.

It is also assumed that the marginal product of each input in (1) is strictly positive at all usage levels and that the use of each input is subject to diminishing returns, so that

$$\left.\begin{array}{lll} \partial f/\partial K > 0 & \partial f/\partial L > 0 & \partial f/\partial A_p > 0 \\ \\ \partial^2 f/\partial K^2 < 0 & \partial^2 f/\partial L^2 < 0 & \partial^2 f/\partial A_p{}^2 < 0 \quad . \end{array}\right\} \quad (2)$$

Nonproductive administrative labor is valued by the firm, and specifically, by the firm's managers, for its direct contribution to managerial utility.[23] Its contribution is recognized by introducing $A_n(t)$ as an argument of the collective utility function U_1 ,[24]

$$U_1 = U_1(R(t),\ D(t),\ M(t),\ A_n(t)). \quad (3)$$

The introduction of administrative labor also requires that the minimum net income constraint be modified to take this into account. Since total labor cost is now $w(t)\cdot L(t) + s(t)\cdot[A_p(t) + A_n(t)]$, the minimum net income constraint becomes

$$\begin{aligned} \pi(t) = (1 - \tau)\{&p(\theta_t)\cdot f(K(t),\ L(t),\ A_p(t)) \\ &- w(t)\cdot L(t) - s(t)\cdot[A_p(t) + A_n(t)] \\ &- M(t) - q(t)\cdot[\delta\cdot K(t)] \geq \pi_o \ , \end{aligned} \quad (4)$$

where θ_t indicates the state of the firm's operating environment at time t (as in section E of chapter three).

Recognizing $A_p(t)$ and $A_n(t)$ as decision variables, and using (1), (3), and (4) to modify the model of the firm (11) in section B of chapter three, the model of the firm is reformulated as the following optimal control problem:

$$\underset{\substack{\{L(t),I(t),\\ M(t),A_p(t),\\ A_n(t)\}}}{\text{maximize}} \int_0^T U_1[p(\theta_t)\cdot f(K(t),L(t),A_p(t));\ (1-\tau)\{p(\theta_t)\cdot f(K(t),L(t),A_p(t)) - w(t)\cdot L(t) - s(t)\cdot[A_p(t) + A_n(t)] - M(t)\} + \tau\cdot q(t)[\delta\cdot K(t)] - q(t)\cdot I(t);\ M(t);\ A_n(t)]e^{-rt}dt + U_2(K(t))e^{-rT} \tag{5}$$

$$\text{subject to}\quad \dot{K}(t) = I(t) - \delta\cdot K(t),\ 0 \le t \le T,\ K(0) \text{ given}$$

$$(1-\tau)\{p(\theta_t)\cdot f(K(t),L(t),A_p(t)) - w(t)\cdot L(t) - s(t)\cdot[A_p(t) + A_n(t)] - M(t) - q(t)\cdot[\delta\cdot K(t)]\} \ge \pi_o\ ,\ 0 \le t \le T$$

$$L(t),K(t),M(t),A_p(t),A_n(t) \ge 0,\ 0 \le t \le T$$

The purpose of the remainder of this section is to characterize the solution to problem (5), and to study its implications for the behavior of the firm in response to changes in the firm's operating environment.

3. Organizational Slack

Since nonproductive administrative labor by definition does not contribute to output, it represents slack to the firm in the sense that when $A_n(t) > 0$, total administrative labor could be reduced by an amount $A_n(t)$ (i.e. the slack could be eliminated) and the level of production, Q(t), would not change. The *degree of organizational slack* at time t is defined to be the amount of nonproductive administrative labor in the firm's personnel organization at time t. The main result of this section demonstrated below is that the degree of organizational slack varies systematically with the state of the firm's operating environment (i.e. over the business cycle).

Before applying Pontryagin's maximum principle to problem (5) and characterizing the resulting necessary conditions, the significance of organizational slack, and its meaning in terms of the efficiency with which the firm's resources are utilized, should be noted. Given the firm's production function (1), if the units of nonproductive administrative labor could be utilized productively, then output would be $Q^* = f(K(t), L(t), A_p(t) + A_n(t))$, which exceeds Q when $A_n(t) > 0$ since by (2), the marginal productivity of the additional 'productive administrative labor' is strictly positive. The output level Q^* can be interpreted as the level of output that would result if all inputs in the firm's employ at time t were utilized in production with maximum technical efficiency. The difference $Q^* - Q$ represents the loss of potential output due to organizational slack and is strictly positive when $A_n(t) > 0$.

Returning to problem (5) and proceeding as before, define the Lagrangian by

$$\begin{aligned} &L_\mu[K, L, I, M, A_p, A_n, \lambda, \mu_1, t] = \\ &\quad H[K, L, I, M, A_p, A_n, \lambda, t] + \mu_1(t)\cdot[(1-\tau)\{p(\theta_t)\cdot \\ &\qquad f(K(t), L(t), A_p(t)) - w(t)\cdot L(t) - s(t)\cdot[A_p(t) \\ &\qquad + A_n(t)] - M(t) - q(t)\cdot[\delta\cdot K(t)]\} - \pi_o] , \end{aligned} \tag{6}$$

where H denotes the Hamiltonian for (5),

$$\begin{aligned} &H[K, L, I, M, A_p, A_n, \lambda, t] = \\ &\qquad\qquad U_1[\quad]e^{-rt} + \lambda(t)\cdot[I(t) - \delta\cdot K(t)] , \end{aligned} \tag{7}$$

in which $U_1[\ ]e^{-rt}$ denotes the integrand in (5).

In order that the time paths $L*(t)$, $I*(t)$, $M*(t)$, $A*_p(t)$, and $A*_n(t)$ provide an optimal solution to problem (5) it is necessary that they satisfy the following conditions:

$\{L*(t), I*(t), M*(t), A*_p(t), A*_n(t)\}$ maximize

$$H[K, L, I, M, A_p, A_n, \lambda, t] \tag{8}$$

subject to the minimum net income constraint and the nonnegativity constraints in (5), $0 \leq t \leq T$

$$\dot{K}(t) = I*(t) - \delta \cdot K(t),\ 0 \leq t \leq T,\ K(0) \text{ given} \tag{9}$$

$$\dot{\lambda}*(t) = -\frac{\partial L_\mu}{\partial K},\ \lambda*(T) = \frac{\partial U_2(K(T))}{\partial K(T)} e^{-rT} \tag{10}$$

Since (9) and (10) are identical to (15) and (16), respectively, in section B of chapter three, they are stated for completeness only and are not considered further in this section.

To characterize the solution to (8), define the Lagrangian $L_{\mu,t}$ by introducing the Lagrange multiplier μ_2 and by appending the product $\mu_2[A_n(t)]$ to L_μ given by (6). It follows that the necessary conditions for an optimal solution to (8) (for each time t) are the following:[25]

$$\left.\begin{aligned} \frac{\partial L_{\mu,t}}{\partial L} = \{\frac{\partial U_1}{\partial R} p(\theta)\frac{\partial f}{\partial L} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w)\}e^{-rt} \\ + \mu_1(1-\tau)(p(\theta)\frac{\partial f}{\partial L} - w) = 0 \end{aligned}\right\} \tag{11}$$

$$\frac{\partial L_{\mu,t}}{\partial I} = \{\frac{\partial U_1}{\partial D}(-q(t))\}e^{-rt} + \lambda(t) = 0 \tag{12}$$

$$\frac{\partial L_{\mu,t}}{\partial M} = \{\frac{\partial U_1}{\partial D}(1-\tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} + \mu_1(1-\tau)(-1) = 0 \tag{13}$$

$$\frac{\partial L_{\mu,t}}{\partial A_p} = \{\frac{\partial U_1}{\partial R} p(\theta)\frac{\partial f}{\partial A_p} + \frac{\partial U_1}{\partial D}(1-\tau)(p(\theta)\frac{\partial f}{\partial A_p} - s)\}e^{-rt} + \mu_1(1-\tau)(p(\theta)\frac{\partial f}{\partial A_p} - s) = 0 \tag{14}$$

$$\frac{\partial L_{\mu,t}}{\partial A_n} = \{\frac{\partial U_1}{\partial D}(1-\tau)(-s) + \frac{\partial U_1}{\partial A_n}\}e^{-rt} + \mu_1(1-\tau)(-s) + \mu_2 = 0 \tag{15}$$

$$\left.\begin{array}{l} (1-\tau)\{p(\theta)f(K,L,A_p) - wL - s(A_p + A_n) - M - q\cdot\delta\cdot K\} \geq \pi_o \\ \mu_1 \geq 0 \\ \mu_1[(1-\tau)\{p(\theta)f(K,L,A_p) - wL - s(A_p + A_n) - M - q\cdot\delta\cdot K\} - \pi_o] = 0 \end{array}\right\} \tag{16}$$

$$A_n(t) \geq 0 \qquad \mu_2 \geq 0 \qquad \mu_2[A_n] = 0 \tag{17}$$

Conditions (11), (12), (13), and (16) have the same interpretation as conditions (18)-(21) in section B of chapter three, and need not be considered separately in this section.

The interpretation of condition (14) and its analogy to (11) are summarized as the following lemma:

Lemma V-1

In equilibrium the collective utility maximizer modeled in (5) will hire more administrative labor and more manufacturing labor than a short run profit maximizer.

Proof

Follows directly from (11), (14), and (2), and the fact that the short run profit maximizer will hire manufacturing labor up to the point at which $w = p(\theta)\frac{\partial f}{\partial L}$ and productive administrative labor up to the point at which $s = p(\theta)\frac{\partial f}{\partial A_p}$. Q.E.D.

Remark

Note that (14) is just (11) with A_p in place of L, so that proving that the firm hires more productive administrative labor than a short run profit maximizer is perfectly analogous to proving that it hires more manufacturing labor. Note also that since the firm hires more productive administrative labor than a short run profit maximizer, the existence of organizational slack (i.e. $A_n(t) > 0$) strengthens the conclusion that 'too much' administrative labor is hired.

The interpretation of conditions (15) and (17), and their analogy to (13) when the possibility that $M(t) = 0$ is admitted, are summarized as the following lemma:

<u>Lemma V-2</u>

In equilibrium if the collective utility maximizer modeled in (5) hires any nonproductive administrative labor, it will do so up to the point at which the marginal rate of substitution between nonproductive administrative labor and dividends, $\frac{\partial U_1/\partial A_n}{\partial U_1/\partial D}$, just equals salary expense net of tax, $(1 - \tau)s$. If no nonproductive administrative labor is hired, it is because the marginal rate of substitution between nonproductive administrative labor and dividends is strictly less than salary expense net of tax (at $A_n = 0$).

<u>Proof</u>

The first statement follows directly from (15) with $\mu_2 = 0$ (since by (17) $\mu_2 = 0$ when $A_n > 0$). The second statement follows directly from (15) with $\mu_2 > 0$ (since by (17) $A_n = 0$ when $\mu_2 > 0$). Q.E.D.

<u>Remark</u>

Note that lemma V-2 provides an interpretation of (15) and (17) that is perfectly analogous to the interpretation of (28) and (30) in section B of chapter three. As in the case of managerial emoluments (according to (28) in section B of chapter three) the firm will, if it hires nonproductive administrative labor, equate the subjective rate of trade off between nonproductive administrative labor and dividends, $\frac{\partial U_1/\partial A_n}{\partial U_1/\partial D}$, with the objective rate of trade off permitted by the market for administrative labor and government tax policy. That is, $(1 - \tau)s$ is the rate at which dividends and nonproductive administrative labor can be traded off within the firm's income statement, given the market-determined salary level s and government tax policy as embodied in τ. Also as in the case of managerial emoluments (according to (30) in section B of chapter three), the firm will not hire any nonproductive administrative labor - there will not be any organizational slack - unless the subjective rate of trade off exceeds the objective rate of trade off over some range of values for A_n.

The third lemma of this section relates the firm's marginal rate of substitution between managerial emoluments and nonproductive administrative labor to the ratio of their unit costs.

<u>Lemma V-3</u>

When the firm modeled in (5) pays managerial emoluments and hires nonproductive administrative labor when it is in equilibrium, the marginal rate of substitution between nonproductive administrative labor and managerial emoluments, $\frac{\partial U_1/\partial A_n}{\partial U_1/\partial M}$, is equal to the unit salary of administrative labor, s.

<u>Proof</u>

Follows directly from (13) and (15). Q.E.D.

Remark 1

Note that the unit cost of administrative labor is s and the unit cost of (a dollar's worth of) managerial emoluments is one, so that s also represents the ratio of the unit costs of these two sources of managerial utility. Thus the equilibrium condition $\frac{\partial U_1/\partial A_n}{\partial U_1/\partial M} = s$ that arises out of lemma V-3 is similar to the neoclassical optimality condition for production that requires that inputs be combined in such a way that the marginal rate of technical substitution between each pair of inputs just equals the ratio of their unit costs. Here 'production' is interpreted in the sense of 'contributing to collective utility'. The point to be emphasized is the important analogy that exists between optimally combining amounts of items that contribute to collective utility so as to maximize a concave multivariate utility function subject to certain constraints and optimally combining amounts of inputs (i.e. sources of output) so as to maximize a concave multivariate production function subject to certain constraints.

Remark 2

Note that in the special case in which s is fixed, lemma V-3 could be restated as: in equilibrium the marginal rate of substitution between expenditures in the form of managerial emoluments and expenditures for nonproductive administrative labor is equal to one.

One of the consequences of lemma V-3 is that an increase in the unit cost of administrative labor will tend to cause the firm to substitute managerial emoluments for (the now relatively more expensive) nonproductive administrative labor, provided the marginal rate of substitution between nonproductive administrative labor and managerial emoluments is decreasing in A_n. This is illustrated in figure V-1. The line AB shows the permissible combinations of M and A_n when all other variables in (5) are held fixed (at their optimal levels). The convex indifference curve shows those combinations of M and A_n that yield the same level of collective utility as the combination (A^*_n, M^*). The convex shape of the indifference curve embodies the decreasing marginal rate of substitution property. When the unit cost of administrative labor is s, the firm will hire an amount A^*_n and will pay managerial emoluments M*. This is the consequence of lemma V-3, which states that the optimal levels A^*_n and M* must occur at a point of tangency such as C in figure V-1. When the unit cost rises to s' (all other variables in (5) held fixed), the line AB rotates clockwise through A to the position AB' since the maximum permissible level of managerial emoluments is unchanged, but the maximum permissible amount of nonproductive administrative labor that can be hired has been reduced. The pure substitution effect of the change in s is determined by holding the utility level fixed, which is accomplished geometrically by drawing a third line parallel to AB' and tangent to the indifference curve. The pure substitution effect, according to which managerial emoluments are substituted for nonproductive administrative labor with the utility level held fixed, is depicted by the 'shift' from C to D in figure V-1. The second portion of the overall effect of a change in s, the income effect, is depicted by the 'shift' from D to E. In the case illustrated in figure V-1,

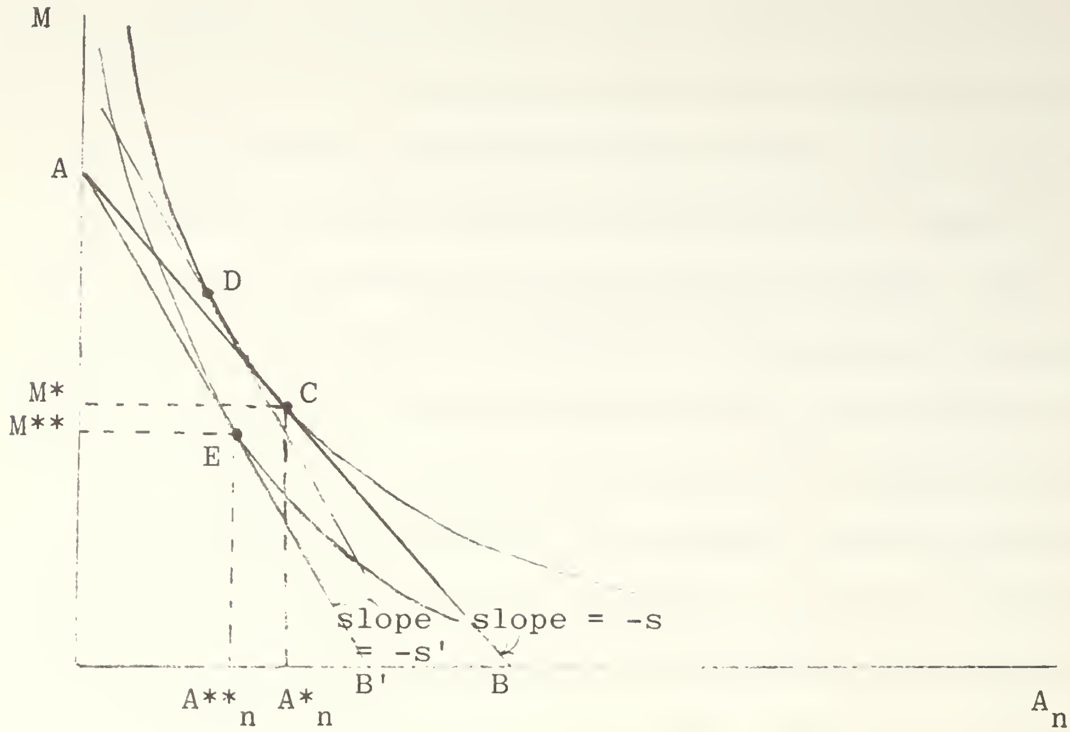

Figure V-1 The Firm's Trade Off Between Managerial Emoluments and Nonproductive Administrative Labor

both managerial emoluments and the amount of nonproductive administrative labor employed by the firm are reduced as a result of the increase in s.

The two effects of a change in s illustrated in figure V-1 can be stated in more formal mathematical terms through the development of the appropriate Hicks-Slutsky-type equation. This is stated as the following theorem.

Theorem V-1

Assume that the utility function U_1 is strictly concave. Holding the amounts of manufacturing labor, productive administrative labor, and investment fixed, when the minimum net income constraint in (5) is binding, the effect of a change in s on the firm's hiring of nonproductive administrative labor can be expressed in the form of a Hicks-Slutsky-type equation as the sum of two effects, one a substitution effect according to which managerial emoluments are substituted for nonproductive administrative labor when s is increased and the other an income effect.

<u>Proof</u>

The appropriate mathematical technique is that used in proving theorem III-4. The technique is applied to equations (13), (15) with $\mu_2 = 0$, and (4) expressed as an equality, with μ_1, M, and A_n each expressed as a function of s and π_o. This leads to the equation[26]

$$[H]\begin{pmatrix} \partial\mu_1/\partial s \\ \partial M/\partial s \\ \partial A_n/\partial s \end{pmatrix} = \begin{pmatrix} (1-\tau)[A_p + A_n] \\ 0 \\ (1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}] \end{pmatrix}, \tag{18}$$

where

$$H = \begin{bmatrix} 0 & -(1-\tau) & -s(1-\tau) \\ -(1-\tau) & (\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 + \frac{\partial^2 U_1}{\partial M^2})e^{-rt} & (\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 s)e^{-rt} \\ -s(1-\tau) & (\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 s)e^{-rt} & (\frac{\partial^2 U_1}{\partial D^2}(1-\tau)^2 s^2 + \frac{\partial^2 U_1}{\partial A_n^2})e^{-rt} \end{bmatrix} \tag{19}$$

From (19) it is straightforward to calculate

$$\det H = -(\frac{\partial^2 U_1}{\partial M^2} s^2(1-\tau)^2 + \frac{\partial^2 U_1}{\partial A_n^2}(1-\tau)^2)e^{-rt} > 0, \tag{20}$$

since $\frac{\partial^2 U_1}{\partial M^2} < 0$ and $\frac{\partial^2 U_1}{\partial A_n^2} < 0$ by the concavity assumption. Differentiating with respect to π_o as in the proof of theorem III-4 leads to the equation

$$[H]\begin{pmatrix}\partial\mu_1/\partial\pi_o\\ \partial M/\partial\pi_o\\ \partial A_n/\partial\pi_o\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad . \tag{21}$$

From (18) and (21) it follows that

$$\partial A_n/\partial s = (1-\tau)[A_p + A_n](\partial A_n/\partial\pi_o) + (1-\tau)[\frac{\partial U_1}{\partial D} + \mu_1 e^{rt}]H^{-1}{}_{33} , \tag{22}$$

where the first term on the right-hand side of (22) is the income effect and the second term is the substitution effect. It follows from (20) that $H^{-1}{}_{33} = -(1-\tau)^2/\det H < 0$, and hence, that the substitution effect is always negative. Q.E.D.

As before, the relationship embodied in the Hicks-Slutsky-type equation (22) can be expressed generically as:

$$\frac{\partial A_n}{\partial s} = (\frac{\partial A_n}{\partial\pi_o})_{s\equiv\text{constant}} + (\frac{\partial A_n}{\partial s})_{\pi_o\equiv\text{constant}} , \tag{23}$$

where the first term on the right-hand side of (23) represents the income effect and the second term represents the substitution effect.

4. Organizational Slack and the Business Cycle

The main result of this section is proved as the following theorem.

<u>Theorem V-2</u>

For the collective utility maximizer modeled in (5), investment, managerial emoluments, and organizational slack vary cyclically, i.e. increasing in the upswing and decreasing in the downswing. The firm's employment of manufacturing labor and productive administrative labor, and hence its output, vary cyclically also, provided $-\frac{R}{\partial U_1/\partial R}\cdot\frac{\partial(\partial U_1/\partial R)}{\partial R} < 1$ at the margin.

Proof

In the proof of theorem III-11 it was shown that $dI/d\theta > 0$ and $dM/d\theta > 0$, and in addition, that $dL/d\theta > 0$, provided $-\frac{R}{\partial U_1/\partial R}\cdot\frac{\partial(\partial U_1/\partial R)}{\partial R} < 1$ at the margin. The proof that $dA_n/d\theta > 0$ is perfectly analogous to the proof (using (15)) that $dM/d\theta > 0$, and the proof that $dA_p/d\theta > 0$, provided $-\frac{R}{\partial U_1/\partial R}\cdot\frac{\partial(\partial U_1/\partial R)}{\partial R} < 1$ at the margin, is perfectly analogous to the proof (using (11)) that $dL/d\theta > 0$, provided the same condition holds. Q.E.D.

The importance of theorem V-2 is twofold in nature. First, it links the phenomenon of organizational slack to the business cycle. During the upswing the profit constraint is more easily satisfied, and one result is that staffs expand and organizational slack develops (or worsens if it is already present). During the downswing the need to satisfy the profit constraint requires that organizational slack be reduced and leads to the sort of cost cutting and staff reductions reported by Williamson and widely reported in the business and financial press when the level of economic activity declines. Second, theorem V-2 extends theorem III-11 to reflect factors internal to the firm.

Thus far in this chapter the firm has been treated as having a single division organizational structure. The remainder of this chapter is concerned with firms having a multidivision organizational structure and with the allocation of the firm's productive resources among these divisions.

C. A MODEL OF THE MULTIDIVISION FIRM

1. Introduction

The purpose of this section is to develop a model of the multidivision firm and to use the model to derive rules for optimally allocating the firm's productive resources among the divisions. In this section it is assumed that all operating decisions, as well as all capital allocation decisions, are made at the headquarters level. The consequences of decentralized decision-making are explored in section D. The purpose of this subsection is to describe briefly the nature of the multidivision firm.

The organizational structure of a typical multidivision firm is illustrated below in figure V-2. At the top of the organizational pyramid is the headquarters, which includes the chairman of the board of directors, the president, one or more vice presidents with functional responsibilities (often the divisions are organized into groups, each of which is headed by a vice president), and supporting staff. This supporting staff typically includes legal staff, corporate planning staff, and financial staff (offices of the treasurer and controller, which in smaller organizations are usually combined into one office). Beneath the headquarters are the divisions, which are the principal operating units of the corporation and which may be viewed as quasi-firms.[27] This notion of a division as a quasi-firm is developed further below.

In the multidivision firm decision-making is typically decentralized. Many of the firm's operating decisions are

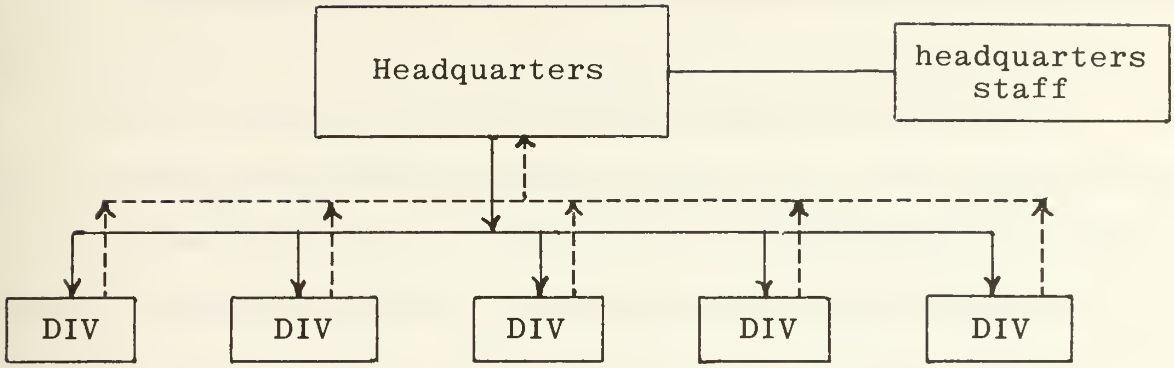

Figure V-2 Organizational Structure of the Multidivision Firm

made at the division level, though decisions concerning investment, new product development, and financial policy are typically made at the headquarters level. In figure V-2 the solid line from headquarters to the divisions indicates the allocation of capital and managerial talent (productive administrative labor) among the divisions and the dashed line indicates the flow of each division's net operating income to the headquarters level. A portion of these funds is used to pay headquarters expenses, to meet the firm's financial obligations (e.g. to pay bond interest), and to pay dividends, and the remainder is allocated among the divisions for investment purposes in accordance with the firm's objectives. Thus, there are two main internal allocation problems: one involving managerial talent (i.e. human capital) and the other involving investment resources (i.e. physical capital).[28] Subject to these allocation decisions by the headquarters, each division determines manufacturing labor and output levels, carries out production and marketing, and remits its net operating income to the headquarters. It is in this sense that each division acts as a quasi-firm.

In this section these quasi-firms are given a minimal degree of discretion. All significant operating decisions are made at the headquarters level. In the next section decision-making is permitted to become decentralized, and it is shown how such decentralization can lead to such problems as loss of internal control, lack of compliance with corporate objectives, and loss of X-efficiency. In developing these results, the model developed in this section will be used for comparative purposes.

In this section and the next the problems introduced by uncertainty, which were explored in chapters three and four, and the role of financial considerations, which was studied in chapter four, are abstracted from in order to deal exclusively with questions related to organizational structure, internal resource allocation, and corporate control. Since the presence of uncertainty was shown in chapters three and four to lead to complexities, but in most cases to results not fundamentally different from those obtained under certainty, and since the multidivision firm's financial operations are typically concentrated at the headquarters level, it is this writer's opinion that neither abstraction detracts seriously from the results obtained in the remainder of this chapter.

2. A Model of the Multidivision Firm

In this subsection it is assumed that all decision-making within the firm is centralized at the headquarters level. It is also assumed that the firm has I divisions, numbered $i = 1, \ldots, I$, each of which produces a single good. Associated with each division is a production function of the form

$$Q_i = f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) , \tag{24}$$

where K_i is the amount of real capital allocated to the division by the headquarters, L_i is the amount of manufacturing labor hired by the division, $A_{p,i}$ is the amount of productive administrative labor hired by the division,[29] and $A^i_{p,H}$ is the amount of headquarters management time (hereafter referred to as 'productive headquarters administrative labor') devoted to problems arising in the i-th division. As before, it is assumed that (24) has a full set of continuous second partial derivatives and that all its first derivatives are strictly positive for all input usage levels.

If the firm's total stock of fixed capital at time t is denoted by $K(t)$ and if its quantity of productive headquarters administrative labor at time t is denoted by $A_{p,H}(t)$, then it follows that

$$\left.\begin{aligned} &\sum_{i=1}^{I} K_i(t) \leq K(t) \\ &\sum_{i=1}^{I} A^i_{p,H}(t) \leq A_{p,H}(t) \end{aligned}\right\} \tag{25}$$

must hold at each time t, $0 \leq t \leq T$, where $K(t)$ is determined along with the optimal allocations, $K_i(t)$, and investment, $I(t)$, and where $A_{p,H}(t)$ is also determined along with the optimal allocations, $A^i_{p,H}(t)$, in the model set out below. While (25) are inequalities, it is shown below that when the firm is in multiperiod equilibrium, (25) will hold as equalities.

Employing (24) as the firm's production functions; adding the constraints (25); expressing total revenue as the sum of the amounts of revenue earned by the I divisions, $R(t) = \sum_{i=1}^{I} p_i(t)\cdot f_i(K_i(t), L_i(t), A_{p,i}(t), A^i_{p,H}(t))$; and redefining net income $\pi(t)$, and total dividends paid, $D(t)$, to reflect the productive administrative labor employed by the divisions, $A_{p,i}(t)$, and the amount of productive headquarters administrative labor employed by the firm, $A_{p,H}(t)$; leads to the following modification of (5) as the model of the centralized multidivision firm:

$$\begin{aligned}
\underset{\substack{\{K_i(t),L_i(t),\\ A_{p,i}(t),\\ A^i_{p,H}(t),A_{p,H}(t),\\ I(t),M(t),A_n(t)\}}}{\text{maximize}} \quad & \int_0^T U_1\Big[\sum_{i=1}^{I} p_i(t)\cdot f_i(K_i(t),L_i(t),A_{p,i}(t),A^i_{p,H}(t));\\
& (1-\tau)\{p_i(t)\cdot f_i(K_i(t),L_i(t),A_{p,i}(t),A^i_{p,H}(t))\\
& -w(t)\sum_{i=1}^{I} L_i(t) - s(t)[A_{p,H}(t) + A_n(t)\\
& + \sum_{i=1}^{I} A_{p,i}(t)] - M(t)\} + \tau\cdot q(t)[\delta\cdot K(t)]\\
& -q(t)\cdot I(t);\ M(t);\ A_n(t)\Big]e^{-rt}dt\\
& + U_2(K(T))e^{-rT} \qquad (26)
\end{aligned}$$

$$\begin{aligned}
\text{subject to} \quad & \dot{K}(t) = I(t) - \delta\cdot K(t),\ 0 \leq t \leq T,\ K(0) \text{ given}\\
& \sum_{i=1}^{I} K_i(t) \leq K(t),\ 0 \leq t \leq T\\
& \sum_{i=1}^{I} A^i_{p,H}(t) \leq A_{p,H}(t),\ 0 \leq t \leq T\\
& (1-\tau)\{\sum_{i=1}^{I} p_i(t)\cdot f_i(K_i(t),L_i(t),A_{p,i}(t),A^i_{p,H}(t))
\end{aligned}$$

$$-w(t)\cdot\sum_{i=1}^{I} L_i(t) - s(t)[A_{p,H}(t)+A_n(t)+\sum_{i=1}^{I} A_{p,i}(t)]$$

$$-M(t) - q(t)\cdot[\delta\cdot K(t)]\} \geq \pi_o,\ 0 \leq t \leq T$$

$$K_i(t), L_i(t), A_{p,i}(t), A^i_{p,H}(t) \geq 0,\ 0\leq t\leq T,\ i=1,\ldots,I$$

$$M(t), A_n(t) \geq 0,\ 0 \leq t \leq T$$

According to the model of the multidivision firm (26), the objective of the firm is to set allocations among the firm's operating divisions of capital, $K_i(t)$, manufacturing labor, $L_i(t)$, productive administrative labor, $A_{p,i}(t)$, and productive headquarters administrative labor, $A^i_{p,H}(t)$, and in addition, to determine levels of productive headquarters administrative labor, $A_{p,H}(t)$, investment, $I(t)$, managerial emoluments, $M(t)$, and nonproductive administrative labor, $A_n(t)$, at each time t, $0 \leq t \leq T$, that lead to maximum discounted collective utility over the firm's planning period, subject to four constraints (in addition to nonnegativity constraints): the familiar net investment constraint; a constraint requiring that the amount of capital allocated not exceed the amount available (at each time t); a constraint requiring that the amount of productive headquarters administrative labor allocated not exceed the amount available (at each time t); and the familiar minimum net income (i.e. profit) constraint. As before, it is assumed that all product and factor markets are perfectly competitive, so that $w(t)$, $s(t)$, $q(t)$, and $p_i(t)$, $i = 1, \ldots, I$, are treated as given by the firm at each time t. Also as before, it is assumed that capital markets are perfect, so that there is a unique rate of interest r at each time t, and further, that r as well as τ and δ

remain constant throughout the planning period. Also as before, U_1 and f_i, $i = 1, \ldots, I$, are assumed to be strictly concave with strictly positive first partial derivatives.

3. Optimal Internal Allocation Rules

The purpose of this subsection is to derive the rules for optimally allocating inputs implied by the model (26) formulated in the previous subsection. The Hamiltonian for (26) is

$$H[K, K_i, L_i, A_{p,i}, A^i_{p,H}, A_{p,H}, I, M, A_n, \lambda, t] = U_1[\ \ ]e^{-rt} + \lambda(t)[I(t) - \delta \cdot K(t)] \quad , \tag{27}$$

where $U_1[\ \ ]e^{-rt}$ denotes the integrand in (26). To find the necessary conditions for an optimal solution to (26), from which the desired allocation rules can be obtained, define the following generalized Lagrangian:

$$\begin{aligned}
&L_\mu[K,K_i,L_i,A_{p,i},A^i_{p,H},A_{p,H},I,M,A_n,\lambda,\mu_1,\mu_2,\mu_3,t] = \\
&\quad H[K,K_i,L_i,A_{p,i},A^i_{p,H},A_{p,H},I,M,A_n,\lambda,t] + \mu_1(t)[K(t) - \sum_{i=1}^{I} K_i(t)] \\
&\quad + \mu_2(t)[A_{p,H}(t) - \sum_{i=1}^{I} A^i_{p,H}(t)] + \mu_3(t)[(1-\tau)\{\sum_{i=1}^{I} p_i(t) \cdot \\
&\quad f_i(K_i(t),L_i(t)A_{p,i}(t),A^i_{p,H}(t)) - w(t) \cdot \sum_{i=1}^{I} L_i(t) - s(t)[A_{p,H}(t) \\
&\quad + A_n(t) + \sum_{i=1}^{I} A_{p,i}(t)] - M(t) - q(t) \cdot [\delta \cdot K(t)]\} - \pi_o] \ ,
\end{aligned} \tag{28}$$

where $H[\ \ ]$ is given by (27).

In order that the time paths $K^*_i(t)$, $L^*_i(t)$, $A^*_{p,i}(t)$, $A^{i*}_{p,H}(t)$, $A^*_{p,H}(t)$, $I^*(t)$, $M^*(t)$, and $A^*_n(t)$ provide an optimal solution to problem (26) it is necessary that they satisfy the following conditions:

$$\{K^*_i(t), L^*_i(t), A^*_{p,i}(t), A^{i*}_{p,H}(t), A^*_{p,H}(t), I^*(t), M^*(t), A^*_n(t)\}$$

$$\text{maximize } H[K, K_i, L_i, A_{p,i}, A^i_{p,H}, A_{p,H}, I, M, A_n, \lambda, t] \tag{29}$$

subject to the allocation constraints (25) and subject also to the minimum profit and nonnegativity constraints in (26), $0 \leq t \leq T$

$$\dot{K}(t) = I^*(t) - \delta \cdot K(t),\ 0 \leq t \leq T,\ K(0) \text{ given} \tag{30}$$

$$\dot{\lambda}^*(t) = -\frac{\partial L_\mu}{\partial K},\ \lambda^*(T) = \frac{\partial U_2(K(T))}{\partial K(T)} e^{-rT}. \tag{31}$$

Since, as before, (30) merely repeats the net investment constraint, only (29) and (31) need be considered explicitly below. Since the implications of M and A_n being zero at optimality have already been considered and since the variables K_i, L_i, $A_{p,i}$, and $A^i_{p,H}$ contribute directly to production, and hence would be expected to be strictly positive at optimality, it is assumed in what follows that each of the decision variables is strictly positive all along its optimal trajectory.

First considering (29), the necessary conditions are the following Kuhn-Tucker conditions:[30]

$$\frac{\partial L_\mu}{\partial K_i} = \{\frac{\partial U_1}{\partial R} p_i \frac{\partial f_i}{\partial K_i} + \frac{\partial U_1}{\partial D}(1-\tau) p_i \frac{\partial f_i}{\partial K_i}\} e^{-rt} - \mu_1 + \mu_3(1-\tau) p_i \frac{\partial f_i}{\partial K_i} = 0 \tag{32}$$

$$\frac{\partial L_\mu}{\partial L_i} = \{\frac{\partial U_1}{\partial R} p_i \frac{\partial f_i}{\partial L_i} + \frac{\partial U_1}{\partial D}(1-\tau)[p_i \frac{\partial f_i}{\partial L_i} - w]\} e^{-rt} + \mu_3(1-\tau)[p_i \frac{\partial f_i}{\partial L_i} - w] = 0 \tag{33}$$

$$\frac{\partial L_\mu}{\partial A_{p,i}} = \{\frac{\partial U_1}{\partial R} p_i \frac{\partial f_i}{\partial A_{p,i}} + \frac{\partial U_1}{\partial D}(1-\tau)[p_i \frac{\partial f_i}{\partial A_{p,i}} - s]\} e^{-rt} + \mu_3(1-\tau)[p_i \frac{\partial f_i}{\partial A_{p,i}} - s] = 0 \tag{34}$$

$$\frac{\partial L_\mu}{\partial A^i_{p,H}} = \{\frac{\partial U_1}{\partial R} p_i \frac{\partial f_i}{\partial A^i_{p,H}} + \frac{\partial U_1}{\partial D}(1-\tau)p_i \frac{\partial f_i}{\partial A^i_{p,H}}\}e^{-rt} - \mu_2 + \mu_3(1-\tau)p_i \frac{\partial f_i}{\partial A^i_{p,H}} = 0 \qquad (35)$$

$$\frac{\partial L_\mu}{\partial A_{p,H}} = \frac{\partial U_1}{\partial D}(1-\tau)(-s)e^{-rt} + \mu_2 + \mu_3(1-\tau)(-s) = 0 \qquad (36)$$

$$\frac{\partial L_\mu}{\partial I} = \frac{\partial U_1}{\partial D}(-q(t))e^{-rt} + \lambda(t) = 0 \qquad (37)$$

$$\frac{\partial L_\mu}{\partial M} = \{\frac{\partial U_1}{\partial D}(1-\tau)(-1) + \frac{\partial U_1}{\partial M}\}e^{-rt} + \mu_3(1-\tau)(-1) = 0 \qquad (38)$$

$$\frac{\partial L_\mu}{\partial A_n} = \{\frac{\partial U_1}{\partial D}(1-\tau)(-s) + \frac{\partial U_1}{\partial A_n}\}e^{-rt} + \mu_3(1-\tau)(-s) = 0 \qquad (39)$$

$$\sum_{i=1}^{I} K_i(t) \leq K(t) \qquad \mu_1 \geq 0 \qquad \mu_1[K(t) - \sum_{i=1}^{I} K_i(t)] = 0 \qquad (40)$$

$$\sum_{i=1}^{I} A^i_{p,H}(t) \leq A_{p,H}(t) \qquad \mu_2 \geq 0 \qquad \mu_2[A_{p,H}(t) - \sum_{i=1}^{I} A^i_{p,H}(t)] = 0 \qquad (41)$$

$$\left.\begin{aligned} &(1-\tau)\{\sum_{i=1}^{I} p_i \cdot f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) - w\sum_{i=1}^{I} L_i - s[A_{p,H} + A_n + \sum_{i=1}^{I} A^i_{p,H}] \\ &\qquad -M - q\cdot\delta\cdot K\} \geq \pi_o \\ &\qquad \mu_3 \geq 0 \\ &\mu_3[(1-\tau)\{\sum_{i=1}^{I} p_i \cdot f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) - w\sum_{i=1}^{I} L_i - s[A_{p,H} + A_n + \sum_{i=1}^{I} A^i_{p,H}] \\ &\qquad -M - q\cdot\delta\cdot K\} - \pi_o] = 0 \end{aligned}\right\} \qquad (42)$$

By comparing necessary conditions (32)-(39) for the model of the multidivision firm (26) with necessary conditions (11)-(15) for the model of the firm (5) having what Williamson calls the unitary form of organization,[31] it is immediately apparent that (33), (37), (38), (34), and (39) are virtually identical to (11)-(15), respectively. In particular, the manufacturing labor allocation rule (11) is, according to (33), applied to each division separately. Similarly, the productive administrative labor allocation rule (14) is, according to (34), applied to each division separately. This result is stated as the following lemma.

Lemma V-4

The multidivision firm modeled in (26) will employ the same rules for optimally allocating manufacturing labor and productive administrative labor to each division as the firm modeled in (5).

The significance of the lemma is twofold. First, lemma V-4 makes clearer the point made earlier that the divisions of a multidivision firm behave like quasi-firms. Second, the lemma leads immediately to the following theorem.

Theorem V-3

The multidivision firm modeled in (26) will allocate to (or, employ in) each of its divisions more manufacturing labor and more productive administrative labor, and hence, will produce more of each of its I goods, than a short run profit maximizer.

Next, consider the necessary conditions (32) and (35), which have no counterparts among (11)-(15). These conditions together with (40) and (41) determine the allocations of physical capital, K_i, and productive headquarters administrative labor, subject to the constraints (25). Rewrite (32) and (35) as

$$p_i\frac{\partial f_i}{\partial K_i}(1-\tau) + p_i\frac{\partial f_i}{\partial K_i}\left[\frac{(\partial U_1/\partial R) + (1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}\right] = \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} \tag{43}$$

and

$$p_i\frac{\partial f_i}{\partial A^i_{p,H}}(1-\tau) + p_i\frac{\partial f_i}{\partial A^i_{p,H}}\left[\frac{(\partial U_1/\partial R) + (1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}\right] = \frac{\mu_2 e^{rt}}{\partial U_1/\partial D}\ . \tag{44}$$

By analogy with equation (40) in chapter three, the left-hand side of (43) can be interpreted as the marginal value of an additional unit of capital allocated to the i-th division,[32] where this marginal value is figured net of tax and where the role of the expression within brackets in (43) is to measure the marginal impact on revenue and the profit constraint in terms of the marginal utility of dividends (i.e. in terms of a 'dividend equivalent'). Similarly, the left-hand side of (44) can be interpreted as the marginal value of an additional unit of productive headquarters administrative labor allocated to the i-th division. Recognizing that the right-hand sides of (43) and (44) are independent of i leads to the following theorem.

<u>Theorem V-4</u>

When the multidivision firm modeled in (26) is in multiperiod equilibrium, the marginal value of an additional unit of capital allocated to any division will be the same for all divisions and the marginal value of an additional unit of productive headquarters administrative labor allocated to any division will be the same for all divisions.

<u>Proof</u>

The first part of the statement follows directly from (43), which must hold for i = 1, ... , I. The second part of the statement follows directly from (44), which also must hold for i = 1, ... , I. Q.E.D.

<u>Remark</u>

The right-hand sides of (43) and (44) are actually marginal rates of substitution. For example, the ratio $\mu_1 e^{rt}/(\partial U_1/\partial D)$ can be interpreted as the marginal rate of substitution between dividends and a unit of capital allocated to any division. Alternatively, the ratio $\mu_1 e^{rt}/(\partial U_1/\partial D)$ can be interpreted as the marginal value of an additional unit of capital allocated to any division, expressed in terms of the marginal impact of such an allocation on total dividends (and therefore, indirectly on collective utility). In either case, dividing $\mu_1 e^{rt}$ by $\partial U_1/\partial D$ converts the nonmeasurable marginal utility expression for value into a ratio (i.e. a marginal rate of substitution) that is, in theory at least, measurable.

The significance of theorem V-4 is that it provides the rules for optimally allocating physical capital and productive headquarters administrative labor among the I divisions of a multidivision firm. In order for these allocations to be optimal it is necessary that the marginal value of an additional unit of each resource be the same in every alternative use for that resource within the firm. Moreover, (43) and (44) yield expressions for these common marginal values in terms of the Lagrange multipliers, or shadow prices, μ_1 and μ_2. The following corollary states that in equilibrium these marginal values must be strictly positive. A later theorem demonstrates the relationship between the internal price of capital, μ_1, and the firm's external cost of capital. Finally, the planning algorithm developed in section D will make special use of the shadow prices μ_1 and μ_2 (as well as of μ_3).

Corollary V-4-1

When the multidivision firm modeled in (26) is in equilibrium, the marginal value of capital $\mu_1 e^{rt}/(\partial U_1/\partial D)$ and the marginal value of productive headquarters administrative labor $\mu_2 e^{rt}/(\partial U_1/\partial D$ are both strictly positive.

Proof

Follows from (42)-(44) and the assumptions that $\partial f_i/\partial K_i > 0$, $\partial f_i/\partial A^i_{p,H} > 0$, $\partial U_1/\partial R > 0$, and $\partial U_1/\partial D > 0$.

Q.E.D.

Corollary V-4-1 leads immediately to the following result.

Corollary V-4-2

When the multidivision firm modeled in (26) is in equilibrium, the available amounts of physical capital and productive headquarters administrative labor at each time t will be fully allocated, i.e. the constraints (25) will hold as equalities.

Proof

It follows from corollary V-4-1 that $\mu_1 > 0$ and $\mu_2 > 0$ at optimality. Hence, from (40) and (41), the constraints (25) must hold as equalities when the firm modeled in (26) is in equilibrium. Q.E.D.

Next, consider (36), which also has no counterpart among (11)-(15). (36) leads immediately to the following theorem.

Theorem V-5

When the multidivision firm modeled in (26) is in equilibrium, it will hire productive headquarters administrative labor up to the point at which the marginal rate of substitution between dividends and productive headquarters administrative labor, $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D + \mu_3 e^{rt}}$, just equals the salary level net of tax, $(1-\tau)s$.

Proof

Rearranging terms in (36) yields

$$\frac{\mu_2 e^{rt}}{\partial U_1/\partial D + \mu_3 e^{rt}} = (1 - \tau)s \ , \qquad (45)$$

which is the desired result. Q.E.D.

The left-hand side of (45) represents the firm's subjective rate of trade off between dividends and productive headquarters administrative labor, while the right-hand side represents the externally imposed rate at which these quantities can be traded off for one another. Thus, according to theorem V-5, the firm will hire productive headquarters administrative labor up to the point at which its internal rate of trade off, $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D + \mu_3 e^{rt}}$, just equals the externally imposed rate, $(1-\tau)s$, at which dividends and productive headquarters administrative labor can be traded off within the firm's income statement. This result is, of course, perfectly analogous to results obtained earlier in chapter three and in section B of this chapter. The next corollary also follows from (36).

Corollary V-5-1

When the multidivision firm modeled in (26) is in equilibrium, the marginal value of an additional unit of productive headquarters administrative labor allocated to any division, $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$, is equal to the after tax marginal cost of an additional unit of productive headquarters administrative labor, $(1-\tau)s\{1 + \frac{\mu_3 e^{rt}}{\partial U_1/\partial D}\}$.

Proof

Rearranging terms in (36) yields

$$\frac{\mu_2 e^{rt}}{\partial U_1/\partial D} = (1 - \tau)s\{1 + \frac{\mu_3 e^{rt}}{\partial U_1/\partial D}\}, \tag{46}$$

which is the desired result. Q.E.D.

Remark

Note that if the profit constraint were not binding, then $\mu_3 = 0$ and the right-hand side of (46) becomes $(1 - \tau)s$, the marginal cost of a unit of productive headquarters administrative labor. Thus, (46) is the familiar marginal value equals marginal cost optimality criterion. Moreover, (46) makes clear that when the profit constraint is binding, the marginal cost of a unit of productive headquarters administrative labor must be adjusted upward to reflect the impact on net income of the hiring of an additional unit of this type of labor, where $(1 - \tau)s\frac{\mu_3 e^{rt}}{\partial U_1/\partial D} > 0$ is the needed adjustment.

Together (35) and (36) lead to the following corollary.

Corollary V-5-2

When the multidivision firm modeled in (26) is in equilibrium, it will hire more productive headquarters administrative labor than a short run profit maximizer.

Proof

Solving (35) and (36) for $\mu_2 e^{rt}$, equating the resulting expressions, and rearranging terms gives

$$\frac{\partial U_1}{\partial R} p_i \frac{\partial f_i}{\partial A^i_{p,H}} + \left(\frac{\partial U_1}{\partial D} + \mu_3 e^{rt}\right)(1-\tau)\left(p_i \frac{\partial f_i}{\partial A^i_{p,H}} - s\right) = 0 \ . \qquad (47)$$

Since $\partial f_i / \partial A^i_{p,H} > 0$, $\partial U_1/\partial R > 0$, and $\partial U_1/\partial D > 0$ by assumption, (47) implies that

$$p_i \frac{\partial f_i}{\partial A^i_{p,H}} < s \ . \qquad (48)$$

The desired result follows from (48) by the assumed strict concavity of f_i .[33] Q.E.D.

The foregoing has been concerned with characterizing the solution to (29). Turning to (31), evaluating the derivative $-\ \partial L_\mu/\partial K$ gives

$$\dot{\lambda} = -\frac{\partial L_\mu}{\partial K} = -\left[\frac{\partial U_1}{\partial D}(\tau q \delta) - \lambda\delta + \mu_1 + \mu_3(1-\tau)(-q\delta)\right]$$

$$= -\left[\frac{\partial U_1}{\partial D} q\delta(-(1-\tau))e^{-rt} + \mu_1 + \mu_3(1-\tau)(-q\delta)\right] \ . \qquad (49)$$

Solving (37) for $\lambda(t)$ and differentiating gives

$$\dot{\lambda} = \frac{\partial U_1}{\partial D}(-\ rq + \dot{q})e^{-rt} \ . \qquad (50)$$

Equating (49) and (50) and solving the resulting expression for $\mu_1 e^{rt}/(\partial U_1/\partial D)$ leads to the following important theorem.

Theorem V-6

When the multidivision firm modeled in (26) is in equilibrium, the marginal value of an additional unit of physical capital allocated to any division is the same for all divisions and is equal to the firm's marginal cost of physical capital.

Proof

The first part of the theorem is merely a restatement of the first part of theorem V-4. To prove the second part of the statement, equate (49) and (50) and solve for $\mu_1 e^{rt}/(\partial U_1/\partial D)$. This procedure yields

$$\frac{\mu_1 e^{rt}}{\partial U_1/\partial D} = q[r + (1-\tau)\delta\{1 + \frac{\mu_3 e^{rt}}{\partial U_1/\partial D}\}] - \dot{q} \ , \qquad (51)$$

where the right-hand side is interpreted as the firm's marginal cost of physical capital and is identically equal to $i(\equiv rq + (1-\tau)q\delta - \dot{q})$ when the profit constraint is not binding (i.e. $\mu_3 = 0$). Q.E.D.

Remark

As (51) makes clear, the firm's marginal cost of capital must be adjusted for the impact the purchase of an additional unit of capital will have on the current period's net income (and indirectly, the impact of such a purchase on discounted collective utility). The adjustment term in (51) is $(1-\tau)q\delta \frac{\mu_3 e^{rt}}{\partial U_1/\partial D}$. Similarly, the marginal value expression (43) contains the adjustment term $p_i\frac{\partial f_i}{\partial K_i}(1-\tau)\frac{\mu_3 e^{rt}}{\partial U_1/\partial D}$. When the profit constraint is not binding, then $\mu_3 = 0$ and (43) and (51) can be combined to yield equation (40) of chapter three (with K_i in place of K).

Theorem V-6 leads to the following corollaries.

Corollary V-6-1

When the multidivision firm modeled in (26) is in equilibrium, it will allocate more physical capital to each division than a short run profit maximizer would.

Proof

Follows directly from (43) and (51), the assumed concavity of each f_i, and the fact that

$$p_i \frac{\partial f_i}{\partial K_i}\left[\frac{(\partial U_1/\partial R) + (1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}\right] > 0 \ . \qquad \text{Q.E.D.}$$

Corollary V-6-2

When the multidivision firm modeled in (26) is in equilibrium, it will allocate more of each input to each division, and hence produce more of each output, than a short run profit maximizer.

Proof

Theorem V-3, corollary V-5-2, and corollary V-6-1.

Q.E.D.

The significance of theorem V-6 is that it extends the necessary condition for optimal investment in physical capital ((40) in chapter three) to the multidivision firm. That is, the marginal value of an additional unit of physical capital for each division must equal the firm's marginal cost of physical capital. Moreover, when the firm's profit constraint is binding, this marginal cost of capital must be adjusted upward to reflect the impact on the firm's net income of enlarging the firm's capital stock. In the planning algorithm developed in the next section the shadow price μ_3 will play a vital role along with μ_1 and μ_2 in achieving the optimal allocation of the firm's productive resources.

4. Section Summary

In this section the model of the single product firm formulated in chapter three was extended. A model of the multidivision firm (26) was developed and rules for optimally allocating inputs among the firm's divisions were derived under the assumption that decision-making within the firm was centralized at the headquarters level.

The main results obtained in this section are the following. The multidivision firm modeled in (26) will in equilibrium allocate more of each input to each division, and hence produce more of each output, than a short run profit maximizer (theorem V-3, corollary V-5-2, and corollary V-6-1, stated collectively as corollary V-6-2). In equilibrium for the multidivision firm modeled in (26), the marginal value of an additional unit of physical capital will be the same for all divisions, and similarly for the marginal value of an additional unit of productive headquarters administrative labor (theorem V-4). In addition, this common marginal value of physical capital is equal to the firm's marginal cost of physical capital when the multidivision firm modeled in (26) is in equilibrium (theorem V-6). In equilibrium the multidivision firm modeled in (26) will hire productive headquarters administrative labor up to the point at which its subjective rate of trading off that type of labor for dividends just equals the externally imposed rate at which those quantities can be traded off for one another within the firm's income statement (theorem V-5).

In addition, it was shown that the Lagrange multipliers, or shadow prices, μ_1, μ_2, and μ_3, play important roles in the resource allocation process. These roles will become more crucial in the next section where decentralized decision-making is permitted and where a planning algorithm based on the necessary conditions (29)-(42) is developed for the decentralized multidivision firm.

D. DECENTRALIZATION IN THE MULTIDIVISION FIRM

1. Introduction

In the previous section it was assumed that decision-making within the multidivision firm was centralized at the headquarters level. In this section decision-making is largely decentralized. Specifically, each division decides how much manufacturing labor, L_i, and how much productive administrative labor, $A_{p,i}$, to hire, and it is assumed that each division hires these amounts directly via the external (to the firm) markets for these types of labor. Each division also decides how much physical capital, K_i, and how much productive headquarters administrative labor, $A^i_{p,H}$, to 'hire', but these amounts must be 'hired' at prices established by the headquarters, which decides how much physical capital and how much productive headquarters administrative labor the firm as a whole will hire.

The next subsection develops a planning algorithm the multidivision firm modeled in (26) could use to ensure optimal input and output decisions by the divisions. The planning algorithm makes important use of the shadow prices μ_1, μ_2, and μ_3 discussed in the previous section. In applying the algorithm, the headquarters would act as a central planning board and would effect changes in input allocations by changing the implicit prices μ_1, μ_2, and μ_3. In the following subsection it is shown that, in general, such a planning algorithm could work only if the headquarters is able to control the divisions, i.e. to force them to establish hiring practices consistent with (29)-(42). In particular, it is shown that if division managers are free to maximize their own utility, then resource allocations that are suboptimal with regard to collective utility are likely to result.

2. A Multiperiod Finite Horizon Planning Algorithm for the Multidivision Firm

The planning algorithm developed in this subsection for the multidivision firm modeled in (26) establishes a price-guided planning procedure.[34] The planning process, as modeled in this subsection, is iterative. This contrasts with the treatment of planning in chapters three and four and earlier in this chapter, in all of which the planning problem was formulated as an optimal control problem, with the characterization of the necessary conditions for an optimal solution to the optimal control problem constituting the rules for optimally allocating productive resources throughout the planning period. Solving the planning problem in this manner presupposes that the firm's planners have complete knowledge of U_1, U_2, and f. In the case of the multidivision firm, the firm's planners would need to determine the production function f_i for each division.

The value of the decentralized planning procedure set out in this subsection - indeed, the value of decentralized planning procedures in general - is the reduction in the amount of information needed by the headquarters of the firm (i.e. the 'central planning board') in carrying out planning.[35] The reduction in informational requirements is made possible by the (correct) use of the information conveyed by the shadow prices μ_1, μ_2, and μ_3. In the planning algorithm set out below the divisions will not need to know U_1, but rather will only need to know the marginal utilities $\partial U_1/\partial R$ and $\partial U_1/\partial D$ and the shadow prices μ_1, μ_2, and μ_3. The headquarters will not need to know the individual production functions f_i, but rather, will only need to know each division's input decisions, K_i, L_i, $A_{p,i}$, and $A^i_{p,H}$, and the division's resulting total revenue R_i.

While the reduced informational requirements is the main advantage of the planning algorithm set out below, the use of such a decentralized, iterative scheme also has important disadvantages. First, because the headquarters lacks complete information concerning each division's production function, internal corporate control can become a problem. As discussed in subsection 3 of this section, division managers may be motivated to pursue their own objectives - in much the same way that corporate managers within headquarters are motivated to pursue their own objectives (at least to some extent) at the expense of shareholders' objectives. As a result, resource allocations may not be optimal with regard to collective utility, and the firm may suffer a loss of X-efficiency.

A second disadvantage concerns the difficulty of establishing equilibrium, feasibility, and convergence properties for planning algorithms.[36] In general, convergence proofs appear to require conditions no weaker than strict concavity of all utility and production functions.[37] Even then, however, exact convergence often cannot be assured.[38] In addition, even when convergence can be proved, in practical applications there remains the problem of determining how many iterations of the planning cycle to have, since iterations are time-consuming and costly and planners typically face both time and budget constraints.[39] Since these more practical considerations normally arise when two or more algorithms for obtaining a solution are compared, and since this subsection is concerned with the development of a single algorithm consistent with the characterization of an optimal solution that was obtained in section C, they are not discussed further here. However, after the planning algorithm has been presented the issues of the existence of an equilibrium plan, the feasibility of intermediate plans, and the convergence of the algorithm are addressed.

The remainder of this subsection is concerned with developing a multiperiod finite horizon planning algorithm for the multidivision firm modeled in (26). The hierarchical levels at which decisions are assumed to be made are shown in table V-1. Each division makes the decisions shown on the left-hand side of table V-1, while headquarters makes the decisions shown on the right-hand side of the table. The purpose of the planning algorithm is to ensure that the decisions are consistent with (29)-(42).

Table V-1 Decisions Made at the Division Level and Decisions Made at the Headquarters Level

Division Level Decisions		Headquarters Level Decisions	
capital input	K_i	prod. HQ admin. labor	$A_{p,H}$
manuf. labor input	L_i	investment	I
admin. labor input	$A_{p,i}$	capital stock	K
prod. HQ admin. labor input	$A^i_{p,H}$	managerial emoluments	M
		nonprod. admin. labor	A_n

The planning process, as described by the planning algorithm set out below, is iterative. A representative cycle of the planning process is shown in figure V-3.

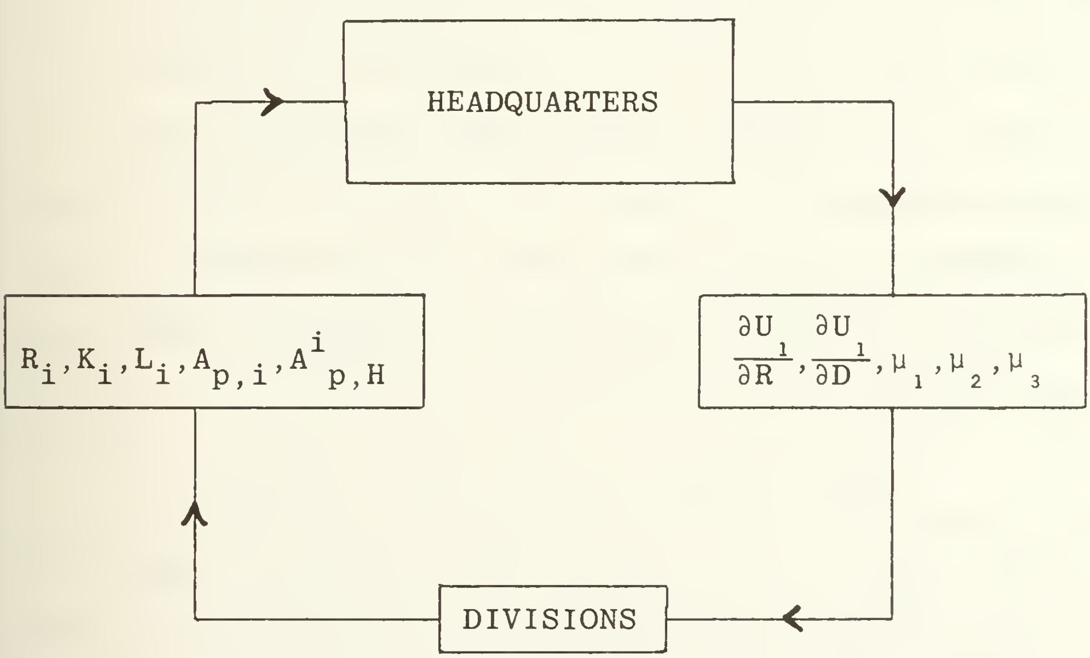

Figure V-3 A Representative Cycle of the Planning Process

The arrows in figure V-3 indicate the direction of the informational flows. As indicated below, the first cycle of the planning process is atypical and additional information, such as the discount rate r, must be furnished the divisions.

Figure V-3 indicates the information transmitted to the divisions from headquarters and the information transmitted to headquarters from the divisions. First, consider the information transmitted to the divisions by headquarters. The headquarters planning staff acts as a central planning board, announcing the revised marginal utilities $\partial U_1/\partial R$ and $\partial U_1/\partial D$ and the revised shadow prices μ_1, μ_2, and μ_3 at each cycle. The divisions, having been told the discount rate r and the tax rate τ in the initial cycle, use the information received from headquarters and (32)-(35) to compute new input usage levels, K_i, L_i, $A_{p,i}$, and $A^i{}_{p,H}$. Then these new input usage levels and (24) can be used to compute the new output level, Q_i. Given price, p_i, total revenue is simply $R_i = p_i Q_i$. It should be emphasized that this is done simultaneously for all t, $0 \leq t \leq T$. Then the division's (revised) proposed time paths of input usage, $K_i(t)$, $L_i(t)$, $A_{p,i}(t)$, and $A^i{}_{p,H}(t)$, and its proposed time path of total revenue (also revised), $R_i(t)$, are sent to headquarters. It also needs to be emphasized that no transactions take place on the basis of this information. Transactions take place only after the iterations of the planning cycle have been completed and the final plan has been approved by headquarters.

In deciding how much of each input to use each division uses the market prices of manufacturing labor, w, and of administrative labor, s, and also uses the implicit prices of physical capital, μ_1, and of productive headquarters administrative labor, μ_2, which are supplied by headquarters. In addition, the implicit price of diminishing net income, μ_3, enters the calculations when the profit constraint is binding. When the profit constraint is not binding (or more correctly, when it was not found to be binding as a result of the previous iteration), then $\mu_3 = 0$ can be transmitted to the divisions, and each division's decisions become, in effect, unconstrained by net income considerations.

Next, consider the information transmitted to headquarters from the divisions and how this information can be used to revise the shadow prices μ_1, μ_2, and μ_3. Note that equation (51), which was obtained from (31) and (37), implies that at optimality

$$\mu_1 = (1 - \tau)q\delta\mu_3 + i(\partial U_1/\partial D)e^{-rt}, \tag{52}$$

where $i \equiv rq + (1 - \tau)q\delta - \dot{q}$. Thus, (52) expresses μ_1 in terms of μ_3 and $\partial U_1/\partial D$ - all other variables in (52) are exogenously determined. Similarly, (36) implies that at optimality

$$\mu_2 = (1 - \tau)s\mu_3 + (1 - \tau)s(\partial U_1/\partial D)e^{-rt}, \tag{53}$$

where only $\partial U_1/\partial D$ (in addition to μ_2 and μ_3) is variable. Thus, (52) and (53) can be used to alter μ_1 and μ_2 in accordance with changes in μ_3 and $\partial U_1/\partial D$. It follows from corollary V-4-2 that at optimality the constraints (25) will hold as equalities, so that at each iteration of the planning process the headquarters planning staff can compute $K(t)$ and $A_{p,H}(t)$ according to

$$\left.\begin{aligned} K(t) &= \sum_{i=1}^{I} K_i(t) \\ A_{p,H}(t) &= \sum_{i=1}^{I} A^i_{p,H}(t) \end{aligned}\right\} \tag{54}$$

That is, corollary V-4-2 makes separate adjustments to (52) and (53) on account of (25) unnecessary.

It remains to be shown how μ_3 is to be adjusted and how $\partial U_1/\partial R$ and $\partial U_1/\partial D$ change. First, using the same values for $\partial U_1/\partial D$ and μ_3 that were sent to the divisions, the headquarters planning staff uses (38) to compute the proposed level of managerial emoluments, M, and uses (39) to compute the proposed level of nonproductive administrative labor, A_n. Then, given $R_i(t)$, $K_i(t)$, $L_i(t)$, $A_{p,i}(t)$, and $A^i_{p,H}(t)$, $0 \leq t \leq T$, $i = 1, \ldots, I$, transmitted by the divisions, it is a straightforward exercise for the headquarters planning staff to use (54) to compute $K(t)$ and $A_{p,H}(t)$ and then to use the following identities to determine $\pi(t)$ and $D(t)$:

$$\begin{aligned} \pi(t) = (1-\tau)\{ &\sum_{i=1}^{I} R_i(t) - w(t)\sum_{i=1}^{I} L_i(t) - s(t)[A_{p,H}(t) + A_n(t) \\ &+ \sum_{i=1}^{I} A_{p,i}(t)] - M(t) - q(t)[\delta\cdot K(t)]\} \end{aligned} \tag{5}$$

$$\begin{aligned} D(t) = (1-\tau)\{ &\sum_{i=1}^{I} R_i(t) - w(t)\sum_{i=1}^{I} L_i(t) - s(t)[A_{p,H}(t) + A_n(t) \\ &+ \sum_{i=1}^{I} A_{p,i}(t)] - M(t)\} + \tau\cdot q(t)[\delta\cdot K(t)] - q(t)\cdot I(t) \ , \end{aligned} \tag{5}$$

where $I(t)$ is computed using $K(t)$, $1 \leq t \leq T$, and the net investment constraint $\dot{K}(t) = I(t) - \delta\cdot K(t)$, i.e. by using $I(t) = \dot{K}(t) + \delta\cdot K(t)$.[40]

Following the above computations, U_1 for each t, $0 \le t \le T$, can be found by substitution. Also, values for $\partial U_1/\partial R$ and $\partial U_1/\partial D$ can be found by substitution into general expressions for these derivatives (computed from U_1). Lastly, an appropriate change in μ_3 needs to be determined. Let ρ denote a positive constant. Then one scheme for adjusting μ_3 is the following:

$$\mu_3(t)^{k+1} = \begin{cases} \mu_3(t)^k + \rho(\pi_0 - \pi(t)), & \text{if } \pi(t) \le \pi_0 \\ \\ 0, & \text{if } \pi(t) > \pi_0 \end{cases} \tag{57}$$

where k denotes the iteration number and where (57) is used for each time t, $0 \le t \le T$. According to (57), when the profit constraint is violated, i.e. $\pi(t) < \pi_0$, the value of μ_3 - the penalty for violating the constraint - is increased. When the constraint is satisfied as an equality, the value of μ_3 is unchanged. When the profit constraint is satisfied as a strict inequality, the value of μ_3 is reduced to zero since in that case the firm's choice of operating policies is unaffected by a marginal increase in π_0 (or marginal decrease in $\pi(t)$). While the choice of ρ is a matter of some concern since it affects the convergence properties of the algorithm, it is not, in general, possible to specify exactly how ρ should be chosen.[41]

For the convenience of the reader, the entire planning algorithm is set out in table V-2. The central portions of the algorithm have already been discussed, but the initial cycle and the stopping rule have not. During the initial cycle the optimality criteria (32)-(35) and values for the discount rate r and the tax rate τ, all of which are used in each cycle, must be communicated to each division. In the simplest case the rules and the values for r and τ are unchanged from the previous year, and as long as this information was stored by each division, the divisions need only be informed that the optimality criteria, r, and τ are unchanged from the previous year.

To begin the planning process it is also necessary that the headquarters planning staff select initial values for $\partial U_1/\partial R$, $\partial U_1/\partial D$, μ_1, μ_2, and μ_3 for each time t. One of the simplest procedures is first, to use the values for years 1 through T of the previous year's final plan as initial values for the same variables for years 0 through T-1 of the current year's plan, e.g. use $\mu_1^*(t)$ from the previous year's final plan as the initial value for $\mu_1(t-1)$, $1 \leq t \leq T$, in the current year's plan, and second, to set the initial values of each of the five variables for the period $T-1 < t \leq T$ in the current year's plan equal to the variable's initial value at time T-1, e.g. $\mu_1(t) = \mu_1(T-1)$, $T-1 < t \leq T$. No special virtues are claimed for this procedure; it is merely one method of initializing the values of the five variables to be transmitted to the divisions. The procedure seems reasonable to this writer since it does take advantage of the overlapping periods covered by plans made in adjacent years, and also since one would hope for a reasonable degree of consistency of plans over time.

Table V-2 The Planning Algorithm

HQ ≡ headquarters D ≡ divisions, $i = 1, \ldots, I$

Step I. [Initialize] HQ sets $k = 1$ (k is a counter for the number of iterations) and also sets $V^0 = 0$. HQ selects ε and K to be used in step IV. HQ selects initial values for $\partial U_1/\partial R$, $\partial U_1/\partial D$, μ_1, μ_2, and μ_3, and informs D of these values. HQ also transmits the optimality criteria (32)-(35) and values for r and τ to D.

Step II. [D Computations] D use (32)-(35), r, τ, $\partial U_1/\partial R$, $\partial U_1/\partial D$, μ_1, μ_2, and μ_3 to compute proposed allocations K_i, L_i, $A_{p,i}$, and $A^i_{p,H}$. From these D compute R_i, and R_i, K_i, L_i, $A_{p,i}$, and $A^i_{p,H}$ are transmitted to HQ.

Step III. [HQ Computations] HQ uses K_i and $A^i_{p,H}$ to compute K and $A_{p,H}$. HQ also uses (38) and (39) to compute proposed M and A_n and (55) and (56) to compute $\pi(t)$ and $D(t)$. Then U_1 is computed for each t, $0 \leq t \leq T$, and $V^k = \int_0^T U_1[\ ]e^{-rt}dt + U_2(\ )e^{-rT}$ is also computed.

Step IV. [Stopping Rule] HQ compares V^k and V^{k-1}. If $|V^k - V^{k-1}| < \varepsilon$ and $\pi(t) \geq \pi_0$, then stop. Otherwise continue. HQ increments k by 1. If $k > K$, then stop. Otherwise continue.

Step V. [Revise Prices] HQ computes new $\partial U_1/\partial R$ and $\partial U_1/\partial D$. HQ uses (57) to revise μ_3 and uses (52) and (53) to revise μ_1 and μ_2. HQ transmits revised $\partial U_1/\partial R$, $\partial U_1/\partial D$, μ_1, μ_2, and μ_3 to D. GO TO STEP II.

The stopping rule, or rule for terminating the algorithm, is also easy to explain. Let V^k denote the value of the objective functional in (26) computed by the headquarters planning staff at the k-th iteration. According to the stopping rule given in table V-2, the search for a 'better' (in the sense of satisfying the constraints in (26) and at the same time yielding a greater value of discounted collective utility) ceases either if the value of the objective functional changes by less than some predetermined amount ε during any iteration or if some predetermined maximum total number of iterations, K, have been performed. The latter condition is needed due to the possibility that, unless certain conditions that are discussed below are met, the algorithm may fail to converge.

The algorithm having been presented, the remainder of this subsection deals with three questions: the existence of an equilibrium plan, the attainment of feasible plans, and the convergence of the planning algorithm. The existence of an equilibrium plan is assured by the following theorem.

<u>Theorem V-7</u>

If there exists an optimal solution to (26) - i.e. an optimal plan covering the period $0 \leq t \leq T$ - then it serves as an equilibrium solution for the planning algorithm in table V-2.

<u>Proof</u>

The optimal plan must satisfy (29)-(42). For values of the decision variables satisfying (29)-(42), it follows that $K_i^{k+1} = K_i^k$, $K^{k+1} = K^k$, $L_i^{k+1} = L_i^k$, etc., and hence that $V^{k+1} = V^k$. Hence, such a set of values for the decision variables constitutes an equilibrium plan. Q.E.D.

It should be noted that no claim can be made as to the uniqueness of the equilibrium plan. In general, uniqueness of the equilibrium plan requires uniqueness of the optimal solution to (26), which has not been assumed here.

The second question concerns feasibility. A desirable quality of planning algorithms is that they produce feasible plans during intermediate cycles, so that if the predetermined maximum number of steps has been reached (say due to slow convergence, or possibly even to failure to converge), the plan produced just prior to termination will at least be feasible. In connection with problem (26), the main concern is the minimum net income constraint, $\pi(t) \geq \pi_O$. Since the shadow price μ_3 constitutes a penalty for violating the profit constraint, one would expect that increasing μ_3 would eventually lead to a feasible plan, provided one exists. This is indeed the case, as proved in the next theorem.

<u>Theorem V-8</u>

Assume that U_1 and f_i, $i = 1, \ldots, I$, are strictly concave. If there exists a feasible solution to (26), and if at any iteration of the planning cycle $\pi(t) < \pi_O$, then in subsequent iterations $\pi(t)$ increases monotonically until feasibility has been achieved.[42]

<u>Proof</u>

It follows from (57) that μ_3 is strictly increasing, i.e. $\mu_3^{k+1} > \mu_3^{k}$, when $\pi(t) < \pi_O$. Hence, the theorem will be proved if it can be shown that $d\pi/d\mu_3 > 0$.

It follows from (55) that

$$\frac{d\pi}{d\mu_3} = (1-\tau)\{\sum_{i=1}^{I}(p_i\frac{\partial f_i}{\partial K_i} - q\delta)\frac{dK_i}{d\mu_3} + \sum_{i=1}^{I}(p_i\frac{\partial f_i}{\partial L_i} - w)\frac{dL_i}{d\mu_3}$$

$$+ \sum_{i=1}^{I}(p_i\frac{\partial f_i}{\partial A_{p,i}} - s)\frac{dA_{p,i}}{d\mu_3} + \sum_{i=1}^{I}(p_i\frac{\partial f_i}{\partial A^i_{p,H}} - s)\frac{dA^i_{p,H}}{d\mu_3}$$

$$- \frac{dM}{d\mu_3} - s\frac{dA_n}{d\mu_3} .$$

Given the concavity of U_1 and f_i, $i = 1, \ldots, I$:

$(p_i\frac{\partial f_i}{\partial K_i} - q\delta)\frac{dK_i}{d\mu_3} > 0$, $i = 1, \ldots, I$, follows from (52) and (32), which is used at step II

$(p_i\frac{\partial f_i}{\partial L_i} - w)\frac{dL_i}{d\mu_3} > 0$, $i = 1, \ldots, I$, follows from (33)

$(p_i\frac{\partial f_i}{\partial A_{p,i}} - s)\frac{dA_{p,i}}{d\mu_3} > 0$, $i = 1, \ldots, I$, follows from (34)

$(p_i\frac{\partial f_i}{\partial A^i_{p,H}} - s)\frac{dA^i_{p,H}}{d\mu_3} > 0$, $i = 1, \ldots, I$, follows from (53) and (35)

$\frac{dM}{d\mu_3} < 0$ follows from (38), which is used at step III

$\frac{dA_n}{d\mu_3} < 0$ follows from (39)

Thus, each of the terms in (58) is strictly positive, and therefore so is $d\pi/d\mu_3$. Q.E.D.

Theorem V-8 is significant not only because it demonstrates that the planning algorithm in table V-2 revises infeasible plans in such a way that net income increases (so that in this sense the revised plan is 'nearer to' feasibility than the previous plan), but also because it suggests how the nonexistence of any feasible plan may be detected. That is, if μ_3 becomes unboundedly large, then this might indicate the lack of any feasible plan (it may also indicate that for some time t, $0 \leq t \leq T$, the maximum attainable level of net income is equal to or marginally less than π_o). Specifically, theorem V-8 leads to the following result.

Corollary V-8-1

If no feasible plan exists, then for some t, $0 \leq t \leq T$, $\mu_3(t)^k \to \infty$ as k becomes large. Moreover, $\pi(t)^k \to \pi(t)^*$ monotonically, where $\pi(t)^*$ denotes the maximum attainable level of net income at time t (and $\pi(t)^* < \pi_o$).

The third question concerning the planning algorithm in table V-2 involves convergence. In general, convergence properties are very difficult to assess. Even in single period planning problems, the strict concavity of the objective function, plus additional concavity/convexity assumptions needed to ensure that the planning problem is a concave programming problem, must be supplemented by additional assumptions, e.g. concerning the size of the adjustment parameter, before even convergence to within some region of proximity to the optimal plan can be proved.[43] In spite of this difficulty, a somewhat weaker stability result can be proved.

Theorem V-9

If U_1 and f_i, i = 1, ... , I, are strictly concave, then for any feasible solution to (26), the levels of total revenue, total dividends paid, managerial emoluments, and nonproductive administrative labor will vary directly with regard to changes in their respective marginal utilities, i.e. $dR/d(\partial U_1/\partial R) > 0$, $dD/d(\partial U_1/\partial D) > 0$, $dM/d(\partial U_1/\partial M) > 0$, and $dA_n/d(\partial U_1/\partial A_n) > 0$.

<u>Proof</u>

Since $R(t) = \sum_{i=1}^{I} R_i(t)$ it follows that

$$\frac{dR}{d(\partial U_1/\partial R)} = \sum_{i=1}^{I} p_i \frac{\partial f_i}{\partial K_i} \frac{dK_i}{d(\partial U_1/\partial R)} + \sum_{i=1}^{I} p_i \frac{\partial f_i}{\partial L_i} \frac{dL_i}{d(\partial U_1/\partial R)}$$

$$+ \sum_{i=1}^{I} p_i \frac{\partial f_i}{\partial A_{p,i}} \frac{dA_{p,i}}{d(\partial U_1/\partial R)} \tag{59}$$

$$+ \sum_{i=1}^{I} p_i \frac{\partial f_i}{\partial A^i_{p,H}} \frac{dA^i_{p,H}}{d(\partial U_1/\partial R)} .$$

By the strict concavity of U_1 and f_i, $i = 1, \ldots, I$, it follows that $\frac{dK_i}{d(\partial U_1/\partial R)} > 0$ by (32), $\frac{dL_i}{d(\partial U_1/\partial R)} > 0$ by (33), $\frac{dA_{p,i}}{d(\partial U_1/\partial R)} > 0$ by (34), and $\frac{dA^i_{p,H}}{d(\partial U_1/\partial R)} > 0$ by (35). Since price and the marginal productivities are strictly positive by assumption, each term in (59) is strictly positive, and therefore so is $\frac{dR}{d(\partial U_1/\partial R)}$.

From (54) and (56),

$$\frac{dD}{d(\partial U_1/\partial D)} = (1-\tau)\{ \sum_{i=1}^{I} (p_i \frac{\partial f_i}{\partial L_i} - w) \frac{dL_i}{d(\partial U_1/\partial D)}$$

$$+ \sum_{i=1}^{I} (p_i \frac{\partial f_i}{\partial A_{p,i}} - s) \frac{dA_{p,i}}{d(\partial U_1/\partial D)}$$

$$+ \sum_{i=1}^{I} (p_i \frac{\partial f_i}{\partial A^i_{p,H}} - s) \frac{dA^i_{p,H}}{d(\partial U_1/\partial D)} \tag{60}$$

$$- \frac{dM}{d(\partial U_1/\partial D)} - s \frac{dA_n}{d(\partial U_1/\partial D)}\}$$

$$+ \sum_{i=1}^{I} [(1-\tau) p_i \frac{\partial f_i}{\partial K_i} + \tau q \delta] \frac{dK_i}{d(\partial U_1/\partial D)} - q \frac{dI}{d(\partial U_1/\partial D)}$$

By the strict concavity of U_1 and f_i, $i = 1, \ldots, I$, it follows from (33)-(35), (38), (39), and (53) that each of the terms within braces in (60) is strictly positive. If it can be shown that the sum of the last two terms in (60) is positive, or at least small in absolute value if it is negative, then $\frac{dD}{d(\partial U_1/\partial D)} > 0$ will follow. Consider an incremental change in any K_i. From the net investment constraint, $dK_i \simeq dI$ and the incremental impact on dividends is

$$[(1-\tau)p_i\frac{\partial f_i}{\partial K_i} + \tau q\delta]dK_i - qdI \simeq [(1-\tau)p_i\frac{\partial f_i}{\partial K_i} + \tau q\delta - q]dK_i \; . \tag{61}$$

By (43) and (51),

$$(1 - \tau)p_i\frac{\partial f_i}{\partial K_i} < i \equiv rq + (1 - \tau)q\delta - \dot{q}$$

or

$$(1 - \tau)p_i\frac{\partial f_i}{\partial K_i} + \tau q\delta < (r + \delta - \dot{q}/q)q < q \; , \tag{62}$$

provided $r + \delta - \dot{q}/q < 1$, which is almost always the case since $r + \delta - \dot{q}/q$ is the firm's pre-tax cost of capital (expressed as a decimal). Hence, the term in brackets in (61) is strictly negative (provided $r + \delta - \dot{q}/q < 1$). From (32) and the concavity of U_1 and f_i,

$$\frac{dK_i}{d(\partial U_1/\partial D)} < 0 \; , \tag{63}$$

so that (61)-(63) imply that the sum of the remaining terms in (60) is at worst negative but small in absolute value. Hence $\frac{dD}{d(\partial U_1/\partial D)} > 0$.

The last two results, $\frac{dM}{d(\partial U_1/\partial M)} > 0$ and $\frac{dA_n}{d(\partial U_1/\partial A_n)} > 0$ follow directly from (38) and (39), respectively, due to the assumed concavity of U_1 and the f_i's. Q.E.D.

What theorem V-9 establishes is that the planning algorithm in table V-2 modifies feasible plans in a manner that promotes stability. That is, an increase in the value of any of the four objectives in (26), as measured by the value of the appropriate marginal utility, sets in motion changes that tend to lead to an increase in the value of the particular argument of U_1 at the next iteration. Clearly, a systematic tendency to cause the opposite result - i.e. a decrease in the value of the argument - would suggest a failure on the part of the algorithm to produce a convergent sequence of plans.

This subsection has presented a multiperiod finite horizon planning algorithm that could be used to determine the optimal internal allocation of productive resources for the multidivision firm modeled in (26). It was shown that an equilibrium plan exists for the algorithm (theorem V-7); that when an intermediate plan is infeasible, the algorithm reallocates resources so as to increase net income and thereby restore feasibility (theorem V-8); and that changes in the marginal values of objectives lead to resource reallocations that cause corresponding changes in the values of the respective arguments of the collective utility function (theorem V-9).

3. Achieving Compliance with Corporate Objectives in the Decentralized Multidivision Firm

In section C and again in the previous subsection it was assumed that the firm's productive resources were allocated in accordance with the firm's objectives, as embodied in the collective utility functions U_1 and U_2 . To the extent that the goals and objectives of division managers conflict with the goals and objectives of top managers and shareholders, it was implicitly assumed that all such conflicts were resolved in the process of obtaining U_1 and U_2, and that once these utility functions had been formulated by the firm's board of directors, division managers would accept the marginal values $\partial U_1/\partial R$ and $\partial U_1/\partial D$ and would employ the optimality criteria (32)-(35) in arriving at their planned input usage levels. Moreover, the planning algorithm in table V-2 is based on such acceptance. The purpose of this subsection is to suggest what may happen - i.e. how the firm's internal allocation of productive resources may be affected - when such acceptance fails to materialize.

Suppose that, instead of employing (32)-(35) in step II in the planning algorithm in table V-2, division managers seek to

$$\underset{\substack{\{K_i, L_i, A_{p,i},\\ A^i_{p,H}, M_i,\\ A_{n,i}\}}}{\text{maximize}} \int_0^T U^i(R_i(t), \pi_i(t), M_i(t), A_{n,i}(t))e^{-rt}dt. \quad (64)$$

That is, suppose each division's management has some control over salary levels, so that $M_i(t) > 0$ is possible, and some control over hiring staff, so that $A_{n,i}(t) > 0$ is possible; and further suppose that each division's managers select input usage levels so as to maximize their own discounted collective utility. According to (64), the i-th division's collective utility at each time t is a function of the division's total revenue,

$$R_i(t) = p_i(t)\cdot f_i(K_i(t),\ L_i(t),\ A_{p,i}(t),\ A^i_{p,H}(t)), \qquad (65)$$

the division's net income,

$$\pi_i(t) = (1-\tau)\{R_i(t) - w(t)\cdot L_i(t) - s(t)[A_{p,i}(t) + A_{n,i}(t)] - M_i(t)\} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} K_i(t) - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D} A^i_{p,H}(t), \qquad (66)$$

and the division's managerial emoluments, $M_i(t)$, and non-productive administrative labor, $A_{n,i}(t)$.

The expression for total revenue (65) is the same as the one used in (26). The expression (66) for the division's net income has not been encountered previously. From (51), $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$ will equal the firm's marginal cost of capital net of tax (for time t) when the multidivision firm modeled in (26) is in equilibrium. By corollary V-5-1, $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$ will equal the firm's marginal cost of productive headquarters administrative labor after tax (for time t) when the multi-division firm modeled in (26) is in equilibrium. Thus, (66) assumes that division managers value net income after tax,[44] rather than before tax, and the last two terms in (66) represent the division's after tax costs of physical capital and productive headquarters administrative labor, respectively. From a resource allocation standpoint, $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$ and $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$ might be explained as internal accounting prices that are charged against the division's revenue for the use of physical capital and productive headquarters administrative labor.[45] Collecting (64)-(66) and recognizing that maximizing (64) is equivalent to maximizing U^i at each time t, it is assumed that each division seeks to

$$\underset{\{K_i, L_i, A_{p,i}, A^i_{p,H}, M_i, A_{n,i}\}}{\text{maximize}} \quad U^i\Big(p_i \cdot f_i(K_i, L_i, A_{p,i}, A^i_{p,H});\ (1-\tau)\{p_i \times f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) - w \cdot L_i - s(A_{p,i} + A_{n,i}) - M_i\} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} K_i - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D} A^i_{p,H};\ M_i;\ A_{n,i}\Big) . \tag{67}$$

subject to $K_i,\ L_i,\ A_{p,i},\ A^i_{p,H},\ M_i,\ A_{n,i} \geq 0$

The necessary conditions for an optimal solution to (67) are the following:[46]

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial K_i} + \frac{\partial U^i}{\partial \pi_i}\{(1-\tau) p_i \frac{\partial f_i}{\partial K_i} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D}\} = 0 \tag{68}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial L_i} + \frac{\partial U^i}{\partial \pi_i}(1-\tau)\{p_i \frac{\partial f_i}{\partial L_i} - w\} = 0 \tag{69}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial A_{p,i}} + \frac{\partial U^i}{\partial \pi_i}(1-\tau)\{p_i \frac{\partial f_i}{\partial A_{p,i}} - s\} = 0 \tag{70}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial A^i_{p,H}} + \frac{\partial U^i}{\partial \pi_i}\{(1-\tau) p_i \frac{\partial f_i}{\partial A^i_{p,H}} - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D}\} = 0 \tag{71}$$

$$\frac{\partial U^i}{\partial \pi_i}(-(1-\tau)) + \frac{\partial U^i}{\partial M_i} \leq 0 \quad [\frac{\partial U^i}{\partial \pi_i}(-(1-\tau)) + \frac{\partial U^i}{\partial M_i}] \cdot M_i = 0 \quad M_i \geq 0 \tag{72}$$

$$\frac{\partial U^i}{\partial \pi_i}(-s(1-\tau)) + \frac{\partial U^i}{\partial A_{n,i}} \leq 0 \quad [\frac{\partial U^i}{\partial \pi_i}(-s(1-\tau)) + \frac{\partial U^i}{\partial A_{n,i}}] \cdot A_{n,i} = 0 \quad A_{n,i} \geq 0 \tag{73}$$

Collectively (68)-(73) can be used to explain what is meant by a loss of internal control and a loss of X-efficiency. In particular, (68)-(71) can be compared with (32)-(35) to determine the implications of the divisions' seeking to maximize their own utility levels, and (72) and (73) can be used to argue that, in general, $M_i(t) > 0$ and $A_{n,i}(t) > 0$.

It is obvious that the input usage levels determined using (68)-(71) will, in general, differ from those determined using (32)-(35). However, a more positive result can be proved.

<u>Theorem V-10</u>

For given rents $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$ and $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$, if $\frac{\partial U^i/\partial R_i}{\partial U^i/\partial \pi_i} > \frac{\partial U_1/\partial R+(1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}$, where the elements of the ratio on the right are transmitted to the division by headquarters, and if f_i, $i = 1, \ldots, I$, are strictly concave, then the division following rules (68)-(71) will employ more of each input, and will therefore produce more output and earn greater total revenue, than it would if it followed (32)-(35).

<u>Proof</u>

Solving (32) and (68) for $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$ gives

$$[1-\tau + \frac{\partial U^i/\partial R_i}{\partial U^i/\partial \pi_i}]p_i\frac{\partial f_i}{\partial K_i} = \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} = [1-\tau + \frac{\partial U_1/\partial R+(1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}] \times p_i\frac{\partial f_i'}{\partial K_i}, \tag{74}$$

where $\partial f_i'/\partial K_i$ denotes the marginal product of capital when (32) is used to determine capital input usage. It follows from (74) that

$$\frac{\partial U^i/\partial R_i}{\partial U^i/\partial \pi_i} > \frac{\partial U_1/\partial R+(1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D} \iff \frac{\partial f_i}{\partial K_i} < \frac{\partial f_i'}{\partial K_i} . \tag{75}$$

But (75) implies by the assumed strict concavity of f_i that $K_i > K_i'$, i.e. that capital input usage is greater under (68) than under (32) for given $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$.

The same procedure applied to (33) and (69), to (34) and (70), and to (35) and (71) - by solving for $\frac{(1-\tau)w}{\partial U_1/\partial D}$, $\frac{(1-\tau)s}{\partial U_1/\partial D}$, and $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$, respectively - leads to analogous results.

By the above, since more of each input is used, more output is produced. Given constant price (or in the more general case, a downward-sloping demand curve and positive marginal revenue), the division's total revenue is greater when it applies (68)-(71) than when it applies (32)-(35), given the above assumptions. Q.E.D.

Corollary V-10-1

Under the assumptions stated in theorem V-10, if one or more divisions use (68)-(71) instead of (32)-(35) in determining input usage levels, then the firm's total revenue will exceed the level consistent with (32)-(35).

Theorem V-11

As long as $\frac{\partial U^i/\partial M_i}{\partial U^i/\partial \pi_i} > 1 - \tau$ for some $M_i > 0$ and as long as $\frac{\partial U^i/\partial A_{n,i}}{\partial U^i/\partial \pi_i} > s(1-\tau)$ for some $A_{n,i} > 0$, then the i-th division will pay strictly positive managerial emoluments and will hire some strictly positive amount of nonproductive administrative labor.

Proof

(72) implies that $\frac{\partial U^i/\partial M_i}{\partial U^i/\partial \pi_i} \leq (1-\tau)$ must hold at optimality.

If $\frac{\partial U^i/\partial M_i}{\partial U^i/\partial \pi_i}$ decreases with increasing M_i, i.e. if the $\pi_i - M_i$

indifference curves have the standard convex shape, then

$\frac{\partial U^i/\partial M_i}{\partial U^i/\partial \pi_i} > (1-\tau)$ when $M_i = 0$. Therefore $M_i = 0$ cannot be

optimal, as (72) would be violated. Hence $M_i > 0$.

A similar argument applied to (73) gives $A_{n,i} > 0$.

Q.E.D.

Corollary V-11-1

Under the assumptions stated in theorem V-10, the firm's collective levels of managerial emoluments and nonproductive administrative labor are greater under (68)-(71) than under (32)-(35).

Proof

Theorem V-11 and (38) and (39). Q.E.D.

Corollary V-11-2

Under the assumptions stated in theorem V-10, the firm's net income is lower when one or more divisions employ (68)-(71) than it is when all divisions employ (32)-(35). As a result, the firm sets a lower dividend payout D(t).

Proof

The first statement follows from corollary V-10-1, theorem V-11, and corollary V-11-1, and the definition of the firm's net income (55). The second statement then follows from (56). Q.E.D.

<u>Theorem V-12</u>

When the profit constraint in (26) is not binding at optimality, the multidivision firm in which one or more divisions seek to maximize their own utility will have greater total revenue and will have greater collective levels of managerial emoluments and nonproductive administrative labor, but will earn lower net income and will pay smaller total dividends, than a multi-division firm of the type modeled in (26).

<u>Proof</u>

When the profit constraint is not binding at optimality, $\mu_3 = 0$ by (42). Then by (46) and (51),

$$\frac{\mu_2 e^{rt}}{\partial U_1/\partial D} = (1-\tau)s \text{ and } \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} = rq + (1-\tau)q\delta - \dot{q} \tag{76}$$

at optimality. But (76) must hold for firms of both types. Hence, the conclusion follows by corollary V-10-1, corollary V-11-1, and corollary V-11-2. Q.E.D.

The significance of theorems V-10, V-11, and V-12 is that the divisions may exhibit a revenue preference and an expense preference relative to the collective preferences embodied in U_1 in a manner analogous to the revenue preference and expense preference relative to the preferences of shareholders exhibited by the firm modeled in (26). The required condition is that

$$\frac{\partial U^i/\partial R_i}{\partial U^i/\partial \pi_i} > \frac{\partial U_1/\partial R + (1-\tau)\mu_3 e^{rt}}{\partial U_1/\partial D}, \tag{77}$$

which has the following interpretation. The left-hand side of (77) can be interpreted as the i-th division's marginal rate of substitution between revenue and net income. The right-hand side of (77) can be interpreted as the marginal rate of substitution between total revenue (adjusted for the impact of a change in revenue on minimum net income) and dividends. When (77) holds, the i-th division can be said to show a preference for revenue relative to net income (and indirectly dividends) that exceeds the firm's preference for revenue relative to dividends. As a result of the firm's inability to enforce its preferences, the divisions' use of (68)-(71) leads to greater total revenue. If somehow the firm's control could be restored, resources could be reallocated in accordance with U_1 and a higher level of discounted collective utility could be achieved.

It should be noted that if the inequality in (77) were reversed, the divisions would underutilize inputs and would therefore produce too little output and earn too little revenue. This is possible, as for example, when the firm's headquarters has designed a compensation scheme that attaches relatively great importance to net income, and at the same time, division managers are motivated to a much greater extent by income considerations than by enhanced promotional opportunities and other factors associated with size, staff, etc., that are held to be the traditional sources of managerial satisfaction. However, in view of the discussions in section A of chapter one and in section G of chapter two, it is the opinion of this writer that the case discussed earlier in this subsection is the more likely, and hence the more interesting, of the two.

Theorems V-10, V-11, and V-12 suggest what can happen when division managers exercise some degree of discretion. Moreover, the theorems suggest that (provided (77) continues to hold) the actual behavior exhibited by the divisions can vary systematically from that which would be expected by headquarters on the basis of (32)-(35), if headquarters had sufficient information with which to determine the input allocations consistent with maximizing discounted collective utility at the firm level. But such information may be very costly and time-consuming to obtain, and this was the main justification for introducing the decentralized planning procedure in the previous subsection. The implication is that decentralized multidivision firms may systematically exhibit X-inefficiency of the type indicated in theorem V-11, namely, excessively large staffs - staffs that are excessive even from the standpoint of the firm's collective utility. There may be organizational slack within each division, as well as within headquarters, and as a result, the firm's total labor force will not be utilized with maximum technical efficiency.

Having demonstrated the possible adverse impact decentralization may have on the efficiency with which productive resources are allocated and utilized within the firm, the remainder of this subsection addresses the issue of how headquarters can attempt to achieve (greater) compliance on the part of the divisions with respect to corporate objectives. It is assumed that (77) holds. Then theorem V-12 indicates that revenue, managerial emoluments, and nonproductive administrative labor exceed, while net income and dividends fall short of, corporate objectives (i.e. their respective collective utility maximizing levels).

To counter the tendencies stated in theorem V-12, the firm might seek to centralize control. Full centralization would be impractical in very large organizations, for it would wipe out the very advantages for which the multidivision form of organization was adopted. Short of this, headquarters could do the following:

- set the salary levels of top managers in each division, and rely on these individuals to control salaries in the lower levels of the division's managerial hierarchy, thereby reducing each M_i
- impose an upper limit on total direct hiring of administrative labor (those individuals who might fill a nonproductive slot) by the division,

$$A_{p,i}(t) + A_{n,i}(t) \leq \overline{A}_i(t) \ . \tag{78}$$

  If nonproductive staff positions could be identified, they could be ordered eliminated. But this procedure may prove costly, so that the somewhat weaker approach embodied in (78) may have to be adopted.
- impose a net income constraint on the division,

$$\pi_i(t) \geq \pi_{o,i} \ . \tag{79}$$

If headquarters sets managerial emoluments for each time t, $M_i(t) = \overline{M}_i(t)$, and if it also applies to each division constraints of the form (78) and (79), then the model of a division (67) can be reformulated as the following mathematical programming problem:

$$\underset{\{K_i, L_i, A_{p,i}, A^i_{p,H}, A_{n,i}\}}{\text{maximize}} \quad U^i(p_i \cdot f_i(K_i, L_i, A_{p,i}, A^i_{p,H}); (1-\tau)\{p_i \times f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) - w \cdot L_i - s(A_{p,i} + A_{n,i})\} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} K_i - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D} A^i_{p,H}; A_{n,i}) . \tag{80}$$

$$\text{subject to} \quad A_{p,i} + A_{n,i} \leq \bar{A}_i$$

$$(1-\tau)\{p_i \cdot f_i(K_i, L_i, A_{p,i}, A^i_{p,H}) - w\, L_i - s(A_{p,i} + A_{n,i})\} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D} K_i - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D} A^i_{p,H} \geq \pi_{o,i}$$

where the nonnegativity constraints have not been stated and where the level of managerial emoluments set by management, $M_i = \bar{M}_i$, has been dropped from the model.

The necessary conditions for an optimal solution to (80) are the following:[47]

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial K_i} + \left(\frac{\partial U^i}{\partial \pi_i} + \lambda_2\right)\left\{(1-\tau)p_i \frac{\partial f_i}{\partial K_i} - \frac{\mu_1 e^{rt}}{\partial U_1/\partial D}\right\} = 0 \tag{81}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial L_i} + \left(\frac{\partial U^i}{\partial \pi_i} + \lambda_2\right)(1-\tau)\left\{p_i \frac{\partial f_i}{\partial L_i} - w\right\} = 0 \tag{82}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial A_{p,i}} + \left(\frac{\partial U^i}{\partial \pi_i} + \lambda_2\right)(1-\tau)\left\{p_i \frac{\partial f_i}{\partial A_{p,i}} - s\right\} - \lambda_1 = 0 \tag{83}$$

$$\frac{\partial U^i}{\partial R_i} p_i \frac{\partial f_i}{\partial A^i_{p,H}} + \left(\frac{\partial U^i}{\partial \pi_i} + \lambda_2\right)\left\{(1-\tau)p_i \frac{\partial f_i}{\partial A^i_{p,H}} - \frac{\mu_2 e^{rt}}{\partial U_1/\partial D}\right\} = 0 \tag{84}$$

$$\left(\frac{\partial U^i}{\partial \pi_i} + \lambda_2\right)(-s(1-\tau)) + \frac{\partial U^i}{\partial A_{n,i}} - \lambda_1 = 0 \tag{85}$$

$$(\bar{A}_i - A_{p,i} - A_{n,i})\lambda_1 = 0 \quad A_{p,i} + A_{n,i} \leq \bar{A}_i \quad \lambda_1 \geq 0 \tag{86}$$

$$(\pi_i(t) - \pi_{o,i})\lambda_2 = 0 \quad \pi_i(t) \geq \pi_{o,i} \quad \lambda_2 \geq 0 \tag{87}$$

Since (81)-(87) make clear that the policy choices of the division are unaffected when the constraints in (80) are not binding at optimality - by (86) and (87), $\lambda_1 = \lambda_2 = 0$ and (81)-(84) reduce to (68)-(71), respectively, and (85) reduces to (73) when $A_{n,i} > 0$ - in what follows it is assumed that headquarters has selected $\overline{A}_i$ and $\pi_{o,i}$ so that both constraints in (80) are binding when the division selects its optimal operating policies.

The Lagrange multipliers λ_1 and λ_2 serve as shadow prices internal to the division. Since λ_1 tends to increase as $\overline{A}_i$ decreases and since λ_2 tends to increase as $\pi_{o,i}$ increases, it follows that the headquarters can achieve greater compliance on the part of the divisions by lowering $A_{n,i}$ and by raising $\pi_{o,i}$. This is proved as the following theorem and corollaries.

Theorem V-13

If U^i and f_i are strictly concave and if the two constraints in (80) are binding at optimality, then the division's optimal usage of each input tends to decrease as the net income objective, $\pi_{o,i}$, is increased and its hiring of nonproductive administrative labor, $A_{n,i}$, also tends to decrease as the hiring ceiling, $\overline{A}_i$, is reduced.

Proof

To prove the first statement, it follows from (81) that

$$\frac{dK_i}{d\pi_{o,i}} = \frac{dK_i}{d\lambda_2}\frac{d\lambda_2}{d\pi_{o,i}}$$

$$= -\frac{\{\ \}}{\frac{\partial^2 U^i}{\partial R_i^2}\left(p_i\frac{\partial f_i}{\partial K_i}\right)^2 + \frac{\partial U^i}{\partial R_i}p_i\frac{\partial^2 f_i}{\partial K_i^2} + \frac{\partial^2 U^i}{\partial \pi_i^2}\{\quad\}^2 + \frac{\partial U^i}{\partial \pi_i}(1-\tau)p_i\frac{\partial^2 f_i}{\partial K_i^2}}\frac{d\lambda_2}{d\pi_{o,i}}$$

< 0 ,

where the terms within braces are the same as in (81). Analogous results follow when the same procedure is applied to (82)-(85).

To prove the second statement, it follows from (85) that

$$\frac{dA_{n,i}}{d\bar{A}_i} = \frac{dA_{n,i}}{d\lambda_1}\frac{d\lambda_1}{d\bar{A}_i} = -\frac{-1}{\frac{\partial^2 U^i}{\partial \pi_i^2}s^2(1-\tau)^2 + \frac{\partial^2 U^i}{\partial A_{n,i}^2}}\frac{d\lambda_1}{d\bar{A}_i} > 0 ,$$

where the secondary effect on the net income constraint has been ignored. Q.E.D.

It immediately follows that

Corollary V-13-1

By imposing on each division a minimum net income constraint, the firm's headquarters alters each division's marginal rate of substitution between revenue and net income in favor of net income, i.e. tending to increase $\frac{\partial U^i/\partial \pi_i}{\partial U^i/\partial R_i}$, and by imposing a constraint on the hiring of administrative labor, the firm's headquarters alters each division's marginal rate of substitution between net income and nonproductive administrative labor in favor of net income, i.e. tending to increase $\frac{\partial U^i/\partial \pi_i}{\partial U^i/\partial A_{n,i}}$.

Proof

It follows from (81) (and similarly from (82)-(85)) that

$$\frac{\partial U^i/\partial \pi_i + \lambda_2}{\partial U^i/\partial R_i} = - \frac{p_i \cdot \partial f_i/\partial K_i}{(1-\tau)p_i \cdot \partial f_i/\partial K_i - \mu_1 e^{rt}/(\partial U_1/\partial D)} ,$$

$$> \frac{\partial U^i/\partial \pi_i}{\partial U^i/\partial R_i} , \text{ when } \lambda_2 > 0 .$$

Similarly, from (85),

$$\frac{\partial U^i/\partial \pi_i + \lambda_2}{\partial U^i/\partial A_{n,i} - \lambda_1} = \frac{1}{s(1-\tau)} > \frac{\partial U^i/\partial \pi_i}{\partial U^i/\partial A_{n,i}} , \text{ when } \lambda_1 > 0 \text{ and } \lambda_2 > 0.$$

Q.E.D.

Corollary V-13-2

By imposing on each division a constraint on the hiring of administrative labor, the firm's headquarters alters the optimal mix of manufacturing labor and productive administrative labor in favor of manufacturing labor, i.e. tending to increase $- dL_i/dA_{p,i}$.

Proof

It follows from (82) and (83) that

$$- \frac{dL_i}{dA_{p,i}} = \frac{\partial f/\partial A_{p,i}}{\partial f/\partial L_i} = \frac{s}{w} + \frac{\lambda_1}{w(1-\tau)(\lambda_2 + \partial U^i/\partial \pi_i)} > \frac{s}{w} , \qquad (88)$$

provided $\lambda_1 > 0$.

Corollary V-13-3

Imposing a minimum net income constraint $\pi_{o,i}$ does not cause the marginal rates of technical substitution between pairs of inputs at optimality to be altered.

Proof

Follows from (88) and from analogous optimality conditions for the other inputs when $\lambda_1 = 0$. Q.E.D.

The significance of theorem V-13 and corollaries is that the headquarters of the firm can push the divisions toward compliance with corporate objectives by imposing a hiring ceiling on administrative labor and also by imposing a net income constraint (theorem V-13). The first approach induces each division to use less administrative labor by imposing an implicit tax λ_1 on its use (corollaries V-13-1 and V-13-2). The second approach also encourages a reduced usage level of administrative labor, but also encourages reduced usage levels of the other inputs by imposing the same implicit proportional tax λ_2 on the use of each input. As a result, marginal rates of technical substitution at optimality are unaltered (corollary V-13-3).

4. Section Summary

This section has explored some of the consequences of decentralization in the multidivision firm modeled in (26). First, a planning algorithm for the firm was developed, and several important properties of the algorithm were derived (theorems V-7, V-8, and V-9). Second, it was assumed that division managers were free to exercise discretion in selecting input usage levels and that they sought to maximize their own utility. Several implications for the efficiency with which the firm's productive resources are allocated within the firm were explored (theorems V-10, V-11, and V-12, plus corollaries). Third, it was shown that one possible step toward achieving compliance on the part of the divisions with corporate objectives might involve the imposition of hiring and profit constraints on the divisions by headquarters. The implications of such constraints for the behavior of the divisions was indicated (theorem V-13).

E. CHAPTER SUMMARY

This chapter has extended the model of the centralized, single division firm developed in chapters three and four of this thesis to incorporate several divisions (and several products) and to permit decentralized decision-making. The focal point of the chapter was the development of a model of the multidivision firm (26) and the use of that model and later variants of it, (67) and (80), to study resource allocation within the firm, to explore the consequences of decentralization when division managers seek to maximize their own utility, and to suggest measures that might be taken by the firm's headquarters to gain greater compliance with corporate objectives on the part of the firm's divisions.

In section B two classes of labor, manufacturing labor and administrative labor, were distinguished and the concept of organizational slack was introduced. It was shown that the effect of a change in the salary level, s, on the firm's employment of nonproductive administrative labor could be expressed in the form of a Hicks-Slutsky-type equation (theorem V-1). It was also shown that the degree of organizational slack varies systematically over the business cycle (or at least, systematically with regard to the state of the firm's immediate operating environment)(theorem V-2).

A model of the multidivision firm (26) was formulated in section C. In the model each division acts as a quasi-firm, hiring its own manufacturing labor and productive administrative labor directly, although at the direction of the firm's headquarters, and hiring capital and productive headquarters administrative labor from the firm's headquarters. It was shown that in equilibrium the firm will allocate more of each input to each division, and hence, will produce more of each output than a short run profit maximizer (theorem V-3, corollaries V-5-2, V-6-1, and V-6-2). It was also shown that in equilibrium the marginal value of an additional unit of physical capital will be the same for all divisions and that the marginal value of an additional unit of productive headquarters administrative labor will also be the same for all divisions (theorem V-4), and further, that the common (internal) marginal value of an additional unit of physical capital will equal the firm's (external) marginal cost of physical capital (theorem V-6).

The consequences of decentralization in the multidivision firm were explored in section D. First, a multiperiod finite horizon planning algorithm was developed for the firm modeled in (26), and it was shown that an equilibrium plan exists (theorem V-7), that when an intermediate plan is infeasible the algorithm iterates toward feasibility (theorem V-8), and that the algorithm exhibits stability in the sense that resource reallocations occur in line with changes in the relative marginal values attached to corporate objectives (theorem V-9). Second, division managers were permitted to seek to maximize their own utility, and it was shown that the divisions could exhibit revenue and expenses biases with regard to corporate objectives analogous to those exhibited by the firm with regard to shareholders' objectives (theorems V-10, V-11, and V-12, plus corollaries). Third, the implications of two approaches to trying to achieve greater compliance on the part of divisions with corporate objectives were derived (theorem V-13).

Taken collectively, this chapter and the two that precede it have dealt with three areas within the theory of the firm that, as the survey of the literature in chapter two of this thesis indicates clearly, have in the past been given only superficial treatment in the theory of the firm literature. Chapter three modeled the behavior of the firm in response to systematic changes in the state of its immediate operating environment - i.e. the business cycle. Chapter four explored the relationship between the firm's operating policy choices and its financial policy choices, indicating clearly the fundamental interrelationships, and did so within a partial equilibrium time-state-preference model. The present chapter modeled internal resource allocation and demonstrated several of the important implications of decentralization for the modern firm.

Chapters three, four, and five constitute the author's basic theoretical model. The remainder of the thesis is concerned with an application of the model within the institutional milieu of the U.S. airframe industry. The representative airframe builder model is developed in chapter seven, and to provide the necessary background discussion, chapter six characterizes the major airframe builders; describes their rather special relationship with their main customer, the U.S. government; and discusses how these firms conduct their short term and long term planning.

CHAPTER FIVE FOOTNOTES

1. Leibenstein, op.cit. As discussed below, the concept of X-efficiency introduced by Leibenstein is actually somewhat broader than the statement in the text suggests. The concept of X-efficiency does indeed involve the efficiency with which resources are allocated and utilized within firms, but Leibenstein is also concerned with the impact of motivational factors and organizational factors on the allocation and utilization of resources, and not just with 'efficiency' in the purely technical sense in which that term is normally used.

2. See, for example, Herendeen, op. cit., pp. 94-95.

3. A recent neoclassical model that treats managerial control at each level of the organizational hierarchy as an intermediate good and incorporates these intermediate goods in the firm's production function can be found in M.J. Beckmann, "Management Production Functions and the Theory of the Firm," Journal of Economic Theory (vol. 14; no. 1; February 1977), pp. 1-18.

4. See section B of chapter two of this thesis.

5. P. Drucker, The Concept of the Corporation (Mentor; New York; 1964), pp. 46-88.

6. A.D. Chandler, Jr., Strategy and Structure (Doubleday; New York; 1966); Monsen and Downs, op. cit.; and O.E. Williamson, Managerial Discretion, Organization Form, op. cit. For an interesting practical account of one firm's experience under different organizational structures see "Parsons Picks Up," Forbes (July 15, 1976).

7. J. Hirshleifer, "Economics of the Divisionalized Firm," Journal of Business (vol. 30; no. 2; April 1957), pp. 96-108; S.A. Allen, III, "Corporate Divisional Relationships in Highly Diversified Firms," in J.W. Lorsch and P.R. Lawrence, eds., Studies in Organization Design (Irwin; Homewood, Ill.; 1970), pp. 16-35; O.E. Williamson, Managerial Discretion, Organization Form, op. cit.; and A.M. Spence, "The Economics of Internal Organization: An Introduction," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 163-172. The Spence paper gives a partial survey of a growing body of literature, the 'economics of internal organization', which constitutes

an important new direction for the theory of the firm. For practical examples see "Group management to control diversity," Business Week (September 15, 1975), and "Two PhDs turn Teledyne into a cash machine," Business Week (November 22, 1976).

8. O.E. Williamson and N. Bhargava, "Assessing and Classifying the Internal Structure and Control Apparatus of the Modern Corporation," in Cowling, Market Structure and Corporate Behavior, op. cit., pp. 125-148.

9. J.C. Emery, Organizational Planning and Control Systems (Macmillan; New York; 1969); Galbraith, The New Industrial State, op. cit.; J. Markham, "Market Structure and Decision-Making in the Large Diversified Firm," in Weston and Ornstein, The Impact of Large Firms on the U.S. Economy, op. cit., ch. 14; J. Bower, "Planning within the Firm," American Economic Review (vol. 60; no. 2 May 1970), pp. 186-194; and Davis, Caccappolo, and Chaudry, op. cit. See also "At Potlatch, nothing happens without a plan," Business Week (November 10, 1975), and "The Opposites: GE Grows While Westinghouse Shrinks," Business Week (January 31, 1977).

10. See Heal, op. cit.; J. Kornai, Mathematical Planning of Structural Decisions, 1st ed. (North Holland; Amsterdam; 1967); and C.J. Bliss, "Prices, Markets and Planning," Economic Journal (vol. 82; no. 325; March 1972), pp. 87-100.

11. K.J. Arrow, "Vertical Integration and Communication," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 173-183, and O.E. Williamson, "The Vertical Integration of Production: Market Failure Considerations," American Economic Review (vol. 61; no. 2; May 1971), pp. 112-123.

12. Spence, op. cit., pp. 165-167.

13. Leibenstein, op. cit. See also M.A. Crew and C.K. Rowley, "On Allocative Efficiency, X-Efficiency and the Measurement of Welfare Loss," Economica (vol. 38; no. 150; May 1971), pp. 199-203, and G.J. Stigler, "The Xistence of X-Efficiency," American Economic Review (vol. 66; no. 1; March 1976), pp. 213-216. See also O.E. Williamson, Managerial Discretion, Organization Form, op. cit., which emphasizes the control aspects associated with organizational structure (and their implications for X-efficiency), and Wood, Economic Analysis of the Corporate Economy, op. cit., pp. 65-66, which reviews the argument that the large diversified corporation is a more efficient (i.e. X-efficient in an intertemporal sense) vehicle for growth than small single product firms.

14. In particular, the value of the multidivision form of organization is that it permits greater coordination and control in large diversified firms, i.e. too much centralization can hurt. O.E. Williamson, Managerial Discretion, Organization Form, op. cit. For a practical example, see "New Spur for a Sluggish Giant," Business Week (March 17, 1975).

15. See Leibenstein, op. cit., pp. 405, 408-409, where the results of various empirical studies are discussed. See also "How a Big Company Controls Its Costs In Good Times and Bad," Wall Street Journal (June 4, 1975). For a practical example of the importance to large firms of effective internal controls, see "Investigating the collapse of W.T. Grant," Business Week (July 19, 1976).

16. O.E. Williamson, Managerial Discretion, Organization Form, op. cit.

17. Spence, op. cit.; Arrow, Vertical Integration and Communication, op. cit.; R. Wilson, "Informational Economies of Scale," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 184-195; S.A. Boorman, "A Combinatorial Optimization Model For Transmission of Job Information Through Contact Networks," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 216-249; H.A. Simon, Models of Man (Wiley; New York; 1957); A.A. Alchian and H. Demsetz, "Production, Information Costs, and Economic Organization," American Economic Review (vol. 62; no. 5; December 1972), pp. 777-795; Monsen and Downs, op. cit.; and O.E. Williamson, Managerial Discretion, Organization Form, op. cit.

18. Spence, op. cit., p. 164.

19. Ibid., pp. 165-167; P. Doeringer and M. Piore, Internal Labor Markets and Manpower Analyses (D.C. Heath; Lexington, Mass.; 1971); J.E. Stiglitz, "The Theory of "Screening," Education, and the Distribution of Income," American Economic Review (vol. 65; no. 3; June 1975), pp. 283-300; A.M. Spence, Market Signaling: Informational Transfer in Hiring and Related Screening Processes (Harvard University Press; Cambridge; 1974); R. Radner, "A Behavioral Model of Cost Reduction," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 196-215; R. Radner and M. Rothschild "On the Allocation of Effort," Journal of Economic Theory (vol. 10; no. 3; June 1975), pp. 358-376; and O.E. Williamson M.L. Wachter, and J.E. Harris, "Understanding the Employment Relation: The Analysis of Idiosyncratic Exchange," Bell Journal of Economics (vol. 6; no. 1; spring 1975), pp. 250-278.

20. Spence, op. cit., pp. 167-169; S.F. Maier and J.H. Vander Weide, "Capital Budgeting in the Decentralized Firm," Management Science (vol. 23; no. 4; December 1976), pp. 433-443; W.T. Carleton, G. Kendall, and S. Tandon, "Application of the Decomposition Principle to the Capital Budgeting Problems in a Decentralized Firm," Journal of Finance (vol. 29; no. 3; June 1974), pp. 815-839; and H.M. Weingartner, Mathematical Programming and Analysis of Capital Budgeting Problems (Markham; Chicago; 1967); J. Bower, Managing the Resource Allocation Process (Graduate School of Business Administration, Harvard University; Cambridge; 1970); and Arrow, Vertical Integration and Communication, op. cit. Calculating the cost of capital for a division of a firm is discussed

in M.J. Gordon and P.J. Halpern, "Cost of Capital for a Division of a Firm," Journal of Finance (vol. 29; no. 4; September 1974), pp. 1153-1163.

21. O.E. Williamson, Managerial Discretion, Organization Form, op. cit., pp. 353-354, makes the same point.

22. This notion of nonproductive administrative labor corresponds to O.E. Williamson's notion of 'slack staff'. O.E. Williamson, Corporate Control and Business Behavior, op. cit., ch. 4. The assumption that the firm employs individuals in staff positions who are completely non-productive may seem extreme. The important point is, as Williamson argues, that corporate staffs may be increased without explicit regard for the potential contribution to production of those hired. Assuming that the productivity of such individuals is zero, rather than some small positive amount, is done in this chapter only for convenience.

23. Ibid., pp. 51-52.

24. Rather than introduce a separate argument of U_1, the managerial emoluments argument could be replaced by the sum of managerial emoluments and nonproductive administrative labor expense, $M(t) + s(t) \cdot A_n(t)$. This latter approach assumes, however, that the marginal rate of substitution between managerial emoluments and nonproductive administrative labor expense is always equal to one. Since (3) does not involve such a restriction, it is preferred to the alternative formulation.

25. It is assumed that L, K, and A_p are strictly positive at each time t along each variable's optimal trajectory since each contributes positively to output. It is also assumed that M is strictly positive at each time t along its optimal trajectory since the possibility that $M(t) = 0$ at some time t was considered in chapter three.

26. Recall that, by assumption, the amount of productive administrative labor, A_p, is held fixed.

27. See, for example, O.E. Williamson, Managerial Discretion, Organization Form, op. cit.

28. In this section and the next it is assumed for convenience that the investment is undertaken at the headquarters level and that the firm's stock of physical capital is allocated among the firm's divisions at each time t. This is more inclusive than merely allowing for the allocation of investment funds - and assuming that physical capital is purchased at the division level - since it allows for the reassignment of plant and equipment from one division to another.

29. Both L_i and $A_{p,i}$ (i.e. the amounts division i is permitted to hire) are determined at the headquarters level.

30. Note that since there is just a single revenue argument of the collective utility function in (26), $\partial U_1/\partial R_i = \partial U_1/\partial R_j$, $i = 1, \ldots, I$; $j = 1, \ldots, I$. That is, the marginal collective utility of revenue is the same for each division (product) when the firm is in equilibrium, or equivalently, in equilibrium the marginal rate of substitution between revenue earned by one division and revenue earned by any other division is one. Hence $\partial U/\partial R$ in (32)-(35) can be written without the subscript i. To allow for the value of diversification - i.e. implying marginal rates of substitution that may differ from one - separate revenue arguments could be introduced into the collective utility function. This is done in the representative airframe builder model formulated in chapter seven of this thesis.

31. Ibid., pp. 344-347.

32. Note that the left-hand side of (43) is identical to the right-hand side of equation (40) in chapter three when $\mu_3 = 0$. Indeed, in obtaining (40) in chapter three it was assumed that the profit constraint was not binding (i.e. that $\mu_3 = 0$).

33. (48) implies that the firm modeled in (26) will allocate more productive headquarters administrative labor to each division than a short run profit maximizer. But (48) must hold for each division, and since, by corollary V-4-2, all productive headquarters administrative labor is allocated among the firm's divisions, the firm modeled in (26) actually hires more productive headquarters administrative labor than a short run profit maximizer.

34. Heal, op. cit., chs. 4-6.

35. Ibid.

36. Ibid.

37. Ibid., ch. 4.

38. Ibid., pp. 101-102.

39. Ibid., ch. 3.

40. This assumes that K(t) arising out of (54) is differentiable for all t, $0 < t < T$. Note that this would follow if each $K_i(t)$ were differentiable for all t, $0 < t < T$. This latter assumption is not untenable in view of the standard assumption that K(t) is differentiable for all t, $0 < t < T$, in the case of the single product firm.

41. Ibid., ch. 4.

42. This assumes that K, the predetermined upper bound on the number of iterations, is large enough that the number of iterations required to achieve feasibility can be performed before the algorithm terminates.

43. <u>Ibid.</u>, ch. 4; and H. Uzawa, "Iterative Methods for Concave Programming," in K.J. Arrow, L. Hurwicz, and H. Uzawa, eds., <u>Studies in Linear and Non-Linear Programming</u> (Stanford University Press; Stanford, Calif.; 1958), pp. 154-165.

44. It should be noted that what follows is not affected materially if $\pi_i(t)$ is defined as net income before tax, rather than after tax. As long as τ is constant, as assumed here, the necessary conditions and the resulting expressions for the marginal rates of substitution between pairs of arguments in (64) are changed only slightly. However, since (32)-(35) are expressed net of tax, the comparisons made below are somewhat more straightforward when $\pi_i(t)$ is defined net of tax.

45. That is, $\frac{\mu_1 e^{rt}}{\partial U_1/\partial D}$ and $\frac{\mu_2 e^{rt}}{\partial U_1/\partial D}$ are rents established by the headquarters management. The use of these prices, or rents, was not discussed previously, though this interpretation is consistent with the model formulated in section C and with the planning algorithm presented in the previous subsection. These prices would be charged against the division's revenue at each iteration of the planning cycle, although no transactions would take place until the final plan had been agreed upon.

46. Only the possibilities that $M_i(t) = 0$ or $A_{n,i}(t) = 0$ are economically interesting here.

47. (81)-(85) follow from differentiating the Lagrangian $L_\lambda = U^i + \lambda_1(\overline{A}_i - A_{p,i} - A_{n,i}) + \lambda_2(\pi_i(t) - \pi_{o,i})$. In addition, there are the two constraints in (80), which for brevity are not repeated. (86) and (87) are the appropriate Kuhn-Tucker conditions for the constraints in (80).

VI. A DESCRIPTION OF THE INTERNAL PLANNING PROCESSES OF THE MAJOR U.S. MILITARY AIRFRAME BUILDERS[1]

A. INTRODUCTION

Over the last five years many articles have appeared in the business literature describing the problems that in recent years have plagued the United States aerospace industry, and in particular, the major military airframe builders [2] - the nine firms listed in Table VI-1 that serve as prime contractors for the production of the tactical, bomber, and support aircraft used by the Air Force, the Marine Corps, and the Navy to carry out their respective missions. There have been reports of an impending shakeout of makers of tactical planes, due mainly to the apparent reduction in the number of contracts for new military tactical aircraft. [3] A prolonged slump in defense spending for weapons systems procurement that began in 1970 and the political debate over the future of the B1 bomber program that delayed - and still threatens to put an end to - its production placed added pressure on the stability of the industry. [4] In addition, commercial aircraft sales fell dramatically as airline passenger traffic leveled off, [5] precipitating a financial crisis for one of the three major producers of both commercial and military aircraft. [6]

More recently, the 1976 national elections and the well-publicized upturn in real defense spending that began the same year have drawn the attention of not only the business community, but also the public ,

Table VI-1 Profile of the Nine Major Airframe Builders by Major Military Program

Company	Military Aircraft in Production [1]	
Boeing	E-3	(airborne command and control) [2]
	Advanced Airborne Command Post [3]	
Fairchild	A-10	(attack)
General Dynamics	F-16	(fighter)
Grumman	E-2	(radar)
	A-6	(attack) / EA-6 (radar)
	F-14	(fighter)
Vought (LTV)	A-7	(attack)
Lockheed	C-130	(transport)
	S-3	(antisubmarine)
	P-3	(antisubmarine)
McDonnell Douglas	F-4	(fighter) [4]
	F-15	(fighter)
	A-4	(attack) [5]
	F-18	(fighter) [6]
Northrop	F-5	(fighter-trainer) [5]
	F-18	(fighter) [6]
Rockwell	B-1	(bomber)

1. Or soon to go into production.

2. Uses the Boeing 707 airframe.

3. Uses the Boeing 747 airframe.

4. Total deliveries of approximately 5000 aircraft.

5. Total deliveries of approximately 3000 aircraft.

6. The F-18 will be produced jointly by McDonnell Douglas and Northrop.

Sources: Company Form 10-K reports for company fiscal year 1975 submitted to the Securities and Exchange Commission.

to the defense industry - those firms that produce the weapons systems on which the nation's defense depends. More importantly, the turnabout in defense spending has brightened the general outlook for the aerospace industry considerably.[7] New arms programs, such as the F-16 to be produced by General Dynamics and the F-18 to be produced jointly by McDonnell Douglas and Northrop, have meant that, for some aerospace firms at least, the future looks very bright. For others, however, the trend toward longer production runs and fewer contracts threatens their continued existence as prime contractors. Indeed, in the absence of a sufficient number of new programs, a shakeout will occur and the victims will be forced to increase foreign sales or to work as subcontractors, or else to drop out of the aircraft end of the aerospace business altogether.

The next section describes these and several other problems that confront the major airframe builders, and in particular, how the uncertainties concerning the future state of product market demand, future resource availabilities and costs, etc., that all firms face are compounded by the technological uncertainties that result from having to push the state of the art each time a new weapons system is developed, and in addition, by a dwindling number of contracts from its principal customer - the Department of Defense - that threatens several firms' survival as prime contractors. In such a business environment, the need for effective corporate planning to somehow deal with these uncertainties is, in the opinion of this writer, critical. The development of a new weapons system normally requires several years or longer from conception to production. In addition, the financial resources necessary to support the research and development effort are scarce - even though the Department of Defense does provide partial financial assistance through the distribution of independent research and development (hereafter referred to as IR&D) funds and in recent years has demonstrated a willingness to fund development programs on a cost-plus basis.[8] For these reasons it is necessary that these firms exercise particular care and forethought in allocating their scarce financial resources. Just as important, each of these firms must also allocate its highly skilled design and engineering talent among existing and proposed future projects. In each case the problem confronting the firm's managers is one of deciding how best to allocate the firm's resources to meet their own objectives and the objectives of the firm's shareholders.[9]

The purpose of this chapter is to provide a description of the long term and short term planning processes of the nine major military airframe builders in the United States. The focal point for the discussion is how these planning processes are designed to ensure an allocation of the firm's fixed and variable resources that is consistent with the firm's goals and objectives. Section C sets out the general objectives the major airframe builders seek to accomplish when they plan their long term strategies and annual operations. Sections D through F discuss in broad terms the internal planning processes of these firms. This description forms the basis for the planning model of the representative airframe builder formulated in the next chapter.

The description represents a synthesis of these firms' planning procedures, rather than an attempt to describe with perfect accuracy how any one firm plans. Though there are certain differences in how these firms plan, these differences are, in the opinion of this writer, differences of detail rather than of substance, and there is sufficient commonality in the ways these firms plan to justify the synthesis attempted here. Section G, which describes the corporate review process that follows the planning processes and Section H, which presents the summary and conclusions complete the chapter.

Before proceeding to the discussion of planning, it may prove helpful to the reader to provide an overview of the major military airframe builders: their distinguishing characteristics, the peculiar problems they face, and their rather special relationship with their main customer, the Department of Defense (hereafter DOD). This is the purpose of the next section.

B. AN OVERVIEW OF THE NINE MAJOR MILITARY AIRFRAME BUILDERS IN THE UNITED STATES

1. Introduction

The United States aerospace industry is composed of approximately 50 major manufacturing firms [10] together with hundreds of other smaller firms that produce parts and auxiliary equipment. [11] The industry is a large contributor to the nation's output and employment. During 1975 aerospace sales made a direct contribution to gross national product of 1.9 percent, and accounted for 2.6

percent of all manufacturing sales and 5.4 percent of durable goods production.[12] Aerospace employment during 1975 averaged 942,000 workers, or approximately 1.5 percent of total civilian employment and 5.1 percent of total employment in manufacturing.[13] Also during 1975, U.S. aerospace firms rung up a record trade surplus of $7 billion, or approximately 75 percent of the total U.S. trade surplus.[14]

The output of the aerospace industry consists chiefly of aircraft, missiles, space systems, parts, and auxiliary equipment. Of these products, civil and military aircraft account for nearly 55 percent of the industry's output.[15] Of all the firms in the aerospace industry, there are nine - Boeing, Fairchild, General Dynamics, Grumman, LTV,[16] Lockheed, McDonnell Douglas, Northrop, and Rockwell - that serve as the prime contractors for all of the major tactical, bomber, and support aircraft used by the military. In addition, three of the nine - Boeing, Lockheed, and McDonnell Douglas - are the principal producers of large commercial aircraft in the United States.[17]

Turning to the buying side, during 1975 the federal government purchased nearly 60 percent of the aerospace industry's output, and over the last decade, the federal government's share of the industry's output has reached as much as 74 percent.[18] This dependence on government sales makes aerospace production susceptible to large swings in the level of government demand, as national policy and economic conditions change[19] and as the nation experiences alternating periods of war and peace.[20] In addition, the business cycle, as it affects national disposable income and the demand for commercial airline travel, causes fluctuations in the demand for commercial aircraft. When the two cycles coincide, as they have in recent years, output and employment within the aerospace industry can fall dramatically.[21]

The remainder of this section characterizes the nine major military airframe builders and explores the major risks associated with these firms' heavy dependence on a single customer.

2. Size and Diversification

The nine major military airframe builders are large multiproduct companies whose sales are, in general, heavily weighted toward aerospace products. There are, however, important differences in the extent to which these companies have diversified away from the aerospace business.

Table VI-2 provides a profile of the nine firms according to the 1975 sales of each and the value of new military contracts won by each during fiscal year 1975. Eight of the nine firms are among the 500 largest industrial corporations in the United States, as ranked according to annual sales by Fortune magazine. The ninth largest, Fairchild Industries, falls within the upper range of the second 500 largest. Six of the firms are among the 100 largest. What the table does not show is that several of the firms have experienced some slippage in their rankings since 1970 due to a shrinkage in orders for military and commercial aircraft.[22] Nevertheless, these firms remain large,[23] and one would expect that they face problems of organization and control similar to those faced by other large firms.

Company	1975 Sales $ Millions	1975 Rank Among the Fortune 1000 [1]	% of 1975 Industry Sales [2]	1975 New Military Contracts $ Millions [3]	1975 Rank Among DOD Contractors [4]
Boeing	3719	43	13.3	1561 (4.0)	2
Fairchild	219	607	0.8	192 (0.5)	34
General Dynamics	2160	98	7.7	1289 (3.3)	6
Grumman	1329	155	4.7	1343 (3.4)	5
Vought (LTV) [5]	525	37	1.9	366 (0.9)	17
Lockheed	3387	50	12.1	2080 (5.3)	1
McDonnell Douglas	3256	52	11.6	1398 (3.5)	4
Northrop	988	205	3.5	620 (1.6)	12
Rockwell	4943	31	17.7	732 (1.9)	10

1. Rankings were determined by Fortune magazine on the basis of each firm's total net sales for 1975.

2. Expressed as a percentage of total industry net sales to final customers, which were $28 billion and which were distributed among aircraft, missiles and space, and non-aerospace as follows: aircraft $15.2 billion; missiles and space $8.1 billion; and non-aerospace $4.7 billion; as reported in Harr, op.cit., p. 11.

3. The number in parentheses represents the value of 1975 new military contracts awarded as a percentage of $39.5 billion, the net value of all new military contracts awarded during fiscal year 1975.

4. Rankings determined by new contract awards during fiscal year 1975.

5. The 1975 sales figure is for the Vought Corp. subsidiary of LTV Corp. The Fortune 500 ranking is the parent company's ranking.

Sources: Fortune (May 1976); Fortune (June 1976); Harr, op.cit.; and Ridder and Heinz, op.cit., appendix J.

Table VI-2 also indicates where the nine firms fall in relation to the other firms that make up the list of the top 100 DOD contractors. Five of the top six DOD contractors are airframe builders, and the nine major airframe builders all fall within the top 34 DOD contractors.[24] The nine firms collectively accounted for almost 25 percent of new military contracts awarded during fiscal year 1975. It is apparent, then, that how well these firms plan and conduct their operations will be of considerable interest to DOD. Not only is DOD one of the biggest buyers of the products of each firm, but also, each firm is among DOD's largest suppliers of durable goods.

Table VI-3 shows how the sales of each of the nine companies are distributed among aircraft, missiles and space, and non-aerospace product lines. Four companies - Boeing, Fairchild, Grumman, and McDonnell Douglas - derive at least 80 percent of total revenue from the sale of aircraft, engines, parts, and auxiliary equipment, and these four companies, as well as Lockheed, each earn at least 90 percent of total revenue from the sale of these items and missiles and space equipment. For these five firms, non-aerospace production constitutes a relatively small part of the firm's total operations. In contrast, General Dynamics, LTV, Northrop, and Rockwell appear much less dependent on aerospace sales, with, for example, the sale of aerospace products amounting to only 12 percent of LTV's 1975 sales. However, even though the percentage contribution of aerospace sales may be relatively small, large variations in the level of such sales might still have a significant impact on the firm's profit and loss statement and on its balance sheet. At the opposite end of the

As Indicated by 1975 Product Line Net Sales
(Millions of Dollars and Percent)

Company	Aircraft [1]	Missiles and Space [2]	Total Aerospace [3]	Non-Aerospace	Consolidated Net Sales [4]
Boeing	3005. [5]	587. [5]	3592. [5]	127. [5]	3719. [5]
% Net Sales	81	16	97	3	100
Fairchild	189.6	16.4	206.0	12.5	218.5
% Net Sales	87	8	94 [6]	6	100
General Dynamics	342.0	326.7	668.7	1491.3	2160.0
% Net Sales	16	15	31	69	100
Grumman [7]	1140.7 [8]	113.4 [8]	1254.1 [8]	74.5	1328.6
% Net Sales	86	9	94 [6]	6	100
LTV	392.2 [9]	119.4 [9]	511.6	3787.2	4312.5
% Net Sales	9	3	12	88	100
Lockheed	2017. [5]	1263. [5]	3280. [5]	107. [5]	3387. [5]
% Net Sales	60	37	97	3	100
McDonnell Douglas	2636. [5]	433. [5]	3069. [5]	187. [5]	3256. [5]
% Net Sales	81	13	94	6	100
Northrop	537.3	0.	537.3	450.8	988.1
% Net Sales	54	0	54	46	100
Rockwell	523. [5]	832. [5]	1467. [5,10]	3476. [5]	4943. [5]
% Net Sales	11	17	30	70	100

1. SIC Code 372.

2. SIC Code 376.

3. Sum of columns 1 and 2.

Table VI-3 (continued)

4. Sum of columns 4 and 5.

5. Rounded to the nearest million.

6. Percentages may not sum correctly due to rounding errors.

7. Intergroup eliminations, which consist primarily of data processing services provided to affiliated companies (Grumman Form 10-K, p. 3), and income other than sales revenue (ibid., p. 7), have been eliminated in order to adjust the sales breakdown provided by Grumman Corp. (ibid., p. 1)- based on sales and other income - to a breakdown on the same basis - sales only - as the other figures provided in the table.

8. Estimates are based on Grumman Corp's published figure of 84% government sales, expressed as a percentage of sales and other income (ibid., p. 4), and the assumption that the government aircraft sales in 1975 were the same percentage of total government sales as in 1974 (90%, as reported in Ridder and Heinz, op.cit., Table 40, p. 205).

9. Very rough estimates which are based on an apportionment of government sales between aircraft and missiles on the basis of total dollar contract backlogs (as of December 31, 1975). See LTV Form 10-K, p. 10.

10. Includes $112 million listed in Rockwell's Form 10-K as "aerospace operations: other".

Sources: Company Form 10-K reports for company fiscal year 1975 and Ridder and Heinz, op.cit., p. 205.

spectrum, if a firm derives 80 percent or more of its revenue from the sale of aircraft and related parts and equipment, then the performance of the company is highly sensitive to variations in the demand for aircraft. For such firms, this greater dependence on sales of aircraft makes the need for a planning organization that can cope with the cyclical fluctuations inherent in the demand for commercial and military aircraft all the more pressing.

The picture of the major airframe builders that has emerged thus far is one of large firms that have diversified away from aerospace production to varying degrees, but that are still very much dependent for their overall success on the success of their aerospace operations. The next two subsections discuss the important characteristics of the aerospace research and development process and the aircraft production process and suggest several important implications for the internal planning processes of the major airframe builders.

3. Aerospace Research and the Aircraft Development Process

It is a characteristic of the aerospace industry that each new product pushes the state of the art. Whether the new product is a new military fighter aircraft capable of reaching greater speeds and carrying heavier payloads than its predecessors or a new commercial passenger jet that can achieve greater fuel economy while meeting more stringent noise standards, the development of the aircraft requires the expenditure of large sums of money for highly skilled scientific and engineering talent. Usually these expenditures must be spread over a long period of time before the aircraft is ready for production. Often seven to 10 years - and sometimes even longer - will elapse between program initiation and completion.[25]

The long and expensive research and development process has important implications for the aerospace industry in general and for the major airframe builders in particular. First, the industry is labor-intensive, employing as many salaried workers as production workers.[26] In 1975 the U.S. aerospace industry employed nearly 20 percent of all U.S. scientists and engineers engaged in research and development, and at times this percentage has been as high as 30 percent. These scientists and engineers and the knowledge and experience they possess constitute a valuable capital resource, the efficient allocation of which is critical to each firm's overall performance.[27] It is of some concern to corporate planners, then, that major programs be time-phased in such a way that the firm's scientific and engineering talent can be kept fully employed in jobs requiring their skills.

A second implication of the special character of the research and development process in the aerospace industry is that large sums of money capital must be raised in order to finance the research and development process. For example, the production of a new commercial jet might require as much as $2 billion in research and development and initial production costs before the firm begins to recover its investment[28] - a sum that far exceeds the net worth of any of the commercial airframe builders.[29] The financial pressures these firms face are, however, mitigated to a great extent by government funding of IR&D, by development contracts that are typically awarded on a cost-plus-fee basis, [30] and, in the case of commercial aircraft development, by the spillover effects of research funded at least in part by IR&D money. Indeed, the aerospace industry is an anomaly among U.S. industries due to the extent to which the government - mainly through DOD and NASA, although the latter has diminished in importance rapidly in recent years as the total space effort has wound down - finances its research and development. [31] As an example of the importance of government funding, Lockheed Aircraft Corporation spent $52.8 million of its own funds on research and development during 1975, but received more than $480 million from DOD for defense-related research and development, testing, and evaluation. [32]

A third implication of the special nature of the aerospace industry's research and development process is the risks - technological, financial, and otherwise - inherent in expending large sums of money on new products that push the state of the art and that, in the case of new weapons systems, will lead to actual production only if the proposed weapons system (i.e. the prototype) survives a winner-take-all competition for the right to go into production. [33] As far as research and development per se is concerned, the main risk is that associated with technological uncertainty, namely, whether the firm will be able to make the required advances in the state of the art within a 'reasonable' period of time and at a 'reasonable' cost in accordance with its contract. [34]

A fourth implication of the special character of the industry's research and development process is that the length of the process often makes it necessary for the firm's planners to look well beyond the five-year outlook provided by the Five Year Defense Plan in order to determine how to allocate defense-related research and development funds. While some guidance is provided by the projects on which DOD will permit IR&D funds to be spent, the airframe builders cannot rely on this source of information alone.[35] Moreover, for commercial markets there is nothing akin to a Five Year Defense Plan. Hence, planning the allocation of research and development funds necessitates projections of military and commercial needs in the next decade and beyond, and, needless to say, these projections involve a high degree of uncertainty. Coping with such uncertainty requires corporate planners to make certain adjustments in the way they plan, and these adjustments are discussed below in Section E.

This subsection has discussed the research and development requirements that underlie aircraft production and some of the important implications of the nature of this process and for each of the firm's planners. The next subsection takes a look inside these firms at the aircraft production process and at what the nature of this process implies for corporate planners.

4. The Aircraft Production Process and the Learning Curve

It has long been recognized by aeronautical engineers that airframe production is characterized by a learning process.[36] As the total number of airframes produced of a particular type increases, the direct labor input - the number of man-hours of labor per airframe -

diminishes.[37] This is due to the fact that, as the airframe is put together, items such as hydraulic systems, fuel lines, electrical wiring, and avionics gear must be installed by hand. As the total number of airframes increases, the production workers who install these items become more proficient - the experience gained on previous airframes has taught them what goes where, so that aircraft drawings need not be consulted as often as on earlier airframes, and has also taught them the best order in which to install the various items.

Studies of airframe production have revealed the shape of the learning curve.[38] The typical learning curve is what is called an '80 percent curve', which means that every time airframe production is doubled, the direct labor input per airframe declines by 20 percent, or equivalently, falls to 80 percent of what it was before production doubled. Such a curve is shown below in Figure VI-1, where it has been assumed that the first airframe requires a direct labor input of 8,000 man-hours.[39]

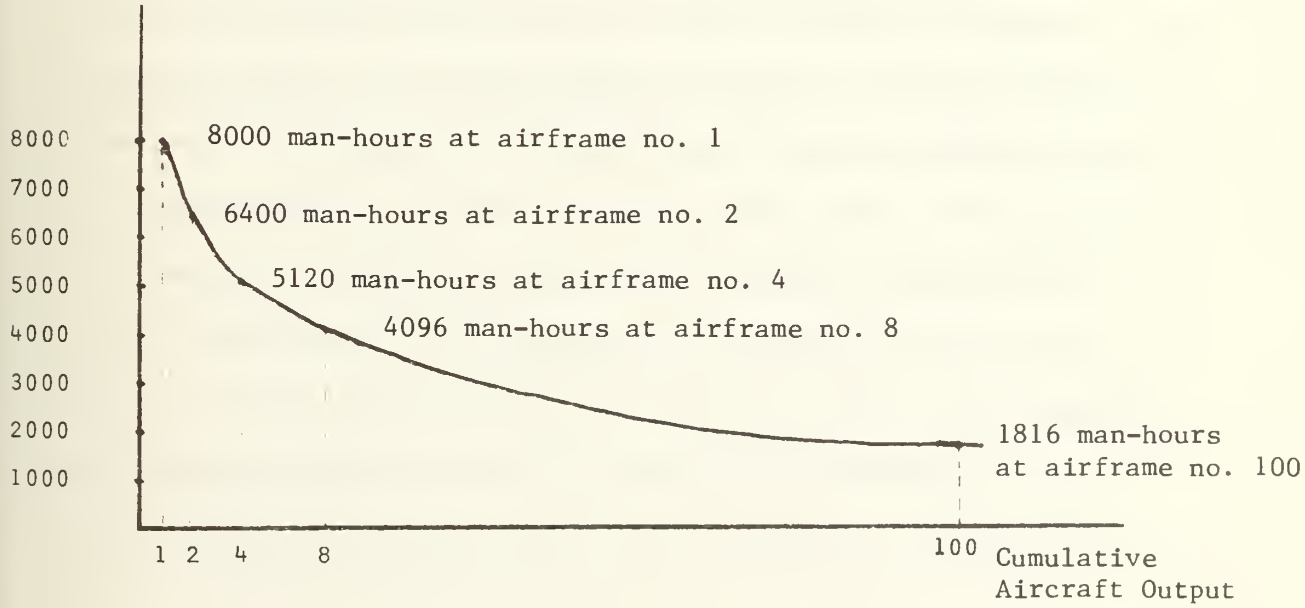

Figure VI-1 80 Percent Learning Curve

It should be emphasized that the learning curve, and the process of learning by doing that it embodies, applies to all airframes singly. Each completely new airframe requires that the learning process begin anew. This is one reason why corporate planning within the major airframe builders proceeds on a more or less airframe-by-airframe basis. This point is developed further in later sections.

The existence of the learning curve has several important implications for corporate planners in addition to the one just mentioned. First, the fall in the direct labor input per airframe means that, barring major design changes that seriously disrupt the learning process or a surge in inflation that sharply increases the cost of labor and other inputs, both the marginal fly-away cost and the average fly-away cost of a particular type of aircraft tend to fall as the cumulative number of units produced increases.[40] This would imply that, from a cost standpoint at least, it is more economical to have a smaller variety of aircraft in order to derive maximum benefit from the learning curve. Thus, it might be argued that producing a common lightweight fighter aircraft that meets the needs of both the Navy and the Air Force, rather than producing a different airplane for each, would enable the government to take maximum advantage of the learning curve. It should be emphasized that such commonality can prove to be more cost-effective (rather than simply less costly) only if a compromise design that will effectively satisfy each service's needs proves feasible.[41]

Second, the existence of a learning curve makes it impractical for the Department of Defense either to have more than one firm build a particular type of aircraft or to switch contractors once production has begun. In the first case, the full benefits of the learning curve could not be derived (i.e. for any given budget, fewer aircraft would be produced), and in the second case, the new contractor would have to begin at the top of the learning curve and a portion of the overall cost savings that had previously been possible would have to be sacrificed. Thus, by the time an airframe builder has won the initial production contract, there is no longer any effective competition on the selling side of the market,[42] and the government-contractor relationship becomes one of bilateral monopoly.[43] The significance of this is explored further below.

Third, the learning curve gives rise to a special problem for the producers of commercial aircraft, which are typically fixed-priced. Before fixing the price and announcing the price to the commercial airlines, the producer must make a careful assessment of the likely future demand for his product, for the lower is the price, the greater is the number of planes that must be sold before the break-even point is reached. An overly optimistic demand projection - say, one that overestimates either the need for additional carrying capacity or the need for replacement aircraft (or both) - can lead the producer to charge a price that implies an unattainably high break-even point, and, as Lockheed's L-1011 experience demonstrates, to intolerably large losses should the expected sales fail to materialize.[44] While the risk of such losses would tend to discourage the firm from setting the break-even point too high, the existence of strong competition among sellers of commercial aircraft, as well as each producer's desire to sell sufficient numbers of aircraft to keep its production lines in continuous operation, push prices in the opposite direction by tending to make producers overly optimistic with regard to how fast they can proceed down the learning curve.

A fourth implication of the existence of the learning curve concerns the relationship between the production of commercial aircraft and the production of military aircraft. If a producer of both commercial and military aircraft can win a contract to produce a military plane that utilizes the same airframe as one of the company's commercial planes,[45] then that company will gain an advantage over its commercial competitors to the extent that it is able to progress down the learning curve more quickly than it could have otherwise. Of course, the military buyer also benefits by saving, not only on research and development costs, but also on a portion of what it would otherwise have cost to produce the military airframes. This is yet one more important aspect of the interface between government sales and commercial sales.

This and preceding subsections have menioned the importance of government sales to the major airframe builders. The next two subsections look more closely at the importance of government sales and at the risks involved in doing work under contract for the government.

5. The Importance of Government Sales

During 1975 almost 60 percent of the aerospace industry's total output was purchased by the federal government.[46] For the nine major airframe builders the proportion of total net sales made to the government was somewhat less, amounting to nearly 55 percent of the total.[47] This high proportion of government sales means that these firms' total sales, and indirectly their profits and overall financial health, are very sensitive to changes in the amount of DOD weapons purchases. A somewhat more informative picture of this dependence emerges when total sales figures are broken down between government sales and commercial sales on a company-by-company basis. This is done below in Table VI-4.

Including sales of aircraft and other items to foreign governments among 'commercial sales' - in accordance with an accounting convention adopted throughout the industry - yields the percentages of government sales in consolidated net sales shown in column six.[48] These percentages range from a low of 38 percent in the case of Boeing and Rockwell to a high of 88 percent in the case of Vought Corp., with seven of the nine firms deriving more than half their revenue from government sales. The main reason Boeing has such a low percentage of government sales is its relatively high proportion - 60 percent - of sales of commercial aircraft. At the opposite end of the aircraft sales spectrum, Grumman Corp. derived 77 percent of its total revenue from sales of aircraft to the government.

The dependence of these firm's total sales on government sales, in general, and on sales of aircraft to the government, in particular, means that changes in the DOD budget can have a significant impact on the sales and levels of employment of these firms.[49] It is not surprising,then, that, as part of the corporate planning

Table VI-4 Proportion of the Nine Major Airframe Builders
1975 Net Sales Made to the Government
(Millions of Dollars and Percent)

Company	Aircraft[1] Comm	Aircraft[1] Gov't	Non-Aircraft[2] Comm	Non-Aircraft[2] Gov't	Total Comm[3]	Total Gov't[4]	Consolidated Net Sales[5]
Boeing[6]	2238.	767.	68.	646.	2306.	1413.	3719.
% Net Sales	60	21	2	17	62	38	100
Fairchild	78.2	111.4	16.4	12.5	94.6	123.9	218.5
% Net Sales	36	51	8	6	43[7]	57	100
General Dynamics	77.6	264.4	757.4	1063.6[8]	832.[6]	1328.[6]	2160.0
% Net Sales	4	12	35	49	39	61	100
Grumman[9]	119.6	1021.1	74.5	113.4	194.1	1134.5	1328.6
% Net Sales	9	77	6	9	15	85[7]	100
Vought (LTV)	49.3[10]	342.9[10]	13.7[10]	119.4[10]	63.0	462.3	525.3
% Net Sales	9	65	3	23	12	88	100
Lockheed[6]	559.	1458.	626.	744.	1185.	2202.	3387.
% Net Sales	17	43	18	22	35	65	100
McDonnell Douglas[6]	1277.	1398.	148.	433.	1425.	1831.	3256.
% Net Sales	39	43	5	13	44	56	100
Northrop	220.9	316.4	154.7[11]	296.1	375.6	612.5	988.1
% Net Sales	22	32	16	30	38	62	100
Rockwell[6]	196.[12]	327.	2869.[12]	1551.[13]	3065.	1878.	4943.
% Net Sales	4	7	58	31	62	38	100

1. SIC Code 372.

2. Mainly missiles and space (SIC Code 376).

3. Sums of columns 1 and 3. Commercial sales include sales to foreign governments, in accordance with the accounting practices of the industry. Entries may not sum correctly due to rounding errors.

4. Sum of columns 2 and 4. Government sales include sales to the U.S. government only. Entries may not sum correctly due to rounding errors

5. Sum of columns 5 and 6.

6. Sales figures are rounded to the nearest million.

7. Percentages may not sum correctly due to rounding errors.

8. Includes sales of $305.7 billion earned on new submarine construction.

9. See footnotes 7 and 8 of Table VI-3.

10. Very rough estimates based on an apportionment of sales given in columns 5 and 6 on the basis of total dollar contract backlogs (as of December 31, 1975). See LTV Form 10-K, op.cit., p. 10.

11. 'Construction' plus 'Other'. See Northrop Form 10-K, op.cit., p. 1.

12. Commercial sales are divided between aircraft and non-aircraft by assuming that the percentage split was the same in 1975 as it was in 1974 (6.4% aircraft - 93.6% non-aircraft; Ridder and Heinz, op.cit., Table 40, p. 205).

13. Primarily 'missiles and space' and 'electronics'.

Sources: Company Form 10-K reports for company fiscal year 1975 and Ridder and Heinz, op.cit., Table 40.

process, these firms carefully prepare environmental forecasts that provide the planning staff with an assessment of the likely political and military environment and what defense policy and military hardware needs are likely to be in such an environment, as far as five to 10 years into the future.

An aerospace company's dependence on government sales, particularly if a large percentage of its sales are derived from a single contract, may subject the firm to a significant termination risk, since the government may terminate a contract for its convenience at any time.[50] This risk and the other risks involved in doing business with the government are discussed in the next subsection.

6. Doing Business with the Government: Risks and Regulations

The recently completed Department of Defense Profit Policy Study (nicknamed Profit '76), which entailed an analysis of defense industry risks and profitability and which led to important revisions in DOD procurement policy,[51] once again raised the question of whether the profits contractors earn on government work are commensurate with the risks they must bear in performing such work. As the Profit '76 study and several earlier studies have noted, working under contract for the government involves certain risks not present in commercial dealings, and the scale of these risks is dependent on the type of contract awarded.[52]

It is not the purpose of this subsection to attempt to determine whether defense work is of a relatively high risk/low return nature. That issue has been debated - often heatedly - many times in the past,[53] and, in the opinion of this writer, the debate is likely to continue well into the future as DOD procurement policy changes and as each side reassesses the relative risks and rewards of government business.[54] One of the complicating factors in the debate is the nonquantifiability of risk. Many studies have listed the risk elements in government contracting,[55] but beyond that, it is very difficult to do more than adopt some surrogate measure - the Profit '76 study employed as a surrogate measure of risk the standard deviation of the firm's rate of return over a ten-year period.[56] Lack of agreement as to the most appropriate measure of risk will, to the extent that different measures lead to different conclusions, help keep the debate alive.

Whether government regulations permit profits sufficient to compensate defense contractors for the risks they face is also important from a planning standpoint. A firm will be willing to invest its own funds in new plant and equipment only if the expected returns from the investment, when adjusted for risk, are judged by top management to be adequate from the standpoint of the firm and its shareholders. If potential returns are felt to be inadequate, then top management will instead allocate the firm's available investment capital to commercial projects.[57]

The remainder of this subsection examines the buyer-seller relationship that exists between the federal government and its prime contractors - a relationship that is conditioned largely by the procurement policies that have been established by the government. The latter part of the subsection discusses how the government's procurement policies - and, in particular, the type of contract it is willing to award - can affect the allocation of risk between buyer and seller. Unlike the studies of risk cited earlier, which either catalogued a long list of risks or else concentrated on developing a single overall measure of risk, this subsection develops a set of risk classifications and clearly distinguishes (for any particular program) between those risk elements associated with the period prior to the award of the first production contract and those risk elements associated with the period following the award of the first production contract. In the opinion of this writer, such a distinction has important implications for corporate planners and top managers who must carefully weigh expected returns and risks before deciding how best to allocate the firm's scarce engineering, scientific, and managerial talent among current and proposed programs.

a. The Buyer-Seller Relationship

Government contracts are awarded and administered under a detailed set of rules spelled out in the Armed Services Procurement Regulation (ASPR), an imposing collection of volumes that totals more than 3000 pages.[58] The provisions of ASPR, together with thousands of additional directives and instructions, detail the conditions that must be met when the contract between buyer and seller, which specifies the obligations of both parties under the procurement agreement, is written. Such an agreement is necessitated by the fact that the procurement of weapons systems does not - indeed, can not - take place via normal commercial market transactions. In commercial markets firms design and develop new products entirely on their own and finance research and development and initial production prior to observing the actual demand for the good. Due to several factors, among them national security considerations, the high cost and long lead time required to develop new aircraft, and all the uncertainties connected with weapons system acquisition,[59] it is not in the best interests of the government to rely on this process for obtaining major weapons systems.[60] The government needs some assurance that it will get what it needs when it needs it and at a reasonable price, and the contractor needs some assurance that the government has a need for the new aircraft it is developing, as well as financial support during the lengthy and costly research and development process. Thus, there is the need for negotiation and contractual relationships to supplant the market place in determining product design, price, etc.

In discussing the buyer-seller relationship that exists between the government and the airframe builders, it is important to distinguish, for any particular program, between the period prior to the award of the initial production contract and the period following that award. The contracting process for a new type of aircraft begins with a Request for Proposals, which the government issues to interested bidders. These contain detailed specifications of the government's requirements. At this stage of the process, and continuing through the building of prototypes for a fly off, there are two or more sellers but only one buyer. The buyer-seller relationship is what economists call a 'monopsony' - two or more sellers competing against one another to sell their output to the sole buyer.[61] In such an environment there is a danger that a contractor will submit an unrealistically low bid or an unrealistically optimistic set of technical specifications in order to increase its probability of winning.[62] This danger increases when there are not enough major contracts to go around, and increases even further when the procuring agency resorts to auctioning - i.e. asking contractors whose bids fall within the competitive range for "best and final" offers.[63] During this portion of the contracting process the main risks the contractor faces are those associated with either losing the competition or else winning the competition but finding it impossible to meet the terms of the contract.[64]

Once the aircraft has gone into production, however, the buyer-seller relationship changes dramatically.[65] The existence of the learning curve precludes further competition among airframe builders, and the government-contractor relationship becomes one of 'monopsony' - a single buyer and a single seller. Production contracts are typically renegotiated on a yearly basis. But once the plane has gone into production, there no longer remain any technical uncertainties. Moreover, the shape of the learning curve is known well enough that labor costs can be estimated fairly accurately, at least over the next year.[66] Though the cost of components purchased from subcontractors can change, the degree of risk associated with cost increases that diminish net income is relatively small. However, there is always a risk that the government will terminate the contract at its convenience, reimbursing the contractor for costs incurred up to the termination date and paying it a pro rata share of the previously negotiated fee, but leaving it on its own to decide what to do with facilities and a labor force for which there is no longer any need. The importance of the termination risk is difficult to assess, but, in the opinion of this writer, is likely to be small in relation to the risks associated with not winning the contract in the first place.

After the contractor has performed the work required under the contract and been paid, any profits it may have earned are subject to scrutiny by the Renegotiation Board.[67] The board averages contractor performance on all contracts on a yearly basis, and if it determines that during the year under review the contractor earned 'excessive' profits, it recaptures the 'excess' for the government. This review process is, however, a one-way street because the contractor has no recourse in the event it believes its profits were too low that year.[68] Moreover, the determination of the reasonableness of each contractor's profits is made on the basis of a set of six criteria[69] that are widely regarded as vague and subjective.[70] Government contractors and independent analysts have criticized the Renegotiation Board's decisions as arbitrary.[71] Kaysen and others believe that the way the government does business, and in particular, the operation of the Renegotiation Board, dulls whatever incentives exist in individual contracts for promoting efficient contractor performance.[72]

This subsection has mentioned several of the risks involved in government contracting. The next subsection examines these and other associated risks.

b. Financial Risks and Business Risks

The term 'risk' is one that is subject to varying interpretations. Some authors treat 'uncertainty' as a synonym for 'risk',[73] while others follow Knight[74] and distinguish 'uncertainty', which is held to be 'elusive and nonmeasurable', from 'risk' which is held to be measurable.[75] Financial and business writers often use the term 'risk' in an all-inclusive manner to encompass all the assorted uncertainties, most of which are not susceptible to measurement, that confront a firm, while financial management textbooks normally aim for a higher degree of precision, often carefully distinguishing between 'business risk', measured, say, as the coefficient of variation of the firm's net operating income, and 'financial risk', measured, say, as the coefficient of variation of the firm's net income (or earnings available to shareholders) or as the probability of bankruptcy.[76] Studies dealing with the specific subject of risk elements in government contracting tend to drift to either of two extremes: either providing a long list of sources of uncertainty[77] or else selecting some single overall measure of risk.[78] One exception is a recent publication of the Aerospace Industries Association of America[79] that categorizes risks that confront aerospace firms into four broad classes. A similar approach is adopted in the first portion of this subsection, although the categories differ somewhat and, in contrast to the earlier approach, a surrogate measure of each of the risks in each category is suggested.

The major types of risk that are encountered by the nine major military airframe builders are discussed below. Many of these risks, as indicated below, are those that affect all government contractors. In what follows a distinction is drawn between *financial risks* and *business risks*, but as the discussion makes clear, government contracting affects both types of risk.

As discussed above, financial risk, which must be borne to some extent by all corporations, encompasses the risk of bankruptcy and the variability in the firm's net income. Surrogate measures for these two components of financial risk are the probability of bankruptcy and the coefficient of variation of net income, respectively.[80] The probability of bankruptcy is, of course, the ultimate risk that any business organization faces, but, in the case of the major airframe builders, this risk is, according to at least one expert, almost insignificant. Kurth has offered empirical evidence in support of his belief in a 'bail-out imperative' that prompts the government to come up with a new program and award it to a prime contractor in deep financial trouble.[81] Kurth would undoubtedly argue that the Navy's modification of the F-14 contract and the government's loan guarantee for Lockheed were merely different manifestations of the same phenomenon.[82] The second component of financial risk is also affected by the way the government does business. For example, Fairchild Industries, Inc., reported to its shareholders that "the transition from the development to the production contract [for the A-10 attack aircraft] substantially impacted 1975 sales and earnings."[83]

What distinguishes financial risks from business risks is that only the former reflects the impact of the firm's financial decisions, and in particular, what proportion of the firm's capital has been raised through the issuance of debt instruments. Both financial risk, as defined above, and business risk, as defined below, reflect the impact of the operating decisions of the firm. In this sense, then, financial risk is the more inclusive term and can be thought of as the firm's overall risk.[84]

Business risks are of six types: (i) technical risks, (ii) bidding risks, (iii) production risks, (iv) cost risks, (v) government dependence risks, and (vi) commercial market risks. Each of these is discussed below.

Technical risks are those associated with pushing the state of the art each time a new military aircraft is developed.[85] Often there are several unknowns to be dealt with, and even if the firm is confident it can solve each technical problem individually, there may remain much uncertainty concerning the time and cost required to accomplish these results and there may also be uncertainty as to how well the new integrated system incorporating all these advances will perform.[86] Since aerospace firms typically earn a greater portion of their sales revenue on research and development work than do firms in other industries,[87] this source of uncertainty is of somewhat greater significance in the aerospace industry. However, in view of DOD's apparent increasing willingness to fund research and development on a cost-plus basis,[88] the impact of technical risks on the firm is correspondingly reduced. One way to measure these risks, while reflecting the importance of government funding, is to estimate the firm's probability of failure to meet the contract's specifications. Since increasing the government's share of the costs makes it less costly for the firm to engage in an additional dollar's worth of research and development, as long as the additional dollar is spent productively, the probability of failure will tend to fall. Reducing this probability is in the firm's interest because failure to meet the contract specifications may result, not only in financial loss, but in loss of reputation as well.

The second category of business risk, bidding risk, is also associated primarily (although not exclusively) with government contracting. Prior to the award of a contract, the firms that intend to bid spend money, some of it their own and the rest of it the government's bid and proposal (B&P) money, preparing the bid. As in the case of technical risks, government funding helps reduce the impact of these risks. Also similar to technical risks, the measurement of bidding risk may be carried out by once again estimating the probability of failure, in this case, the probability of failing to win the contract.

The third class of business risk, production risk, is associated with fluctuations in the levels of demand for the firm's product that lead to volatility in manpower requirements. One major source of fluctuation in demand for military aircraft is the rapid buildup and the rapid phasing out that accompany the start and completion, respectively, of a major government contract.[89] Such fluctuations may force the firm through successive periods of layoffs and rehirings, both of which involve substantial direct and indirect costs. For example, where labor unions exist, one would expect on the basis of economic theory [90] that unions would press for higher wages in order to obtain for their workers a risk premium to compensate them for the risk of being laid off. A more serious problem from the company's standpoint is the threat of not being able to rehire previously laid off skilled scientific or engineering talent or skilled line managers because they were hired by other firms, or worse yet, because they left the industry. Any measure of the volatility of aerospace employment, such as the coefficient of variation of the number of employee-hours per week over some specified time period, might serve as a measure of this type of risk.

The fourth category of business risk, cost risk, involves the contractor's possible failure to produce the item within target cost. This category of risk overlaps with technical risk, since the greater is the technical complexity of the item - i.e. the greater are the technical risks - the greater is the risk that actual costs will exceed target cost. As discussed below, when research and development contracts are of the cost-plus form, the government assumes most of the cost risk. On production contracts, on which the technical risks are normally much smaller than on research and development contracts, but on which the government normally insists on a fixed price, there is also cost risk. This risk is induced by such factors as production delays, design changes ordered by the government that lead to cost increases that are not fully reimbursed, and increases in the cost of inputs due to general inflation that are not fully covered in the contract. Also, as discussed below, the use of fixed-price contracts for the production phase of a major program forces a larger share of the cost risk to fall on the contractor.[91] As far as the measurement problem is concerned, cost risk on any contract or project could be measured either by the standard deviation of the probability distribution of actual cost about target cost - estimated, say, on the basis of historical data - or more simply, as the estimated probability that target cost will be exceeded.

Two other aspects of cost risk should be noted. First, to reduce bid risk the contractor may submit an unrealistically low bid, thereby increasing its cost risk. Second, the use of subcontractors can also have an impact on cost risk in two offsetting ways.[92] On the one hand, the greater use of subcontractors relaxes the prime contractor's direct control over those phases of research and development and/or production that have been contracted out, while on the other hand, a portion of the prime contractor's overall cost risk can be transferred to the subcontractor if the latter accepts the work on fixed-price basis.[93]

The fifth category of business risk, government dependence risk, also overlaps with the other categories of business risk. Government dependence risk is caused by a contractor's having to sell high technology military aircraft to (or at least through) a single buyer. This imposes certain risks in research and development work because the government must be satisfied with the product before it will authorize production, and in addition, because the contracting authority may alter its requirements as the weapons system evolves, ordering the contractor to modify components, which in turn may have a ripple effect on other components in the system. A second aspect of government dependence risk is the volatility of government funding, which can have a strong impact on a contractor's production risk. The fall in DOD spending following the Vietnam war peak is one of the main factors responsible for the current overcapacity in the aerospace industry.[94] In addition, each of the above factors can also compound cost risk. A third aspect of government dependence risk is termination risk - the probability that the government may terminate a contract for convenience. Such termination may be due to a lack of funding, or to political pressures such as those threatening the B1 bomber program, or more simply, to altered priorities. In any case, the very limited possibilities for, and the very high cost in terms of direct outlays and loss of efficiency of, converting the production facilities to some alternative use[95] mean that layoffs follow, the contractor's business base and earnings shrink, and the amount of unused capacity increases. A fourth aspect of government dependence risk is that associated with the present very real possibility that, even if real defense spending continues to increase, the number of major contracts, and hence, the number of airframe builders needed to serve as prime contractors, may diminish.[96] As far as measurement of government dependence risk is concerned, one might adopt some measure of the volatility of a contractor's government sales, as, for example, the standard deviation of government sales (about a trend) over some specified period.

The last category of risks includes those risks that are strictly commercial, and as such, can be interpreted and measured in the same manner as Van Horne's business risks. In the case of the three firms producing commercial passenger aircraft, however, it may be more meaningful to attach risks to specific programs. These are of essentially two types: the risk (i.e. the probability) of not reaching the break-even number of units sold for a particular type of aircraft and the risk (i.e. the probability) of a disastrous accident that will tarnish the company's image and might thereby detract from future sales. The first of these also reflects the risks associated with mistiming the introduction of a new commercial aircraft.[97]

One method of dealing with these substantial financial and business risks is for the airframe builders to cooperate through joint ventures.[98] This approach is superior from the standpoint of the firms involved to the more traditional prime contractor - subcontractor form of business relationship in that research and development are shared more equally and each producer shares in the production and marketing of the aircraft. Hence, in a joint venture each producer is a prime contractor,[99] and as will be argued below, it is prime contracting rather than subcontracting, and the prestige that accompanies the successful development and production of a new high technology aircraft that is one of the primary sources of satisfaction for the managers of these firms.[100]

c. The Government - Airframe Contractor Relationship: Allocation of Risks by Contract Type

Having indicated the major sources of risk, the discussion will deal next with the question of risk-sharing between the government and the contractor. At one extreme, the government could provide all the fixed capital (i.e. plant and equipment) and all the working capital (i.e. short term funding for inventories and work in process) and pay the airframe contractor a fixed fee for managing these assets. In this case the government would assume the larger share of business risk. At the opposite extreme, the contractor could provide all its own fixed and working capital and accordingly assume all the business risk. In reality, the government does furnish some fixed capital,[101] although it appears to be trying to phase out its plant ownership role in the aerospace industry.[102] The government also funds a large portion of the major airframe builders' working capital requirements by providing progress payments,[103] though with interest now an allowable cost, the extent of government funding of working capital requirements may decrease.[104]

One of the most important mechanisms by which the government is able to shift risk between itself and the airframe builder is by its selection of the type of contract for a particular procurement.[105] Several studies have examined the relationship between contract type, the extent of the risks borne by the contractor, and contractor performance.[106] Procurement contracts are of four basic types: firm-fixed-price (FFP), fixed-price-incentive (FPI), cost-plus-incentive-fee (CPIF), and cost-plus-fixed-fee (CPFF).[107] As implied by the fee ranges set by the Department of Defense,[108] the government's share of overall risk (and, in particular, its share of cost risk) is greatest under CPFF contracts, somewhat less under CPIF contracts, less yet under FPI contracts, and least under FFP contracts. Correspondingly, the contractor's share of the risk and its fee become greater as the government's share of the risk falls.[109]

A third mechanism by which risk is shifted is via government contract provisions regarding warranties.[110] The purpose of a warranty is to protect the buyer in the event the item turns out to be defective. The warranty typically specifies the extent of the producer's liability for repairing or replacing defective items. Several comparisons of government contract provisions regarding warranties with commercial warranties have shown government warranties, in general, to be more demanding.[111] The existence of more stringent warranties has the effect of increasing the share of technical risk, and hence the share of overall financial risk, borne by the firm, since ceteris paribus the more exacting are the standards, the greater is the likelihood they cannot be met, and consequently, the greater is the likelihood the contractor will suffer some sort of financial penalty.

This subsection has examined the government-airframe contractor relationship and has discussed the major risks airframe contractors face and how government procurement policy affects the sharing of these risks with its prime contractors. The next subsection looks at the commercial side of the airframe builders' business, and in particular, at their attempts to diversify in order to reduce their dependence on government sales.

7. Diversification: Balancing Government Business and Commercial Business

The previous subsection discussed some of the major differences between the buyer-seller relationship that exists between the government and a prime contractor and the buyer-seller relationship that is typical of commercial markets. The subsection went on to point out the risks associated with this special relationship, and in particular, the risks a contractor faces as it becomes increasingly dependent on government sales. While commercial ventures also pose certain risks, some of which are of great magnitude, many of the major airframe builders have increased their efforts to diversify into non-aerospace commercial ventures in recent years.[112]

One reason offered to explain this desire to diversify is the relatively low profitability and the relatively high risks of government business. A second reason is the limited growth potential provided by government sales during recent years as real defense spending fell. Table VI-5 summarizes the recent profitability and growth experience of the nine major military airframe builders.

As the table shows, the median profitability, whether measured by the average return on equity or the average return on total capital, as well as the median net profit margin and the average annual sales growth, for the nine firms were below the respective median values both for the aerospace industry as a whole ('industry median') and for all industries taken collectively ('all industries median').[113] If one views a corporation as a business entity that exists primarily for the benefit of its shareholders, then, of the three indicators of profitability shown in Table VI-5, average return on equity is the most appropriate. Therefore, on the basis of this measure and the profitability figures provided,[114] one must conclude that whatever differences exist between the profitability of the major airframe builders and the profitability of other aerospace firms and firms

Company	Average Return on Equity [1,2]	Average Return on Total Capital [2,3]	Net Profit Margin [4]	Average Annual Sales Growth [5]
Boeing	6.8	5.8	2.3	1.5
Fairchild	6.2	5.0	1.9	-2.8
General Dynamics	13.3	10.5	4.0	-3.3
Grumman	9.3	3.5	2.1	2.3
LTV	12.3	5.8	0.6	6.5
Lockheed	46.8 [6]	5.9	1.2	4.8
McDonnell Douglas	12.6	9.9	3.2	2.3
Northrop	11.9	8.8	2.4	9.4
Rockwell	12.8	9.0	2.4	10.0
Median	12.3	5.9	2.3	2.3
Industry Median [7]	12.6	8.8	2.9	5.8
All Industries Median	12.7	9.1	4.6	11.8

1. Assumes that all "common stock equivalents" - convertible bonds, convertible preferred stocks, warrants, and stock options - have been converted into common shares.

2. Percentage computed as a five-year average of the returns computed for 1972 through 1975 and the 12-month period ending with the latest (as of December 1976) available quarterly report.

3. Percentage return on combination of stockholders' equity, long term debt, minority stockholders' equity in consolidated subsidiaries, and accumulated deferred taxes and investment tax credits. The numerator in the calculation is the sum of net income, minority interests in net income, and estimated after tax interest paid on long term debt.

4. Net profits for the latest (as of December 1976) 12 months divided by net sales for the same period.

5. Computed in the following manner: the difference between average annual net sales for the period 1972 through 1975 and the most recent 12-month period and average annual net sales for the period 1967 through 1971, expressed as a five-year annually compounded rate of growth. The averaging done in the calculation is intended to smooth out short run distortions.

6. Lockheed's very large avarage return on equity is an anomaly caused by that firm's very small equity (or net worth).

7. Industry median differs from the industry median listed in Forbes because Forbes did not include LTV among the aerospace firms.

Source: Forbes (January 1, 1977), pp. 39, 133, and 154.

in other industries are not significant.[115] It is equally clear from the figures provided in Table VI-5 that the major airframe builders have grown more slowly than other firms. Moreover, Rockwell International, which grew the fastest, was also the most active in acquiring other firms outside the aerospace industry.[116]

One direction in which the major military airframe builders might choose to diversify is the production of commercial jet aircraft. Indeed, three of the nine already dominate the world market.[117] The technological complementarity of military and commercial aircraft would tend to make such diversification appear attractive. Also, the character of the production processes is similar enough that managerial and productive expertise could also be transferred rather easily - certainly more easily than to, say, automobiles or food products. However, the demand for commercial aircraft is highly cyclical[118] and, despite the evident need for new jetliners,[119] the present outlook for commercial jet aircraft sales is clouded by the severe financial problems afflicting the nation's airlines[120] and the apparent peaking out of the growth of airline passenger traffic.[121] Moreover, the financial risks are enormous, with outlays for research and development and initial production amounting to as much as $2 billion before the producer begins to recover its investment.[122] The recent entry of foreign producers, supported by the vast financial resources of their governments, has greatly increased the competitive pressures faced by U.S. firms.[123] Thus, the opportunities for diversification in this direction are, in the opinion of this writer, virtually nonexistent.

Diversification into commercial non-aircraft product lines has also been carried out, and the major airframe builders have generally been more successful in those ventures than involve the transference of the technological expertise developed in their aerospace operations.[124] However, as Table VI-6 indicates, these commercial non-aircraft ventures have, in four cases, recently acted as a net drain on corporate net earnings.[125] Admittedly, some of these losses are due to the recent recession, rather than to the firm's basic inability to develop profitable commercial non-aircraft lines of business. However, the fact remains that the managerial skills required to oversee an organization that develops and produces a relatively small number of high technology products that it markets to only a small number of select customers are different from those required to mass produce and to market on a wide scale consumer-oriented goods and services. As a result, in trying to diversify into commercial non-aircraft ventures, the major military airframe builders have to be very careful where they invest their money,[126] and as the experience of Rockwell International would seem to indicate, diversification by external means (i.e. by taking over established firms) is preferable to diversification by internal means, since the former approach brings experienced managers into the firm and brings an established marketing network under its control.[127] More seriously, the limited opportunities for profitable diversification, when coupled with the relative inflexibility of the plant and equipment these firms operate, may have the effect of forcing these firms to accept (what they may regard as) subnormal profits without recourse to the avenue of relief open to firms in traditional economic theory - the ability to costlessly switch industries.[128]

Company	Aircraft[1] Comm	Gov't	Non-Aircraft[2] Comm	Gov't	Total Comm[3]	Gov't[4]	Consolidated Net Income[5]
Boeing	55.1[6]		21.2[6]		- 6[6]	- 6[6]	76.3
% Net Earnings	72		28		-	-	100
Fairchild	5.7[6]		(2.7)[7]	0.2	- 6[6]	- 6[6]	3.2
% Net Earnings	145		NM[8]	7	-	-	100
General Dynamics	7.1	14.6	35.9	23.5	43.0	38.1	81.1
% Net Earnings	9	18	44	29	53	47	100
Grumman[9]	5.6	21.9[10]	(4.0)	-	1.6	21.9	23.5
% Net Earnings	24	93	NM	-	7	93	100
LTV[11]	26.1[6,12]		(32.5)	-	- 6[6]	- 6[6]	(6.4)
% Net Earnings	NM		NM	-	-	-	100
Lockheed	(52)[13]	84[13]	12[13]	1[13]	(40)[13]	85[13]	45[13]
% Net Earnings	NM	187	27	2	NM	189	100
McDonnell Douglas	- 6[6]	- 6[6]	- 6[6]	- 6[6]	- 6[6]	- 6[6]	85.6
% Net Earnings	-	-	-	-	-	-	100
Northrop	23.8[6]		(1.1)[14]	2.0	- 6[6]	- 6[6]	24.7
% Net Earnings	96		NM	8	-	-	100
Rockwell	7.3[6]		32.2	62.1	- 6[6]	- 6[6]	101.6
% Net Earnings	7		32	61	-	-	100

1. SIC Code 372.

2. Mainly missiles and space (SIC Code 376).

3. Sum of columns 1 and 3. Entries may not sum correctly due to rounding errors.

4. Sum of columns 2 and 4. Entries may not sum correctly due to rounding errors.

5. Sum of columns 5 and 6. Net income as listed in column 7 is figured net of interest and administrative expenses and net of federal income taxes, but before extraordinary items, for all companies except LTV. Due to a substantial tax-loss carryforward, LTV reported net income of $13.1 million for 1975 though without the carryforward the company would have shown a loss.

6. Breakdown between government and commercial cannot be determined from the company's Form 10-K.

7. Parentheses indicate a loss.

8. Not meaningful.

9. See footnotes 7 and 8 of Table VI-3.

10. Includes space.

11. Breakdown provided for LTV Corp. only because figures for Vought Corp. were not available.

12. Adjusted net income for Vought Corp., which includes net income earned from commercial aircraft business.

13. Rounded to the nearest million.

14. 'Construction' plus 'Other'.

Sources: Company Form 10-K reports for company fiscal year 1975.

In the course of interviews with executives of the major military airframe builders, the author was told that these firms would like to diversify into new product lines in order to reduce their dependence on the government.[129] These firms generally view government business as relatively risky and relatively less profitable than commercial business, and they see diversification as one way to reduce the overall risks they face.[130] Yet, as this subsection has tried to point out, diversification into commercial markets poses special problems for many of these firms. It is not surprising, then, that many of them have also pursued a different approach to risk reduction, namely, the expansion of foreign markets for their goods and the development of cooperative ventures with foreign producers. These developments are discussed in the next subsection.

8. Foreign Sales and Foreign Competition

As sales of military aircraft to the U.S. government fell following the Vietnam war and as domestic sales of commercial aircraft fell during the recent recession, one of the factors that helped sustain the U.S. aerospace industry was foreign sales of military aircraft. During 1975, for example, foreign sales of military aircraft and other aerospace products amounted to $2.5 billion[131] and accounted for 350,000 jobs and for seven percent of U.S. exports.[132] As domestic opportunities diminish, the major airframe builders have tried to expand foreign sales.[133] Such sales can, however, lead to political difficulties[134] since, even if the sales are made by the manufacturer directly to the foreign government, they must be reviewed by various U.S. government agencies.[135] In addition, the foreign buyer may insist on certain conditions, such as the sharing of production with one or more firms in that country[136] or a guaranteed purchase by U.S. buyers of a certain amount of that country's exports,[137] as part of the deal.

In addition to the difficulties associated with having to make various concessions to foreign governments in order to sell airplanes, the U.S. airframe builders are meeting with increased competition from foreign builders of both commercial and military aircraft. Most of these foreign competitiors are supported financially by their governments.[138] Since many of the foreign commercial airlines are government-owned, the foreign government can direct its airline to buy domestically produced aircraft.[139] A third factor making for increased foreign competition is the multinational pooling of efforts, which permits the sharing of heavy development costs[140] and which can also serve to expand the 'guaranteed market' for a particular foreign aircraft.[141] These factors place the major U.S. airframe builders at a disadvantage, and in order to counter the risk of erosion of their foreign markets, several U.S. firms have recently entered into joint ventures with foreign aerospace firms.[142]

9. Summary

This section has provided an overview of the nine major military airframe builders in the United States. The discussion has focused not only on the firms themselves, but also on the environment within which they operate, and in particular, on their relationship with their principal customer, the United States government, on the financial and business risks they must bear, and also on the increasing foreign pressures they face. With the material presented in this section as a background, the remainder of the chapter describes how these firms conduct their long term and short term planning.

C. BACKGROUND TO THE PLANNING PROCESS; THE OBJECTIVES OF THE FIRM

1. The Planning Process

In light of the many risks associated with the aerospace business that were discussed in the previous section, as well as the apparent reduction in the number of major military aircraft programs and the intensifying foreign competition for both military and commercial aircraft sales, the following maxim culled from the office wall of an aerospace planning executive seems to this writer an appropriate way to begin this section:

"The company that doesn't plan for its future isn't likely to have one."

The overall purpose of the corporate and divisional planning conducted by the major airframe builders may be stated succinctly as follows: to allocate the company's scarce productive resources - manpower, facilities, and skilled managerial, engineering, and technical talent - and its scarce financial resources among existing and potential commercial product lines and among existing and potential government contracts in accordance with goals and objectives of the company. The 'existing' in the above statement refers mainly to *short term,* or *operational*, *planning*, with its emphasis on the firm's most efficient use of its current stocks of capital resources - both physical capital in the form of plant and equipment and human capital in the form of the knowledge and skills embodied in the firm's managers, engineers, and scientists - and its emphasis on carrying out production to meet current commitments in the most efficient manner - in terms of minimizing production costs while maintaining product quality and contract performance (in an attempt to generate maximum sustainable earnings). The 'potential' in the above statement refers mainly to *long term*, or *strategic, planning*, with its emphasis on new business - and the most effective use of research and development funds, engineers, and scientists in order to develop the expertise necessary to develop new products and secure new government contracts - and its emphasis on the most efficient use of the firm's financial resources to purchase new production facilities and new equipment and to start up or acquire new businesses.

Planning, then, takes place on two levels: short term planning, for which the time period involved is typically one year, although some of the major airframe builders carry out operational planning over longer periods, and long term planning, for which the time period involved is typically five years, although some of the major airframe builders carry out strategic planning over longer periods.[143] As described below, consistency between the long term plan and the short term plan is achieved by first formulating the long term plan and then using the first year of the long plan (or the first two years or five years if that is the firm's short term planning period) as the basis for the short term plan.

At both the strategic and the operational levels, planning is done iteratively. The nine major airframe builders are organized as multidivision companies, with one or more divisions producing aircraft and other aerospace products [144] and several divisions producing non-aerospace products, as illustrated in Figure VI-2.[145] Decision-making and much of the responsibility for planning are decentralized, although major decisions, such as those requiring capital investment, and strategic and operational plans must be cleared with the company's headquarters, which maintains its own planning staff - sometimes consisting of just one individual - and which ensures that the plans of the various divisions, when amalgamated, are consistent with the company's goals and objectives. Achieving this consistency may require several iterations between division and headquarters until the latter is satisfied with the former's plans.

The headquarters planning staff has an additional responsibility that is critical to the planning process. The corporate planning staff prepares annually an environmental forecast, which, as the name implies, characterizes the firm's operating environment over a period of years at least as long as the strategic planning period. Each division inputs information relating to its own area of expertise to the corporate planning staff, which gathers additional information relating to such areas as the general state of the economy

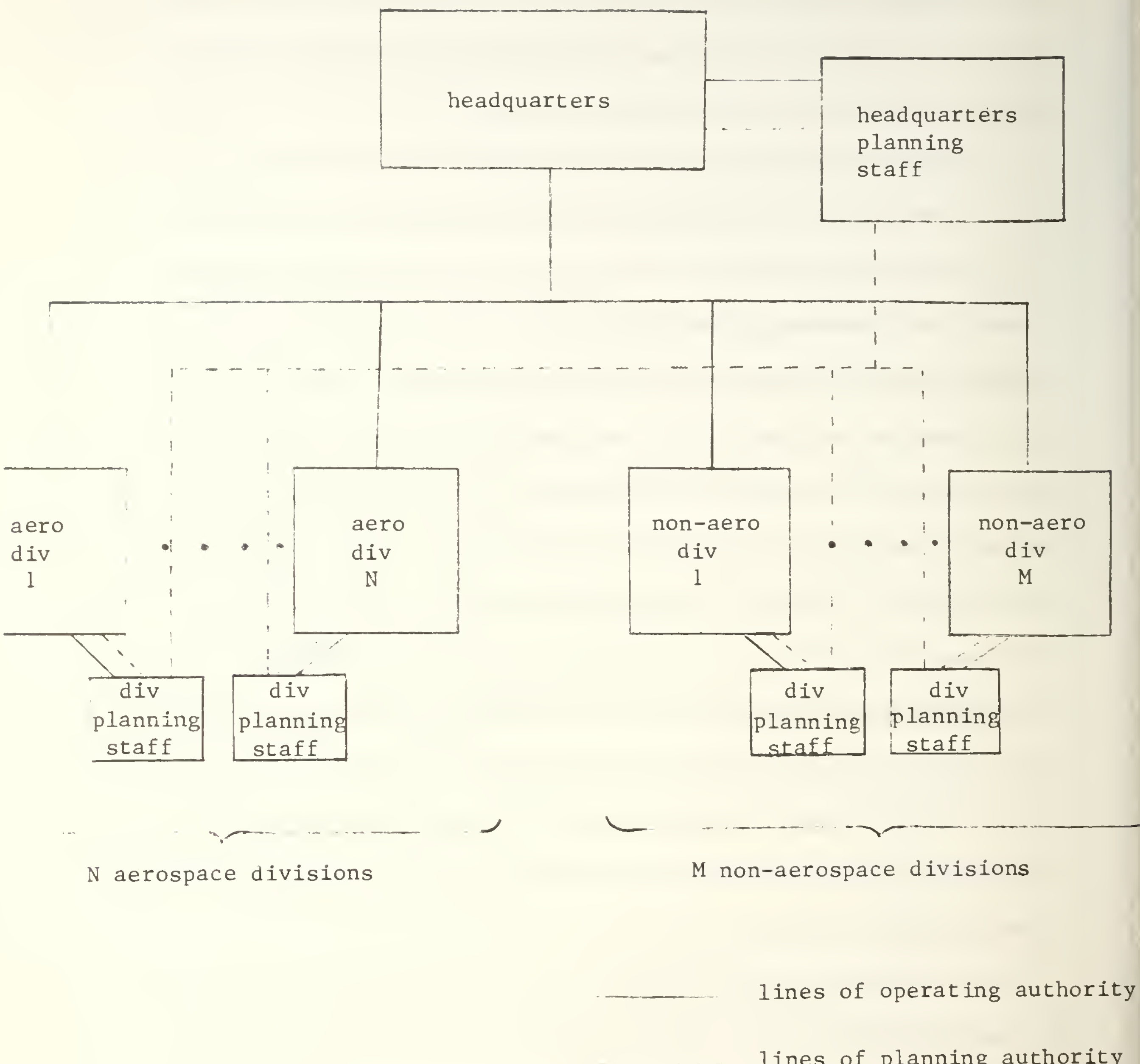

Figure VI-2 The Headquarters-Division Planning Relationship in future years, the future political environment and its likely impact on future defense budgets, etc., and prepares the final document(s). No claim is made as to the perfect accuracy of these projections, but rather, the environmental forecast is intended as a summary statement by the company of what it regards as reasonable assessments of future conditons likely to have an impact on the demand for the company's products - i.e. assessments on which the corporate planning staff would expect the divisions to base their strategic and operational plans.

The preparation and use of the environmental forecast are discussed in greater detail below, where section D focuses on the forecast itself and sections E and F discuss long term planning and short term planning, respectively, for which the environmental forecast provides important input data. The remainder of this section concentrates on the objectives of the firm, which, like the environmental forecast, underlie the planning process.

2. The Objectives of the Firm

Though a company's objectives may not be stated explicitly at the outset of the planning cycle, the acceptance or rejection of divisional objectives and plans is based to a large extent on how well these objectives and plans support the company's objectives. Indeed, the divisional long term and short term plans finally accepted by headquarters and the specific goals they contain - a sales or output goal, a profit goal, etc. - are a reflection of corporate objectives. This subsection discusses the broad objectives of the nine major military airframe builders. While recognizing that all corporations do not share identical goals, the author believes that there is sufficient commonality among the nine firms as to broad objectives to warrant the general treatment undertaken here.

Before describing the objectives of these firms, it will help to make that discussion more meaningful if the sources of the firms' objectives are discussed first. According to the traditional theories of the firm,[146] the objective of the firm is that of its shareholders, and the firm acts so as to maximize shareholder utility, which, under the appropriate assumptions, reduces to maximizing the stock market value of the firm's equity. According to an alternative point of view, as expressed by the managerial theories,[147] the objectives of the firm are set by top management, and profitability or the stock market value of equity affect managers only as constraints on their discretion to pursue alternative objectives. According to a third point of view, that of the behavioralists,[148] the firm's objectives are established through an internal bargaining process that takes place among the various special interest groups, for example, the labor force, the marketing staff, technical staff, shareholders, top management, middle management, etc., that compromise the firm and its 'owners'.

Based on personal interviews with executives of the nine firms, it is the belief of this writer that none of the three views discussed in the preceding paragraph is entirely correct. Rather, it is the author's view that the objectives of each firm are set by top management, principally the president and the chairman of the board of directors, together with the other members of the board of directors.[149] It is up to top management to weigh the specific objectives of the various special interest groups, to resolve any conflicts that might arise, and to ensure compliance on the part of these groups with regard to the firm's established objectives. In particular, it is the board of directors, which includes representatives of shareholders and top management, rather than the shareholders themselves, that sets the firm's dividend policy and that also makes the major financial and investment decisions that affect the firm's future ability to pay dividends. It is also the board of directors that sets the compensation levels for top management. Of course, the relative weights assigned to a particular set of objectives could vary considerably from one firm to another, depending, for example, on the degree of influence of one or more key shareholders,[150] and could also vary over time for any one firm. Yet, the role of top management in establishing objectives would imply that, to the extent that the objectives of managers conflict with those of shareholders, shareholders' goals are not likely to be followed exclusively (in contrast to what the traditional theories have implied), and the role of the key shareholders in establishing the objectives of the firm would also imply that managers' goals are not likely to be followed exclusively either (in contrast to what the managerial theories have implied). Moreover, in establishing objectives, top management and other directors can take into account the desires of the various special interest groups within the firm, as suggested in the behavioralist approach, but in a manner suggestive of greater consistency in overall objectives over time than the behavioralists have implied.[151]

Broadly, the objectives of the nine major military airframe builders, as interpreted by this writer, fall into five classes: (i) sales objectives; (ii) a profit, or earnings, objective; (iii) a product quality, or in the case of weapons systems, weapons system performance, objective; (iv) a backlog, or new business, objective; and (v) a managerial emoluments objective. The first class consists of multiple objectives in order to reflect managements' desire to balance government and commercial business, while the other four classes consist of a single objective each. The remainder of this section is devoted to a discussion of these five classes of objectives.

The sales objectives reflect top managements' interest in size and diversification. In a model of the typical DOD airframe contractor to be developed in a subsequent paper, four sales objectives will be specified, one each for sales of aircraft and related parts and equipment to the government, sales of other products to the government, sales of aircraft and related parts and equipment to commercial buyers, and sales of other commercial goods. Alternatively, given the initial levels of these quantities, the four objectives could be restated in equivalent form as growth objectives. The reason for stating four sales objectives, rather than merely having two - one for government sales and one for commercial sales - or one - combining all sales figures into a single measure - is so that diversification between aircraft and non-aircraft products, as well as diversification between government and non-government sales, can be represented. As discussed in the previous section, several of the nine firms have in recent years tried to develop new commercial non-aerospace ventures and two of the nine, LTV and Rockwell, are already widely diversified away from government sales and away from aircraft sales. In addition, all have some non-aircraft sales to the government - chiefly missiles and space equipment or ships. They are interested in diversifying in the two general directions mentioned above, though as David Lewis, Chairman of General Dynamics, recently made very clear, these firms are going to continue to actively seek government contracts to produce military aircraft,[152] and diversification is going to take place through the expansion of sales in commercial and non-aircraft ventures and not through the intentional contraction of their airframe business. It is felt by this writer that the value of increased sales in each of the four product areas as well as the importance of relative increases in non-government and non-aircraft sales - i.e. diversification - are best captured by stating multiple sales objectives.

The second objective, which relates to profits, is important for at least two reasons. Profits serve as an index of the efficiency with which management employs the firm's assets. Also, profits serve an important financial function. They represent the surplus of revenue over costs that may be used to pay dividends, and thereby satisfy the owners of the firm's equity shares; and, after dividends have been paid, the remainder represents retained earnings that may be used to finance new investment in plant and equipment or to acquire other firms.

The third objective, maintaining high product quality and strong weapons system performance, is highly important to the managers of these firms, many of whom have engineering backgrounds and many of whose families, for example, the McDonnells and the Rockwells, have been in the aerospace business for generations. Product quality is so important also because each firm's managers want their company's name associated with technical excellence. Not only does such a reputation help foster a favorable public attitude toward the firm, but it also helps the company maintain its position as a prime contractor [153] and can contribute to sales of its commercial products that bear the company's name.[154] More importantly, high product quality and strong weapons system performance contribute to the firm's long run profitability.

The fourth objective, enlarging the business backlog, is particularly important in the case of aircraft sales, where production lead times are normally several months or more and where a temporary shutting down of a production line could cost several million dollars. A larger backlog provides some security and, as discussed below in section E, pushes the firm's going-out-of-business curve outward and makes the task of long term planning somewhat easier.

The last objective, managerial emoluments, reflects managements' interest in its own level of compensation. This includes not only salary, which is fully taxable, but also the perquisites, such as stock options, the earnings on which are taxed at the lower capital gains rate (provided, of course, the securities are held long enough to qualify for special tax treatment), and expense accounts, company cars, etc., which are not taxable.

In the next chapter the four sales objectives and the profit, product quality, backlog, and managerial emoluments objectives will be used as arguments of a managerial utility function that will constitute the objective function in the mathematical programming formulation of the typical DOD airframe builder's planning problem. For the purposes of this descriptive chapter, however, all these objectives will remain in the background. As part of the long term and short term planning processes described below, top management evaluates proposed projects in terms of the five classes of objectives,[155] and an important part of the two planning processes is the formulation of divisional goals and objectives, which top management reviews carefully and which, once approved by top management, are the focal point around which the divisonal plans are structured.

D. THE ENVIRONMENTAL FORECAST

1. The Corporate Planning Cycle

The corporate planning cycle for each of the nine major military airframe builders consists of the following three primary phases: preparation of the environmental forecast, development of the corporate long range plan, and specification of the corporate operating plan. These phases occur sequentially and together they span the company's entire fiscal year.[156] That is, planning for fiscal year T and beyond takes place throughout fiscal year T-1.

The primary phases of the corporate planning cycle are illustrated in Figure VI-3. Each company's fiscal year is divided into four quarters. During the first quarter of year T-1 the environmental forecast is prepared. During the second and third quarters of year T-1 various long range planning studies are carried out and reviewed, culminating in the company's long range plan for years T and beyond. During the fourth quarter of year T-1 the operating plan for year T is established, essentially by specifying the first year of the long range plan in greater detail.

Qtr I Year T-1	Qtr II Year T-1	Qtr III Year T-1	Qtr IV Year T-1	Year T
prepare environmental forecast	formulate long range plan for Year T and beyond		specify Year T operating plan	implementYear operating pla begin a new planning cycl

Figure VI-3 The Corporate Planning Cycle

2. The Environmental Forecast: Its Purpose and Its Structure

The publication of the company's environmental forecast and its distribution among the firm's operating divisions initiates the planning cycle. The form and content of the environmental forecast may vary from one company to another. In some cases the forecast is a formal document that carefully analyzes and weighs political, social, economic, and business factors that may in the future have an impact on the firm. In other cases it is an informal paper prepared by a small staff who gather information and supporting data from a variety of public and private sources to highlight those significant factors likely to affect the firm in the future. In either case the purpose of the environmental forecast is the same.

The purpose of the environmental forecast is, as its name suggests, to summarize within a single document the company's prognosis of the most likely external business environment within which it can expect to operate over its long term planning horizon. More importantly, the environmental forecast establishes a common set of assumptions that, when used in the next two phases of the planning cycle, give the plans of the individual operating divisions a vital degree of consistency that would otherwise be lacking.

The environmental forecast is written in conjunction with the firm's operating divisions, each of which is asked to submit to the corporate planning staff early in the first quarter information within its area of expertise that is relevant to its planning problem. Such a procedure has two main advantages. First, the divisions are involved in the planning process very early in the planning cycle and, since they provide much of the information on which the environmental forecast is based, they are more likely to view the environmental forecast's projections as reasonable than they would if the projections had been developed by the corporate planning staff without consultation. Second, the corporate planning staff can utilize the marketing expertise that is available in the divisions, thus enabling them to spend more time on analysis, rather than on data collection.

The structure of the environmental forecast varies from one company to another, depending on the needs of divisional planners.

Generally, the forecast treats three main subject areas. First, it describes the international environment and how such factors as international political tensions and international economic trends are likely to affect world demand for military aircraft. Second, it discusses the domestic political and economic environment and projects the size of the defense budget over the long term planning horizon. Of particular concern to the company is how changes in the defense budget might affect the company's current military programs as well as any future military programs on which the company is planning to bid. Third, it discusses factors relevant to the specific product markets, both military and commercial, in which the company sells its goods. For example, if the company produces commercial aircraft, such factors as the expected future growth of airline passenger traffic, the expected future growth of air cargo shipments, the expected future impact of fuel price changes, and regulatory trends, would, to the extent that meaningful projections can be made, give planners in the commercial aircraft division a good planning base from which to work.

A useful by-product of the process leading to the environmental forecast is a set of analyses, one for each operating division, of the strengths and weaknesses of the company's operations. Each such analysis assesses such factors as the strengths and weaknesses of that division's products in relation to the products of the firm's major competitors (in the case of commercial products) and problems that might arise in connection with government contracts (in the case of military weapons systems). The analysis of the division's strengths and weaknesses is used by the divisional planning staff in conjunction with the environmental forecast that comes down from the corporate planning staff near the end of the first quarter to formulate the division's long term plan.

E. LONG TERM PLANNING: PORTFOLIO SELECTION AND THE GOING-OUT-OF-BUSINESS CURVE

1. Introduction

The second phase of the corporate planning cycle consists of long term planning. The purpose of long term, or strategic, planning is to determine the corporation's business strategy over the long term planning horizon - typically a period of five years' duration. The long term planning process leads to a long term plan for each of the firm's operating divisions that is consistent with the corporation's goals and objectives and that spells out that division's role - i.e. its business strategy - in meeting the company's goals and objectives.

This section describes the long term planning process that is followed within the nine major military airframe builders. At the outset it should be noted that long term planning, as practiced by these firms, is not designed to lead necessarily to an 'optimum' plan. The planning process is an iterative one, as described below, though the iterations are designed to achieve feasibility and robustness rather than to ensure optimality. During the long term planning process headquarters and the operating divisions search for a long term plan - essentially a collection of business strategies together with the facilities, financial, research and development, and manpower requirements needed to support them - that is *feasible* in the sense of leading to the attainment of corporate goals and objectives if the 'expected' environment (as described in the environmental forecast) materializes and that is also *robust* in the sense that the plan will also permit the firm's goals and objectives to be attained if the business environment in general, and market conditions in particular, should vary from what is expected in a manner that top management perceives as reasonable.

The fact that these firms do not strive for a plan that is optimal - in the sense that it leads to a higher level of managerial utility, a higher stock market value of the firm's equity, or a higher value of some other function or quantity than any other feasible plan - is due to at least two factors, each of which was mentioned by several of the executives interviewed by the author. First, gathering the information required to formulate the long term planning problem as an optimization problem, say as a mathematical programming problem, would, in the opinions of virtually all the executives interviewed, be prohibitively costly in terms of time and money.[157] Second, even if the problem could be formulated in a manner acceptable to top management, a solution would have to be obtained, and in the opinion of most of the executives interviewed, the size and complexity of the problem would make it prohibitively costly to find the solution.[158, 159] These cost considerations would imply, if they are correct, that even though the long term plans that are developed are not necessarily optimal, the planning process itself may be optimal - in the sense that, of all the procedures available for achieving any particular set of feasible and robust plans, it involves the least cost.[160]

2. Long Term Planning at the Divisional Level[161]

At approximately the same time the corporate planning staff releases the environmental forecast - late in the first quarter or early in the second quarter - top management distributes among the company's operating divisions a statement of the corporation's long term goals and objectives and a set of strategic guidelines. From this point onward in the corporate planning cycle, the primary organizational responsibility for planning shifts from headquarters to the operating divisions. The corporate planning staff continues to be involved in the planning process, but its role involves coordinating the planning efforts of the divisions, analyzing the plans of the divisions, and amalgamating the plans of the divisions for review by top management. The long term plans themselves are prepared at the divisional level.

The long term planning process as carried out at the divisional level is illustrated in Figure VI-4. The process begins with the environmental forecast that establishes the projected future states of the firm's operating environment and with the statement of corporate goals and objectives and the set of strategic guidelines handed down by top management. The environmental forecast together with the set of strategic guidelines specify the constraints imposed by

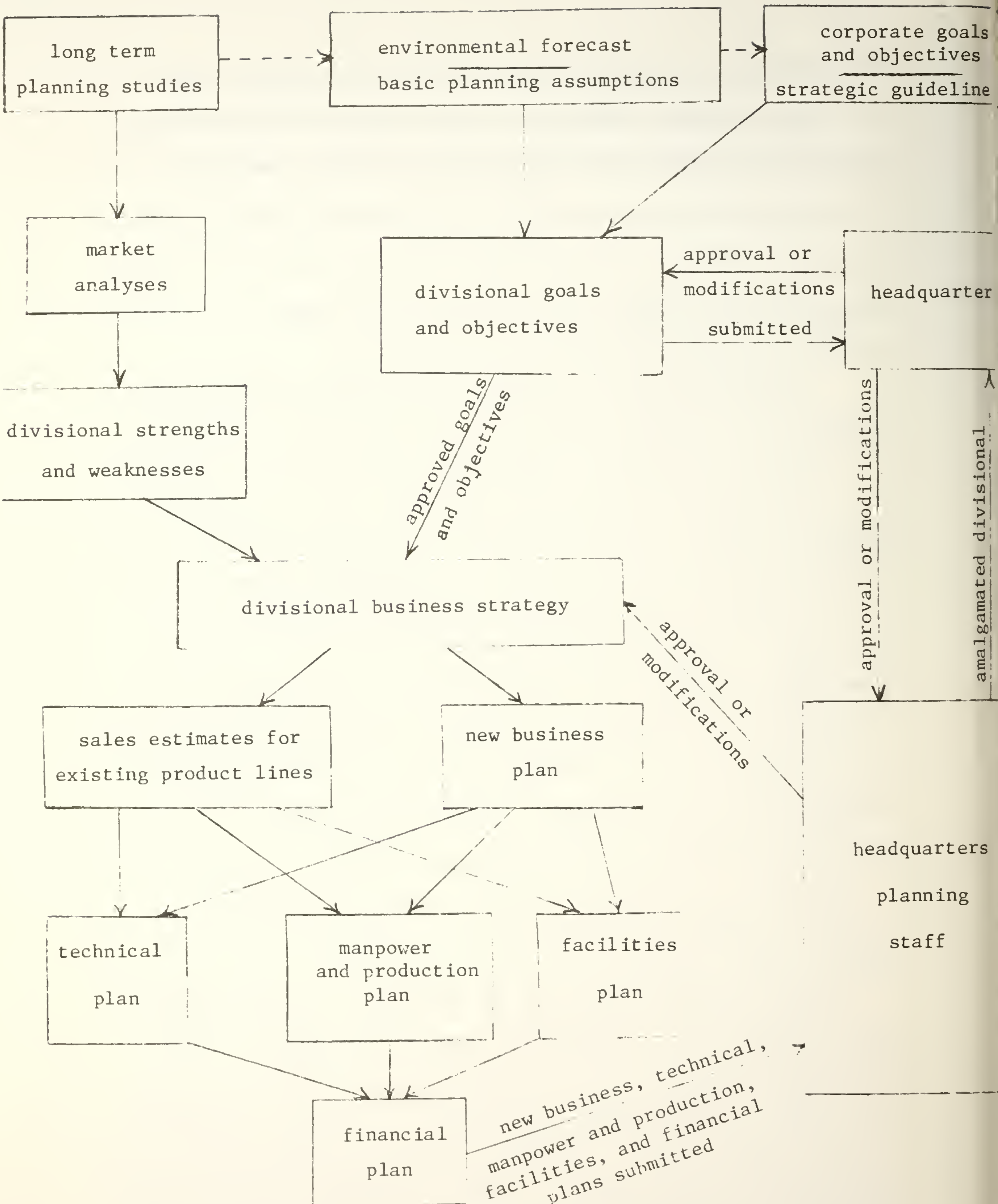

Figure VI-4 The Divisional Long Term Planning Cycle top management, within which the division must develop its business strategy.

An important part of the long term planning process is the preparation of long term planning studies, most of which are carried out at the divisional level. As indicated in Figure VI-4, the divisional long term planning studies, which are initiated in the first quarter, are the basis for the division's input to the environmental forecast, which, in turn, is carefully considered by top management when it formulates the corporation's goals and objectives and the strategic guidelines. That is, the goals, objectives, and guidelines do reflect what top management perceives as the firm's likely future operating environment.

Even after the environmental forecast has been distributed, the divisional long term planning studies continue. As indicated in Figure VI-4,the later long term planning studies generally narrow in focus, providing specific market analyses, e.g. the state of future demand in specific product markets, and detailing the strengths and weaknesses of the division vis-à-vis the division's major competitors in each of its product markets.

Concomitant with the preparation of the planning studies is the formulation of the division's goals and objectives. As indicated in Figure VI-4, the two major inputs used by divisional managers are the environmental forecast and top management's statement of corporate goals and objectives. The divisional goals and objectives are submitted to top management, which must approve them before long term planning can proceed any further.

After the divisional goals and objectives have been approved and the division's long term marketing studies and its assessment of its strengths and weaknesses have been completed, the division's management prepares the division's long term business strategy for achieving its goals and objectives, and by implication, for achieving the goals and objectives of the corporation. The business strategy covers such items as product design, pricing policy, development of new customer relationships, etc., for existing product lines as well as its strategies concerning new product lines, i.e. in which direction(s) it intends to diversify, what new product lines it intends to develop, when the new products should be introduced to the market place, for what upcoming government programs it intends to bid, etc. From the divisional business strategy flows sales estimates for existing product lines and the new business plan.

Generally, the business strategy is developed and sales estimates are generated and the new business plan is formulated by product line and by government contract. Often the corporation's principal operating units will consist of several subdivisions each, with each subdivision responsible for one or more individual product lines. One of the tasks assigned to the planning staff of the division - i.e. the principal operating unit [162] - is to integrate the various sales estimates provided by these subdivisions with the new business plan for the division in order to establish the following plans: (i) the technical plan, which lists the technical requirements and the size of the technical staff and the amount of funds needed to support the division's research and development effort; (ii) the manpower and production plan, which lists planned output and the numbers and cost of production and managerial personnel needed to meet these targets; and (iii) the facilities plan, which lists the types and amounts of plant and equipment needed and the costs of maintaining existing facilities and investing in new facilities.[163] These three plans, together with sales estimates and the new business plan, are used to generate the division's long term financial plan.

Once completed, the financial plan, together with the new business, technical, manpower and production, and facilities plans, are submitted to the corporate planning staff, which reviews the division's plans. If the corporate planning staff is not satisfied, for example, because the plan contains inconsistencies or because the division appears to have deviated too far from the assumptions contained in the environmental forecast, it sends the plan back to the division. If the corporate planning staff accepts the plan, it then amalgamates the plan with the plans of all the other operating divisions. The amalgamated plans form the provisional corporate plan, which is submitted to top management for its approval. Top management reviews the provisional corporate plan as well as summaries of the divisional plans and checks for consistency with corporate goals and objectives. Specifically, top management checks the provisional corporate plan for feasibility - i.e. will the plan enable the corporation's goals and objectives to be attained? - and for robustness - i.e. if the plan is carried out, but environmental conditions change from what has been anticipated, how seriously will the corporation's ability to meet its goals be impaired? Top management is also likely to check for other desirable characteristics, such as flexibility - i.e. is the plan sufficiently flexible that certain actions can be postponed without having severe consequences for other parts of the plan, or does the plan call for substantial irreversible investments very early in the planning period? If any of the divisional plans require modification, they are returned to the division. As indicated in Figure VI-4, the division will probably have to alter its business strategy. Once the long term plan has been accepted by headquarters, the long term planning phase of the corporate planning cycle has been completed.

The above description of the long term planning process was intentionally kept general in order to give the reader an intuitive feel for the overall process. The remainder of this section looks at important aspects of this overall process more closely.

3. Long Term Planning: Government Sales and the Going-Out-Of-Business Curve

The description of the long term planning process provided in the preceding subsection did not distinguish between government sales and commercial sales. In Figure VI-4 government sales and commercial sales were lumped together in the blocks labeled 'sales estimates for existing product lines' and 'new business plan'. Yet, because of the important differences between doing business with the government and doing business with commercial customers, which were discussed in section B, the long term planning process must treat these two classes of sales somewhat differently. This subsection discusses long term planning for government sales, and the next discusses long term planning for commercial sales.

Long term planning, as it relates to government sales, involves the two types of planning indicated in Figure VI-4: (i) planning for the future resource requirements implied under existing government production contracts and under anticipated follow-on government production contracts and (ii) determining the upcoming government programs on which to bid and planning for the resources - particularly technical staff and research and development facilities requirements - required to launch a successful bid for each. One of the devices planners use to illustrate both aspects of government sales planning is what is called the going-out-of-business curve, an example of which is the heavy curve in Figure VI-5. The going-out-of-business curve shows what would happen to government sales if current programs were not extended beyond their present termination dates and if no new programs were won. If the viability of the division were heavily dependent on government sales, then a lack of extensions and an absence of new programs could literally drive the division out of existence (and hence the curve's name). The sales impact of new programs the division

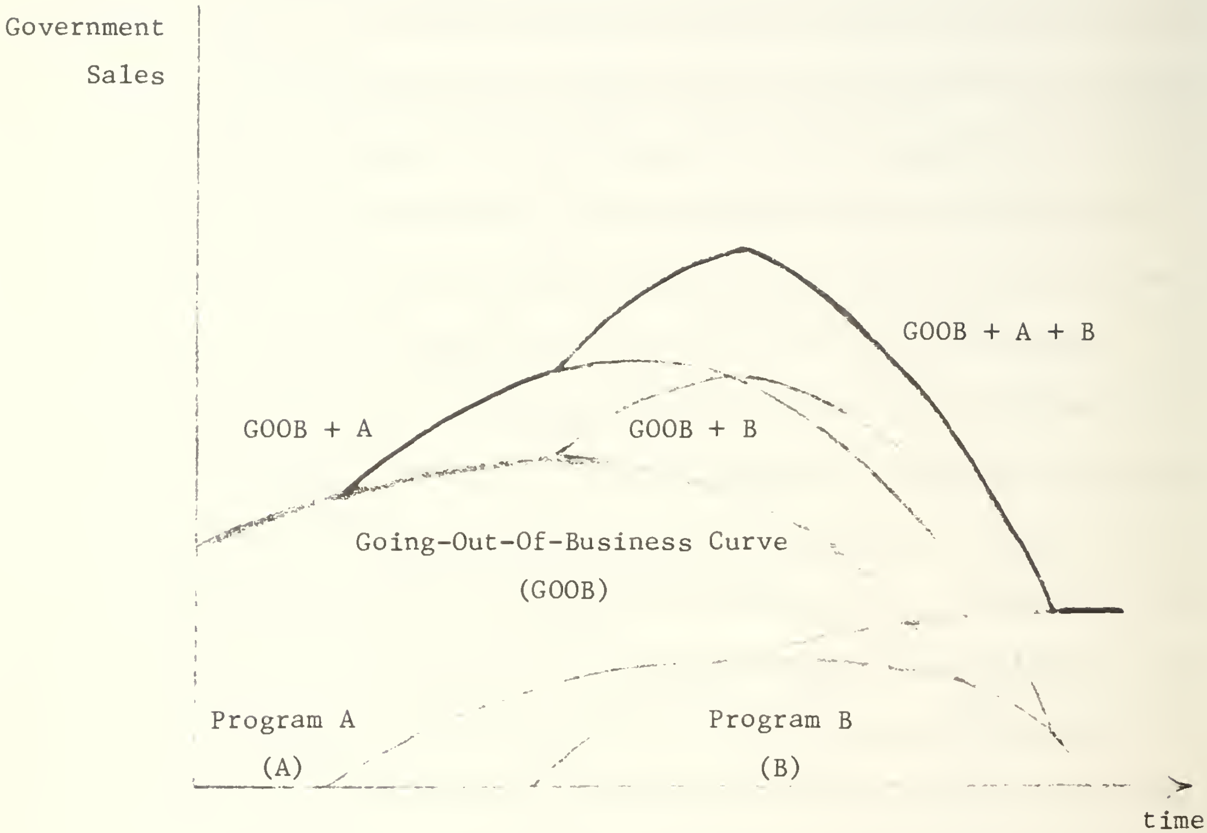

Figure VI-5 Going-Out-Of-Business Curve With Overlays hopes to win and contract extensions it hopes to be granted can be overlaid, as shown in the figure, to provide a graphical representation of government sales for each year covered by the long term plan. Each of the modified going-out-of-business curves shows the growth pattern of government sales over the long term planning period, contingent upon contract extensions and/or new programs.

The type of planning that is done for existing government programs - those that underlie the going-out-of-business curve - depends on whether the program is in the production stage or in the development and testing stage. For production contracts, planning is relatively easy since the required technical advances have already been made, and in the case of aircraft production, the division is likely to have estimated the position of the learning curve. Production planning, which is based largely on projections of future needs supplied to the contractor by the government, is concerned mainly with manpower and facilities needs. In aircraft production the direct labor input per airplane will fall as the cumulative number of aircraft produced increases due to the learning curve effect discussed in section B. A second factor that needs to be considered in production planning is the program's peak production rate - seldom do programs call for a uniform production rate - for this can affect the need for plant floor space and storage space for inventories of parts and equipment. A third factor is the program's termination risk. If termination occurs, the divison will find itself with excess capacity unless some contingency use for the facilities has been planned. However, according to the executives interviewed by the author, estimation of the termination risk is so highly subjective as to be, in practice, of little use. In addition, the overcapacity that exists throughout much of the industry [164] and the limited alternative uses to which these production facilities can be put, imply that termination will in most cases result in additional sparce capacity until such time as a new program can be won.

For development and testing contracts the planning required is more difficult than for production contracts because the division and the firm are interested in making the required technical advances within the required time and within the projected costs. The prediction of the resources - mainly engineering and design personnel, development facilities, and the facilities and manpower required to assemble the test and evaluation aircraft - needed to develop the product called for under the contract is inherently more difficult than projecting production resource requirements and costs. Even though development and testing contracts are typically of the cost-plus variety, thereby reducing the firm's share of the overall cost risk below what it would be under a fixed price contract, the contractor is anxious that its customer - the government - be satisfied with the final product. It is important that the product perform well enough and that its cost of production be kept low enough that production funding will not be threatened.

Indeed, several executives told the author that their companies would make additional improvements not called for in the contract, possibly even when there was virtually no chance of immediate reimbursement for costs incurred, in order to improve the product's performance and thereby increase the likelihood of a larger and longer production run.

The type of planning that must be carried out for new programs - planning that involves the allocation of scientific and research talent, the allocation of IR&D funds plus the company's own research and development funds, and the allocation of engineering and design personnel and manpower and facilities to be used in the development of such items as prototypes that will be flown in a competition to determine the winner of relatively more lucrative production contracts - is more difficult than the planning that is conducted for existing programs. The uncertainities and business pressures are normally greater since the division is interested not only in meeting the specifications of the research and prototype development contracts assigned to it by headquarters, but also in placing the company in an advantageous position for any follow-on contracts.

The efficient allocation of research and development funds and personnel is of critical importance to the firm because of the long lead times required for research and development and also because of the small number of new major programs. In planning their research and development program, the division and the company must have some conception of what the government's needs will be many years in advance. For this reason, long term planning studies often try to look out beyond the Department of Defense's Five Year Defense Plan in order to predict the military's needs far enough in advance to permit the necessary research to be undertaken.[165] In other words, one of the responsibilities with which divisional planners are charged is the responsibility of allocating 'pure research' moneys - some of which are reimbursed through IR&D awards - in such a way that the company (and the division) will have accumulated a sufficient store of technical and scientific skills by the time a new program is formally announced to be competitive in its attempt to win the program.

In allocating the research and development resources over proposed new programs, divisional planners must analyze the risks and potential returns associated with possible new programs and allocate the resources to those programs most likely to lead to accomplishment of the division's goals and objectives. Generally, it is impossible for a company to bid competitively on every major new program, so selectivity is required, and typically, the decision as to the major new programs the company is going to try to win will be made by top management and will be incorporated within the company's and within the appropriate division's goals and objectives. It is then up to that division's managers and planners to allocate the division's research and development resources to meet those goals and objectives.[166]

4. Long Term Planning: Commercial Sales

As in the case of government sales, planning for commerical sales is best treated as two types of planning: (i) planning resource requirements for existing product lines and (ii) planning resource requirements for new business, including both new products, e.g. a new generation of commercial passenger aircraft, and new lines of business, e.g. the acquisition of another company as part of a diversification strategy. Both aspects of commercial sales planning are discussed below.

In general, the factors that must be considered when planning commercial sales are different from those that must be considered when planning government sales. Variations in government sales are due to changes in the needs of the sole customer or to changes in the sole customer's ability to purchase weapons that result from changes in the levels of Congressional funding of major weapons system programs, which in turn can often be traced to political factors, i.e. to the political cycle. In contrast, commercial sales are made to a variety of customers who select from among a variety of products that can meet their needs and whose ability to pay is largely affected by the condition of their balance sheets and by the state of demand for their goods and services, which in turn can be traced to a variety of economic factors, i.e. to the business cycle.

Long term commercial sales planning for existing product lines requires a careful analysis of each competitor's products, and unlike sales of existing products to the government, which take place under conditions of bilateral monopoly, commercial markets, such as those for commercial passenger aircraft, are usually served by a number of producers, each of which carefully watches its market share - its percentage of total market sales. In the case of commercial aircraft, planning for production is similar to planning for the production of military aircraft in that the learning curve effect must be taken into account. However, whereas a single contract for military aircraft will specify the delivery schedule, and by implication, the production schedule, for that aircraft for a year, the production schedule for commercial aircraft is typically less certain since the number of customers is much greater; since the continued ability of each purchaser to pay for the planes it has agreed to buy is somewhat less certain; since each commercial customer's purchase of a particular aircraft is more easily postponed both because commercial aircraft can be leased and because the introduction of a new commercial aircraft does not normally have associated with it the sense of urgency that typically accompanies the introduction of a new military aircraft; and since a potential sale can be lost to one of the commercial producer's competitors. Hence long term planning for sales of commercial aircraft, as well as for sales of other commercial products, requires contingency planning in the form of 'high' and 'low' sales estimates for each year, in addition to the 'best' estimate of future sales for each year. In the case of commercial aircraft, the best estimates are obtained after careful analyses of trends in first, commercial airline passenger traffic and second, in the future demand for air cargo shipments have been performed. These analyses are used to project future commercial airline fleet requirements for new aircraft, and in light of financial projections, to predict the likely demand for new aircraft in each year over the long term planning period. The 'high' and 'low' estimates may be obtained by applying some simple rule of thumb, such as 'add 10 percent to the best estimate to obtain the high estimate and subtract 10 percent from the best estimate to obtain the low estimate for each year'. Alternatively, high and low estimates may be obtained by varying the assumptions on which the best estimates were based and by applying the same estimation procedures to an 'optimistic' set of assumptions and to a 'pessimistic' set of assumptions. As illustrated below by Figure VI-6, this results in three sets of demand projections for each year, one for each of three states of nature: the expected state, as specified in the environmental forecast, the optimistic state, and the pessimistic state.

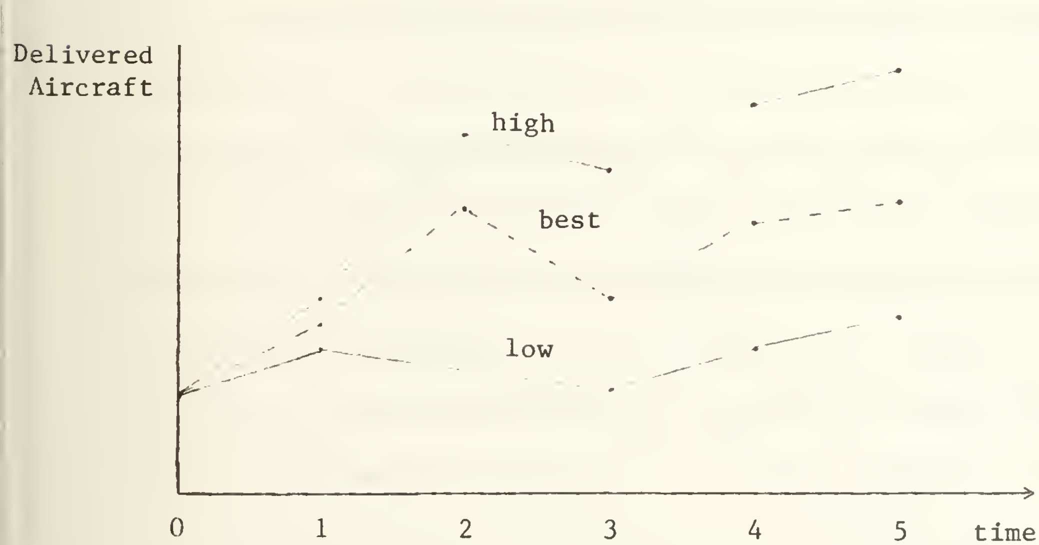

Figure VI-6 Projections of Long Term Demand for Commercial Aircraft

One of top management's more difficult decisions, which can greatly affect the long term plan, is the decision regarding what to do when the demand for a particular commercial airplane has been very weak and threatens to remain weak for some time. Commercial aircraft sales are generally made on a contractual basis, but when the financial positions of the commercial airlines worsen seriously, as they have in recent years, the airlines are loath to enter into long term contracts.[167] In the absence of long term contracts or some other indication of likely future demand, the commercial airplane maker has essentially three choices: (i) to continue production at a low rate and hope that future sales will materialize,[168] (ii) to introduce product variations, such as a 'stretch version' of the airplane - to try to increase sales,[169] or (iii) to close down the production line. The first two choices involve additional costs and potentially large financial risks,[170] while the third may require a massive write-off that can adversely affect the firm's financial health over a period of several years.[171]

Planning for new commercial business involves, as in the case of planning for new government programs, considerably greater uncertainty than planning for sales of existing products. In the case of a new generation of commercial passenger aircraft, there are the risks discussed in section B. The initial investment required often exceeds the company's net worth, and if the introduction of the new plane has been poorly timed or if potential demand for such a plane has been seriously overestimated, the company's viability may be threatened. In planning for diversification into new product lines there are several factors that need to be considered. These include the product line's compatibility - in terms of the nature of the product and the requirements for marketing it - with existing product lines, the need to hire production and staff workers, the need to purchase equipment, and the financial resources that will have to be tied up in the new product line. In addition, if diversification is to take place by external means, there is the problem of selecting a takeover candidate and launching a successful bid for a controlling interest in the firm. Normally these decisions and the required financial arrangements are made at the headquarters level, although the formulation of the technical, manpower and production, facilities, and financial plans involving the new product line are the responsibility of the division to which the new product line has been assigned.

Once the sales estimates and new business plans for both commercial sales and government sales have been prepared and reviewed by divisional management, divisional planners can derive the technical plan, the manpower and production plan, and the facilities plan for the division as a whole, and then from these, the division's financial plan. From there the long term planning process proceeds as described in subsection 2 of this section.

5. Long Term Planning: Human Capital and Fixed Capital

The preceding subsection explored several of the important planning considerations that underlie the blocks labeled 'sales estimates for existing product lines' and 'new business plan' in Figure VI-4. This subsection deals with the next stage in the divisional long term planning process, and in particular, with the technical plan and with the facilities plan. The former is primarily concerned with the allocation of the division's scientific and engineering talent - human capital - whereas the latter is mainly concerned with the allocation of plant and equipment - fixed capital.

The economic literature in recent years has contained many books and papers that have explored the economic significance of human capital.[172] The purpose of this section is not to review that literature, but rather, it is first, to characterize human capital and to indicate its importance to the major military airframe builders, and second, to indicate how the existence and importance of human capital affect the long term planning process in these firms.

Following Schultz and others, a firm's 'human capital' is defined as the scientific and technical knowledge and skills embodied in the scientists, engineers, designers, and technicians who work for the firm.[173] In contrast, fixed capital consists of durable goods such as plant and equipment.[174] Both types of capital are valued by the firm for the services they provide. In the case of human capital, there are the services provided during the research and development phases of a weapons system program, and in the case of fixed capital, there are the services provided during the production phase of the program.[175] In addition, both types of capital normally require an investment on the part of the firm. Fixed capital is normally purchased and maintained by the firm, although firms often lease plant and equipment,[176] while human capital must be accumulated through research and development activities funded, at least in part, by the firm, although firms can in a sense lease human capital by hiring engineers and scientists who have worked on similar projects for other aerospace firms.[177]

There are also important differences between human capital and fixed capital. First, human capital tends to accumulate through use (i.e. experience) as skills and techniques are acquired and as valuable lessons are learned from past mistakes, whereas fixed capital tends to deteriorate through use. Second, the typical airframe builder tends to enjoy greater flexibility in its use of human capital than in its use of fixed capital. Human capital can be gained or disposed of through the hiring or laying off, respectively, of an individual, whereas fixed capital, which comes in larger units, typically requires much greater cash outlays when acquired and, because of the limited alternative uses for much of the equipment, often can be sold only at a loss. Third, knowledge and skills are transferable so that improvements in human capital can be transmitted from one individual to another, whereas embodying technological improvements in machinery more often requires that a completely new machine be built. Fourth, due to differences in individual learning ability and due also to the importance of recent experience on similar projects to the learning effect that underlies the growth of human capital, the quality of human capital about to be hired by a firm is generally more difficult for a manager to evaluate than the quality of a piece of equipment for which engineering specifications and various test data are available.[178] Fifth and last, it is possible to order a particular piece of equipment or a plant meeting certain specifications, but to hire an individual with the requisite scientific background and experience can involve a costly and time-consuming search process. It should be noted that the last two factors are two of the major reasons why the major airframe builders are anxious to stabilize their scientific, engineering, design, and technical staffs.

The importance of human capital to the major airframe builders lies in the role each firm's scientists, engineers, designers, and technicians play during the research and development phases of a major weapons program. As discussed in section B, the development of a new generation of aircraft typically calls for several advances in the state of the art. Second, the military buyer typically outlines its needs for a new weapons system, but leaves it up to the several firms willing to enter the competition for the program to submit specific designs. Similarly, the development of a prototype requires a great deal of engineering and design work, and as the performance and overall effectiveness - i.e. the quality - of what is produced weighs so heavily in the final decision,[179] the firm's success in winning new programs is heavily dependent on how well its scientists, engineers, designers and technicians perform.

The implications of the critical role played by human capital for the long term planning process of the major airframe builders are first, that scientists, engineers, designers, and technicians are treated as a class of labor distinct from production workers, and second, that these highly skilled workers are treated more like fixed capital than like labor. That is, planners make a conscious effort to time phase major programs in such a way that the staff embodying human capital can remain fully employed and reasonably stable over time.[180] It may also mean that, on occasion, a firm will bid on a program or on a piece of a program, at least in part, because it needs work to provide stable employment opportunities for its scientific and technical staff.

Due to the overcapacity throughout much of the airframe industry, which was discussed in section B, as well as the paucity of new major weapons programs, it is the allocation of human capital, rather than the allocation of fixed capital that is the more critical capital allocation problem facing these firms' planners. In particular, the decline in aircraft sales since 1968 has forced over 70,000 scientists, engineers, and technicians to leave the industry,[181] many of them for good, and this decline in the pool of available talent has made the airframe builders increasingly reluctant to release these people for fear that in the future they might not be able to rehire sufficient numbers when the need arises. While under more favorable business conditions, with demand pressing against capacity, the airframe builders would also face the problem of having to decide how to allocate spare productive capacity and the decision as to whether to expand productive capacity, the fixed capital allocation problem is not a pressing one at the present time.

6. Long Term Planning as a Portfolio Selection Problem

Up to this point in this section, the discussion has been mainly descriptive, rather than analytical, and has focused on long term planning at the divisional level. This subsection considers long term planning at the headquarters level and suggests that, in reviewing the amalgamated divisional plans, top management approaches the long term planning problem in much the same way that an individual investor approaches the problem of selecting the portfolio of securities that, in terms of his relative risk aversion, provides the proper balance of risk and return.

As indicated in Figure VI-4, once the division has completed its financial plan, which lists the financial requirements implied by its technical, manpower and production, and facilities plans, it submits these four plans together with the new business plan to the headquarters planning staff for their review. The headquarters planning staff may require that certain divisions modify their plans. After the required modifications have been made to the satisfaction of the corporate planning staff, the divisional long term plans are amalgamated into a provisional corporate long term plan, which is typically broken down into a corporate new business plan, a corporate technical plan, a corporate manpower and production plan, a corporate facilities plan, and a corporate financial plan, each of which is, with the exception of the corporate financial plan, an amalgamation of the respective divisional plans.

In addition, there may be one or more divisions that perform support, rather than operational, duties. For example, there may be a computer services division that provides for all the data processing needs of the operating divisions but does not sell to outside users. Even if it sells to outside users, these sales may be peripheral to the division's support function. In that case, the headquarters planning staff may find it more efficient from a planning standpoint to have the support division estimate only the external demand (if any) for its services and to estimate the internal demand for the support services at the same time it amalgamates the plans of the operating divisions. External and internal demands for support services would then be combined and the various provisional corporate plans would be revised accordingly. Finally, the provisional corporate plans are prepared in summary format for review by top management and are then forwarded for their scrutiny.

One of top management's greatest concerns is the corporate financial plan, which indicates the financial needs of the corporation over the long term as well as the anticipated impact of the new business, technical, manpower and production, and facilities plans on the firm's financial statements. In addition, since all funds generated internally that are available for distribution as dividends or for reinvestment are allocated by top management, and since all funds that are raised externally must be raised through the issuance of debt or equity financial instruments or through a loan agreement with one or more banks, in either case handled through the corporate controller, it is necessary to add top management's plans for obtaining the financial resources - i.e. the money capital - needed to fund the activities of the operating divisions to the amalgamated divisional financial plans.

In reviewing the corporate financial plan and the other four plans that comprise the overall corporate long term plan, top management faces a multiperiod portfolio selection problem. Associated with each of the product lines that the divisions plan to continue and also with each of the new business ventures planned are (i) an expected return - a contribution to corporate net operating income - for each year out to the long term planning horizon and (ii) various risks, which fall within the categories discussed in section B. These risks can have a major impact on the corporation's overall financial risk. In addition, the product lines and projects must also be evaluated in terms of the contribution of each to the other objectives of the corporation. The firm's present financial health as well as its capacity for borrowing limit the extent to which current commitments can be undertaken, thereby imposing a "budget constraint," while current commitments will affect future earnings and future borrowing capacity, which in turn will limit the extent to which future commitments can be undertaken.

The foregoing suggests what the author believes could prove to be an interesting analytical approach to understanding the typical airframe builder's long term planning problem. A somewhat different, though not inconsistent, approach is adopted in the next chapter.

7. Summary

This section has described the typical airframe builder's long term planning process - the second of the three phases of the corporate planning cycle. Key elements in the planning process were highlighted, the special treatment accorded human capital was discussed, the headquarters planning staff's role in coordinating divisional planning and in amalgamating divisional plans in order to prepare an overall corporate long term plan for review by top management was described, and the formulation of the corporate long term financial plan was discussed. In addition, an analytical approach to modeling top management's role in the long term planning process was suggested.

The next section describes the short term planning process and indicates how consistency between the long term plan just discussed and the short term plan is achieved.

F. SHORT TERM PLANNING: ANNUAL BUDGET PREPARATION

1. Introduction

The third phase of the corporate planning cycle involves the preparation of the short term, or operating, plan, which is generally carried out for the first year of the long term planning period only.[182] The annual operating plan, which for reasons of consistency, is derived from the corporate long term plan, provides much greater detail for the period covered than does the long term plan.

The annual operating plan is presented in the form of a detailed budget.[183] Unlike the long term plan, which is mainly concerned with the allocation of capital resources and with the development of strategies and plans whose main impact will be on sales and earnings five years or more into the future, the short term plan is mainly concerned with the allocation of variable resources and with the development of strategies and plans that will largely determine sales and earnings within the next year. Whereas the long term plan is heavily concerned with developing new product lines to replace those that will eventually be phased out and with winning new programs to replace those that are due to expire, the short term plan is heavily concerned with controlling direct costs and overhead cost for the firm's current commercial and military product lines. Also unlike the long term plan, which generally summarizes projections on a year-by-year basis,[184] the short term plan is presented on a month-by-month basis.

This section describes the short term planning process that is followed within the nine major military airframe builders. As was the case with the long term planning process, most of the short term planning takes place at the divisional level, with the headquarters planning staff once again coordinating divisional planning efforts and with top management once again carefully reviewing the final product of this phase of the planning cycle.

2. Short Term Planning at the Divisional Level

After top management has approved the corporate long term plan - normally late in the third quarter - the short term planning process can begin. This process begins much as the long term planning process was begun, namely, with the distribution of the corporation's short term goals and objectives by top management. These short term goals and objectives are generally more specific than the long term goals and objectives and are normally accompanied by a set of specific guidelines for divisional short term planning. For example, the long term plan is likely to specify an overall profit margin, i.e. the ratio of net operating income to net sales, for each year, while the short term plan is likely to specify the profit margin (or more specifically, the 'contribution margin') for each product for the coming year. To ensure consistency with the corporate long term plan, these goals and objectives and guidelines sent out from headquarters are based on the approved corporate long term plan. As was the case with long term planning, from this point in the short term planning process onward, most of the actual planning takes place at the divisional level.

Figure VI-7 illustrates the short term planning process as it is typically carried out at the divisional level. The process begins with a statement of the assumptions on which the divisional short term plan is to be based and with the formulation of the division's short term goals and objectives. The planning assumptions flow directly from the corporate and divisional long term plans, and the goals and objectives follow from the divisional long term goals and objectives embodied in the corporate long term plan and the more specific corporate short term goals and objectives and planning guidelines supplied to the division. As in the case of the long term planning process, headquarters approval of the division's goals and objectives is required, although such approval is less problematical than in the long term case because the basic direction and strategies for the corporation and for the division have already been mapped out during the long term planning process.

Once the division's short term goals and objectives and planning assumptions have been approved, the goals and objectives together with the divisional business strategies approved as part of the long term planning process are used by divisional management to formulate the division's short term business strategies - the collection of policies that specify the division's competitive position vis-à-vis its competitors in each of the commercial markets in which it sells and its approach to contract negotiations with the government. It needs to be emphasized that great care is

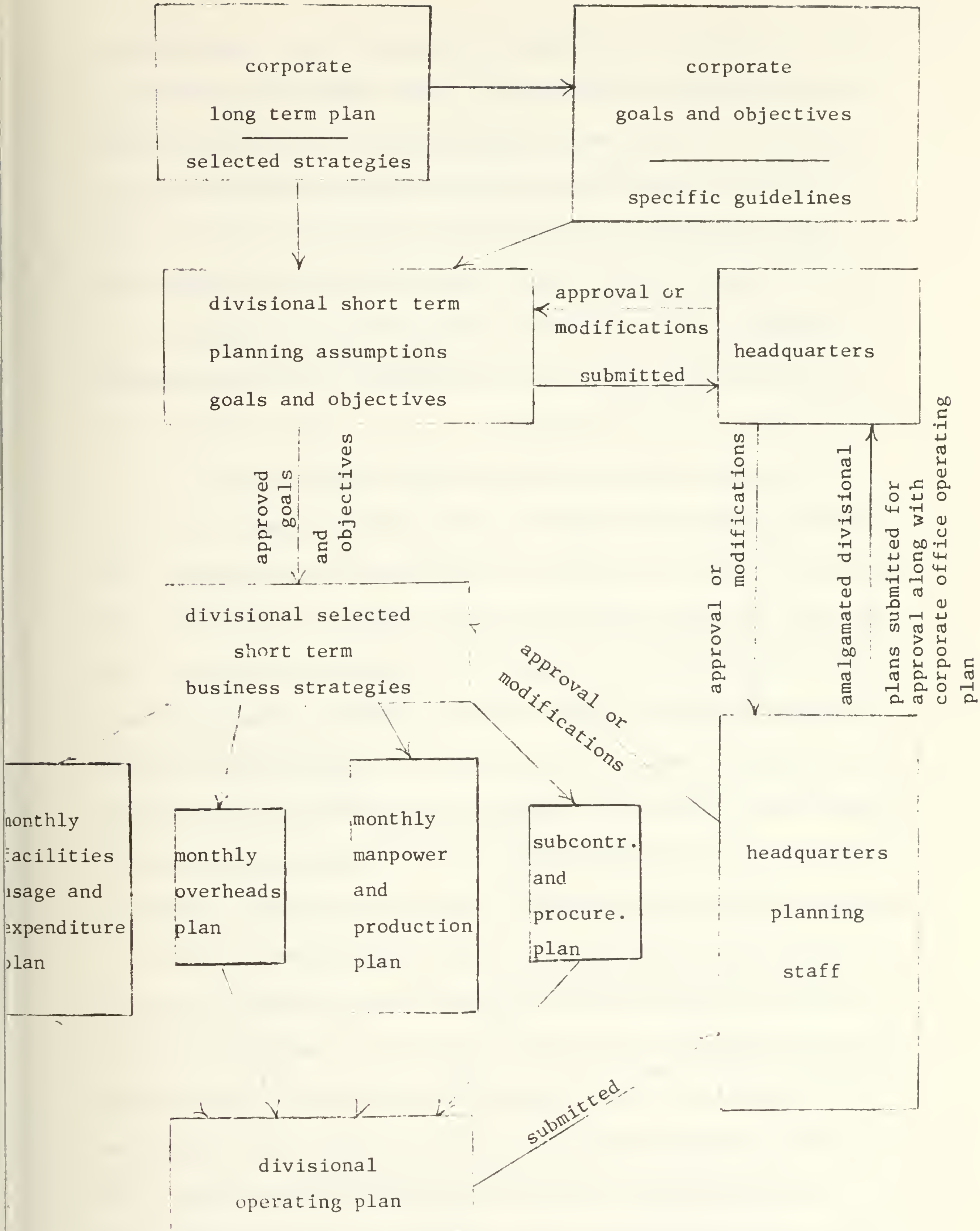

Figure VI-7 The Divisional Short Term Planning Cycle taken to ensure that these short term strategies are consistent with the long term strategies that underlie the long term plan. For example, not altering the design of a particular product, say a particular type of aircraft, may enable the division to avoid the expense involved in design changes, and thereby boost the division's overall profit margin, though the failure to 'modernize' the airplane may hurt future sales and profits in the event competitive pressures or a change in government regulations force these changes to be made hurriedly.

After establishing the division's short term business strategies, divisional management directs the division's planning staff to collect the information needed to determine the actual resource requirements of the division and to plan how these requirements will be met. This part of the short term planning process results in several component plans, which, for convenience, have been grouped into four component plans in Figure VI-7. The monthly facilities usage and expenditure plan indicates the plant and equipment allocated to each product line - whether government or commercial - for each of the twelve months covered by the operating plan. It also indicates the time of arrival and use of new equipment that will be purchased and the beginning of operation and use(s) of new plant that will become available during the year. The monthly overheads plan indicates the monthly overhead rates to be applied to direct labor to determine the cost of goods sold. The plan gives a breakdown as to general and administrative costs, the cost of facilities, research and development, etc. The monthly manpower and production plan indicates monthly output rates, the delivery of commercial and military aircraft on a monthly basis, direct (i.e. for production) labor requirements (numbers of personnel and their cost) on a monthly basis, and the projected monthly hiring/firing quotas. The subcontracting and procurement plan indicates subcontractors for government programs and details the firm's need for inputs other than labor on a monthly basis, along with the anticipated costs of these inputs.

The four plans just described form the division's annual operating plan, which is submitted to the headquarters planning staff after it has been approved by the division's management. As in the long term planning process, the headquarters planning staff may return the plan for modifications. Finally, after all divisional short term plans have been accepted, they are amalgamated into a corporate operating plan. Also incorporated into this plan is the annual operating plan for the corporate office. The corporate short term plan is submitted to top management for its review, and once this plan has been approved, the short term planning process ends.

At the conclusion of the short term planning process, the corporate planning cycle begins again. At the same time, a corporate review process begins, during which the just completed annual operating plan will be reviewed on a quarterly, or possibly on a semiannual, basis in light of actual operating results. The next section describes this review process. The remainder of this section looks more closely at certain important aspects of the short term planning process just described.

3. Short Term Planning: Government Sales, Contract Performance, and Cost Controls

Much of the short term planning associated with the sale of goods and services to the government can be based on the provisions of the contracts that were negotiated prior to the start of the short term planning process. In particular, for such items as military aircraft, government contracts generally specify product performance standards, a monthly delivery schedule, the price per plane (since, as pointed out in section B, production contracts are typically of the fixed price variety), the method by which the fee will be determined, etc. Such information proves helpful to the contractor during the preparation of the monthly manpower and production plan.

In addition, the government negotiates the overhead rates to be applied on all contracts, [185] and one of the by-products of this process is an estimate of reimbursable overhead costs. Under certain circumstances, such as those surrounding either an audit by the General Accounting Office or a general management review carried out by, or at the direction of, the administrative contracting officer, the government can issue specific directives to a contractor concerning its overheads. [186] Such information can prove helpful to the contractor when it formulates its monthly overhead plan.

Since contractor performance (or effort) weighs heavily in determining the fee to be earned, [187] the division must carefully plan the allocation of labor over government contracts during the short term planning process. Where prices are fixed, as they are on most development contracts as well as on production contracts, control of costs is critical in order to prevent costs from rising and consuming part or all of the fee. Even under cost plus contracts, cost control is important, for even though costs now play a smaller role in determining the size of the fee, the contractor's productive efficiency, and in particular, improvements in productivity, are now included as a factor in determining the fee.[188]

A second important performance factor is the contractor's performance in meeting delivery schedules. Where the number of subcontractors and suppliers is large, the problem of trying to control the scheduling of production and delivery becomes more difficult. A subcontractor making a late delivery can cause the production schedule to slip, and this can lead to a late delivery, for which the prime contractor is likely to be penalized. Controlling subcontractor performance often necessitates placing teams of people, analogous to the government's plant representative offices, in the subcontractor's plants, and this in turn requires that manpower and dollars be allocated for that purpose.[189]

In short term planning connected with government sales, then, the emphasis is placed on allocating personnel and dollars so as to ensure satisfactory contract performance. In particular, short term planning for government sales emphasizes control of production and overhead costs and the control of production rates in order to meet delivery schedules. Similarly, as the next subsection makes clear, short term planning for commercial sales also displays a decided cost and productivity emphasis.

4. Short Term Planning: Commercial Sales and Operating Efficiency

Short term planning for commercial sales is, in some important respects, more difficult than short term planning for government sales, although the basic thrust of the planning process is essentially the same. In each case short term planning is concerned with the employment levels of the variable inputs needed to satisfy the firm's production commitments. What tends to make commercial sales planning somewhat more difficult is the greater uncertainty concerning actual product demand. On the other hand, the difficulties associated with trying to predict resource requirements for research and development projects, which tend to complicate the task of government sales planning, generally have a smaller impact on the commercial side of the business. Overall, then, the relative difficulty of these two aspects of short term planning, and by implication, the relative amounts of time, manpower, and money that must be allocated to each aspect of short term planning, is largely dependent on the proportion of government sales made under research and development contracts.

In the case of commercial aircraft sales, the short term planning problem is more complicated than the one for military aircraft sales. While commercial aircraft are sold on a contractual basis, the limited financial resources of the commercial airlines make contract cancellations or delivery postponements more likely than in the case of military aircraft sales.[190] A second problem associated with commercial aircraft sales concerns advance payments. If such payments are made at all on a commercial contract, they are generally much lower than progress payments made by the government under military contracts.[191] In addition, when the financial positions of commercial airlines worsen, they become loath to enter into long term contracts,[192] preferring instead to buy aircraft on much shorter notice once they have generated sufficient financial resources with which to make the purchase. Assuming the decision has been made (as part of the long term planning process) to continue production of the airplane or of some modified version, a decision concerning the production rate for the coming year must be made. This production rate must satisfy the constraint on the minimum feasible production rate, which is imposed by the technical conditions of production.

In contrast to the special planning problems associated with commerical aircraft sales, planning for the sales of other commerical products generally does not lead to problems any different from those faced by the airframe builders' non-aerospace commercial competitors. The main difference, in terms of planning, between these sales and sales of commercial aircraft is in inventory planning. In planning commercial aircraft sales, production schedules are set to conform as closely as possible to the delivery schedule for firm orders so that inventories of completed aircraft can be held near zero. In planning for other commercial sales, for which inventories of finished goods are a normal part of doing business, one of the purposes of short term planning is to determine the appropriate (in light of demand and cost considerations) inventory levels.

Overall, the short term planning process is designed to achieve operating efficiency in the area of commercial sales. As was the case with government sales, short term planning of commercial operations focuses on variable inputs such as the labor used in the production process and seeks to determine employment levels consistent with cost minimization.

5. Summary

The short term planning process is concerned mainly with the allocation of variable inputs among existing and about-to-be-introduced product lines and with the efficient use of the firm's existing capital resources, in contrast to the long term planning process, which is mainly concerned with the acquisition or disposal of capital resources, the winning of new government contracts, and the development of new commercial lines of business. Because the short term planning process begins once the long term planning process has been completed and uses the long term plan as a planning base, short term planning normally requires just one quarter, as opposed to the two quarters usually required for long term planning. Using the long term plan as a basis for the short term plan also ensures consistency between the two plans.

One of the important outputs of the short term planning process is a set of provisional quarterly financial statements for the corporation. That is, once top management has approved the corporate operating plan submitted by the headquarters planning staff (see Figure VI-7), a provisional profit and loss statement and a provisional balance sheet can be prepared for each quarter - or for each month, if so desired - of the corporate fiscal year. These provisional financial statements are used by top management during the fiscal year to check the progress of the corporation toward its long term and short term financial goals and objectives.

While the completion of the corporate operating plan and the preparation of the provisional corporate financial statements mark the end of the corporate planning cycle, there remains a very important follow-on to the corporate planning cycle, namely, the corporate review(s). The next section describes the corporate review process, the purpose of which is to measure periodically the performance of the divisions against the respective divisional plans and to modify the divisional and corporate operating plans in light of operating experience.

G. THE CORPORATE REVIEW PROCESS

Though substantial resources are devoted to formulating the corporate and divisional plans, the uncertainties associated with estimates of product demand, input costs and availabilities, etc., in the future, as well as unforeseen circumstances, such as an oil embargo, an unexpected sharp decline in the demand for airline passenger travel, or an unanticipated contract termination, will cause actual performance to deviate from the projections outlined in the corporate and divisional operating plans. Thus, it has been deemed necessary by the managements of the major airframe builders to conduct formal periodic reviews - normally on a quarterly basis, though in some cases, on a semi-annual basis - to measure these deviations, and if possible, to determine their cause so that the divisional and corporate operating plans for the remainder of the fiscal year can be adjusted in accordance with the firm's operating experience.

The existence of the review process is indicative of the fact that the corporate planning and review process, when considered as a whole, is an adaptive process. That is, the planning and review process exhibits feedback control in that each periodic corporate review causes information on divisional performance to be generated that is used by top management and divisional managers to modify their operating plans. In addition, the corporate reviews enable top management and the headquarters planning staff to reevaluate periodically the operating environment and performance of each division. The information collected and the evaluations performed as part of the corporate reviews for the third and fourth quarters of the fiscal year are, to the extent that they enable corporate planners to evaluate the current state of demand in each of the firm's product markets, helpful to the headquarters planning staff at the time it prepares the environmental forecast.

Figure VI-8 shows the corporate planning and review cycle for arbitrary year T. The whole cycle spans a period approximately nine quarters in length. The planning for year T takes place the previous year. The corporate review for each quarter takes place soon after the end of the quarter, with the time lag determined by how much time is required to collect and process the data needed to carry out the review. The fourth quarter review is different from the three earlier reviews in that, not only can the full year's performance be measured against the whole operating plan, but also the review of the year's performance forms the basis for the annual report to the firm's shareholders.

Each quarterly review contains the following basic elements. First, the actual results - sales by product line, levels of resource usage, net operating income for the division, etc. - are summarized for both the quarter and the year to date. The actuals are compared with the approved operating plan. Usually this is done simply by listing the actual figure next to the projection - and significant variations are noted. Second, based on the actuals to date, an up-to-date forecast for the balance of the fiscal year is presented, along with a statement of any changes that may have been made in the underlying assumptions since the operating plan was approved. Third, the status of major military

Qtr I	Qtr II	Qtr III	Qtr IV	Qtr I	Qtr II	Qtr III	Qtr IV	Qtr I
Yr T-1	Yr T-1	Yr T-1	Yr T-1	Yr T	Yr T	Yr T	Yr T	Yr T + 1
environ. forecast	long range plan Year T and beyond		Year T operating plan		Qtr I Year T review	Qtr II first half Year T reviews	Qtr III three-qtrs Year T reviews	Qtr IV full year Year T reviews

Figure VI-8 The Corporate Planning and Review Cycle and commercial programs is reviewed, with special emphasis placed on problems - existing or potential - that might in the future require the attention of top management,[193] e.g. those problems that might call for large expenditures or major schedule changes and that could affect other divisions of the company. Fourth, the status of capital expenditure programs and any cost or schedule changes to them are reviewed. Fifth, research and development activities, their cost, and significant accomplishments are reviewed. Sixth, efforts directed toward winning new military programs that are near to bidding are reviewed. For each of the above elements of the review process, top management reviews the record to date and suggests appropriate modifications of strategies, expenditure levels, etc. In particular, one or more of the divisional operating plans may be changed at the direction of top management in light of that division's performance to date.

The corporate review process, then, is a device that enables top management to review periodically the performance of the corporation's operating divisions and to determine what changes need to be made, in light of each division's experience to date, to each division's operating plan in order to improve the likelihood that by the end of the fiscal year the corporation's goals and objectives for the year will have been met.

H. CHAPTER SUMMARY

The nine major military airframe builders in the United States are large diversified organizations whose performance is influenced to a significant degree by variations in the size and number of major military aircraft programs. The business environment within which these firms operate is characterized by several different types of risk, some of which arise out of their special relationship with their main customer, the U.S. government, and some of which are peculiar to the highly technical and specialized nature of their aerospace business. In addition, the apparent decline in the number of major military aircraft programs, the stagnant demand for commercial passenger jet aircraft, and the increasing foreign competition from firms financed by their governments, have limited the opportunities for growth in these firms' traditional markets, and more importantly, have threatened the structure of the industry.

Due in part to the nature of the aerospace business and in part to their dependence on the sale of high technology aircraft to the government, these firms have had to evolve a long term planning process capable of coping with the risks they face. These firms must plan their allocation of human capital on the basis of very limited information, knowing that the penalty for guessing incorrectly may include the failure to win a major program. They must allocate research expenditures so as to develop the technology that will be needed by weapons systems that may not reach the development stage for a decade or more. Thoughout the research and development process they must compete with one another for the favor of their main customer, the U.S. government, and for the relatively more lucrative production contracts that secure for each of their recipients a place among the select group of prime government contractors.

Their commercial business, particularly if it includes the sale of commercial passenger aircraft, can also entail substantial risks. The number of potential customers is greater, but the financial strength of each is not only weaker than the government's, but also susceptible to the swings of the business cycle. Since the cost of developing a new generation of commercial aircraft may exceed the developing company's net worth, the risk of ultimate financial failure cannot be ignored. Moreover, in recent years a severe recession and an increase in foreign competition have intensified these risks.

The substantial risks attendant upon their aerospace operations and the limited growth opportunities provided by the markets for commercial and military aircraft have caused many of the major airframe builders to look outside the industry for opportunities to diversify and grow. But the limited alternative uses to which their fixed capital can be put and the nature of their aerospace business - the production and marketing of highly sophisticated products to a relatively small number of commercial and government buyers - have forced these firms to be very selective with regard to how they choose to diversify.

Similarly, the nature of the aerospace business and the dependence of these firms on sales of aircraft to the government give rise to special problems in short term planning. Due to the high costs of temporarily shutting down a production line or of maintaining an inventory of completed aircraft, production schedules must be set to conform as closely as possible to anticipated delivery schedules. Due to the importance of meeting the specifications of government production contracts, most of which are granted on a fixed price basis, it is necessary that production schedules and production and overhead costs be carefully controlled.

All the factors mentioned above combine to make the typical airframe builder's task of planning a difficult one, the requirements of which are, in the opinion of this writer, more demanding than those faced by the majority of non-aerospace firms. This chapter has described how the major airframe builders conduct their long term and short term planning. The discussion has highlighted the important aspects of each process and has indicated the relationship between the two. The chapter has also drawn attention to the important differences between doing business with the government and doing business with commercial customers and has pointed out the implications of these differences for the long term and short term planning processes of the nine major military airframe builders.

This chapter has described the institutional milieu within which the major airframe builders operate. In the next chapter the basic theoretical model developed in chapters three through five of this thesis is modified and a model of a representative airframe builder is formulated. The model is developed as an analytical model of the planning cycle described in this chapter. The model is used to characterize the optimal operating and financial policies of the representative airframe builder and to study the impact on the behavior of these firms of the government's progress payments policy. The model is also used to examine several procurement policy issues raised in this chapter, such as the likely impact on the behavior of these firms of the procurement policy changes resulting from the recent Profit '76 study.

CHAPTER SIX FOOTNOTES

1. This chapter is based in part on personal interviews conducted at The Boeing Company, Fairchild Republic Company (a division of Fairchild Industries, Inc.), the Convair Division of General Dynamics Corporation, Grumman Corporation, Vought Corporation (formally LTV Aerospace Corporation and a wholly owned subsidiary of the LTV Corporation), Lockheed Aircraft Corporation, McDonnell Douglas Corporation, Northrop Corporation, and Rockwell International Corporation. The author would like to thank the executives with whom he spoke for their generous assistance, though he alone accepts full responsibility for any errors that may have been committed in describing these firms' planning processes.

2. The term 'airframe' refers to the body of the airplane without its engines. For the information of the reader, the two major producers of engines that are installed in the airframes produced by the firms listed in footnote 1 are United Technologies Corp. (through its Pratt & Whitney Aircraft Div.) and General Electric Co.

3. See "A shakeout for U.S. fighter-plane makers," Business Week (June 9, 1975), and "General Dynamics: Winning in the Aerospace Game," Business Week (May 3, 1976). According to these articles, the threatened firms are Fairchild, Grumman, and Vought (LTV). However, Joseph G. Gavin, Jr., President of Grumman Corp., has stated publicly that he expects follow-on orders to carry F-14 production beyond the currently projected 1980 termination date. See "The New Face of the Defense Industry," Business Week (January 10, 1977), p. 55. A recent study conducted jointly by the Department of Defense and the Office of Management and Budget concludes, however, that there is excess capacity in the aircraft industry costing the Department of Defense approximately $400 million per year to maintain and recommends that the industry be consolidated. See "Washington Roundup," Aviation Week & Space Technology (January 24, 1977), p. 11.

4. See "Conferees Vote Money for the B1 Bomber And for Nuclear-Powered 'Strike' Cruiser," Wall Street Journal (July 28, 1975); "B1 Decision Is Delayed by Senate Panel In Passing Defense Appropriations Bill," Wall Street Journal (July 22, 1976); and "Conferees Vote to Put B1 Bomber Funds On Tight Rein Until After Inauguration," Wall Street Journal (September 1, 1976).

5. See "Early Revival Unlikely As Jumbo-Plane Sales Continue to Languish," Wall Street Journal (August 10, 1976), and "Aerospace and Defense," Forbes (January 1, 1977), p.136. There are some signs, however, that the demand for commercial aircraft, and in particular, the Boeing 727, is beginning to pick up. Revised noise standards and the need for replacement aircraft are at least partly responsible. See "Airlines give Boeing a one-shot boom," Business Week (October 11, 1976). The long term outlook is also brightening, though many airlines will require a period of sustained profitability if they are to be able to satisfy their need for new aircraft. See "Billions and billions and billions to grab for," The Economist (September 11, 1976); "Time To Fasten Seat Belts?," Forbes (October 15, 1976); and "Nation's Airlines Face A Key Problem: How To Pay for New Airplanes," Wall Street Journal (October 22, 1976).

6. See "Lockheed Woes Increase as Plan For Rescue Fails," Wall Street Journal (March 3, 1975), and "Biting the bullet on the TriStar," Business Week (April 12, 1976). However, indications are that the patient is on the mend. See "The Fabulous Invalid," Forbes (August 15, 1976), and "Lockheed Restructuring Voted by Owners Of Common; Debt-Holder Approval Seen," Wall Street Journal (September 30, 1976).

7. See The New Face of the Defense Industry, op.cit., and also Aerospace and Defense, op.cit.

8. That is, 'total package procurement', in which companies were forced to accept a single fixed price contract covering both development and production, is no longer part of the Department of Defense's procurement policy. The current DOD policy regarding major system acquisitions is outlined in Department of Defense Directive 5000.1, "Major System Acquisitions" (January 18, 1977) and in Department of Defense Directive 5000.2, "Major System Acquisition Process" (January 18, 1977).

9. And, in at least one case, in accordance with the wishes of the firm's debt-holders. See footnote 6.

10. K.G. Harr, Jr., "A Short Course in Aerospace Economics 1976," Aerospace (September 1976), p. 11.

11. Ridder and Heinz classify 634 firms as belonging to the aerospace industry based on the industry classifications provided by Dun and Bradstreet's Million Dollar Directory, Standard and Poor's Register of Corporations, and several other

sources. See W.C. Ridder and M.K. Heinz, "Structure, Conduct, and Performance of the United States Aerospace Industry," unpublished M.S. thesis (Naval Postgraduate School; Monterey, California; March 1976). Ridder and Heinz provide an interesting historical perspective on the industry as well as a careful analysis of the industry's structure, conduct, and performance. An earlier study of the aerospace industry that carefully examined the major airframe builders is H.O. Stekler, _The Structure and Performance of the Aerospace Industry_ (University of California Press; Berkley; 1965). After allowing for mergers and after excluding Martin Marietta, which no longer builds aircraft, Stekler's list of the major airframe builders is identical to the list of firms in footnote 1. _Ibid_., p. 47. A second study by Carroll considers these firms as well as the missile frame builders. S.L. Carroll, "The Airframe Industry," unpublished Ph.D. dissertation (Harvard University; Cambridge, Mass,; August 1970).

12. Harr, _op.cit_., p. 12

13. _Ibid_., p. 12.

14. _Ibid_., p. 12.

15. _Ibid_., p. 11.

16. Strictly speaking, LTV is not an aerospace firm. Because aerospace sales constitute approximately 12 percent of total sales (see Table VI-3 below), whereas steel operations contribute 39 percent and meat and food products contribute 48 percent, as reported in the company's Form 10-K for its fiscal year 1975, the company is usually classified as a 'conglomerate', rather than as a member of any single industry (for example, see "Multicompanies," _Forbes_ (January 1, 1977), p. 101). The firm does, however, participate in the industry through its Vought Corp. subsidiary, and if the significance to LTV of this participation were to be judged in terms of profits, rather than sales, then, in 1975 at least, aerospace production would become preeminent (see Table VI-6 below).

17. These firms also play very important roles in the international market for commercial aircraft. The Aerospace Industries Association of America estimates that roughly four of every five planes flown commercially world-wide are American-made. Harr, _op.cit_., p. 7

18. _Ibid._, p. 7.

19. See, for example, "Cruise Missile's Future Is Mainly Up to Carter; Its Potential Is Great," _Wall Street Journal_ (January 3, 1977), and "Defense Budget of $110.1 Billion Proposes Big Weapons Rise With Little Fat to Cut," _Wall Street Journal_ (January 18, 1977).

20. These factors, and their impact on the defense industry, are discussed in M.L. Weidenbaum, _The Economics of Peacetime Defense_ (Praeger; New York; 1974).

21. Since 1968 total employment within the aerospace industry has fallen by more than one-third. Moreover, between 1968 and 1975 more than 70,000 highly skilled jobs - scientists, engineers, and technicians - were lost. Harr, _op.cit._, p. 12.

22. For example, in the Fortune 500 ranking for 1970, Boeing was ranked 17, Lockheed was ranked 33, and McDonnell Douglas was ranked 44. See "The Fortune Directory of the 500 Largest Industrial Corporations," _Fortune_ (May 1971).

23. One added indication of this is the fact that, if Vought Corp. were ranked separately from the rest of LTV, it would rank number 330 in the FORTUNE 500.

24. Six of the top 10 are airframe builders. Of the remaining four top 10 DOD contractors, United Technologies Corp. (no. 3), through its Pratt & Whitney Aircraft Division, has contracts to build engines for three of the four new fighters; General Electric Co. (no. 7) has the engine contract for the F-18; and Litton Industries Inc. (no. 8) and Hughes Aircraft Co. (no. 9) are also important suppliers of aerospace products, See Ridder and Heinz, _op.cit._, Appendix J, p. 400.

25. Harr, _op.cit._, p. 11.

26. _Ibid._, p. 12.

27. It will be argued below that it is human capital - in the form of the knowledge and experience embodied in skilled engineers and scientists - rather than physical capital - in the form of plant and equipment, some of which is owned by the government - that is the scarcer of the two components of the firm's total capital resources and that is, consequently, of greater concern to each firm's strategic planners who must plan the allocation of these capital resources.

28. Ibid., p. 14.

29. D.E. Raphael of the Stanford Research Institute believes that the aerospace industry faces an impending widespread capital shortage, and he estimates that the working capital requirements of these firms will rise from $5.9 billion in 1975 to $8.8 billion in 1980 and to $15 billion in 1985. D.E. Raphael quoted in The New Face of the Defense Industry, op.cit., p. 53.

30. Ibid., p. 52, and Department of Defense, Defense Procurement Circular Number 76-3 (Washington, D.C.; September 1, 1976), p. 12.

31. Comparative figures are provided in "Where Private Industry Puts Its Research Money," Business Week (June 28, 1976).

32. Ibid., p. 65. Even though DOD funds a large proportion of defense-related research, it does not finance 100 percent of the research, and on a project-by-project basis each of the airframe builders is risking large sums of money. For example, Boeing Co. spent $41 million of its own money, in addition to $95.2 million supplied by DOD, for research and development connected with the YC-14, the new short take-off-and-landing transport being developed for the Air Force (in competition with McDonnell Douglas's YC-15). Ibid., p. 66.

33. The references that set out current DOD policy regarding major weapons system acquisition are given in footnote 8.

34. The implications of the type of contract for risk-sharing between the government and the contractor are discussed below in subsection 6.

35. One aerospace executive told the author that his company estimated that approximately 56 percent of IR&D funds were spent on projects that would never result in fruitful military applications, and that, of the remainder, only one quarter (i.e. 11 percent of the total) would be spent on developing weapons systems that his company would produce (the other three quarters being spent on projects that would lose out to other firms).

36. One of the earliest articles on the subject was T.P. Wright, "Factors Affecting the Cost of Airplanes," Journal of the Aeronautical Sciences (vol. 3; no. 4; February 1936), pp. 122-128. See also K. Hartley, "The Learning Curve and Its Application to the Aircraft Industry," Journal of Industrial Economics (vol. 13; no. 2; March 1965), pp. 122-128. The

existence of the learning curve has been taken into account in production and cost planning by military, as well as by industry, planners. For a survey of Air Force applications see H. Asher, "Cost-Quantity Relationships in the Airframe Industry," R-291 (The RAND Corporation; Santa Monica, CA; 1956). It should be noted that the learning curve phenomenon is not unique to the airframe industry. For other industries in which it applies see W.Z. Hirsch, "Firm Progress Ratios," Econometrica (vol 24; no. 2; April 1956) pp. 136-143. In addition, the phenomenon of the learning curve also has applications at the macroeconomic level. See P.J. Verdoorn, "Complementarity and Long-Range Projections," Econometrica (vol 24; no. 4; October 1956), pp. 429-450, and K.J. Arrow, "The Economic Implications of Learning by Doing," Review of Economic Studies (vol. 29; 1962), pp. 155-173.

37. An interesting general discussion of learning curves is provided in S.C. Webb, Managerial Economics (Houghton Mifflin; Boston; 1976), ch. 17.

38. Hartley, op.cit., p. 122, and Webb, op.cit., p. 251.

39. The equation of the learning curve in Figure 1 is

$$\ell = 8000\ x^{-0.32193},$$

where ℓ is the direct labor input per airframe and x is the cumulative number of airframes produced. More generally, a learning curve satisfies an equation of the form

$$\ell = ax^{b},$$

where a is the direct labor input of the first airframe produced and $b = \log_2 p$, where p is the percent of learning expressed as a decimal and where $\log_2$ signifies a logarithm to the base 2. Note that the shape of the learning curve implies that the learning process is subject to steadily diminishing returns.

40. Though inflation might cause the cost per airframe measured in current dollars to increase - if rising unit input costs more than offset the effect of improved labor efficiency - the cost per airframe would still fall when measured in term of dollars of constant purchasing power (i.e. in real terms).

41. In essence, the controversy surrounding the Navy's decision to procure the F-18 despite congressional pressure to procure the F-17, which was a modification of the F-16 selected previously by the Air Force, was the result of this sort of disagreement as to whether the additional costs incurred in selecting a different design could be justified on the grounds of improved effectiveness.

42. The actual point beyond which a major program becomes virtually nontransferable probably lies somewhere before the award of the first production contract, but after the selection of the winner of the prototype competition. That is, during the final development and the test and evaluation stages of the program, the firm that won the prototype competition develops the finished product. Since the technology developed and the experience accumulated during these stages cannot be transferred costlessly, at some point the potential costs of transferring the program become so high as to in effect preclude a change of prime contractor.

43. That is, a market for a product characterized by a single buyer (the Department of Defense through one of its services) and a single seller (the contractor). Bilateral monopoly is discussed in most elementary price theory textbooks. For example, see R. Sherman, The Economics of Industry (Little, Brown and Company; Boston; 1974), pp. 283-287.

44. See "Lockheed Sets L-1011 Charge Of $515 Million," Wall Street Journal (March 31, 1976).

45. The current competition between Boeing and McDonnell Douglas for a contract worth approximately $2 billion to build midair refueling tankers is an example. The Boeing entry will utilize the 747 airframe, while the McDonnell Douglas entry will utilize the DC 10 airframe. (The Lockheed entry - a derivative of the L-1011 - has already been eliminated.) Aerospace and Defense, op.cit., p. 136.

46. Harr, op.cit., p. 7.

47. This latter figure was computed by treating Vought Corp. as if it were an aerospace firm separate from LTV Corp.

48. This procedure has a strong impact on Northrop Corp.'s government sales. If sales to foreign governments were included among 'government sales', then the share of government sales in Northrop Corp.'s total sales would exceed 80 percent. The main reason for this large difference is that the primary market for Northrop Corp.'s F-5 fighter is foreign governments.

49. See footnotes 21 and 22.

50. For example, sales of the A-10 aircraft formed 23% of Fairchild Industries's total sales during 1975. Fairchild Industries Form 10-K, op.cit., p. 6.

51. Defense Procurement Circular No. 76-3, op.cit. The DOD's primary motive was to encourage defense contractors to increase their investment in plant and cost-saving new equipment. The Profit '76 study recommended four major changes designed to accomplish this. First, interest, including the imputed interest on contractor-owned facilities, became an allowable cost. Second, the contractor's level of investment in facilities was introduced as a factor into the weighted guidelines that government contracting officers must follow in negotiating a profit objective with the contractor. Third, risk will be weighed more heavily and cost will be weighed less heavily in negotiating profit levels. Fourth, productivity improvements were introduced into, and past contractor performance was deleted from, the list of guidlines used to determine profit levels. It should be noted that the policy changes will be less favorable to the airframe builders than they will be to shipbuilders and other government contractors, according to Brig. General James W. Stansberry, USAF, Director, Profit '76, quoted in "Pentagon Drafts Policy to Spur Spending By Defense Contractors on New Facilities," Wall Street Journal (July 6, 1976).

52. Ibid., pp. 12-15; Aerospace Industries Association of America, Risk Elements in Government Contracting (Washington, D.C.; October 1970), pp. 6-9; Aerospace Profits vs. Risks (Washington, D.C.; June 1971), ch. 4; and J.R. Fox, Arming America: How the U.S. Buys Weapons (Division of Research, Harvard Business School; Boston; 1974), pp.236-240.

53. The argument that risks are high and returns are low in the aerospace industry in relation to other industries is made in Harr, op.cit., p. 12, and in J.K. Brown and G.S. Stothoff, The Defense Industry: Some Perspectives from the Financial Community, (Division of Management Research, The Conference Board; New York; 1976). The opposite view is expressed in Weidenbaum, op.cit., pp. 69-70, which cites a GAO study of aerospace profitability over the period 1966-1969, and in The New Face of the Defense Industry, op.cit., p. 56. For a discussion of these articles see the next footnote.

54. One of the practical problems encountered in analyzing the question of the sufficiency of the returns earned by aerospace firms is the period of time covered by the analysis. The years 1966-1969 covered by the GAO study referred to in footnote 53 preceded the post-Vietnam slump in defense spending, and much of the empirical evidence cited in the Business Week article ("The New Face of the Defense Industry") is based on the same period. In this regard the Brown and Stothoff study, which focuses on the period 1965-1974, reports statistical results that are less biased.

55. For example, Risk Elements in Government Contracting, op.cit.

56. Profit '76 Summary Report (U.S. Government Printing Office; Washington, D.C.; December 7, 1976.)

57. The apparent preference of contractors for investing in facilities to be used in commercial production, rather than to support government production, was the major justification for the Profit '76 study. Defense Procurement Circular No. 76-3, op.cit., p. i.

58. An overview of the procurement process is provided in S.J. Evans, H.J. Margulis, and H.B. Yoshpe, Procurement (Industrial College of the Armed Forces; Washington, D.C.; 1968). Several excellent analyses of the weapons acquisition process have been performed. The classic studies are M.J. Peck and F.M. Scherer, The Weapons Acquisition Process: An Economic Analysis (Division of Research, Harvard Business School; Boston; 1962), and F.M. Scherer, The Weapons Acquisition Process: Economic Incentives, (Division of Research, Harvard Business School, Boston; 1964). An interesting follow-on of these studies is Fox, op.cit. An interesting discussion of the differences between government-contractor transactions and commercial transactions can be found in J.F. Gorgol, The Military-Industrial Firm (Prager; New York; 1972), ch. 2.

59. See Peck and Scherer, op.cit., ch. 3, and J.M. Suarez, "Profits and Performance of Aerospace Defense Contractors," Journal of Economic Issues (vol. 10; no. 2; June 1976), pp. 386-402, for more on the non-market character of the weapons acquisition process.

60. It should be noted, however, that prior to World War II this commercial-like process was heavily relied on to generate new ideas for military aircraft. Ibid., ch. 4.

61. See Aerospace Industries Association of America, Monopsony: A Fundamental Problem in Government Procurement (Washington, D.C.; May 1973) and Stanford Research Institute, "The Industry-Government Aerospace Relationship," two volumes (Menlo Park, CA; May 1963).

62. See Fox, op.cit., pp. 256-257, 467-471. An interesting theoretical discussion of the bidding process can be found in D.P. Baron, "Incentive Contracts and Competitive Bidding," American Economic Review (vol. 62; no. 3; June 1972), pp. 384-394; C.C. Blaydon and P.W. Marshall, "Incentive Contracts and Competitive Bidding: Comment," American Economic Review (vol. 64; no. 6; December 1974), pp. 1070-1071; and D.P. Baron, "Incentive Contracts and Competitive Bidding: Reply," American Economic Review (vol 64; no. 6; December 1974), pp. 1072-1073.

63. Harr, op.cit., p. 13.

64. Due to the abandonment of 'total package procurement' this latter risk has been reduced substantially in recent years. See The New Face of the Defense Industry, op.cit., p. 52, and footnote 8.

65. See footnote 42.

66. See the references listed in footnote 38.

67. See A.M. Agapos and L.E. Gallaway, "Defense Profits and the Renegotiation Board in the Aerospace Industry," Journal of Political Economy (vol. 78; no. 5; September/October 1970), pp. 1093-1105; Weidenbaum, op.cit., pp. 70-72; and J.F. Weston, ed., Procurement and Profit Renegotiation (Wadsworth; San Francisco; 1966).

68. Weidenbaum, op.cit., p. 72.

69. The six factors are the following: the efficiency of the contractor, the reasonableness of cost and profits, the amount and source of public and private capital employed, the extent of risk assumed, the nature and the extent of the contribution to the defense effort, and the character of the business. Ibid., p. 71.

70. Ibid., p. 72, and Harr, op.cit., p. 13.

71. Weidenbaum, op.cit., p. 72. Weidenbaum argues that the board's preoccupation with profits rather than costs is not in the taxpayer's best interests since cost levels and cost overruns are so much greater in magnitude than profits. He argues that the board should pay greater attention to the reasonableness of contractor costs.

72. C. Kaysen, "Improving the Efficiency of Military Research and Development," in E. Mansfield, ed., Defense, Science, and Public Policy (W.W. Norton; New York; 1968), p. 119. See also Harr, op.cit., p. 13, and Weidenbaum, op.cit., p. 70.

73. See, for example, J. Hirshleifer, Investment, Interest, and Capital (Prentice-Hall; Englewood Cliffs, N.J.; 1970), p. 215; Aerospace Profits vs. Risks, op.cit., p. 2; and D. Vickers, The Theory of the Firm: Production, Capital, and Finance (McGraw-Hill; New York; 1968), p. 7.

74. F.H. Knight, Risk, Uncertainty and Profit (Houghton Mifflin; New York; 1921).

75. G.C. Philippatos, Financial Management Theory and Techniques (Holden-Day; San Francisco; 1971), pp. 69-70.

76. Ibid., p. 70, and J.C. Van Horne, Financial Management and Policy, 2nd ed. (Prentice-Hall; Englewood Cliffs, N.J.; 1971), pp. 46, 198-200.

77. See, for example, Risk Elements in Government Contracting, op.cit.

78. As, for example, the Profit '76 Study. See footnote 56 for reference.

79. Aerospace Profits vs. Risks, op.cit., pp. 2-4.

80. These measures are the ones suggested by Van Horne. See footnote 76 for page references.

81. J.R. Kurth, "Why We Buy the Weapons We Do," Foreign Policy (no. 11; summer 1973), pp. 43-46, or J.R. Kurth, "Aerospace Production Lines and American Defense Spending," in S. Rosen, ed., Testing the Theory of the Military-Industrial Complex (D.C. Heath; Lexington, Mass.; 1973), pp. 142-144.

82. For a view of the weapons procurement process contrary to Kurth's see A. Kanter and S.J. Thorson, "The Weapons Procurement Process: Choosing Among Competing Theories," in Rosen, op.cit., pp. 157-196.

83. Fairchild Industries, Inc., 1975 Annual Report (Fairchild Industries, Inc; Germantown, MD.), p. 3.

84. Aerospace Profits vs. Risks, op.cit., p. 2, makes the same point, although it adopts 'the probability of obtaining profits substantially below a competitive average' as the definition of financial risk and suggests 'a firm's dispersion (as measured by standard deviation, coefficient of variation, or skewness) in the rate of return from its trended mean' as the best statistical measure of overall risk. Ibid., p. 10. The Profit '76 study chose the standard deviation of the firm's rate of return from its mean over a ten-year period as its measure of the firm's financial risk. See footnote 56 for a reference.

85. These might also be called research and development risks. Ibid., pp. 2-3. See also Harr, op.cit., p. 13. For a practical example of these risks see "A plague of faulty fighter engines," Business Week (August 25, 1975), and "Grumman Confirms Engine Problems Of F14 Navy Plane," Wall Street Journal (May 21, 1976).

86. See Aerospace Profits vs. Risks, op.cit., pp. 2-3. In particular, the failure to make the required advance in one area, for example, designing a radar system of the required size and weight, may necessitate design changes in other parts of the system, for example, redesigning other aircraft components to make them smaller and lighter to compensate for the excessive size and weight of the radar system.

87. Ibid., p. 2.

88. Defense Procurement Circular No. 76-3, op.cit., p. 12.

89. Over time the impact of this source of risk may be dulled by what Kurth calls the 'follow-on imperative': about the time one major government contract phases out another one phases in. See Kurth, Why We Buy the Weapons We Do, op.cit., pp. 38-42, or Kurth, Aerospace Production Lines and American Defense Spending, op.cit., pp. 139-142, for supporting evidence. The reference provided in footnote 82 takes a position contrary to Kurth's. However, the observed pattern of follow-on awards may not, in the opinion of this writer, be the result of government policy designed to help prospective have-nots, but rather, may simply reflect the significant advantages - such as grasp of related technology, trained labor force, available production facilities, etc. - a contractor has in bidding on follow-on contracts.

90. See Sherman, op.cit., pp. 153-154.

91. Defense Procurement Circular No. 76-3, op.cit., pp. 12-15.

92. Ibid., p. 14.

93. The twin problems of incurring greater overall cost risk through subcontracting, while at the same time shifting cost risk onto subcontractors, become somewhat greater when work is subcontracted on an international basis, as it has been on the F-16.

94. Business Week estimates that overcapacity in the U.S. aerospace industry as of January 1977 might have been as high as 40%. The New Face of the Defense Industry, op.cit., p. 58. The impact of such overcapacity is partly mitigated by the fact that much of the overcapacity is in government-owned plants. Ibid., p. 58. See also Aerospace Profits vs. Risks, op.cit., pp. 5-7.

95. Harr, op.cit., p. 13, and Aerospace Profits vs. Risks, op.cit., p. 5.

96. See General Dynamics: Winning in the Aerospace Game, op.cit., and A shakeout for U.S. fighter-plane makers, op.cit.

97. See N. Rosenberg "On technological expectations," Economic Journal (vol. 86; no. 343; September 1976), pp. 523-535. Introducing an airplane 'too soon' would give competitors an opportunity to observe market demand and to modify their aircraft to suit better the needs of potential buyers, while introducing it 'too late' would let the competitors capture a dominant position in the market place. Exactly this sort of problem confronts Boeing and McDonnell Douglas and their decisions as to when to introduce the next generation of commercial jet aircraft. See "The Next Commercial Jet . . . If," Business Week (April 12, 1976).

98. As, for example, Northrop's and McDonnell Douglas's joint venture on the F-18. Joint commercial ventures across international boundaries are also likely. See "I'm McDonnell Dassault, buy me," The Economist (August 21, 1976), and "Free-world partners plan jets for the 1980s", Business Week, (August 30, 1976).

99. In the case of the McDonnell Douglas - Northrop joint venture, each will act as a prime contractor on a different version of the same basic aircraft - McDonnell Douglas on the U.S. Navy version and Northrop on the land-based foreign version of the F-18. See A shakeout for U.S. fighter-plane makers, op.cit., and The New Face of the Defense Industry, op.cit.

100. This was made clear to the author in the course of interviews with executives of the nine firms listed in footnote 1, and in particular, during his interview with Joseph G. Gavin, Jr., President of Grumman Corp.

101. Government-furnished fixed capital accounts for less than 20 percent of the aerospace industry's total fixed capital. Aerospace Profits vs. Risks, op.cit., pp. 5-6.

102. This is one of the intentions of the Profit '76 study's recommendations, namely, to get defense contractors to purchase their own plant and equipment. The New Face of the Defense Industry, op.cit., p. 56.

103. Working capital requirements, expressed per dollar of sales, are higher in the aerospace industry than in other durable goods industries because of the long lead times for development and the high cost of skilled engineering and technical talent. The government funds at least one-half of the aerospace industry's working capital requirements. Aerospace Profits vs. Risks, op.cit., pp. 5-6.

104. The desirability of government-furnished capital is a question debated among the military airframe builders. On the one hand, Northrop believes that contractors should own all their own facilities and bear all the financial and business risks - even to the extent of doing development work under fixed-price contracts - and receive greater profits accordingly, while on the other hand, Grumman believes that DOD should provide a large portion of the capital and shoulder a large share of the risks, particularly those associated with research and development. The New Face of the Defense Industry, op.cit., p. 58. Based on personal interviews, this writer's conclusion is that the Grumman viewpoint is shared by most, but not all, of the other major military airframe builders.

105. See Aerospace Profits vs. Risks, op.cit., pp. 10-11; Risk Elements in Government Contracting, op.cit., ch. 1; and F.T. Moore, "Incentive Contracts," in S. Enke, ed., Defense Management (Prentice-Hall; Englewood Cliffs, N.J.; 1967), ch. 12.

106. Ibid., ch. 12; O.W. Williamson, "The Economics of Defense Contracting: Incentives and Performance," in R.N. McKean,ed., Issues in Defense Economics (Columbia University Press; New York; 1967), pp. 217-256; F.M. Scherer, "The Theory of Contractual Incentives for Cost Reduction," Quarterly Journal of Economics (vol. 78; no. 2; May 1964, pp. 257-280; J.J. McCall, "The Simple Economics of Incentive Contracting," American Economic Review (vol. 60; no. 5; December 1970), pp. 837-846; and M.E. Canes, "The Simple Economics of Incentive Contracting: Note," American Economic Review (vol. 65; no. 3; June 1975), pp. 478-483. The Scherer paper is particularly noteworthy because it offers empirical evidence in support of the hypothesis that defense contractors are risk averse. Scherer, The Theory of Contractual Incentives for Cost Reduction, op.cit., pp. 273-276.

107. These basic contract types, as well as several variations, are discussed in Evans, Margulis, and Yoshpe, op.cit.

108. Defense Procurement Circular No. 76-3, op.cit., pp. 12-15. See also Aerospace Profits vs. Risks, op.cit., p. 10.

109. An extreme case in which virtually all risk was borne by the contractor was the 'total package procurement' policy introduced by Robert McNamara when he was Secretary of Defense. Under total package procurement, companies were forced to bid on a fixed-price contract covering both development and production, and, as Lockheed's experience on the C-5A transport contract and Grumman's experience on the F-14 contract attest, the contractor's risk of severe financial loss due to such factors as inflation and unforseen costs were intolerably high, and as a result, total package procurement has been

abandoned in favor of separate contracts for development and production, with the former normally on a cost-plus basis and with the latter normally on a FPI basis for the early stages of production. Defense Procurement Circular No. 76-3, op.cit., p. 12.

110. Ibid., p. 11

111. Ibid., p. 11, and Risk Elements in Government Contracting, op.cit., ch. 2.

112. For example, Fairchild Industries's attempts to develop its communications business (see "A Last Run For The Money," Forbes (May 15, 1976)) and Rockwell International's acquisition of Admiral Corp. and many other commercially oriented companies (see "Rockwell walks a rough road to profits," Business Week (November 3, 1975) and "Rockwell's surprising winner: Collins Radio," Business Week (November 15, 1976)). In addition, General Dynamics recently announced its intention to look for potential non-aerospace commercial acquisitions (see "General Dynamics Sees Bright Future On Strength of Tanker, Fighter Projects," Wall Street Journal (January 27, 1977)).

113. The difference between the median values of the average return on equity is so much smaller than the difference between the median values for the average return on total capital because a significant portion of the major airframe builders' total capital is provided by the government and because the major airframe builders tend to have higher debt-equity ratios than firms in other industries. In addition, the difference in Table VI-5 between median return on total capital for the airframe builders and for all industries probably understates the true difference because 'total capital' in the table excludes human capital, of which the aerospace industry has proportionately more than other industries.

114. More rigorously, a difference of medians test was performed. See W.L. Hays, Statistics (Holt, Rinehart and Winston; New York; 1973), pp. 194-197. Testing the null hypothesis that the average rate of return on equity for the nine major airframe builders has the same distribution as the average rate of return on equity for the other eight aerospace firms included by Forbes (Aerospace and Defense, op.cit., p. 133) against the alternative hypothesis that the other eight firms have a higher median return yielded a critical (at the .05 level) score of six. Since the 'other' sample had only five values above the grand median, the null hypothesis could not be rejected. Since, by inspection, the industry median and the all-industry median are not significantly different, the conclusion stated in the text follows.

115. Several other studies have reached the same conclusion. For example, see Weidenbaum, op.cit., pp. 69-70. It should be emphasized that this conclusion carries no implication regarding the question of whether profits are adequate in relation to risks. Further, it should be noted that if either of the other two measures of profitability in Table VI-5 are used as the basis of comparison - as they often are in studies sponsored by the aerospace industry - then the opposite conclusion is drawn, namely, that aerospace profits are significantly lower than profits in other industries. See Aerospace Profits vs. Risks, op. cit., pp. 13-17.

116. See Rockwell walks a rough road to profits, op.cit., for a discussion of these acquisitions and the growth motive that lay behind them.

117. This dominance is, of course, one factor that tends to discourage potential entrants.

118. See R.C. Fraser, A.D. Donheiser, and T.G. Miller, Jr., Civil Aviation Development: A Policy and Operations Analysis (Praeger; New York; 1972), pp. 9-12.

119. Both to replace older, less fuel efficient aircraft and to meet new federal noise standards. See Harr, op.cit., p. 14, and Aerospace and Defense, op.cit., p. 136.

120. See Nation's Airlines Face A Key Problem: How To Pay for New Planes, op.cit. The problems, financial and otherwise, that confront the commercial aircraft end of the aerospace industry are discussed in R.C. Fraser, A.D. Donheiser, and T.G. Miller, Jr., op.cit.

121. See The Next Commercial Jet . . . If, op.cit.

122. Harr, op.cit., p. 14. As a result, losses can be large. See Lockheed Sets L-1011 Charge Of $515 Million, op.cit.

123. Harr, op.cit., pp. 14-15. The significance of foreign sales and foreign competition is discussed in the next subsection.

124. Ibid., pp. 15-16.

125. For example, Fairchild Industries is experiencing large losses in trying to start up its communications business and plans to use the profits it hopes to earn on its A-10 contract to pay these start-up costs. See A Last Run For The Money, op.cit. As a second example, Lockheed has experienced large losses on its L-1011 TriStar program, but due to its profitable defense business, is able to meet bond interest payments. "Haack at Lockheed proclaims an upturn," Business Week (June 28, 1976.)

126. This point was made by several of the executives interviewed by the author. Problems these firms face in trying to diversify into commercial markets are discussed in J.S. Gilmore and D.C. Coddington, Defense Industry Diversification (U.S. Arms Control and Disarmament Agency; Washington, D.C.; January 1966).

127. See Rockwell walks a rough road to profits, op.cit., and Rockwell's surprising winner: Collins Radio, op.cit. An earlier study by Gilmore and Coddington reached the opposite conclusion, namely, that aerospace firms favor growth by internal means. Gilmore and Coddington, op.cit. However, their study covered a time period in which the growth prospects in these firms' traditional markets were excellent. Since the managers of these firms were preoccupied with developments in their traditional markets, it is not surprising that Gilmore and Coddington found that the degree of diversification undertaken by these firms was insignificant in terms of its impact on company sales and profits.

128. Aerospace Profits vs. Risks, op.cit., p. 3.

129. During interviews conducted by the author, executives of several of the firms that are more heavily dependent on government sales expressed a desire to see their companies expand their commercial operations enough to attain a 50-50 sales split between government and non-government business. David S. Lewis, Chairman of General Dynamics Corp., has also stated publicly his company's goal of a 50-50 sales split. See "General Dynamics renews its Pentagon romance," Business Week (February 3, 1975).

130. However, as several aerospace executives have recognized, government sales can provide stability when commercial demand weakens - provided the business cycle and the political cycle do not cause military sales and commercial sales to turn down simultaneously. Ibid., pp. 58-59.

131. Harr, op.cit., p. 16.

132. "Anatomy of the Arms Trade," Newsweek (September 6, 1976).

133. See, for example, L. Kraar, "Grumman Still Flies For Navy, But It Is Selling the World," Fortune (February 1976). Over the last decade Northrop Corp.'s major product has been the F-5, the market for which has been almost entirely overseas. See "The New Adventures of Tom Jones," The New York Times (September 19, 1976).

134. See "Belgium Joins Others, Picks U.S.-Built F-16," Wall Street Journal (June 9, 1975); "The Politics Of The F-16," Forbes (December 15, 1976); and "NATO Defense Chiefs Agree in Principle To Buy AWACS if Financing Is Settled," Wall Street Journal (December 9, 1976).

135. Harr, op.cit., p. 16. For a practical example, see "Iran Seeks 300 General Dynamics F-16s, Near Double of What U.S. Agreed to Sell," Wall Street Journal (September 13, 1976).

136. See "Lockheed Signs $1.03 Billion Agreement With Canada for Planes, Related Work," Wall Street Journal (July 22, 1976); Belgium Joins Others, Picks U.S.-Built F-16, op.cit.; and NATO Defense Chiefs Agree in Principle To Buy AWACS if Financing Is Settled, op. cit.

137. See "Buying guns to sell planes," Business Week (June 23, 1975).

138. See "European Members of NATO Strive to Build Weapons Industry to Compete With U.S. Firms," Wall Street Journal (November 3, 1976).

139. Harr, op.cit., p. 15. The foreign-owned airlines have also contributed greatly to overcapacity on international routes, and, to the extent that such overcapacity has hurt the U.S. international airlines financially, this may have had a detrimental impact on commercial aircraft sales of the U.S. aerospace industry. See T. O'Hanlon, "The Mess That Made Beggars of Pan Am and T.W.A.," Fortune (October 1974).

140. See "Air Transportation: The Real Issues," Government Executive (October 1976), for a discussion of the consequences of this pooling.

141. Harr, op.cit., p. 15.

142. See Free-world partners plan jets for the 1980s, op.cit.

143. These longer periods are, in one case, a short term planning period of five years and a long term planning period of ten years, and in the other case, a short term planning period of two years and a long term planning period of seven years. The other seven firms use the one year and five year time horizons stated in the text.

144. In many cases there is more than one division, as for example, McDonnell Aircraft Company, which produces mainly military aircraft, and Douglas Aircraft Company, which produces mainly commercial aircraft (and which was a separate company until taken over by McDonnell in 1965) of McDonnell Douglas Corp.

145. In several cases, such as LTV's Vought Corp., the aerospace operations are centralized in a wholly owned subsidiary, rather than a division, and the parent company is a holding company. For the purposes of this chapter, the distinction between a division and a wholly owned subsidiary is not an important one since it does not affect the corporate planning process.

146. These are discussed in chapter two of this thesis.

147. See section G in chapter two of this thesis.

148. See section H in chapter two of this thesis.

149. For the eight of the nine firms that have a company president - General Dynamics has instead three executive vice presidents with specific area responsibilities within which each serves in the same capacity as the president of a subsidiary would (see General Dynamics: Winning in the Aerospace Game, op.cit.) - that individual (and often one or more other top executives) sits on the board of directors.

150. Major shareholders are particularly influential at McDonnell Douglas and Rockwell, where they hold top management positions, including chairman of the board of directors, and at General Dynamics, where the major shareholder personally recruited the chairman of the board of directors. See Rockwell walks a rough road to profits, op.cit., which describes Chairman Willard F. Rockwell's role in determining Rockwell International's objectives, and General Dynamics: Winning in the Aerospace Game, op.cit., which describes the influence of the firm's largest stockholder, Henry Crown.

151. To make the author's view of the typical airframe builder's objectives more clear, it is his belief that each of the three theories - traditional, managerial, and behavorial - has something to contribute to the overall understanding of these firms' objectives, though any one of the three on its own gives an incomplete picture.

152. General Dynamics renews its Pentagon romance, op.cit.

153. The question of weapons system quality and the preferences of the U.S. government with regard to quality, cost, and development time are discussed in Peck and Scherer, op.cit., ch. 10.

154. This carryover effect is probably stronger the greater is the technological complementarity between the particular commercial product and the firm's high technology military aircraft, e.g. it is likely to be stronger for commercial aircraft than for such items as refrigerators or canoes.

155. The connection between proposed projects and managerial emoluments may appear somewhat tenuous. In many cases, however, a portion of managerial compensation is based on an incentive compensation scheme, so that proposed projects can affect compensation through their impact on the company's performance, For example, Boeing has an incentive compensation plan. See The Boeing Company Form 10-K, op. cit., p. 14 and Exhibit 15.

156. For eight of the nine firms - Rockwell, whose fiscal year ends September 30, is the exception - the fiscal year parallels the calendar year. Unfortunately, the one exception makes it necessary to describe the planning cycle in terms of quarters (of the fiscal year), rather than in terms of calendar months.

157. For example, if the objective is held to be expected utility maximization, then specifying the appropriate utility function involves theoretical, as well as practical, difficulties. See G.M. Heal, The Theory of Economic Planning (American Elsevier; New York; 1973), ch. 2.

158. For example, formulating the planning problem as a nonlinear programming problem that contained an objective function that reflected not only the objectives discussed in section C, but also the existence of uncertainty, and that also contained the many constraints needed to characterize the real-world planning problem, might lead to any one, or possibly several, of the problems often encountered in trying to solve large scale nonlinear programming problems. See H.M. Wagner, Principles of Operations Research, 2nd ed. (Prentice-Hall; Englewood Cliffs, N.J.; 1975), chs. 14-15.

159. A third reason could be added to the two already mentioned: a basic distrust of planning models. Several of the planning executives interviewed by the author were steadfast in their belief that planning models of any kind - whether of the mathematical programming variety, of the simulation variety, or of some other variety - would disrupt, rather than promote, the long term and short term planning process.

160. The notion of a planning process that is optimal in the sense of being most cost effective, rather than in the sense of leading to an optimal solution to the planning problem, is analogous to Baumol's and Quandt's optimally imperfect

rules of thumb for business decisions. See W.J. Baumol and R.E. Quandt, "Rules of Thumb and Optimally Imperfect Decisions," American Economic Review (vol. 54; no. 2; March 1964), pp. 23-46.

161. The terms 'division' and 'divisional', it should be re-emphasized, are used to refer to the principal operating units of the corporation. These principal operating units are variously referred to as companies (e.g. Douglas Aircraft Company and McDonnell Aircraft Company of McDonnell Douglas Corp.), as divisions (e.g. Convair Division and Fort Worth Division of General Dynamics Corp.), as subsidiaires (e.g. Vought Corp of LTV Corp. and Grumman Aerospace Corp. of Grumman Corp.), and as groups (e.g. Admiral Group of Rockwell International Corp.). Often the principal operating units will themselves have divisions, but in what follows the focus is on the principal operating units, and the terms 'division' and 'divisional' refer to these units only and not to their subdivisions.

162. See the previous footnote.

163. Note that the three plans outline the division's needs for three classes of resources. The technical plan deals with human capital resources; the manpower and production plan deals essentially with labor resources (although managerial talent also contains a large human capital component); and the facilities plan deals with physical capital resources.

164. See footnotes 3 and 94.

165. It is almost universially accepted within the industry that once a new weapons program appears in the Five Year Defense Plan it is generally too late to begin the research and development process for that program.

166. Also, as will be pointed out in the next section, it is the division's responsibility, in formualting the operating plan, to allocate sufficient manpower and funds to form the required bid and proposal teams for those new programs on which the company (through the division) intends to bid.

167. Early Revival Unlikely As Jumbo-Plane Sales Continue to Languish, op.cit.

168. Boeing, McDonnell Douglas, and Lockheed are doing this, but Boeing's 747 production line and McDonnell Douglas's DC-10 production line were each operating at approximately 20% of capacity at the end of 1976. Ibid.

169. See "Swissair Seeks to Launch New DC9 Model With Order to McDonnell Douglas Corp,." Wall Street Journal (January 20, 1977).

170. For example, an airplane that costs $30 million to build will involve an interest cost of $250,000 for every month it remains unsold (assuming an annual interest rate of 10 percent.)

171. See, for example, Lockheed Sets L-1011 Charge Of $515 Million, op.cit.

172. See, for example, T.W. Schultz, Investment in Human Capital (Free Press; New York; 1971); R.A. Wykstra, ed., Human Capital Formation and Manpower Development (Free Press; New York; 1971); B.F. Kiker, ed., Investment in Human Capital (University of South Carolina Press; Columbia, S.C.; 1971); F. Welch, "Education in Production," Journal of Political Economy (vol. 78; no. 1; January-February 1970), pp. 35-59; and G.S. Becker, Human Capital, 2nd ed. (Columbia University Press; New York; 1975).

173. Schultz, op.cit., ch. 3.; B.F. Kiker, "The Historical Roots of the Concept of Human Capital," in Kiker, op.cit., pp. 51-77; and Becker, op.cit., ch. II. A broader definition of human capital would also include the skills and know-how embodied in the firm's production workers, but since the focal point of this section is long term planning, and in particular, the allocation of scientists, engineers, designers, and technicians, the narrower definition provided in the text seems to this writer more appropriate.

174. The distinction between fixed capital and human capital, as well as the distinction between these types of capital and other types of capital, are discussed in the papers cited in footnote 172.

175. Under the wider definition of human capital, which was mentioned in footnote 173, one would have to include also the services of human capital (embodied in production workers) that are provided during the production phase of the program. This particular flow of human capital services underlies the learning curve discussed in section B.

176. This is particularly important in the airframe industry, where, as discussed in section B, a significant portion of the total fixed capital is provided by the government.

177. One of the consequences of the human capital embodied in aerospace engineers and scientists may be the existence of a segmented labor market for persons embodying these skills and knowledge. This body of theory is discussed in G.G. Cain, "The Challenge of Segmented Labor Market Theories to Orthodox Theory: A Survey," Journal of Economic Literature (vol.14; no. 4; December 1976), pp. 1215-1257.

178. The difficulties and costs associated with trying to evaluate a prospective employee's stock of human capital are discussed in J.G. Riley, "Information, Screening and Human Capital," American Economic Review (vol. 66; no. 2; May 1976), pp. 254-260.

179. And this is likely to become increasingly important as the Department of Defense implements its new design-to-cost policy. The policy is outlined in several DOD and service instructions beginning with Department of Defense Directive 5000.28, "Design to Cost" (May 23, 1975). The concept of design to cost is explained in J.J. Bennett, "Design to Cost", Commander's Digest (vol. 19; no. 17; August 12, 1976).

180. These firms' reluctance to lay off key engineering personnel, for example, has led to accusations of hoarding of engineering personnel. Several studies have provided evidence that engineering talent is being wasted in jobs that require only routine skills. See Peck and Scherer, op.cit., pp. 515-517. This supposed 'hoarding' may, in the opinion of this writer, still be less costly to the firm than a policy of hiring and firing due to the potentially high costs of searching for the required talent.

181. Harr, op.cit., p. 12.

182. The two exceptions are noted in footnote 143. In each of these cases, however, the first year's operating plan is given in the greatest detail and is presented in the form of a budget.

183. The budget preparation process is described in management accounting textbooks. See, for example, R.N. Anthony and G.A. Welsch, Fundamentals of Management Accounting (Irwin; Homewood, Ill; 1974), ch. 11.

184. In some cases, however, the projections for the first few years of the long term plan are broken out on a quarter-by-quarter basis.

185. The overhead rate is a ratio that is applied to the cost of an hour of direct labor in order to allocate indirect costs, such as general and administrative expenses, depreciation and maintenance, utilities, etc., over the goods produced. Often several different overhead rates are used. For example, government procurement regulations favor the following three: a manufacturing overhead rate, an engineering overhead rate, and a general and administrative expenses overhead rate. See Defense Procurement

Circular No. 76-3, op.cit., p. 11. A general discussion of overhead rates and their computation can be found in Anthony and Welsch, op.cit., pp. 70-74. Using their terminology, the contractor and the government negotiate a 'predetermined overhead rate' for each overhead cost pool once a year. The evaluation of overheads as part of determining contractor fees is discussed in Defense Procurement Circular No. 76-3, op.cit., pp. 11-12. The apparent tendency for contractors to try to include indirect labor as direct labor in order to reduce the overhead rates and appear more efficient than they really are is argued in Peck and Scherer, op.cit., pp. 517-519.

186. This is not meant to suggest that such 'assistance' is always welcomed by the contractor.

187. Recently, as one result of the Profit '76 study, the weight attached to contractor performance in determining the fee to be earned on a contract has been reduced from 65% to 50%. See Defense Procurement Circular No. 76-3, op.cit., p. 2.

188. Ibid., pp. i-ii.

189. Due to the importance of meeting delivery schedules, there may be a tendency for firms to overman. See Peck and Scherer, op.cit., pp. 516-517. Such overmanning, to the extent that it reduces the risk of late delivery (and poor contract performance) and to the extent that the cost of overmanning is borne by the government, constitutes a transfer of risk from the contractor to the government.

190. See Early Revival Unlikely As Jumbo-Plane Sales Continue to Languish, op.cit.

191. Several executives interviewed by the author indicated that, even when commercial demand is strong, these advance payments seldom exceed 25% of production costs, as opposed to the government's provision of progress payments covering 80% of (allowable) costs.

192. Ibid.

193. This is not meant to imply that divisional managers always wait until the corporate review to indicate problem areas, although this may happen. Normally, serious problems are called to the attention of top management as they arise, and the corporate review process is one place where top management can become forewarned of potential problem areas.

VII. A MODEL OF A REPRESENTATIVE AIRFRAME BUILDER

A. INTRODUCTION

Chapter six characterized the nine major military airframe builders in the United States and described their long term and short term planning processes. The purpose of this chapter is to develop a model of a representative U.S. military airframe builder that can be used to study several of the issues raised in chapter six, such as how the desire of these firms to maintain stable employment for their skilled scientific and engineering talent affects their behavior. The development of the airframe builder model proceeds via several modifications of the author's basic theoretical model described and analyzed in chapters three through five of this thesis.

While there have been many studies of both the U.S. aerospace industry[1] and the government-contractor relationship during the weapons acquisition process,[2] there have been few attempts to model a defense contractor. Scherer, McCall, and Baron have developed models of contractor behavior that are mainly concerned with the effects that incentive contracts have on contractor bidding and contractor performance.[3] All three models assume that the firm is of the traditional type.[4] Gorgol has developed a simulation model[5] and Jones has specified a production-investment-finance model of a representative contractor.[6] Both these studies were mainly concerned with model formulation - indeed, both carefully tried to formulate a 'realistic' model. Gorgol's study yielded few analytical results, as it was that author's intention to try to simulate actual behavior, rather than to explore changes in contractor behavior in response to changes in government policy. Jones's study did not deal with analytical issues as it is that author's intention to treat these topics in later papers.[7]

In this chapter, two versions of the airframe builder model are developed, one without progress payments and the other with progress payments, and the operating and financial policies suggested by the two models are compared in order to evaluate the impact of the government's policy of granting progress payments to cover a portion of a contractor's costs under ongoing production contracts. The chapter begins with the development of the airframe builder model without progress payments. The model employs the time-state-preference approach to modeling uncertainty and takes into account contractor risk aversion.[8] The model is used in section C to derive a representative airframe builder's optimal operating and financial policies. In section D progress payments are incorporated into the model. The implications of progress payments are derived by comparing the optimal operating and financial policies obtained from this version of the model with those obtained in section C. The airframe builder model is also used to study the impact of other aspects of government procurement policy on contractor behavior. In particular, several of the implications of the new design-to-cost policy[9] and the recent Profit '76 study[10] are explored in section E.

B. THE AIRFRAME BUILDER MODEL

The purpose of this section is to develop the basic model of a representative airframe builder for the special case in which progress payments are not needed because the flow of cash from the government to the contractor matches perfectly the recognition of revenue and profit by the contractor. The model described below is an extension of the model of the expected collective utility maximizer set out and analyzed in chapters three through five of this thesis. As described below, the modifications to the theoretical model are made in accordance with the analysis of the planning processes of the major military airframe builders presented in chapter six. The further generalization of the model to incorporate progress payments is carried out in section D.

As in chapters three through five, it is assumed that the firm has a finite planning horizon of length T periods. As indicated in chapter six, in most cases T = 5 years, though in one case T = 7 years and in another case T = 10 years. For this reason T is left arbitrary. In addition, T is measured in discrete time units. It is convenient to treat both short term planning and long term planning within the same basic model, so that the units in which T is measured are referred to as 'periods'. It is to be understood that the discrete 'period' stands for 'year' in the case of long term planning and for 'quarter' in the case of short term planning.

As in chapter four, it is assumed that at each time t there are S distinguishable states of nature.[11] These are designated s = 1, ..., S. The time periods are designated

$t = 0, 1, \ldots, T$, where $t = 0$ denotes the present, at which the state of nature is assumed known with certainty. Henceforth, the double subscript t,s, $1 \leq t \leq T$, $1 \leq s \leq S$, will designate the state of nature s at time t.

The states of nature s call for further explanation. The possible states of nature at each time t are meant to reflect the different possible actions by the government or by competitors that might somehow affect the firm in question, as well as the different possible states of commercial demand (i.e. the business cycle), the different possible acts of nature that might affect the firm (e.g. a flood or a fire destroying a plant), and other conditions of the firm's operating environment that might have a nonnegligible impact on the firm. It is assumed that the alternative states of nature lie beyond the firm's control (which, as discussed in section K of chapter two, is a standard assumption made in employing the time-state-preference framework), although it is assumed that the possible states at each time t are known and that the firm can attach a (possibly subjective) probability to each.

In terms of the representative airframe builder's planning process, the states of nature s correspond to the different scenarios that are identified in (or at least, might be identified as a result of) the environmental forecast. The planning process takes these alternative states of nature into account through the development of contingency plans, which set out the actions to be taken by the firm contingent upon the realization of any particular state of nature (i.e. scenario) s at time t.

During the planning period the firm will work under many different government contracts, some of which it has already won and others of which it will bid on successfully. In addition, there will be contracts on which it will bid unsuccessfully. In what follows it is important to distinguish among these contracts because each involves a specific output, a distinctive technology, and hence a separate decision problem for the firm.[12] Since the government contracting process treats manufacturing contracts differently from research and development, test and evaluation contracts, as discussed in chapter six, it is also important to distinguish between these two classes of contracts.[13] In formulating the model the subscript c is used to distinguish among contracts. The aerospace research and development contracts under which the firm is working and on which it intends to bid are numbered $c = 1, \ldots, C_1$. The aerospace manufacturing contracts under which the firm is working and on which it intends to bid are numbered $c = C_1 + 1, \ldots, C_1 + C_2$. The non-aerospace government contracts under which the firm is working and on which it intends to bid are numbered $c = C_1 + C_2 + 1, \ldots, C_1 + C_2 + C_3$. In what follows the set of aerospace research and development contracts in force at time t in state s is denoted by $C_{1,t,s}$. Similarly, the sets of aerospace manufacturing contracts and non-aerospace government contracts in force at time t in state s are denoted by $C_{2,t,s}$ and $C_{3,t,s}$, respectively, and the collection of all government contracts in force at time t in state s is denoted by $\cup\{C_{i,t,s}\}$.

In this chapter it is assumed that the market for contingent output claims is incomplete.[14] It is also assumed that for each period t, $0 \leq t \leq T$, and for each possible state of nature s at each time t, a (possibly imperfectly competitive) market will exist for each of the contractor's commercial goods and a perfect market will exist for each of the inputs employed by the firm. Thus, in the model of the representative airframe builder developed below, the contractor will be able to select alternative output levels for each date and state (t, s) for each good and will also be able to select alternative input usage levels for each date and state for each input it employs. But the incompleteness assumption means that these are alternative output and input levels only. In the model developed below no trading in contingent claims of any sort takes place. As in the model presented in section E of chapter four it is assumed that inputs for date t and state s are purchased and output for date t and state s is produced and sold at time t only after state s has been realized.

On the basis of the discussion of the objectives of the airframe builders in section C of chapter six, the objective functional of the model of the firm is assumed to take the form of the maximization of discounted expected utility, where the discounting takes place over the period $t = 0$ to $t = T$; where the expectation is taken over the states of nature (at each time t); and where utility is expressed as a function of several arguments that reflect the sources of satisfaction to these firms. The first subsection formulates the objective functional.

1. The Objective Functional

As discussed in section C of chapter six, the objectives of the representative military airframe builder can be grouped into five classes: sales, net income, weapons system effectiveness, contract backlogs, and managerial emoluments.

To take into account the interest of these firms in diversification, the following four sales goals are specified in the model: total revenue earned on aerospace sales to the government, R_{GA}; total revenue earned on sales of other goods to the government, R_{GO}; total revenue earned on commercial aerospace sales, R_{CA}; and total revenue earned on sales of other commercial products, R_{CO}. In addition, there is a net income goal, π; a set of weapons system effectiveness/contract performance goals, ${}_{c}\hat{E}$;[15] a backlogs goal, $\hat{B}$; and a managerial emoluments goal, M. The sales, net income, and managerial emoluments goals are stated for each time period and each state of nature, and the weapons system effectiveness/contract performance goal is stated as a set of contract-specific goals for each time period and each state of nature. Because contract backlogs in any one period are directly related to government sales in future periods, the contract backlogs goal is stated as of the planning horizon $t = T$ to avoid redundancy. To allow for the value to the firm of commercial sales, net income, etc., beyond the planning horizon, the terminal capital stock, $K(T)$, also needs to be allowed for in the objective functional.[16]

In the model of the representative military airframe builder the objective functional is:

$$\max \{ \sum_{t=1}^{T} E_s[U_1(R_{GA,t,s}; R_{GO,t,s}; R_{CA,t,s}; R_{CO,t,s};$$
$$\pi_{t,s}; [{}_c\hat{E}_{t,s}]; M_{t,s})](\frac{1}{1+r})^t + E_s[U_2(K_{T,s}, \hat{B}_{T,s})] \quad (1)$$
$$\times (\frac{1}{1+r})^T \}$$

where E_s denotes expectation with respect to the states of nature; where U_1 and U_2 are assumed to be concave and twice differentiable with strictly positive first partial derivatives; where r is the exogenously determined rate of discount (which is assumed to remain constant);[17] where the brackets around ${}_c\hat{E}_{t,s}$ denote a vector the arguments of which pertain to the respective contracts in force at time t in state s; and where the arguments of the utility functions are as defined above.

2. The Constraints

In the model under development, the objective of the representative military airframe builder is to select the values of certain decision variables, which have yet to be specified, so as to maximize (1) subject to certain constraints, which also need to be specified. These constraints, which are formulated in this subsection, define the permissible ranges of values of the decision variables and also relate the decision variables to the arguments of the utility functions in (1).

One of the firm's decisions involves the selection of input levels. The productive resources of the firm are divided into the following five classes:[18] capital furnished by the contractor, which is denoted by K^C; capital furnished by the government,[19] which is denoted by K^G; manufacturing labor, which is denoted by L^M; administrative labor, which is denoted by L^A; and engineering and scientific labor, which is denoted by L^E. In the model K^G is treated as exogenously determined. In particular, it is one of the government's procurement policy parameters.

Let $\overline{K}^C$ denote the available stock of contractor-furnished capital and let $\overline{K}^G$ denote the available stock of government-furnished capital. Since the amount of capital allocated to production cannot exceed capacity, the constraints

$$\left.\begin{aligned} K^C{}_{t,s} &\leq \overline{K}^C{}_{t,s} \\ K^G{}_{t,s} &\leq \overline{K}^G{}_{t,s} \end{aligned}\right\} \qquad (2)$$

must be satisfied for each state s and each time t. It is assumed that capital of each type is homogeneous; but it is not required that contractor-furnished capital and government-furnished capital be perfectly substitutable for one another in production under each government contract. It should be noted that government-furnished capital can be used only in production under government contracts; it is provided to the contractor rent-free and its depreciation does not constitute a cost to the contractor. Contractor-furnished capital can be used in either government or commercial production; its use involves a cost to the contractor, and both depreciation and imputed interest are allowable costs under government contracts.[20] It is assumed that contractor-furnished capital depreciates at a constant percentage rate, δ, so that the constraint

$$\Delta\bar{K}^{C}_{t,s} \equiv \bar{K}^{C}_{t,s} - \bar{K}^{C}_{t-1,s'} = I_{t,s} - \delta \cdot \bar{K}^{C}_{t,s} \tag{3}$$

must be satisfied for each state s and each time t, where $\bar{K}^{C}_{t,s}$ denotes contractor-furnished capital at time t in state s; where s' is used to denote the possible states of nature at time t - 1 in order to distinguish them from the possible states of nature at time t; where it is assumed that investment is made at the beginning of the period and that depreciation is reckoned on the basis of the current period's capital stock; and where $I_{t,s}$ denotes the contractor's investment in capital at time t in state s.

The firm's choice of input mix is determined at least in part by the technology of production. Following Jones, the technological relationship between inputs and outputs for production under government contracts is treated as contract specific.[21] That is, there is a separate production function for each contract. The general production function for each government contract is denoted by

$$\begin{aligned} F_C([{}_cQ_{t,s}];[{}_c\hat{E}_{t,s}];[{}_cL^M_{t,s}];[{}_cL^A_{t,s}]; \\ [{}_cL^E_{t,s}];[{}_cK^C_{t,s}];[{}_cK^G_{t,s}]) = 0 \ , \end{aligned} \tag{4}$$

where

[ ] denotes a vector

$c = 1, \ldots, C_1, C_1 + 1, \ldots, C_1 + C_2, C_1 + C_2 + 1, \ldots,$

$\quad C_1 + C_2 + C_3$

$t = 1, \ldots, T$

$s = 1, \ldots, S$

${}_cQ_{t,s}$ denotes output under contract c at time t in state s

${}_c\hat{E}_{t,s}$ denotes overall product effectiveness/contract performance under contract c at time t in state s[22]

The quantity ${}_cL^M_{t,s}$ denotes the amount of manufacturing labor allocated to contract c at time t in state of nature s, and the other input variables are interpreted analogously. It is assumed that the production function (4) for each contract c has a full set of continuous second partial derivatives. There is a production function of the form (4) for each government contract $c \in U\{C_{i,t,s}\}$. According to (4), the quantity of output at each time t and in each state of nature s is defined as an implicit function of the alternative amounts of output possible at different times t and in alternative states of nature s and also of the amounts of manufacturing labor, administrative labor, engineering and scientific labor, contractor-furnished capital, and government-furnished capital assigned at each time t and in each state s to meet the terms of that contract. It is assumed that F_C is defined in such a way that inputs applied in production in period t cannot be transformed into period t - 1 or earlier period outputs (i.e. that the use of inputs precedes output).[23]

It is assumed that the firm produces two commercial products, one aerospace and the other non-aerospace. Thus, in addition to (4), there are two other production functions:

$$\left.\begin{aligned} &F_R([{}_RQ_{t,s}];[{}_RL^M_{t,s}];[{}_RL^A_{t,s}];[{}_RL^E_{t,s}];[{}_RK^C_{t,s}]) = 0 \\ &F_N([{}_NQ_{t,s}];[{}_NL^M_{t,s}];[{}_NL^A_{t,s}];[{}_NL^E_{t,s}];[{}_NK^C_{t,s}]) = 0 \ , \end{aligned}\right\} \quad (5)$$

where the subscript R denotes aerospace production and the subscript N denotes non-aerospace production and where it is assumed that each of F_R and F_N has a full set of continuous second partial derivatives. Note that the difference between (4) and (5) is that the former include government-furnished capital and the latter do not.[24]

The firm's choice of capital inputs must satisfy (2), which may be reexpressed as:

$$\left.\begin{aligned} &\sum_{U\{C_{i,t,s}\}} {}_cK^C_{t,s} + {}_RK^C_{t,s} + {}_NK^C_{t,s} \leq \overline{K}^C_{t,s} \\ &\sum_{U\{C_{i,t,s}\}} {}_cK^G_{t,s} \leq \overline{K}^G_{t,s} \end{aligned}\right\} \quad (6)$$

where it is required that both constraints be satisfied at each time t and in each state of nature s, and where each sum is taken over all government contracts in force at time t in state s. In addition, it was noted in chapter six that the airframe builders try to provide stable employment for their engineers, scientists, and administrative labor. One method of incorporating this into the model is to formulate the following constraints:

$$\left.\begin{array}{l} \sum_{U\{C_{i,t,s}\}} {}_{C}L^{A}_{t,s} + {}_{R}L^{A}_{t,s} + {}_{N}L^{A}_{t,s} = \bar{L}^{A} \\ \\ \sum_{U\{C_{i,t,s}\}} {}_{C}L^{E}_{t,s} + {}_{R}L^{E}_{t,s} + {}_{N}L^{E}_{t,s} = \bar{L}^{E} \end{array}\right\} \qquad (7)$$

where $\bar{L}^A$ and $\bar{L}^E$ denote the exogenously determined constant levels of administrative labor and engineering and scientific labor, respectively; where (7) is required to hold at each time t and in each state of nature s; and where each sum is taken over all government contracts in force at time t in state s. The implications of (7) are explored below in section E.[25]

Turning next to the financial constraints, expressions must be formulated for the various key entries, such as net income, that appear in the firm's income statement, balance sheet, statement of retained earnings, and statement of sources and uses of cash. The discussion begins with the development of expressions for revenue earned under government contracts.

The level of sales earned under government contracts is partly dependent on the type of contract. Research and development contracts are typically of the cost-plus-fixed-fee (CPFF) variety, while production contracts are typically of the fixed-price-incentive (FPI) variety.[26] In each case allowable costs include the opportunity cost of fixed capital used in meeting the contract, where a 'cost of money rate' determined by the government is used in calculating this imputed cost.[27] Denote this cost of money rate by $\bar{i}_{t,s}$, which is exogenously determined in the model below.

Under research and development contracts total revenue equals total allowable costs plus the fixed fee. Total cost under a research and development contract at time t and in state of nature s, which is denoted by ${}_cC_{t,s}$ is given by

$$\begin{aligned} {}_cC_{t,s} &= w_M \cdot {}_cL^M_{t,s} + w_A \cdot {}_cL^A_{t,s} + w_E \cdot {}_cL^E_{t,s} \\ &\quad + (\bar{i}_{t,s} + \delta) q_{t,s} \cdot {}_cK^C_{t,s} \,, \end{aligned} \tag{8}$$

where w_M, w_A, and w_E are the exogenously determined unit costs of manufacturing labor, administrative labor, and engineering and scientific labor, respectively; where $q_{t,s}$ is the unit price of capital goods at time t in state s; and where $\delta \cdot q_{t,s} \cdot {}_cK^C_{t,s}$ represents depreciation expense (figured on a replacement cost basis) charged to the contract.[28] In addition, there is a fee ${}_c\bar{\pi}_{t,s}$, which the contractor perceives as being set by the government. In general, the contractor cannot be assured of being fully reimbursed for all its costs. Some portion will normally be disallowed. Therefore, even if the contractor is motivated to increase revenue, and by implication total cost, as Williamson and others have argued,[29] the contractor's policy choices are restricted by the possibility that a portion of the costs already incurred will be disallowed (because disallowed costs decrease net income). Let ${}_c\gamma_{t,s}$ denote the fraction of costs that the contractor perceives as allowable on contract c at time t and in state of nature s. It is reasonable to assume not only that ${}_c\gamma_{t,s}$ depends on actual cost, but that it should, at least beyond the contract's target cost, C*, decrease at a decreasing rate as a function of total cost:

$$\left.\begin{aligned} {}_c\gamma_{t,s}({}_cC_{t,s}) &\begin{cases} = 1, \text{ if } C \leq C^* \\ < 1, \text{ if } C > C^* \end{cases} \\ \frac{d\gamma}{dC} &\begin{cases} = 0, \text{ if } C \leq C^* \\ < 0, \text{ if } C > C^* \end{cases} \\ \frac{d^2\gamma}{dC^2} &\begin{cases} = 0, \text{ if } C \leq C^* \\ < 0, \text{ if } C > C^* \end{cases} \end{aligned}\right\} \tag{9}$$

It follows, then, that total revenue earned under a research and development contract, ${}_cR_{t,s}$, is given by

$${}_cR_{t,s} = {}_c\gamma_{t,s} \cdot {}_cC_{t,s} + {}_c\bar{\pi}_{t,s} , \tag{10}$$

where ${}_cC_{t,s}$ is given by (8) and ${}_c\gamma_{t,s}$ satisfies (9), and the fee net of disallowed costs, ${}_c\pi_{t,s}$, is given by

$${}_c\pi_{t,s} = {}_c\bar{\pi}_{t,s} - [1 - {}_c\gamma_{t,s}({}_cC_{t,s})]{}_cC_{t,s} , \tag{11}$$

where the expression in brackets represents that portion of total actual costs that are disallowed.

Turning next to production contracts, two cases must be distinguished: initial production contracts, for which bidding takes place,[30] and follow-on production contracts, for which bidding does not take place. In the case of initial production contracts, the contractor makes a bid ${}_cC^*_{t,s}$, where the subscript t is included for consistency with the notation adopted earlier in this section. Recall that states of nature were assumed to be defined in such a way that the potential actions of competitors (e.g. other firms bidding on the same contract) were taken into account. Assuming that a firm never bids on a contract that it does not wish to win, the bid ${}_cC^*_{t,s}$ in state of nature s is defined to be the maximum bid that will ensure the firm of winning the contract *in state of nature s*, i.e. given the state of the environment, bids by competitors, etc., that characterize state s. For subsequent production contracts there is again a target cost ${}_cC^*_{t,s}$ that is dependent on the state of nature. But these target costs are determined through negotiation between the contractor and the government, rather than through competitive bidding. Hence, even though the same symbol ${}_cC^*_{t,s}$ is used (for notational convenience), the underlying meaning is different according to whether it pertains to the first production contract or to a follow-on production contract.

All production contracts are assumed to be of the FPI type.[31] The contract stipulates a target cost ${}_cC^*_{t,s}$ and a target fee expressed as a proportion ${}_c\alpha_{t,s}$ of the target cost. Thus, the target fee is ${}_c\bar{\pi}_{t,s} = {}_c\alpha_{t,s} \cdot {}_cC^*_{t,s}$. The contract also specifies a sharing ratio, ${}_c\beta_{t,s}$, which gives the firm's share of overruns and underruns.[32] Following Scherer, absolute floors and ceilings on the maximum amounts of overruns and underruns for which the government is prepared to share responsibility are ignored,[33] although an implicit ceiling is built into the proportion of allowable costs function ${}_c\gamma_{t,s}$.[34] For each production contract, the constants ${}_c\alpha_{t,s}$ and ${}_c\beta_{t,s}$ are treated as exogenously determined,[35] and the function ${}_c\gamma_{t,s}({}_cC_{t,s})$ is again assumed exogenously determined and to satisfy (9). With actual costs, ${}_cC_{t,s}$, given by (8), total revenue earned on a production contract is given by

$$
\begin{aligned}
{}_cR_{t,s} = {}&{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + {}_c\alpha_{t,s} \cdot {}_cC^*_{t,s} \\
&+ {}_c\beta_{t,s}[{}_cC^*_{t,s} - {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s}] ,
\end{aligned} \tag{12}
$$

and the fee net of disallowed costs, ${}_c\pi_{t,s}$, is given by

$$
\begin{aligned}
{}_c\pi_{t,s} = {}&{}_c\alpha_{t,s} \cdot {}_cC^*_{t,s} \\
&+ {}_c\beta_{t,s}[{}_cC^*_{t,s} - {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s}] \\
&- [1 - {}_c\gamma_{t,s}({}_cC_{t,s})] \cdot {}_cC_{t,s}
\end{aligned} \tag{13}
$$

where it follows from (9) that ${}_c\gamma_{t,s} \equiv 1$ when ${}_cC_{t,s} \leq {}_cC^*_{t,s}$ in both (12) and (13). According to (12), total revenue is equal to costs paid by the government, $\gamma(C) \cdot C$, plus the fee received from the government, $\alpha C^* + \beta[C - \gamma(C) \cdot C]$, where the first term is the target fee and the second term is the incentive adjustment. But the actual fee, or contribution to the firm's operating income, is, according to (13), equal to the fee received from the government less the amount of disallowed costs, $[1 - \gamma(C)] \cdot C$.[36]

If it is assumed that all non-aerospace government contracts are of the FPI type, then (12) and (13) give the revenue and net fee, respectively, earned on those contracts. In addition, there are two commercial products, one aerospace and the other non-aerospace. Assume that for each quantity demanded at time t and in state s is a function of the product's price for that date and state. Assuming that each demand function is invertible (for each date and state), then the time-state-dependent revenue levels for these products satisfy

$$\left.\begin{aligned} R_{CA,t,s}({}_RQ_{t,s}) &= {}_RQ_{t,s} \cdot {}_Rp_{t,s}({}_RQ_{t,s}) \\ R_{CO,t,s}({}_NQ_{t,s}) &= {}_NQ_{t,s} \cdot {}_Np_{t,s}({}_NQ_{t,s}) , \end{aligned}\right\} \tag{14}$$

where ${}_RQ_{t,s}$ and ${}_NQ_{t,s}$ are the same as in the production functions (5) and where it is assumed that the demand functions ${}_Rp_{t,s}$ and ${}_Np_{t,s}$ are such that both $R_{CA,t,s}$ and $R_{CO,t,s}$ are strictly concave functions of ${}_RQ_{t,s}$ and ${}_NQ_{t,s}$, respectively. The expressions (14) for $R_{CA,t,s}$ and $R_{CO,t,s}$ express two of the arguments of the objective functional (1) in terms of the decision variables ${}_RQ_{t,s}$ and ${}_NQ_{t,s}$. The other two revenue arguments of the objective functional can be expressed in terms of (10) and (12) as

$$\begin{aligned} R_{GA,t,s} = & \sum_{C_{1,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + {}_c\overline{\pi}_{t,s}] \\ & + \sum_{C_{2,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + {}_c\alpha_{t,s} \cdot {}_cC^*_{t,s} \\ & + {}_c\beta_{t,s}\{{}_cC^*_{t,s} - {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s}\}] \end{aligned} \tag{15}$$

and

$$\begin{aligned} R_{GO,t,s} = & \sum_{C_{3,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + {}_c\alpha_{t,s} \cdot {}_cC^*_{t,s} \\ & + {}_c\beta_{t,s}\{{}_cC^*_{t,s} - {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s}\}] , \end{aligned} \tag{16}$$

where the sums in (15) and (16) are taken over those contracts of each type that are in force at time t in state s.

Turning next to the firm's financial statements, a balance sheet for the representative airframe builder is shown in table VII-1. Cash on hand, $C_{t,s}$, is assumed to equal some minimum level, $\overline{C}_{t,s}$, needed to fund transactions, plus some additional amount (possibly zero), $\hat{C}_{t,s}$, held

Table VII-1 Representative Airframe Builder's Balance Sheet

Assets		Liabilities	
Cash	$C_{t,s}$	Debt	$B_{t,s}$
Inventories	$V_{t,s}$	Equity	$E_{t,s}$
Fixed Assets	$q_{t,s} \cdot K_{t,s}$		
Total Assets	$\underline{\underline{C_{t,s}+V_{t,s}+q_{t,s} \cdot K_{t,s}}}$	Total Liabilities and Stockholders' Equity	$\underline{\underline{B_{t,s}+E_{t,s}}}$

as precautionary balances. It is further assumed that the transactions demand for cash can be expressed as a function of the different output levels, so that

$$C_{t,s} = \overline{C}_{t,s}([{}_cQ_{t,s}],\ {}_RQ_{t,s},\ {}_NQ_{t,s}) + \hat{C}_{t,s} \ , \tag{17}$$

where $[{}_cQ_{t,s}]$ denotes the vector of outputs under government contracts and where ${}_cQ_{t,s} \equiv 0$ for those contracts c not in force at time t in state s. It is assumed that

$$\partial C_{t,s}/\partial_c Q_{t,s} > 0 \text{ and } \partial C_{t,s}/\partial_k Q_{t,s} > 0 \ , \tag{18}$$

where $c = 1, \ldots, C_1+C_2+C_3$, and where $k = R, N$, so that transactions balance requirements rise monotonically with output levels. It is also assumed that the minimum-cost value of inventories held at time t in state s is some function of the output levels at time t in state s,

$$V_{t,s} = V_{t,s}([{}_cQ_{t,s}],\ {}_RQ_{t,s},\ {}_NQ_{t,s}) \ . \tag{19}$$

It is assumed that

$$\partial V_{t,s}/\partial_c Q_{t,s} > 0 \text{ and } \partial V_{t,s}/\partial_k Q_{t,s} > 0 \ , \tag{20}$$

where $c = 1, \ldots, C_1+C_2+C_3$ and where $k = R, N$, so that required inventories (or working capital) increase monotonically with output levels.

Finally for the assets side of the balance sheet, fixed assets are contractor-owned fixed assets, $\overline{K}^C_{t,s}$ in (3) and (6), so that

$$K_{t,s} = \overline{K}^C_{t,s} \, . \tag{21}$$

As assumed throughout this thesis, fixed assets are listed in the firm's balance sheet on a replacement cost basis. The physical stock of fixed capital, $K_{t,s}$, is thus valued at the current unit cost of fixed capital, $q_{t,s}$. The fixed assets entry in table VII-1 represents the replacement cost of contractor-owned productive capacity only, but as indicated below, is affected by the availability of government-furnished capital.

Turning to the liabilities side of the balance sheet in table VII-1, the quantity of debt outstanding at time t in state s is denoted by $B_{t,s}$. Associated with debt is an average interest cost $i_{t,s}$,[37] so that total interest expense per period is given by the product $i_{t,s} \cdot B_{t,s}$. New debt issues or redemptions are denoted by $Y_{t,s}$, which satisfies the identity

$$Y_{t,s} = B_{t,s} - B_{t-1,s'} \equiv \Delta B_{t,s} \, , \tag{22}$$

where s' is used to denote the possible states of nature at time t - 1 in order to distinguish them from the possible states of nature at time t. Assuming all new debt issues or redemptions to be handled simultaneously, $\Delta B_{t,s} > 0$

in (22) implies net issues and $\Delta B_{t,s} < 0$ in (22) implies net redemptions. It is further assumed, as in chapter four, that the average rate of interest on debt is a function of both the amount of debt and the change in the amount of debt,

$$i_{t,s} = i_{t,s}(B_{t,s}, Y_{t,s}) \, , \tag{23}$$

where

$$\partial i_{t,s}/\partial B_{t,s} > 0 \text{ and } \partial i_{t,s}/\partial Y_{t,s} > 0 \, . \tag{24}$$

The amount of equity at time t in state s is denoted by $E_{t,s}$, which is the sum of contributed capital $_K E_{t,s}$, and retained earnings, $_R E_{t,s}$. It is assumed that the firm sets its dividend policy at the beginning of the planning period, thereby establishing a dividend per period per share that remains fixed throughout the planning period. Let d denote this dividend per period per share and let $n_{t,s}$ denote the number of equity shares outstanding at time t in state s. Then total dividends paid at time t in state s are given by the identity

$$D_{t,s} = d \cdot n_{t,s} \, . \tag{25}$$

New equity issues or redemptions at time t in state s are denoted by $Z_{t,s}$, which satisfies the identity

$$Z_{t,s} = n_{t,s} - n_{t-1,s}, \equiv \Delta n_{t,s} \, . \tag{26}$$

Note that (25) and (26) require that shares be issued or redeemed at the beginning of the period, so that new shareholders are entitled to receive dividends in the period in which they purchase their shares. If the issue price/redemption price per share at time t in state s is denoted by $v_{t,s}$, then it follows from (26) that the book value of contributed capital at time t in state s is given by the identity

$$K^E_{t,s} = K^E_{t-1,s'} + Z_{t,s} \cdot v_{t,s} . \tag{27}$$

Denoting retained earnings at time t in state s by $e_{t,s}$, the accumulated stock of retained earnings at time t in state s satisfies the identity

$$R^E_{t,s} = R^E_{t-1,s'} + e_{t,s} \tag{28}$$

and, combining (27) and (28), equity at time t in state s satisfies the identity

$$\begin{aligned} E_{t,s} &= K^E_{t,s} + R^E_{t,s} \\ &= K^E_{t-1,s'} + Z_{t,s} \cdot v_{t,s} + R^E_{t-1,s'} + e_{t,s} \\ &= E_{t-1,s'} + Z_{t,s} \cdot v_{t,s} + e_{t,s} , \end{aligned} \tag{29}$$

where $Z_{t,s}$ is a decision variable and $e_{t,s}$ is determined by the firm's choice of operating and financial policies, as described below.

The importance of the balance sheet identity between assets and liabilities is summarized in the following constraint, which must hold at each state and date:

$$\begin{aligned} &\overline{C}_{t,s}([{}_cQ_{t,s}], {}_RQ_{t,s}, {}_NQ_{t,s}) + \hat{C}_{t,s} + q_{t,s} \overline{K}^C_{t,s} \\ &\quad + V_{t,s}([{}_cQ_{t,s}], {}_RQ_{t,s}, {}_NQ_{t,s}) = B_{t,s} + E_{t,s} . \end{aligned} \tag{30}$$

Turning next to the representative airframe builder's income statement and statement of retained earnings, which are illustrated in table VII-2, an expression for net income for the period and state of nature, $\pi_{t,s}$, and an expression

Table VII-2 Representative Airframe Builder's Income Statement and Statement of Retained Earnings

Income Statement

Sales revenue:	
Government sales revenue	$R_{GA,t,s}+R_{GO,t,s}$
Commercial sales revenue	$R_{CA,t,s}+R_{CO,t,s}$
Total Sales Revenue	$R_{GA,t,s}+R_{GO,t,s}+R_{CA,t,s}+R_{CO,t,s}$
Expenses:	
Labor $w_M L^M_{t,s}+w_A L^A_{t,s}+w_E L^E_{t,s}$	
Emoluments $M_{t,s}$	
Depreciation $q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}$	
Interest $i_{t,s}\cdot B_{t,s}$	
Total Expenses	$w_M L^M_{t,s}+w_A L^A_{t,s}+w_E L^E_{t,s}+M_{t,s}$ $+q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}+i_{t,s}\, B_{t,s}$
Pretax Income $\{\quad\}$	$\equiv\sum_i R_{i,t,s}-\sum_j w_j L^j_{t,s}-M_{t,s}$ $-q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}-i_{t,s}\cdot B_{t,s}$
Income tax	$\tau\cdot\{\quad\}$
Net Income	$\pi_{t,s}\equiv(1-\tau)\{\sum_i R_{i,t,s}-\sum_j w_j L^j_{t,s}-M_{t,s}$ $-q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}-i_{t,s}\cdot B_{t,s}\}$

Statement of Retained Earnings

Beginning balance, retained earnings	$R^E_{t-1,s'}$
Add net income for the year	$\pi_{t,s}$
Total	$R^E_{t-1,s'}+\pi_{t,s}$
Less total dividends paid during the year	$d\cdot n_{t,s}$
Ending balance, retained earnings $R^E_{t,s}$	$= R^E_{t-1,s'}+\pi_{t,s}-d\cdot n_{t,s}$

for retained earnings for the period and state of nature, $e_{t,s}$, must be calculated. In the income statement in table VII-2 the quantities $L^M_{t,s}$, $L^A_{t,s}$, and $L^E_{t,s}$ are defined as:

$$\left.\begin{aligned} L^M_{t,s} &= \sum_{U\{C_{i,t,s}\}} {}_CL^M_{t,s} + {}_RL^M_{t,s} + {}_NL^M_{t,s} \\ L^A_{t,s} &= \sum_{U\{C_{i,t,s}\}} {}_CL^A_{t,s} + {}_RL^A_{t,s} + {}_NL^A_{t,s} \\ L^E_{t,s} &= \sum_{U\{C_{i,t,s}\}} {}_CL^E_{t,s} + {}_RL^E_{t,s} + {}_NL^E_{t,s} \end{aligned}\right\} . \tag{31}$$

Also in that income statement, the sum $\sum_i R_{i,t,s}$ is taken over i = GA, GO, CA, CO, and the sum $\sum_j w_j L^j_{t,s}$ is taken over j = M, A, E. It follows from the income statement that net income at time t in state of nature s, which is one of the arguments of the objective functional (1), is given by

$$\begin{aligned} \pi_{t,s} = (1-\tau)\{&R_{GA,t,s} + R_{GO,t,s} + R_{CA,t,s} \\ &+ R_{CO,t,s} - w_M L^M_{t,s} - w_A L^A_{t,s} \\ &- w_E L^E_{t,s} - M_{t,s} - q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s} \\ &- i_{t,s} \cdot B_{t,s}\} \end{aligned} \tag{32}$$

Note that $\pi_{t,s}$ as defined by (32) incorporates fees earned on government contracts, as expressed by (11) and (13). From the statement of retained earnings in table VII-2 it follows that the net addition to retained earnings in period t and state of nature s satisfies the identity:

$$e_{t,s} = \pi_{t,s} - d\cdot n_{t,s} \; . \tag{33}$$

Turning next to the representative airframe builder's statement of sources and uses of cash, which is illustrated in table VII-3, the various factors that contribute to and draw on the stock of cash are listed in order to determine the net change in the firm's stock of cash during period t and state s. For convenience, table VII-3 was prepared under the assumptions that there were no redemptions of debt or of equity shares and that inventories were increased. It is easily checked that the accounting identity expressing the net change in the firm's stock of cash holds when there are net redemptions of either or both types of securities and when inventories are decreased. It should be noted that in constructing table VII-3 it has been assumed that each weapons system contract can be treated as a sequence of contracts, one per period, under which the contractor receives full payment for costs and for a portion of the overall contract fee as compensation for work completed that period. That is, in this version of the model it is assumed that the flow of cash from the government to the contractor matches perfectly the recognition of revenue and profit by the contractor. The impact of progress payments, which actually cover only a portion of the costs incurred during the period and the impact of not paying the fee until the entire contract has been completed are considered below in section D. Under the assumptions of this section, the change in the firm's stock of cash at time t in state of nature s satisfies the identity:

$$
\left.\begin{aligned}
C_{t,s} - C_{t-1,s'} = (1-\tau)\{&R_{GA,t,s}+R_{GO,t,s}+R_{CA,t,s}+R_{CO,t,s} \\
&-w_M L^M_{t,s}-w_A L^A_{t,s}-w_E L^E_{t,s}-M_{t,s}-i_{t,s}\cdot B_{t,s}\} \\
&+\tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}+Y_{t,s}+Z_{t,s}\cdot v_{t,s}-d\cdot n_{t,s} \\
&-q_{t,s}\cdot I_{t,s}-V_{t,s}+V_{t-1,s'}\ .
\end{aligned}\right\} \tag{34}
$$

There remains one argument of the objective functional (1) that requires further explanation. Total contract backlogs at time T and in state s, $\hat{B}_{T,s}$, are dependent on the government's willingness to continue purchasing the items currently produced - i.e. to not terminate the contracts. The government's attitude is embodied in the states of nature s. Given the government's willingness to continue the contract, it is assumed that the total backlog under the contract is responsive to both the unit cost and the effectiveness of the item. Assuming that backlogs occur only under aerospace manufacturing and non-aerospace contracts, total contract backlogs (measured in dollars) at time T and in state s can be expressed as

$$
\begin{aligned}
\hat{B}_{T,s} = \sum_{C_{2,T,s} \cup C_{3,T,s}} &({}_cR_{T,s}/{}_cQ_{T,s})\cdot \\
&{}_c\hat{B}_{T,s}[{}_cR_{T,s}/{}_cQ_{T,s},\ {}_c\hat{E}_{T,s}]\ ,
\end{aligned} \tag{35}
$$

where the sum is taken over all aerospace manufacturing and non-aerospace contracts in force at time T; where ${}_c\hat{B}_{T,s}$ represents total backlogs (measured in physical units) under contract c in state s and where ${}_c\hat{B}_{T,s} \equiv 0$ for those states s that correspond to contract termination; where ${}_cR_{T,s}/{}_cQ_{T,s}$ represents unit cost to the government (i.e. revenue per unit to the contractor); where for convenience

Sources of cash:

From operations:

Sales revenue $\sum_i R_{i,t,s}$

Total expenses and taxes $(1-\tau)\{\sum_j w_j L^j_{t,s} + M_{t,s} + q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s} + i_{t,s}\cdot B_{t,s}\} + \tau\sum_i R_{i,t,s}$

Adjustment for noncash outlay $q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}$

Cash outflow for expenses $(1-\tau)\{\sum_j w_j L^j_{t,s} + M_{t,s} + i_{t,s}\cdot B_{t,s}\} + \tau\{\sum_i R_{i,t,s} - q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}\}$

Total cash generated by operations $(1-\tau)\{\sum_i R_{i,t,s} - \sum_j w_j L^j_{t,s} - M_{t,s} - i_{t,s}\cdot B_{t,s}\} + \tau q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}$

From other sources:

New debt issues $Y_{t,s}$

New equity issues $Z_{t,s}\cdot v_{t,s}$

Total cash generated $(1-\tau)\{\sum_i R_{i,t,s} - \sum_j w_j L^j_{t,s} - M_{t,s} - i_{t,s}\cdot B_{t,s}\} + \tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s} + Y_{t.s} + Z_{t,s}\cdot v_{t,s}$

Uses of cash:

To pay dividends $d\cdot n_{t,s}$

To purchase capital goods $q_{t,s}\cdot I_{t,s}$

To increase inventories $\underline{V_{t,s} - V_{t-1,s'}}$

Total cash applied $d\cdot n_{t,s} + q_{t,s}\cdot I_{t,s} + V_{t,s} - V_{t-1,s'}$

Increase (decrease) in stock of cash $C_{t,s} - C_{t-1,s'} \equiv (1-\tau)\{\sum_i R_{i,t,s} - \sum_j w_j L^j_{t,s} - M_{t,s} - i_{t,s}\cdot B_{t,s}\}$

$$+\tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s} + Y_{t,s} + Z_{t,s}\cdot v_{t,s} - d\cdot n_{t,s} - q_{t,s}\cdot I_{t,s}$$

$$-V_{t,s} + V_{t-1,s'}$$

it has been assumed that the dollar value of the physical contract backlog is computed by multiplying the physical backlog times the terminal period (and state s) unit cost; and where it is assumed that

$$\partial\hat{B}/\partial(R/Q) < 0 \text{ and } \partial\hat{B}/\partial E > 0 \ . \tag{36}$$

According to (36), backlogs are directly related to effectiveness and inversely related to unit cost. The function ${}_c\hat{B}_{T,s}$ might be interpreted as a bivariate *quasi-demand function*.[38] Since ${}_cQ_{T,s}$ is set by the government, each term in the sum in (35) expresses the backlog (both in physical terms and in dollar terms) as a function of ${}_cR_{T,s}$ and ${}_c\hat{E}_{T,s}$.

3. The Completed Model

Collecting (1), (3)-(6), (8), (14)-(16), (22), (23), (26), (29), and (30)-(35), the planning model of the representative military airframe builder is formulated as the following stochastic optimal control problem:

$$\text{maximize}\{\sum_{t=1}^{T} E_s[U_1(R_{GA,t,s};R_{GO,t,s};R_{CA,t,s};R_{CO,t,s};$$

$$\text{(37a)} \quad \pi_{t,s};[{}_c\hat{E}_{t,s}];M_{t,s})](\tfrac{1}{1+r})^t + E_s[U_2(\bar{K}^C{}_{T,s},\hat{B}_{T,s})](\tfrac{1}{1+r})^T\} \tag{37}$$

$$\text{subject to } R_{GA,t,s} = \sum_{C_{1,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s} + {}_c\bar{\pi}_{t,s}]$$

$$\text{(37b)} \quad + \sum_{C_{2,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s} + {}_c\alpha_{t,s}\cdot {}_cC^*{}_{t,s}$$

$$+ {}_c\beta_{t,s}\{{}_cC^*{}_{t,s} - {}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s}\}]$$

(37c) $$R_{GO,t,s} = \sum_{C_{3,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}$$
$$+{}_c\beta_{t,s}\{{}_cC^*_{t,s}-{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}\}]$$

(37d) $${}_cC_{t,s} = w_M\cdot{}_cL^M_{t,s}+w_A\cdot{}_cL^A_{t,s}+w_E\cdot{}_cL^E_{t,s}+(\bar{i}_{t,s}+\delta)\times$$
$$q_{t,s}\cdot{}_cK^C_{t,s}$$

(37e) $$R_{CA,t,s}({}_RQ_{t,s}) = {}_RQ_{t,s}\cdot{}_Rp_{t,s}({}_RQ_{t,s})$$

(37f) $$R_{CO,t,s}({}_NQ_{t,s}) = {}_NQ_{t,s}\cdot{}_Np_{t,s}({}_NQ_{t,s})$$

(37g) $$\pi_{t,s} = (1-\tau)[R_{GA,t,s}+R_{GO,t,s}+R_{CA,t,s}+R_{CO,t,s}$$
$$-w_M\{\sum_{U\{C_{i,t,s}\}}{}_cL^M_{t,s}+{}_RL^M_{t,s}+{}_NL^M_{t,s}\}$$
$$-w_A\{\sum_{U\{C_{i,t,s}\}}{}_cL^A_{t,s}+{}_RL^A_{t,s}+{}_NL^A_{t,s}\}$$
$$-w_E\{\sum_{U\{C_{i,t,s}\}}{}_cL^E_{t,s}+{}_RL^E_{t,s}+{}_NL^E_{t,s}\}$$
$$-M_{t,s}-q_{t,s}\cdot\delta\cdot\bar{K}^C_{t,s}-i_{t,s}(B_{t,s},Y_{t,s})\cdot B_{t,s}]$$

(37h) $$\hat{B}_{T,s} = \sum_{C_{2,T,s}UC_{3,T,s}} ({}_cR_{T,s}/{}_cQ_{T,s})\times$$
$${}_c\hat{B}_{T,s}[{}_cR_{T,s}/{}_cQ_{T,s},{}_c\hat{E}_{T,s}]$$

(37i) $$\bar{K}^C_{t,s} = (\frac{1}{1+\delta})\bar{K}^C_{t-1,s'}+(\frac{1}{1+\delta})I_{t,s}$$

(37j) $$F_c([{}_cQ_{t,s}];[{}_c\hat{E}_{t,s}];[{}_cL^M_{t,s}];[{}_cL^A_{t,s}];$$
$$[{}_cL^E_{t,s}];[{}_cK^C_{t,s}];[{}_cK^G_{t,s}]) = 0$$

(37k) $$F_R([{}_RQ_{t,s}];[{}_RL^M_{t,s}]:[{}_RL^A_{t,s}];[{}_RL^E_{t,s}];$$
$$[{}_RK^C_{t,s}]) = 0$$

(37l) $$F_N([{}_NQ_{t,s}];[{}_NL^M{}_{t,s}];[{}_NL^A{}_{t,s}];[{}_NL^E{}_{t,s}];[{}_NK^C{}_{t,s}]) = 0$$

(37m) $$\sum_{U\{C_{i,t,s}\}} {}_cK^C{}_{t,s} + {}_RK^C{}_{t,s} + {}_NK^C{}_{t,s} \leq \overline{K}^C{}_{t,s}$$

(37n) $$\sum_{U\{C_{i,t,s}\}} {}_cK^G{}_{t,s} \leq \overline{K}^G{}_{t,s}$$

(37o) $$B_{t,s} = B_{t-1,s'} + Y_{t,s}$$

(37p) $$n_{t,s} = n_{t-1,s'} + Z_{t,s}$$

(37q) $$E_{t,s} = E_{t-1,s'} + Z_{t,s} \cdot v_{t,s} + \pi_{t,s} - d \cdot n_{t,s}$$

(37r) $$\overline{C}_{t,s}([{}_cQ_{t,s}], {}_RQ_{t,s}, {}_NQ_{t,s}) + \hat{C}_{t,s} + q_{t,s} \cdot \overline{K}^C{}_{t,s}$$
$$+ V_{t,s}([{}_cQ_{t,s}], {}_RQ_{t,s}, {}_NQ_{t,s}) = B_{t,s} + E_{t,s}$$

(37s) $$C_{t,s} = C_{t-1,s'} + (1-\tau)[R_{GA,t,s} + R_{GO,t,s} + R_{CA,t,s} + R_{CO,t,s}$$
$$- w_M\{\sum_{U\{C_{i,t,s}\}} {}_cL^M{}_{t,s} + {}_RL^M{}_{t,s} + {}_NL^M{}_{t,s}\}$$
$$- w_A\{\sum_{U\{C_{i,t,s}\}} {}_cL^A{}_{t,s} + {}_RL^A{}_{t,s} + {}_NL^A{}_{t,s}\}$$
$$- w_E\{\sum_{U\{C_{i,t,s}\}} {}_cL^E{}_{t,s} + {}_RL^E{}_{t,s} + {}_NL^E{}_{t,s}\}$$
$$- M_{t,s} - i_{t,s}(B_{t,s}, Y_{t,s}) \cdot B_{t,s}] + \tau \cdot q_{t,s} \cdot \delta \cdot \overline{K}^C{}_{t,s}$$
$$+ Y_{t,s} + Z_{t,s} \cdot v_{t,s} - d \cdot n_{t,s} - q_{t,s} \cdot I_{t,s} - V_{t,s} + V_{t-1,s'}$$

$${}_cL^M{}_{t,s}, {}_RL^M{}_{t,s}, {}_NL^M{}_{t,s}, {}_cL^A{}_{t,s}, {}_RL^A{}_{t,s}, {}_NL^A{}_{t,s},$$
$${}_cL^E{}_{t,s}, {}_RL^E{}_{t,s}, {}_NL^E{}_{t,s}, {}_cK^C{}_{t,s}, {}_RK^C{}_{t,s}, {}_NK^C{}_{t,s},$$
$${}_cK^G{}_{t,s}, {}_RQ_{t,s}, {}_NQ_{t,s}, {}_c\hat{E}_{t,s}, M_{t,s}, \hat{C}_{t,s}, V_{t,s},$$
$$K_{t,s}, B_{t,s}, n_{t,s} \geq 0$$

where

denotes a vector

$c = 1, \ldots, C_1, C_1 + 1, \ldots, C_1 + C_2, C_1 + C_2 + 1, \ldots, C_1 + C_2 + C_3$

$t = 1, \ldots, T$

$s = 1, \ldots, S$

and where the boundary conditions on the first order difference equations are analogous to those in chapter four and so are left unstated.

The decision variables and the exogenously determined variables in the model (37) are listed in table VII-4.

Table VII-4 Decision Variables and Exogenous Variables in the Airframe Builder Model

Decision Variables:

(i) labor inputs ${}_cL^M_{t,s}$, ${}_RL^M_{t,s}$, ${}_NL^M_{t,s}$, ${}_cL^A_{t,s}$, ${}_RL^A_{t,s}$, ${}_NL^A_{t,s}$, ${}_cL^E_{t,s}$, ${}_RL^E_{t,s}$, ${}_NL^E_{t,s}$

(ii) capital usage ${}_cK^C_{t,s}$, ${}_RK^C_{t,s}$, ${}_NK^C_{t,s}$, ${}_cK^G_{t,s}$

(iii) commercial outputs ${}_RQ_{t,s}$, ${}_NQ_{t,s}$

(iv) effectiveness/performance ${}_c\hat{E}_{t,s}$

(v) gross investment $I_{t,s}$

(vi) managerial emoluments $M_{t,s}$

(vii) new issues/redemptions $Y_{t,s}$, $Z_{t,s}$

(viii) precautionary cash balances $\hat{C}_{t,s}$

Exogenously Determined Variables/Parameters:

(i) fee on R & D contracts ${}_c\bar{\pi}_{t,s}$

(ii) target cost on production contracts ${}_cC^*_{t,s}$

(iii) production contract parameters ${}_c\alpha_{t,s}$, ${}_c\beta_{t,s}$

(iv) allowable cost function ${}_c\gamma_{t,s}$

Table VII-4 (contd)

(v) government contract output levels ${}_{c}Q_{t,s}$

(vi) government procurement policy parameters $\bar{i}_{t,s}$, $\bar{K}^{G}_{t,s}$

(vii) wage rates w_M , w_A , w_E

(viii) rate of depreciation δ

(ix) unit prices of capital goods and equity $q_{t,s}$, $v_{t,s}$

(x) discount rate and tax rate r , τ

(xi) firm's dividend policy d

In this section the planning model of the representative military airframe builder was formulated as a stochastic optimal control problem. The model (37) developed in this section is an extension of the basic theoretical model developed and analyzed in chapters three through five, modified in accordance with the discussion of the internal planning processes of the major military airframe builders presented in chapter six. In formulating the model (37) uncertainty was taken into account by adopting the time-state-preference framework. In addition, government procurement policy was taken into account by developing contract-specific revenue functions that incorporate procurement policy parameters. The remainder of this chapter is concerned with several procurement policy issues. In particular, the impact of progress payments will be evaluated in section D by comparing the policy implications of the model (37) developed in this section with the policy implications of the model developed in section D, which incorporates progress payments.

C. THE REPRESENTATIVE AIRFRAME BUILDER'S OPTIMAL OPERATING AND FINANCIAL POLICIES

1. Introduction

The purpose of this section is to characterize the representative airframe builder's optimal operating and financial policies, as implied by the model of a representative airframe builder formulated in the previous section. Subsection 2 characterizes the optimal operating policies - those concerned with input mix, output mix, and investment decisions - and subsection 3 characterizes the optimal financial policies - those concerned with cash management and leverage policy decisions.

Before characterizing the solution to (37), some comments should be made concerning the methodology to be employed. The military airframe builder model (37) was formulated as a stochastic optimal control problem, with uncertainty modeled using the time-state-preference framework and with time modeled using discrete time periods. The advantage of employing the time-state-preference framework is, as noted in chapter four, that the optimization along any time-state chain is formally equivalent to optimization under certainty. By applying the time-state-preference framework, optimization techniques appropriate to decision-making under certainty may be applied to each of these time-state chains, with uncertainty taken into account by weighting the policy decisions appropriate to each chain by the probability of occurrence of the sequence of states that distinguish the chain. Therefore, the optimal policies implied by (37) can be characterized by proceeding in a manner that parallels the development of the discrete time version (since time is measured in discrete units in (37)) of Pontryagin's maximum principle.

Proceeding to the characterization of the optimal solution to (37), define the following generalized Lagrangian:[39]

$$L_\lambda = \sum_{t=1}^{T}\sum_{s=1}^{S} \phi_{t,s}\, U_1[R_{GA,t,s}({}_cL^M{}_{t,s}, {}_cL^A{}_{t,s}, {}_cL^E{}_{t,s}, {}_cK^C{}_{t,s}); \quad (38)$$

$$R_{GO,t,s}({}_cL^M{}_{t,s}, {}_cL^A{}_{t,s}, {}_cL^E{}_{t,s}, {}_cK^C{}_{t,s}); R_{CA,t,s}({}_RQ_{t,s});$$

$$R_{CO,t,s}({}_NQ_{t,s}); \pi_{t,s}({}_cL^M{}_{t,s}, {}_cL^A{}_{t,s}, {}_cL^E{}_{t,s}, {}_RL^M{}_{t,s},$$

$${}_RL^A{}_{t,s}, {}_RL^E{}_{t,s}, {}_NL^M{}_{t,s}, {}_NL^A{}_{t,s}, {}_NL^E{}_{t,s}, {}_cK^C{}_{t,s}, {}_RQ_{t,s},$$

$${}_NQ_{t,s}, M_{t,s}, \overline{K}^C{}_{t,s}, B_{t,s}, Y_{t,s}); [{}_c\hat{E}_{t,s}]; M_{t,s}]\left(\frac{1}{1+r}\right)^t$$

$$+ \sum_{s=1}^{S} \phi_{T,s} U_2[\overline{K}^C{}_{T,s}; \hat{B}_{T,s}\ ({}_cL^M{}_{T,s}, {}_cL^A{}_{T,s}, {}_cL^E{}_{T,s}, {}_cK^C{}_{T,s},$$

$${}_c\hat{E}_{T,s})]\left(\frac{1}{1+r}\right)^T + \sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S} \lambda_{1,t,s,s'}[\overline{K}^C{}_{t,s} - \left(\frac{1}{1+\delta}\right) \times$$

$$\overline{K}^C{}_{t-1,s'} - \left(\frac{1}{1+\delta}\right) I_{t,s}] + \sum_{c=1}^{C_1+C_2+C_3} \lambda_{2,c}\{F_c([{}_cQ_{t,s}]; [{}_c\hat{E}_{t,s}];$$

$$[{}_cL^M{}_{t,s}]; [{}_cL^A{}_{t,s}]; [{}_cL^E{}_{t,s}]; [{}_cK^C{}_{t,s}]; [{}_cK^G{}_{t,s}])\}$$

$$+\lambda_3\{F_R([{}_RQ_{t,s}]; [{}_RL^M{}_{t,s}]; [{}_RL^A{}_{t,s}]; [{}_RL^E{}_{t,s}]; [{}_RK^C{}_{t,s}])\}$$

$$+\lambda_4\{F_N([{}_NQ_{t,s}]; [{}_NL^M{}_{t,s}]; [{}_NL^A{}_{t,s}]; [{}_NL^E{}_{t,s}]; [{}_NK^C{}_{t,s}])\}$$

$$+ \sum_{t=1}^{T}\sum_{s=1}^{S} \lambda_{5,t,s}[\overline{K}^C{}_{t,s} - \sum_{U\{C_{i,t,s}\}} {}_cK^C{}_{t,s} - {}_RK^C{}_{t,s} - {}_NK^C{}_{t,s}]$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{6,t,s}[\overline{K}^{G}_{t,s}-_{U\{C_{i,t,s}\}}{}^{\Sigma}{}_{c}K^{G}_{t,s}]$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{7,t,s,s'}[B_{t,s}-B_{t-1,s'}-Y_{t,s}]$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{8,t,s,s'}[n_{t,s}-n_{t-1,s'}-Z_{t,s}]$$

$$+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{9,t,s,s'}[E_{t,s}-E_{t-1,s'}-Z_{t,s}\cdot v_{t,s}$$

$$-\pi_{t,s}({}_{c}L^{M}_{t,s},{}_{c}L^{A}_{t,s},{}_{c}L^{E}_{t,s},{}_{R}L^{M}_{t,s},{}_{R}L^{A}_{t,s},{}_{R}L^{E}_{t,s},$$

$${}_{N}L^{M}_{t,s},{}_{N}L^{A}_{t,s},{}_{N}L^{E}_{t,s},{}_{c}K^{C}_{t,s},{}_{R}Q_{t,s},{}_{N}Q_{t,s},M_{t,s},$$

$$\overline{K}^{C}_{t,s},B_{t,s},Y_{t,s})+d\cdot n_{t,s}]+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{10,t,s}[B_{t,s}$$

$$+E_{t,s}-\overline{C}_{t,s}([{}_{c}Q_{t,s}],{}_{R}Q_{t,s},{}_{N}Q_{t,s})-\hat{C}_{t,s}-q_{t,s}\overline{K}^{C}_{t,s}$$

$$-V_{t,s}([{}_{c}Q_{t,s}],{}_{R}Q_{t,s},{}_{N}Q_{t,s})]+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{11,t,s,s'}$$

$$[\overline{C}_{t,s}({}_{R}Q_{t,s},{}_{N}Q_{t,s})+\hat{C}_{t,s}-\overline{C}_{t-1,s'}({}_{R}Q_{t-1,s'},{}_{N}Q_{t-1,s'})$$

$$-\hat{C}_{t-1,s'}-\pi_{t,s}({}_{c}L^{M}_{t,s},{}_{c}L^{A}_{t,s},{}_{c}L^{E}_{t,s},{}_{R}L^{M}_{t,s},{}_{R}L^{A}_{t,s},$$

$${}_{R}L^{E}_{t,s},{}_{N}L^{M}_{t,s},{}_{N}L^{A}_{t,s},{}_{N}L^{E}_{t,s},{}_{c}K^{C}_{t,s},{}_{R}Q_{t,s},{}_{N}Q_{t,s},$$

$$M_{t,s},\overline{K}^{C}_{t,s},B_{t,s},Y_{t,s})-q_{t,s}\cdot\delta\cdot\overline{K}^{C}_{t,s}-Y_{t,s}-Z_{t,s}\cdot v_{t,s}$$

$$+d\cdot n_{t,s}+q_{t,s}\cdot I_{t,s}+V_{t,s}-V_{t-1,s'}]+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{12,t,s}\hat{C}_{t,s}\ ,$$

where (37b) and (37d) were used to define $R_{GA,t,s}$ as a function of ${}_{c}L^{M}_{t,s}$, ${}_{c}L^{A}_{t,s}$, ${}_{c}L^{E}_{t,s}$, and $K^{C}_{t,s}$; where $R_{GO,t,s}$, $R_{CA,t,s}$, $R_{CO,t,s}$, $\pi_{t,s}$, $\overline{C}_{t,s}$, and $V_{t,s}$ are similarly defined as functions of the decision variables and the state variables; and where the λ's denote Lagrange multipliers. Due to the large number of equality constraints in (37), only the signs of $\lambda_{5,t,s}$, $\lambda_{6,t,s}$, and $\lambda_{12,t,s}$ can be discerned on the basis of Kuhn-Tucker theory. At optimality, $\lambda_{5,t,s} \geq 0$, $\lambda_{6,t,s} \geq 0$, and $\lambda_{12,t,s} \geq 0$ for all s and t. For the convenience of the reader, a complete list of the symbols used in this chapter is provided in the appendix that immediately follows the footnotes to this chapter.

2. Optimal Operating Policies

The necessary conditions for an optimal solution to (37) are obtained by differentiating (38) and by setting out the appropriate Kuhn-Tucker conditions. In this subsection these necessary conditions are used to examine the following six operating policies: (i) allocation of labor inputs, (ii) determination of optimal size of stock of contractor-furnished capital, (iii) allocation of capital inputs, (iv) determination of commercial outputs, (v) determination of effectiveness/performance levels under government contracts, and (vi) payment of managerial emoluments.

For any aerospace contract c at time t, 0 < t < T, and in state of nature s and for any of the three types of labor j = M, A, E, the following necessary condition must be satisfied:

$$\phi_{t,s}\left\{\frac{\partial U_1}{\partial R_{GA,t,s}}\cdot\frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}} + \frac{\partial U_1}{\partial \pi_{t,s}}\cdot\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}}\right\}\left(\frac{1}{1+r}\right)^t + \lambda_{2,c}\frac{\partial F_c}{\partial_c L^j_{t,s}}$$
$$- \sum_{s'=1}^{S}\lambda_{9,t,s,s'}\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} - \sum_{s'=1}^{S}\lambda_{11,t,s,s'}\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} = 0 \tag{39}$$

$$\text{or } \{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}} (\frac{1}{1+r})^t\} \frac{\partial R_{GA,t,s}}{\partial {}_cL^j{}_{t,s}} + \{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t$$

$$- \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \frac{\partial \pi_{t,s}}{\partial {}_cL^j{}_{t,s}} + \lambda_{2,c} \frac{\partial F_c}{\partial {}_cL^j{}_{t,s}} = 0 \tag{40}$$

The terms $\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}} (\frac{1}{1+r})^t$ and $\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t$ are both probability-adjusted and time-weighted marginal utilities, reflecting the stochastic multiperiod nature of the optimization. It is easily verified that (39) and (40) also must hold for non-aerospace government contracts, i.e. when GO is substituted for GA.

Before utilizing (40) to determine labor input allocation rules, two points should be noted. First, the sum $\sum_{s'=1}^{S} \lambda_{9,t,s,s'}$ in (40) reflects the impact an increase in labor usage will have on the firm's stockholders' equity at time t in state s, depending on what stockholders' equity was in state s' at time t - 1, and similarly, the sum $\sum_{s'=1}^{S} \lambda_{11,t,s,s'}$ in (40) reflects the impact an increase in labor usage will have on cash flow at time t in state s, depending on what cash flow was in state s' at time t - 1. The allocation of labor at time t thus depends partly on interperiod considerations, and the impact of increased labor usage on expected utility is adjusted to take into account its impact on stockholders' equity and cash flow.

The first term in (39) is the partial derivative of the objective functional in (37) with respect to ${}_cL^j{}_{t,s}$. Hence this term can be interpreted as the *marginal value* of an additional unit of type j labor working on contract c during period t and state of nature s, where it is understood that marginal value is measured in terms of discounted expected collective marginal utility. Rewriting the difference of the first and third terms in (39) as the sum of a marginal revenue effect and a marginal net income effect, in which the latter is adjusted to take into account the stockholders' equity and cash flow effects discussed above, leads to (40). The sum of the first two terms in (40) can be interpreted as the *net marginal value* of an increase in ${}_cL^j_{t,s}$, where marginal value is measured in the manner just described and where the 'net' is understood to mean 'net of the stockholders' equity and cash flow impact'.

Interpreting the first two terms in (40) collectively as the net marginal value of an additional unit of labor of type j allocated to contract c at time t in state of nature s, (40) leads to the following labor input allocation rules:

<u>Theorem VII-1</u>

For contract c at time t in state of nature s, the airframe builder should allocate the different types of labor so that the marginal rate of technical substitution between any pair just equals the ratio of their net marginal values.

<u>Proof</u>

Denote the two types of labor by j and k. For given c, t, and s there are two conditions like (40), one for j and the other for k. Solving each for λ_2, equating the two expressions, and rewriting yields

$$-\frac{\partial\, {}_cL^k_{t,s}}{\partial\, {}_cL^j_{t,s}} = \frac{\{\quad\}\partial R_{GA,t,s}/\partial\, {}_cL^j_{t,s} + \{\quad\}\partial \pi_{t,s}/\partial\, {}_cL^j_{t,s}}{\{\quad\}\partial R_{GA,t,s}/\partial\, {}_cL^k_{t,s} + \{\quad\}\partial \pi_{t,s}/\partial\, {}_cL^k_{t,s}}, \qquad (41)$$

where the terms within braces in (41) are the same as those in (40). Moreover, the same result holds when GO is substituted for GA. Q.E.D.

Remark

The right-hand side of (41) represents the ratio of the net marginal values of the two different types of labor. Each net marginal value is expressed in terms of a sum of discounted probability-weighted marginal utilities, where the marginal utilities reflect the impact on total revenue and net income of increased labor usage. Condition (41) is really analogous, then, to the neoclassical input allocation rule, according to which the marginal rate of technical substitution between each pair of inputs must equal the ratio of the unit costs of the two inputs.[40]

Corollary VII-1-1

For contract c and labor of type j at time t, the airframe builder should allocate type j labor between states of nature s and s' in such a way that the marginal rate of technical substitution between the state-specific amounts of type j labor just equals the ratio of their net marginal values in the two states.

Proof

Following the steps in the proof of theorem VII-1 leads to the following expression:

$$- \frac{\partial_c L^j_{t,s'}}{\partial_c L^j_{t,s}} = \frac{\{\phi_{t,s} \cdot \frac{\partial U_1}{\partial R_{GA,t,s}} \cdot (\frac{1}{1+r})^t\} \partial R_{GA,t,s} / \partial_c L^j_{t,s} +}{\{\phi_{t,s'} \cdot \frac{\partial U_1}{\partial R_{GA,t,s'}} \cdot (\frac{1}{1+r})^t\} \partial R_{GA,t,s'} / \partial_c L^j_{t,s'} +}$$

$$\frac{\{\phi_{t,s} \cdot \frac{\partial U_1}{\partial \pi_{t,s}} \cdot (\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s,s''} + \lambda_{11,t,s,s''})\} \times}{\{\phi_{t,s'} \frac{\partial U_1}{\partial \pi_{t,s'}} \cdot (\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s',s''} + \lambda_{11,t,s',s''})\} \times} \qquad (42)$$

$$\frac{\partial \pi_{t,s} / \partial_c L^j_{t,s}}{\partial \pi_{t,s'} / \partial_c L^j_{t,s'}} ,$$

where the right-hand side of (42) is written on three lines but represents one fraction. Moreover, the same result holds when GO is substituted for GA. Q.E.D.

Remark

As would be expected in a stochastic optimization, the net marginal values in (42) incorporate the corporate planners' (possibly subjective) state probabilities $\phi_{t,s}$ and $\phi_{t,s'}$. Put simply, the value to the firm of any particular allocation of labor across states of nature at time t is dependent on the probability distribution over the possible states of nature at time t. Also, as in the case of (41), each of the net marginal values on the right-hand side of (42) is expressed as the sum of discounted probability-weighted marginal utilities, and each sum reflects the impact on total revenue and net income, and indirectly on expected collective utility, of increased labor usage.

Corollary VII-1-2

For a contract c that is ongoing at times t and t - 1, for a state s that may obtain at times t and t - 1, and for labor of type j, the airframe builder should allocate labor usage intertemporally in such a way that the intertemporal marginal rate of technical substitution just equals the ratio of the net marginal values of type j labor at times t and t - 1.[41]

Proof

Following the steps in the proof of theorem VII-1 leads to the following expression:

$$-\frac{\partial_c L^j_{t-1,s}}{\partial_c L^j_{t,s}} = \frac{\{\phi_{t,s}\cdot\frac{\partial U_1}{\partial R_{GA,t,s}}\cdot(\frac{1}{1+r})^t\}\partial R_{GA,t,s}/\partial_c L^j_{t,s} +}{\{\phi_{t-1,s}\cdot\frac{\partial U_1}{\partial R_{GA,t-1,s}}\cdot(\frac{1}{1+r})^{t-1}\}\partial R_{GA,t-1,s}/\partial_c L^j_{t-1,s} +}$$

$$\frac{\{\phi_{t,s}\cdot\frac{\partial U_1}{\partial \pi_{t,s}}\cdot(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} +}{\{\phi_{t-1,s}\cdot\frac{\partial U_1}{\partial \pi_{t-1,s}}\cdot(\frac{1}{1+r})^{t-1} - \sum_{s'=1}^{S}(\lambda_{9,t-1,s,s'} +} \qquad (43)$$

$$\frac{\lambda_{11,t,s,s'})\}\partial \pi_{t,s}/\partial_c L^j_{t,s}}{\lambda_{11,t-1,s,s'})\}\partial \pi_{t-1,s}/\partial_c L^j_{t-1,s}},$$

where the right-hand side of (43) is written on three lines but represents one fraction. Moreover, the same result holds when GO is substituted for GA. Q.E.D.

Remark

The right-hand side of (43) is analogous to the right-hand side of (42), where the net marginal values in the former pertain to different time periods (but to the same state of nature s in each) and where the net marginal values in the latter pertain to different states (but to the same period t).

Thus far equation (40) has been used to characterize the representative airframe builder's allocation of labor inputs. The three allocation results summarized as theorem VII-1 and corollaries VII-1-1 and VII-1-2 concern relative allocations. An interesting allocation question suggested by the discussion of the basic theoretical model in chapters three and four is whether government contractors exhibit an upward cost bias, as so many writers have suggested,[42] by utilizing labor beyond the point at which a short run profit maximizer would - in the hope of increasing revenue and thereby increasing expected utility. As proved in the following theorem, the representative airframe builder modeled in (37) does indeed exhibit such an upward cost bias.

<u>Theorem VII-2</u>

If the utility function U_1 is strictly concave with respect to total revenue and net income, then the representative military airframe builder modeled in (37) utilizes labor of each type under each government contract beyond the short run profit maximizing level.

<u>Proof</u>

An expected utility of profit maximizer would select the labor input level ${}_cL^j{}_{t,s}$ for which

$$\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \frac{\partial \pi_{t,s}}{\partial {}_cL^j{}_{t,s}} + \lambda_{2,c} \frac{\partial F_c}{\partial {}_cL^j{}_{t,s}} = 0 \; . \qquad (44)$$

Since $\partial U_1/\partial R > 0$ by assumption and since $\partial R/\partial L > 0$ must hold at optimality, it follows from (40) that the expression on the left-hand side of (44) must be strictly negative at optimality for the firm modeled in (37). Note that since $\partial R/\partial L > 0$ whenever $\partial \pi/\partial L \geq 0$, it follows that the firm modeled in (37) would never use less labor than a short run profit maximizer. But then, since it can be shown that $\lambda_{2,c}(\partial^2 F_c/\partial({}_cL^j{}_{t,s})^2) > 0$ at optimality,[43] it follows that

$$\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial {}_cL^j{}_{t,s}} \qquad (45)$$

must be smaller at optimality for the representative airframe builder modeled in (37) than for a short run profit maximizer. Moreover, if $\partial \pi_{t,s}/\partial {}_cL^j{}_{t,s} < 0$, then it follows from the fact that $\lambda_{2,c}\cdot\partial F_c/\partial {}_cL^j{}_{t,s} > 0$ at optimality that the expression within braces in (44) and in (45) must be strictly positive at optimality. Then differentiating (45) with respect to ${}_cL^j{}_{t,s}$ gives

$$\phi_{t,s}\frac{\partial^2 U_1}{\partial \pi_{t,s}{}^2}(\frac{1}{1+r})^t(\frac{\partial \pi_{t,s}}{\partial {}_cL^j{}_{t,s}})^2+\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t$$

$$- \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial^2 \pi_{t,s}}{\partial({}_cL^j{}_{t,s})^2} < 0 ,$$

where the inequality follows from the assumed concavity of U_1, provided $\partial^2 \pi_{t,s}/\partial({}_cL^j{}_{t,s})^2 < 0$, which, together with the observation that (45) is smaller for the firm modeled in (37) than for a short run profit maximizer, implies that the representative airframe builder utilizes labor beyond the short run profit maximizing level. Thus, in order to complete the proof of the theorem it remains to be shown that $\partial\pi_{t,s}/\partial_c L^j_{t,s} < 0$ and $\partial^2\pi_{t,s}/\partial_c {L^j_{t,s}}^2 \leq 0$ for each government contract c. To simplify the notation the subscripts c, t, and s are not explicitly written. To simplify the exposition, what follows is stated in terms of an aerospace production contract. Replacing GA by GO extends the proof to non-aerospace (production) contracts and setting $\beta = 0$ extends the proof to aerospace research and development contracts.

From (8), (15), and (32),

$$\frac{\partial\pi_{t,s}}{\partial L^j} = (1-\tau)\{\frac{\partial R_{GA}}{\partial L^j} - w_j\} = (1-\tau)\{\frac{\partial}{\partial L^j}[(1-\beta)\gamma(C)\cdot C+K] - w_j\}$$

$$= (1-\tau)\{(1-\beta)[\gamma'(C)\cdot w_j C+\gamma(C)\cdot w_j] - w_j\} < 0, \qquad (46)$$

where K is independent of j and where the inequality follows from (9). Differentiating (46) with respect to L^j gives

$$\frac{\partial^2\pi_{t,s}}{\partial {L^j}^2} = (1-\tau)(1-\beta)[\gamma''(C)\cdot {w_j}^2 C + 2\gamma'(C){w_j}^2] \leq 0, \qquad (47)$$

where the inequality follows from (9). Q.E.D.

Turning next to the allocation of labor across ongoing government production contracts in the terminal period t = T, the following necessary condition must be satisfied:

$$\phi_{T,s}\{\frac{\partial U_1}{\partial R_{GA,T,s}} \frac{\partial R_{GA,T,s}}{\partial_c L^j_{T,s}} + \frac{\partial U_1}{\partial \pi_{T,s}} \cdot \frac{\partial \pi_{T,s}}{\partial_c L^j_{T,s}} + \frac{\partial U_2}{\partial \hat{B}_{T,s}} \times$$

$$\frac{\partial \hat{B}_{T,s}}{\partial_c L^j_{T,s}}\} (\frac{1}{1+r})^T + \lambda_{2,c} \frac{\partial F_c}{\partial_c L^j_{T,s}} - \sum_{s'=1}^{S} (\lambda_{9,T,s,s'} \qquad (48)$$

$$+\lambda_{11,T,s,s'})\frac{\partial \pi_{T,s}}{\partial_c L^j_{T,s}} = 0 \ ,$$

which differs from (40) due to the effect on total contract backlogs of a change in ${}_cL^j_{T,s}$, which is reflected in $\frac{\partial U_2}{\partial \hat{B}_{T,s}} \cdot \frac{\partial \hat{B}_{T,s}}{\partial {}_cL^j_{T,s}}$. As in the case of (40), (48) holds when GO is substituted for GA, i.e. it holds regardless of the type of production contract. From (8), (12), and (36) it follows that $\frac{\partial U_2}{\partial \hat{B}_{T,s}} \cdot \frac{\partial \hat{B}_{T,s}}{\partial {}_cL^j_{T,s}} < 0$, so that the expression on the left-hand side of (40) is strictly positive when $t = T$. The following corollary to theorem VII-2 states the intuitively apparent result that the sensitivity of backlogs to unit cost R/Q tends to restrain the airframe builder's cost bias.

Corollary VII-2-1

If the utility function U_1 is strictly concave with respect to each of its arguments $R_{GA,t,s}$, $R_{GO,t,s}$, and $\pi_{t,s}$, then the sensitivity of backlogs to unit cost R/Q has the effect of reducing the contractor's labor usage under government contracts.

Proof

It follows from the fact that $\partial U_2/\partial \hat{B}_{T,s} > 0$ and $\partial \hat{B}_{T,s}/\partial {}_cL^j_{T,s} < 0$ that the sensitivity of backlogs to unit cost cannot cause an increase in labor usage under government contracts. But then, since $\lambda_{2,c}(\partial^2 F_c/\partial({}_cL^j_{T,s})^2) > 0$ at optimality, it follows that

$$\{\phi_{T,s} \frac{\partial U_1}{\partial R_{GA,T,s}}(\frac{1}{1+r})^T\}\frac{\partial R_{GA,T,s}}{\partial {}_cL^j_{T,s}} + \{\phi_{T,s} \frac{\partial U_1}{\partial \pi_{T,s}}(\frac{1}{1+r})^T - \sum_{s'=1}^{S}(\lambda_{9,T,s,s'} + \lambda_{11,T,s,s'})\}\frac{\partial \pi_{T,s}}{\partial {}_cL^j_{T,s}} \tag{49}$$

must be greater at optimality than it would be in the absence of any relationship between backlogs and terminal period unit costs. Then differentiating (49) with respect to ${}_cL^j_{T,s}$ yields

$$\phi_{T,s}\left(\frac{1}{1+r}\right)^T\left\{\frac{\partial^2 U_1}{\partial R_{GA,T,s}^2}\cdot\left(\frac{\partial R_{GA,T,s}}{\partial\, {}_cL^j_{T,s}}\right)^2 + \frac{\partial U_1}{\partial R_{GA,T,s}}\cdot\frac{\partial^2 R_{GA,T,s}}{\partial({}_cL^j_{T,s})^2}\right.$$

$$+ \frac{\partial^2 U_1}{\partial \pi_{T,s}^2}\left(\frac{\partial \pi_{T,s}}{\partial\, {}_cL^j_{T,s}}\right)^2 + \left\{ \phi_{T,s}\frac{\partial U_1}{\partial \pi_{T,s}}\left(\frac{1}{1+r}\right)^T \right. \qquad (50)$$

$$\left. - \sum_{s'=1}^{S}(\lambda_{9,T,s,s'}+\lambda_{11,T,s,s'})\right\}\frac{\partial^2 \pi_{T,s}}{\partial({}_cL^j_{T,s})} < 0 ,$$

which follows from theorem VII-2 and the assumed strict concavity of U_1.[44] Q.E.D.

The practical import of corollary VII-2-1 is that, even if government contractors exhibit an upward cost bias, this upward cost bias is restrained to some extent if contractors perceive a relationship between unit cost under ongoing contracts and total contract backlogs as of time t = T. The theorem and corollary suggest that this restraining effect would be heightened if government procurement policy were to impose a relationship between contract performance and future contract awards.[45] If a suitable measure of past performance could be devised,[46] then corollary VII-2-1 suggests that government procurement policy should make contract awards dependent on contractor past performance in order to provide an incentive for efficient contractor performance.

Turning next to the allocation of labor to the production of commercial goods, the following necessary condition must be satisfied:

$$\left\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}\left(\frac{1}{1+r}\right)^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\right\} \times \frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}} + \lambda_\ell \frac{\partial F_k}{\partial_k L^j_{t,s}} = 0 , \tag{51}$$

where j = M, A, E; where k = R denotes aerospace production and k = N denotes non-aerospace production; and where ℓ = 3 when k = R and ℓ = 4 when k = N. The next theorem and accompanying corollaries follow from (51).

Theorem VII-3

For production of type k at time t in state of nature s, the representative airframe builder should allocate labor so that the marginal rate of technical substitution between any pair of types of labor just equals the ratio of their respective wage rates. Thus the neoclassical criterion for optimally allocating labor inputs continues to hold.

Proof

Denote the two types of labor by j and j'. Solve each of two versions of (51), one expressed in terms of j and the other expressed in terms of j', for λ_ℓ, equate the resulting expressions, and rearrange terms to obtain

$$-\frac{\partial_k L^{j'}_{t,s}}{\partial_k L^j_{t,s}} = \frac{\partial \pi_{t,s}/\partial_k L^j_{t,s}}{\partial \pi_{t,s}/\partial_k L^{j'}_{t,s}} . \tag{52}$$

It follows from (31) and (32) that

$$\frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}} = -(1-\tau)w_j \text{ and } \frac{\partial \pi_{t,s}}{\partial_k L^{j'}_{t,s}} = -(1-\tau)w_{j'} . \tag{53}$$

Substituting (53) into (52) and simplifying yields

$$-\frac{\partial_k L^{j'}_{t,s}}{\partial_k L^j_{t,s}} = \frac{w_j}{w_{j'}} . \tag{54}$$

Q.E.D.

<u>Remark</u>

Note that (54) is really just the neoclassical criterion for an optimal labor input mix. Thus, as was found in the analysis of the basic theoretical model in chapter three, the firm is motivated to select the cost-minimizing input mix for commercial production.

<u>Corollary VII-3-1</u>

For production of type k and labor of type j at time t, the airframe builder should allocate type j labor between states of nature s and s' in such a way that the marginal rate of technical substitution between the state-specific amounts of type j labor just equals the ratio of their respective net marginal contributions to expected collective utility of an additional dollar of net income in state s(s').

<u>Proof</u>

Fix k, j, and t in (51) for two states of nature s and s'. Solve each expression for λ_ℓ, equate the resulting expressions, and rearrange terms to obtain:

$$- \frac{\partial_k L^j_{t,s'}}{\partial_k L^j_{t,s}} = \frac{\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s,s''}}{\{\phi_{t,s'} \frac{\partial U_1}{\partial \pi_{t,s'}}(\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s',s''}}$$

$$\frac{+ \lambda_{11,t,s,s''})\} \frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}}}{+ \lambda_{11,t,s',s''})\} \frac{\partial \pi_{t,s'}}{\partial_k L^j_{t,s'}}}, \tag{55}$$

where the expression on the right-hand side is written on two lines but represents a single fraction. It follows from (31) and (32) that

$$\frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}} = \frac{\partial \pi_{t,s'}}{\partial_k L^j_{t,s'}} = -(1-\tau)w_j , \tag{56}$$

so that (55) simplifies to

$$- \frac{\partial_k L^j_{t,s'}}{\partial_k L^j_{t,s}} = \frac{\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s,s''}+\lambda_{11,t,s,s''})\}}{\{\phi_{t,s'} \frac{\partial U_1}{\partial \pi_{t,s'}}(\frac{1}{1+r})^t - \sum_{s''=1}^{S} (\lambda_{9,t,s',s''}+\lambda_{11,t,s',s''})\}},$$

where the numerator and denominator in (57) can be interpreted as the respective net marginal contributions to expected collective utility of an additional dollar of net income.

Q.E.D.

Remark 1

The numerator and denominator in (57) can be interpreted as the net marginal contribution to expected collective utility of an additional dollar of net income for the following reason. The term $\phi_{t,s}(\partial U_1/\partial \pi_{t,s})/(1+r)^t$

represents the partial derivative of the objective functional in (37) with respect to $\pi_{t,s}$ while $-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})$ once again reflects the stockholders' equity and cash flow impacts of a change in $\pi_{t,s}$. Allowing for the two effects simultaneously leads to the interpretation of the difference within braces in numerator and denominator in (57) as a 'net marginal contribution to expected collective utility'.

Remark 2

Corollary VII-3-1 represents the logical extension of the neoclassical criterion for combining inputs optimally to a stochastic environment.

Corollary VII-3-2

For production of type k, labor of type j, and state of nature s that might obtain at times t and t - 1, the air-frame builder should allocate type j labor between periods t and t - 1 in such a way that the marginal rate of technical substitution between the time-specific amounts of type j labor just equals the ratio of their respective net marginal contributions to expected collective utility of an additional dollar of net income during period t(t-1).

Proof

Fix k, j, and s in (51) for two time periods t and t - 1, and proceed as in the proof of corollary VII-3-1 to obtain:

$$-\frac{\partial_k L^j_{t-1,s}}{\partial_k L^j_{t,s}} = \frac{\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^{t} - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}}{\{\phi_{t-1,s}\frac{\partial U_1}{\partial \pi_{t-1,s}}(\frac{1}{1+r})^{t-1} - \sum_{s'=1}^{S}(\lambda_{9,t-1,s,s'}+\lambda_{11,t-1,s,s'})\}}$$

Q.E.D.

Remark

Note that for the proof of corollary VII-3-2, (56) still applies, but with $\partial\pi_{t-1,s}/\partial_k L^j{}_{t-1,s}$ in place of $\partial\pi_{t,s'}/\partial_k L^j{}_{t,s'}$.

The second set of operating policies that are to be considered in this section concern the airframe builder's determination of the optimal stock of contractor-furnished capital for each period and state, $\overline{K}^C{}_{t,s}$, and accordingly, from (3), its optimal investment policy for each period and state, $I_{t,s}$. Two cases are considered: $0 < t < T$ and $t = T$.

First, suppose $0 < t < T$. It follows from differentiating (38) that the optimal stocks, $\overline{K}^C{}_{t,s}$, must satisfy

$$\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} \frac{\partial \pi_{t,s}}{\partial \overline{K}^C{}_{t,s}} \left(\frac{1}{1+r}\right)^t + \left[\sum_{s'=1}^{S} \lambda_{1,t,s,s'} - \left(\frac{1}{1+\delta}\right) \sum_{s'=1}^{S} \lambda_{1,t+1,s',s}\right]$$
$$+\lambda_{5,t,s} - \lambda_{10,t,s} \cdot q_{t,s} + \left\{\sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\right\} \times \tag{58}$$
$$\left(-\frac{\partial \pi_{t,s}}{\partial \overline{K}^C{}_{t,s}}\right) - \sum_{s'=1}^{S} \lambda_{11,t,s,s'} q_{t,s} \delta = 0 .$$

It also follows from differentiating (38) that the optimal investment levels, $I_{t,s}$, must satisfy

$$\left(\sum_{s'=1}^{S} \lambda_{1,t,s,s'}\right)\left(-\frac{1}{1+\delta}\right) + \left(\sum_{s'=1}^{S} \lambda_{11,t,s,s'}\right)(q_{t,s}) = 0. \tag{59}$$

Equation (59) can be rewritten as

$$\sum_{s'=1}^{S} \lambda_{1,t,s,s'} = (1+\delta) q_{t,s} \sum_{s'=1}^{S} \lambda_{11,t,s,s'} . \tag{60}$$

It follows from (32) that

$$\frac{\partial \pi_{t,s}}{\partial \overline{K}^{C}_{t,s}} = -(1-\tau)q_{t,s}\cdot\delta \ . \tag{61}$$

Substituting (60) and (61) into (58) and rearranging terms yields

$$\lambda_{5,t,s} = \{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \times$$

$$(1-\tau)q_{t,s}\delta + (\frac{1}{1+\delta})\sum_{s'=1}^{S}\lambda_{1,t+1,s',s} + \lambda_{10,t,s}q_{t,s} \tag{62}$$

$$-(\sum_{s'=1}^{S}\lambda_{11,t,s,s'})q_{t,s} \ .$$

Next an interpretation is given to the equilibrium condition (62).

From (37m) it follows that $\lambda_{5,t,s}$ measures the marginal value in terms of discounted expected collective utility of an additional unit of contractor-furnished capital at time t in state of nature s. The first term on the right-hand side of (62) measures the immediate impact on net income of an additional unit of capital, once again adjusted for the indirect impact on stockholders' equity and cash flow. This interpretation follows from (61). The second term on the right-hand side of (62) measures the value of an additional unit of capital this period in terms of reduced capital needs next period, where $(\frac{1}{1+\delta})$ of each unit will be available next period and where its value in terms of the next period's needs is dependent on the state obtaining next period (hence the sum

$\sum_{s'=1}^{S} \lambda_{1,t+1,s',s})$. The third term on the right-hand side of (62) can be interpreted as a measure of the balance sheet effect of an additional unit of capital. The addition of $q_{t,s}$ value units to the assets side of the balance sheet in table VII-1 necessitates an equal increment on the liabilities and stockholders' equity side - i.e. the investment must be financed somehow. The fourth term on the right-hand side of (62) is interpretable as the direct cash flow effect of the outflow of $q_{t,s}$ in cash to purchase the additional unit of capital. Collecting these results, (62) is seen to be the familiar marginal value equals marginal cost necessary condition for optimal capital investment. The important point is that in equilibrium the Lagrange multiplier $\lambda_{5,t,s}$ measures the marginal value of an additional unit of capital and is numerically equal to the marginal cost of an additional unit of capital (since in equilibrium these quantities must be equal). Moreover, since this marginal cost is strictly positive, except in unusual circumstances, it may be inferred that $\lambda_{5,t,s} > 0$ for all t and s.[47]

Equilibrium condition (62) is appropriate provided $0 < t < T$. For $t = T$ the equilibrium condition takes a somewhat different form. It follows from differentiating (38) that the terminal capital stocks, $\overline{K}^C_{T,s}$ must satisfy

$$\phi_{T,s}\left\{\frac{\partial U_1}{\partial \pi_{T,s}} \frac{\partial \pi_{T,s}}{\partial \overline{K}^C_{T,s}} + \frac{\partial U_2}{\partial \overline{K}^C_{T,s}}\right\}\left(\frac{1}{1+r}\right)^T + \sum_{s'=1}^{S} \lambda_{1,T,s,s'} \tag{63}$$

$$+ \lambda_{5,T,s} - \lambda_{10,T,s}q_{T,s} + \left\{\sum_{s'=1}^{S}(\lambda_{9,T,s,s'} + \lambda_{11,T,s,s'})\right\}\times$$

$$(-\frac{\partial\pi_{T,s}}{\partial\bar{K}^C_{T,s}}) - \sum_{s'=1}^{S} \lambda_{11,T,s,s'} q_{T,s}\delta = 0 ,$$

where $\phi_{T,s} \frac{\partial U_2}{\partial\bar{K}^C_{T,s}} (\frac{1}{1+r})^T$ has replaced $-(\frac{1}{1+\delta}) \sum_{s'=1}^{S} \lambda_{1,t+1,s',s}$ in (58) as a measure of the value of an additional unit of capital in terms of future needs. Equations (59), (60), and (61) continue to hold, but with t = T. Using (60) and (61), each with t = T, to substitute into (63) yields the following equilibrium condition for optimal investment in the terminal period:

$$\lambda_{5,T,s} = \{\phi_{T,s} \frac{\partial U_1}{\partial\pi_{T,s}} (\frac{1}{1+r})^T - \sum_{s'=1}^{S} (\lambda_{9,T,s,s'} + \lambda_{11,T,s,s'})\} \times$$

$$(1-\tau) q_{T,s}\delta - \phi_{T,s} \frac{\partial U_2}{\partial\bar{K}^C_{T,s}} (\frac{1}{1+r})^T + \lambda_{10,T,s}\, q_{T,s} \qquad (64)$$

$$- (\sum_{s'=1}^{S} \lambda_{11,T,s,s'}) q_{T,s} .$$

Since (64) is clearly analogous to (62) term-by-term, it is not necessary to give it a separate interpretation.

Turning next to the optimal allocation of capital inputs, the following necessary conditions must be satisfied for contractor-furnished capital at time t, 0 < t < T:

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R_{k,t,s}} (\frac{1}{1+r})^t\} \frac{\partial R_{k,t,s}}{\partial_c K^C_{t,s}} + \{\phi_{t,s} \frac{\partial U_1}{\partial\pi_{t,s}} (\frac{1}{1+r})^t$$

$$- \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \frac{\partial\pi_{t,s}}{\partial_c K^C_{t,s}} + \lambda_{2,c} \frac{\partial F_c}{\partial_c K^C_{t,s}} \qquad (65)$$

$$- \lambda_{5,t,s} = 0 ,$$

where k = GA, GO, and

$$\lambda_{\ell} \frac{\partial F_k}{\partial_k K^C_{t,s}} - \lambda_{5,t,s} = 0 , \tag{66}$$

where k = R, N and where $\ell = 3$ when k = R and $\ell = 4$ when k = N. Note that (65) and (66) are analogous to (40) and (51), respectively. Note also that when t = T it is necessary to take the backlog effect into account and (65) must be modified in a manner similar to that used to modify (40) to obtain (48).

Necessary condition (65) applies to contractor-furnished capital allocated to government contracts while necessary condition (66) applies to contractor-furnished capital allocated to commercial production. In comparing the two, one notes the first two terms in (65) that have no counterpart in (66). These two terms reflect the impact of a change in the amount of contractor-furnished capital allocated to a government contract that arises from the fact that interest and depreciation are allowable costs. The existence of such an effect can be seen by examining (37b), (37c), and (37d). A change in $_cK^C_{t,s}$ in (37d) has a direct effect on $_cC_{t,s}$, which in turn affects $R_{GA,t,s}$ in (37b) or $R_{GO,t,s}$ in (37c), depending on the type of contract. Thus, a change in the amount of contractor-furnished capital allocated to a government contract has revenue effects that must be taken into account by the contractor at the time the capital input allocation decision is made.

As noted above, in equilibrium $\lambda_{5,t,s}$ is numerically equal to the marginal cost of contractor-furnished capital.

The difference

$$\lambda_{5,t,s} - \{\quad\} \frac{\partial R_{k,t,s}}{\partial_c K^C_{t,s}} - \{\quad\} \frac{\partial \pi_{t,s}}{\partial_c K^C_{t,s}}, \tag{67}$$

where the terms within braces are the same as in (65), can be interpreted as the *net marginal cost of capital allocated to government contracts*, where the 'net' is understood to mean 'net of the revenue-related affects arising out of the allowability of depreciation and interest expense under government contracts'. It should be noted that the net marginal cost of capital allocated to government contracts given by (67) is positive at optimality, since, as implied by lemma VII-1, which is proved below, $\lambda_{2,c}(\partial F_c/\partial_c K^C_{t,s}) > 0$ in (65).

Given the above interpretation of (67), the following theorem and corollaries follow directly from (65) and (66). It is assumed in what follows that $0 < t < T$. Perfectly analogous results can be derived for the special case $t = T$.

Theorem VII-4

For contract c at time t, the airframe builder should allocate contractor-furnished capital between states of nature s and s' in such a way that the marginal rate of technical substitution between the state-specific amounts just equals the ratio of their net marginal costs in the two states.

Proof

Fix c and t in (65). Specify two states s and s'. Solve the respective versions of (65) for $\lambda_{2,c} \dfrac{\partial F_c}{\partial_c K^C_{t,s}}$ and divide one by the other to obtain

$$-\frac{\partial_c K^C_{t,s'}}{\partial_c K^C_{t,s}} = \frac{\lambda_{5,t,s} - \{\ \}\partial R_{k,t,s}/\partial_c K^C_{t,s} - \{\ \}\partial \pi_{t,s}/\partial_c K^C_{t,s}}{\lambda_{5,t,s'} - \{\ \}\partial R_{k,t,s'}/\partial_c K^C_{t,s'} - \{\ \}\partial \pi_{t,s'}/\partial_c K^C_{t,s'}}, \quad (68)$$

which is the desired result. Q.E.D.

Corollary VII-4-1

For contract c and state of nature s that might obtain at times t and t - 1, the airframe builder should allocate contractor-furnished capital between periods t and t - 1 in such a way that the intertemporal marginal rate of technical substitution just equals the ratio of the net marginal costs of contractor-furnished capital under contract c at times t and t - 1.

Proof

Fix c and s in (65). Specify two time periods t and t - 1 at which state s might obtain and proceed as in the proof of theorem VII-4 to obtain

$$-\frac{\partial_c K^C_{t-1,s}}{\partial_c K^C_{t,s}} = \quad (69)$$

$$\frac{\lambda_{5,t,s} - \{\ \}\partial R_{k,t,s}/\partial_c K^C_{t,s} - \{\ \}\partial \pi_{t,s}/\partial_c K^C_{t,s}}{\lambda_{5,t-1,s} - \{\ \}\partial R_{k,t-1,s}/\partial_c K^C_{t-1,s} - \{\ \}\partial \pi_{t-1,s}/\partial_c K^C_{t-1,s}},$$

which is analogous to (68). Q.E.D.

Corollary VII-4-2

For commercial production of type k, the airframe builder should allocate capital between states of nature s and s' at time t in such a way that the marginal rate of technical substitution between the state-specific amounts just equals the ratio of their marginal costs in the two states.

Proof

Use (66) and proceed as in the proof of theorem VII-4 to obtain

$$- \frac{\partial_c K^C_{t,s'}}{\partial_c K^C_{t,s}} = \frac{\lambda_{5,t,s}}{\lambda_{5,t,s'}} . \tag{70}$$

Q.E.D.

Corollary VII-4-3

For commercial production of type k and state s that might obtain at times t and t - 1, the airframe builder should allocate capital intertemporally in such a way that the intertemporal marginal rate of technical substitution just equals the ratio of the marginal costs of capital at times t and t - 1.

Proof

Use (66) and proceed as in the proof of corollary VII-4-1 to obtain

$$- \frac{\partial_c K^C_{t-1,s}}{\partial_c K^C_{t,s}} = \frac{\lambda_{5,t,s}}{\lambda_{5,t-1,s}} . \tag{71}$$

Q.E.D.

The capital allocation rules given in theorem VII-4 and corollaries VII-4-1, VII-4-2, and VII-4-3 are analogous to the labor input allocation rules given in corollaries VII-1-1 and VII-1-2. Additional rules for trading off contractor-furnished capital and labor in both government production and commercial production could be developed. The resulting equilibrium conditions would be perfectly analogous to (68)-(71), and so are not stated explicitly here.

Necessary conditions (65) and (66) apply to contractor-furnished capital only. In addition, government-furnished capital must be allocated in accordance with the following necessary condition:

$$\lambda_{2,c} \frac{\partial F_c}{\partial_c K^G_{t,s}} - \lambda_{6,t,s} = 0 , \qquad (72)$$

where $\lambda_{6,t,s} \geq 0$ is analogous to $\lambda_{5,t,s}$ and measures the marginal value in terms of discounted expected collective utility of an additional unit of government-furnished capital at time t in state of nature s.[48] (72) could be used to derive allocation rules for government-furnished capital that are perfectly analogous to (70) and (71). Due to this analogy, these results are not derived explicitly here. A somewhat more interesting allocation question concerns the trade off at the margin between contractor-furnished capital and government-furnished capital for each ongoing government contract c. The following theorem concerns this trade off:

Theorem VII-5

For contract c at time t in state of nature s, the airframe builder should combine contractor-furnished capital and government-furnished capital in such a way that their marginal rate of technical substitution just equals the ratio of their net marginal values.

Remark

Both $\lambda_{5,t,s}$ and $\lambda_{6,t,s}$ measure the marginal value of an additional unit of capital allocated in an optimal manner over the firm's government contracts (and in the case of $\lambda_{5,t,s}$, also over the firm's commercial products). For government-furnished capital this marginal value is measured in terms of the improvement in expected collective utility resulting from the unit of capital's direct contribution to production. For contractor-furnished capital there is not only this direct effect, but also an indirect effect due to the fact that imputed interest is now an allowable cost. Hence, reallocating a unit of contractor-furnished capital from contract c to its next best alternative use involves a cost in terms of the effect on expected collective utility of the immediate impact on imputed interest. Therefore, in determining the optimal mix of government-furnished capital and contractor-furnished capital under any government contract c, this effect must be taken into consideration. Consequently, as demonstrated in the proof of theorem VII-5, the value of a unit of contractor-furnished capital must be figured net of this effect.

Proof of Theorem VII-5

Solving (65) and (72) for $\lambda_{2,c}$, equating the resulting expressions, and rearranging terms leads to the expression,

$$- \frac{\partial_c K^G_{t,s}}{\partial_c K^C_{t,s}} = \qquad (73)$$

$$\frac{\lambda_{5,t,s} - \{\ \} \partial R_{k,t,s} / \partial_c K^C_{t,s} - \{\ \} \partial \pi_{t,s} / \partial_c K^C_{t,s}}{\lambda_{6,t,s}} ,$$

where the terms in braces are the same as those in (65). The numerator and denominator in (73) are interpreted as the net marginal values of contractor-furnished capital and government-furnished capital, respectively, as discussed in the above remark. Q.E.D.

Corollary VII-5-1

If imputed interest and depreciation were not allowable costs, then the airframe builder would achieve the optimal combination of contractor-furnished capital and government-furnished capital for each contract c at time t and state of nature s by combining them so that their marginal rate of technical substitution just equals the ratio of their marginal values, $\lambda_{5,t,s}/\lambda_{6,t,s}$.

Proof

If imputed interest and depreciation were not allowable costs, then it follows from (8), (15), (16), and (32) that $\partial R_{k,t,s}/\partial {}_cK^C{}_{t,s} = \partial \pi_{t,s}/\partial {}_cK^C{}_{t,s} \equiv 0$. Then from (73), the marginal rate of technical substitution equals $\lambda_{5,t,s}/\lambda_{6,t,s}$.

Q.E.D.

Corollary VII-5-2

If at time t and state of nature s government-furnished capital were in such plentiful supply that its marginal value were zero, and if the marginal rate of technical substitution between government-furnished capital and contractor-furnished capital were always finite, then at optimality the net marginal value of contractor-furnished capital would be zero for each contract c (at time t in state of nature s). Alternatively, the marginal value in production of an additional unit of contractor-furnished capital, $\lambda_{5,t,s}$, would equal the sum of the marginal revenue-related effects arising out of the allowability of depreciation and interest expense under government contracts, $\{\ \}\partial R_{k,t,s}/\partial_c K^C_{t,s} + \{\ \}\partial \pi_{t,s}/\partial_c K^C_{t,s}$.

<u>Proof</u>

By hypothesis $\lambda_{6,t,s} = 0$ for some t and some s. Rewriting (73) as

$$\lambda_{6,t,s}\left(-\frac{\partial_c K^G_{t,s}}{\partial_c K^C_{t,s}}\right) = \lambda_{5,t,s} - \{\ \}\frac{\partial R_{k,t,s}}{\partial_c K^C_{t,s}} - \{\ \}\frac{\partial \pi_{t,s}}{\partial_c K^C_{t,s}} \qquad (74)$$

leads to the main result. Solving (74) for $\lambda_{5,t,s}$ leads to the alternative interpretation of the main result. Q.E.D.

The significance of theorem VII-5 is that it establishes the optimality criterion for combining government-furnished capital and contractor-furnished capital for production under government contracts. Corollary VII-5-1 demonstrates that the optimal trade offs established in theorem VII-5 are sensitive to government policy regarding the allowability of interest cost, and corollary VII-5-2 demonstrates that the optimal trade offs are also sensitive to the availability of government-furnished capital. While neither of these results is counterintuitive, the fact that they were derived from the airframe builder model (37) is supportive of the model's validity.

The fourth set of operating policies that are to be explored in this subsection involve the determination of the optimal commercial outputs. At optimality the commercial output levels ${}_RQ_{t,s}$ and ${}_NQ_{t,s}$ must satisfy the following necessary condition:

$$\begin{aligned}
&\phi_{t,s}\{\frac{\partial U_1}{\partial R_{j,t,s}}\,\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} + \frac{\partial U_1}{\partial \pi_{t,s}}\cdot\frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}}\}(\frac{1}{1+r})^t + \lambda_\ell\,\frac{\partial F_k}{\partial_k Q_{t,s}} \\
&+ \sum_{s'=1}^{S} \lambda_{9,t,s,s'}(-\,\frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}}) - \lambda_{10,t,s}\{\frac{\partial \overline{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}\} \\
&+ \sum_{s'=1}^{S} \lambda_{11,t,s,s'}\{\frac{\partial \overline{C}_{t,s}}{\partial_k Q_{t,s}} - \frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}\} \\
&- \sum_{s'=1}^{S} \lambda_{11,t+1,s',s}\{\frac{\partial \overline{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}\} = 0\ ,
\end{aligned} \tag{75}$$

where $(j,\ k,\ \ell) = (CA,\ R,\ 3)$ or $(CO,\ N,\ 4)$. Rewriting (75) in terms of the derivatives of $R_{j,t,s}$, $\pi_{t,s}$, $\overline{C}_{t,s}$, $V_{t,s}$, and F_k yields the following equation:

$$\begin{aligned}
&\{\phi_{t,s}\,\frac{\partial U_1}{\partial R_{j,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} \\
&\quad + \{\phi_{t,s}\,\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times \\
&\qquad \frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} - \{\lambda_{10,t,s}-[\sum_{s'=1}^{S}\lambda_{11,t,s,s'}-\sum_{s'=1}^{S}\lambda_{11,t+1,s',s}]\}\times \\
&\qquad (\frac{\partial \overline{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}) + \lambda_\ell\,\frac{\partial F_k}{\partial_k Q_{t,s}} = 0\ .
\end{aligned} \tag{76}$$

The Lagrange multiplier sum $\sum_{s'=1}^{S}\lambda_{11,t,s,s'}$ was encountered in (40), where it was interpreted in terms of the impact of operating policy on cash flow at time t in state s, depending on what cash flow was in state s' at time t - 1. In the case of (76) the operating policy in question concerns setting the optimal levels of the commercial outputs. The value of an additional unit of output affects cash flow through its impact on the current period's net income, and the severity of this effect depends on the cash flow (i.e. the availability of cash from) the previous period, which in turn depends on which particular state obtained that period. The Lagrange multiplier sum $\sum_{s'=1}^{S} \lambda_{9,t,s,s'}$ can be given an analogous interpretation. The difference of Lagrange multiplier sums in (76)

$$\sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} \tag{77}$$

has not been encountered previously in this chapter. The difference (77) reflects the fact that an increase in current period output increases both the transactions demand for cash $\overline{C}$ and the required inventories of work in process that are included in V, and that the severity of the resulting effects on the behavior of the firm are dependent on cash available from state s' at time t - 1; but that ceteris paribus the greater accumulation of cash this period reduces the strain of meeting the transactions and inventory cash needs next period, and that the importance of this reduction for the behavior of the firm depends on which state obtains next period. To capture both these interperiod effects in the optimization requires the difference of Lagrange multiplier sums (77).

The foregoing comments suggest that the first three terms in (76) can be interpreted collectively as the marginal value to the firm - expressed in terms of discounted expected collective utility - of an additional unit of commercial output of type k, k = R, N. This marginal value consists of three components. The first is the incremental value transmitted through a change in total revenue, { }($\partial R/\partial Q$). The second is the incremental value transmitted through a change in net income, { }($\partial \pi/\partial Q$). The third is the incremental value transmitted through a change in the cash balance and working capital requirements, { }($\partial \overline{C}/\partial Q + \partial V/\partial Q$). Given this interpretation of the sum of the first three terms in (76), the following theorem is easily proved.

Theorem VII-6

For commercial output of type k at time t, the airframe builder should establish production levels for states s and s' such that the marginal rate of transformation[49] between the state-specific amounts of output just equals the ratio of their marginal values (or marginal contributions to expected collective utility) in the two states. For commercial output of type k and for a state of nature s that might obtain at times t and t - 1, the airframe builder should establish intertemporal production levels such that the marginal rate of transformation between time-specific amounts of output just equals the ratio of their marginal values in the two periods.

Proof

To prove the first statement, fix t and use (76) to obtain expressions for $\lambda_\ell(\partial F_k/\partial {}_kQ_{t,s})$ and $\lambda_\ell(\partial F_k/\partial {}_kQ_{t,s'})$. Divide the former by the latter to obtain an expression for the marginal rate of transformation, $-\partial {}_kQ_{t,s}/\partial {}_kQ_{t,s'}$.

To prove the second statement, proceed in the same manner, but fixing s rather than t. Q.E.D.

Note that theorem VII-6 contains results that are analogous to the results contained in theorem VII-3 and corollaries VII-3-1 and VII-3-2. Collectively, these theorems and corollaries extend the neoclassical optimality rules to the case of the expected collective utility maximizer, as discussed in chapter four. Theorem VII-6 is particularly important because it demonstrates the connection between financial requirements, and in particular, cash balance and working capital requirements, and the firm's output decision. The next theorem demonstrates this connection somewhat more forcefully.

<u>Theorem VII-7</u>

In deciding how much commercial output of type k to produce at time t in state of nature s, the airframe builder will compare after-tax marginal production cost with the following sum of three terms: the unit of output's marginal contribution to net income plus the unit of output's marginal contribution to total revenue adjusted for cash flow effects, less the implied cost impact of the marginal increase in cash balance and working capital requirements. Moreover, this sum will equal after-tax marginal production cost when the airframe builder is in equilibrium.

<u>Remark</u>

The exact nature of this 'sum', which is closely related to the concept of the marginal value of output defined above, is brought out clearly in the proof of the theorem.

Proof of Theorem VII-7

Fixing t and s, using (51) to obtain an expression for $\lambda_\ell(\partial F_k/\partial_k L^j_{t,s})$ and (76) to obtain an expression for $\lambda_\ell(\partial F_k/\partial_k Q_{t,s})$, and dividing the former by the latter yields

$$-\frac{\partial_k Q_{t,s}}{\partial_k L^j_{t,s}} = \frac{\partial F_k/\partial_k L^j_{t,s}}{\partial F_k/\partial_k Q_{t,s}} = \frac{A}{B}\,, \tag{78}$$

where

$$A = \{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}} \tag{79}$$

$$B = \{\phi_{t,s}\frac{\partial U_1}{\partial R_{j,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} + \{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t$$

$$- \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} - \{\lambda_{10,t,s} \tag{80}$$

$$- [\sum_{s'=1}^{S}\lambda_{11,t,s,s'} - \sum_{s'=1}^{S}\lambda_{11,t+1,s',s}]\}(\frac{\partial \bar{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}})\,.$$

From (31) and (32) it follows that

$$\frac{\partial \pi_{t,s}}{\partial_k L^j_{t,s}} = -(1-\tau)w_j\,. \tag{81}$$

Substituting (81) into (79) and rearranging terms in (78) yields

$$\frac{(1-\tau)w_j}{\partial_k Q_{t,s}/\partial_k L^j_{t,s}} = \frac{B}{\{\phi_{t,s}\cdot\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}}\,, \tag{82}$$

and using (80) to substitute for B in (82) yields

$$\frac{(1-\tau)w_j}{\partial_k Q_{t,s}/\partial_k L^j_{t,s}} = \frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}}$$

$$+ \frac{\{\phi_{t,s} \frac{\partial U_1}{\partial R_{j,t,s}} (\frac{1}{1+r})^t\}}{\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}} \times$$

$$\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} \tag{83}$$

$$- \frac{\{\lambda_{10,t,s} - [\sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s}]\}}{\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}} \times$$

$$(\frac{\partial \overline{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}) \ .$$

The left-hand side of (83) is interpretable as the after-tax marginal production cost of commercial output of type k, and the right-hand side of (83) consists of the sum of the three effects in the order in which they were listed in the statement of the theorem. Q.E.D.

The significance of theorem VII-7 is that the equilibrium condition (83) demonstrates clearly the role that financial factors play in determining the airframe builder's commercial output levels. In particular, the output decision would not be independent of financial considerations unless

$$\lambda_{10,t,s} - [\sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s}] = 0 \text{ and}$$

$$\sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'}) = 0.$$

In general, it cannot be assured that either of these conditions will hold at optimality. This is demonstrated, and its implications for the behavior of the representative airframe builder are explored further, in the next subsection.

The fifth set of operating policies that are to be considered in this section involve the determination of the optimal effectiveness/performance levels under government contracts. For each government contract c at each time t and in state of nature s, the effectiveness levels ${}_c\hat{E}_{t,s}$ must satisfy the following necessary condition:

$$\phi_{t,s} \frac{\partial U_1}{\partial {}_c\hat{E}_{t,s}} \left(\frac{1}{1+r}\right)^t + \lambda_{2,c} \frac{\partial F_c}{\partial {}_c\hat{E}_{t,s}} = 0 \; . \tag{84}$$

Necessary condition (84) leads immediately to the following theorem:

<u>Theorem VII-8</u>

For contract c at time t, $0 < t < T$, and states of nature s and s', the airframe builder should set state-specific effectiveness levels ${}_c\hat{E}_{t,s}$ and ${}_c\hat{E}_{t,s'}$ in such a way that their marginal rate of transformation just equals the ratio of their probability-weighted marginal utilities. For contract c and state of nature s that might obtain during periods t and t - 1, $0 < t < T$, the airframe builder should set intertemporal effectiveness levels ${}_c\hat{E}_{t,s}$ and ${}_c\hat{E}_{t-1,s}$ in such a way that their marginal rate of transformation just equals the ratio of their discounted probability-weighted marginal utilities.

Proof

Proceeding as before from (84) yields

$$-\frac{\partial_c\hat{E}_{t,s'}}{\partial_c\hat{E}_{t,s}} = \frac{\phi_{t,s}\dfrac{\partial U_1}{\partial_c\hat{E}_{t,s}}\left(\dfrac{1}{1+r}\right)^t}{\phi_{t,s'}\dfrac{\partial U_1}{\partial_c\hat{E}_{t,s'}}\left(\dfrac{1}{1+r}\right)^t} = \frac{\phi_{t,s}\ \partial U_1/\partial_c\hat{E}_{t,s}}{\phi_{t,s'}\ \partial U_1/\partial_c\hat{E}_{t,s'}} \tag{85}$$

and

$$-\frac{\partial_c\hat{E}_{t-1,s}}{\partial_c\hat{E}_{t,s}} = \frac{\phi_{t,s}(\partial U_1/\partial_c\hat{E}_{t,s})/(1+r)^t}{\phi_{t-1,s}(\partial U_1/\partial_c\hat{E}_{t-1,s})/(1+r)^{t-1}} \ . \tag{86}$$

Q.E.D.

Remark

If t = T in the first statement in theorem VII-8, then the expressions in the numerator and denominator of (85) must be adjusted for the impact of effectiveness under ongoing government contracts on backlogs. For t = T, it follows from (38) that (84) must be replaced by

$$\phi_{T,s}\frac{\partial U_1}{\partial_c\hat{E}_{T,s}}\left(\frac{1}{1+r}\right)^T + \phi_{T,s}\frac{\partial U_2}{\partial\hat{B}_{T,s}}\cdot\frac{\partial\hat{B}_{T,s}}{\partial_c\hat{E}_{T,s}}\left(\frac{1}{1+r}\right)^T + \lambda_{2,c}\frac{\partial F_c}{\partial_c\hat{E}_{T,s}} = 0 \ ,$$

from which it follows that

$$\phi_{T,s}\frac{\partial U_2}{\partial\hat{B}_{T,s}}\ \frac{\partial\hat{B}_{T,s}}{\partial_c\hat{E}_{T,s}} \tag{87}$$

must be added to numerator and denominator (with s' in place of s in the latter) to obtain the appropriate optimality criterion for the terminal period. When t = T in the second statement in theorem VII-8, the product of (87) and $1/(1+r)^T$

must be added to the numerator in (86) to allow for the backlog effect.

Theorem VII-8 contains the decision rules for achieving the optimal effectiveness trade offs across states and across dates. The next theorem presents a related result concerning the optimal allocation of labor under government contracts expressed in terms of marginal weapons system effectiveness/contract performance. But first a lemma, which will aid in the interpretation of the theorem, is proved.

<u>Lemma VII-1</u>

Adopting the convention that for each contract-specific production function F_c,

$$\frac{\partial F_c}{\partial_c Q_{t,s}} > 0 \ , \ \frac{\partial F_c}{\partial_c \hat{E}_{t,s}} > 0 \ , \ \frac{\partial F_c}{\partial_c L^j{}_{t,s}} < 0 \ , \ \frac{\partial F_c}{\partial_c K^{j'}{}_{t,s}} < 0 \ , \tag{88}$$

for all t and for all s, where j = M, A, E, and where j' = C, G,[50] it follows that

$$\lambda_{2,c} < 0 \tag{89}$$

for each government contract c. Moreover, if

$$\frac{\partial U_1/\partial R_{GA,t,s}}{\partial U_1/\partial \pi_{t,s}} \geq (1-\tau)\{\frac{1}{(1-\beta)[\gamma'(C)\cdot C+\gamma(C)]} - 1\} \tag{90}$$

at optimality,[51] then it follows that the cash flow impact of labor, $-\{\sum_{s'=1}^{S}(\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}(\partial\pi_{t,s}/\partial_c L^j{}_{t,s})$ in (40), is negative and also that the net marginal value of labor, $\{\ \ \}(\partial R_{GA,t,s}/\partial_c L^j{}_{t,s}) + \{\ \ \}(\partial\pi_{t,s}/\partial_c L^j{}_{t,s})$ in (40), is negative.

Proof

It follows from (88), from the assumption that $\partial U_1/\partial_c\hat{E}_{t,s} > 0$, and from (84) that $\lambda_{2,c} < 0$ for each government contract c.

Turning to the second statement, note that

$$\frac{\partial U_1/\partial R_{GA,t,s}}{\partial U_1/\partial \pi_{t,s}} \geq (1-\tau)\{\frac{1}{(1-\beta)[\gamma'(C)\cdot C+\gamma(C)]} - 1\}$$

$$\Longleftrightarrow \phi_{t,s}\{\frac{\partial U_1}{\partial R_{GA,t,s}}\cdot\frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}} + \frac{\partial U_1}{\partial \pi_{t,s}}\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}}\}(\frac{1}{1+r})^t \geq 0 \qquad (91)$$

follows from (8), (15), and (32).[52] It then follows from (39) and (88)-(91) that

$$-\{\sum_{s'=1}^{S}(\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}(\partial\pi_{t,s}/\partial_c L^j_{t,s}) < 0 \ . \qquad (92)$$

It also follows from (88) and (89) that

$\{\quad\}(\partial R_{GA,t,s}/\partial_c L^j_{t,s})+\{\quad\}(\partial\pi_{t,s}/\partial_c L^j_{t,s}) < 0$ in (40). Q.E.D

Remark

The left-hand side of (90) is interpreted as the contractor's marginal rate of substitution between net income and revenue earned on the sale of aerospace products to the government (and on aerospace research and development carried out for the government). The right-hand side of (90) is interpreted as the rate at which net income and government aerospace sales revenue can be traded off for one another.[53] (90) requires, then, that the contractor's subjective rate of trading off net income and government aerospace sales revenue be at least as great as the (objective) rate at which these quantities can be traded off within the contractor's income statement.

Theorem VII-9

For contract c and for labor of type j at time t and state of nature s, the amount of labor of type j allocated to work under contract c should be such that the marginal contribution of labor of type j to weapons system effectiveness/contract performance under contract c just equals the negative of the ratio of the discounted probability-weighted marginal utility of effectiveness/performance under contract c to the net marginal value of labor reckoned in terms of its contribution to total revenue and net income.

Proof

Without loss of generality, assume that the contract c is an aerospace contract. Solving (40) for $\lambda_{2,c}(\partial F_c/\partial_c L^j_{t,s})$ and (84) for $\lambda_{2,c}(\partial F_c/\partial_c \hat{E}_{t,s})$, dividing the former by the latter, applying the implicit function theorem, and multiplying through by -1 yields

$$\frac{\partial_c \hat{E}_{t,s}}{\partial_c L^j_{t,s}} = - \frac{\{\}\partial R_{GA,t,s}/\partial_c L^j_{t,s} + \{\ \}\partial \pi_{t,s}/\partial_c L^j_{t,s}}{\phi_{t,s}(\partial U_1/\partial_c \hat{E}_{t,s})/(1+r)^t}, \tag{93}$$

where the expressions within braces in the numerator of (93) are the same as those within braces in (40). Q.E.D.

Remark

Note that the expressions on both sides of (93) are positive since, by lemma VII-1, the numerator on the right-hand side of (93) is negative (while the denominator is positive). Theorem VII-9 implies that at equilibrium the airframe builder's use of each type of labor under each government contract will have been expanded to the point at which the marginal effectiveness/performance product of labor, $\partial_c \hat{E}_{t,s}/\partial_c L^j_{t,s}$, or the rate at which labor units can be 'transformed into' increased effectiveness/performance, just equals the subjective collective rate of trade off between net income and effectiveness (in each case discounted and probability-weighted) embodied in the right-hand side of (93). Because product effectiveness/contract performance does not appear in neoclassical models of the firm, (93) has no counterpart in the neoclassical theory of the firm.

The sixth set of operating policies that are to be discussed in this section concern the airframe builder's payment of managerial emoluments. Managerial emoluments paid at time t in state of nature s must satisfy the necessary condition

$$\phi_{t,s}\left[\frac{\partial U_1}{\partial \pi_{t,s}} \frac{\partial \pi_{t,s}}{\partial M_{t,s}} + \frac{\partial U_1}{\partial M_{t,s}}\right]\left(\frac{1}{1+r}\right)^t + \left\{\sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\right\} \times \left(- \frac{\partial \pi_{t,s}}{\partial M_{t,s}}\right) = 0 \ . \tag{94}$$

(94) leads to the following theorem concerning the firm's payment of managerial emoluments when it is in equilibrium.

<u>Theorem VII-10</u>

When the airframe builder modeled in (37) is in equilibrium, it will pay a level of managerial emoluments that equates the collective marginal rate of substitution between net income and managerial emoluments to one minus the tax rate.

<u>Proof</u>

It follows from (32) that

$$\frac{\partial \pi_{t,s}}{\partial M_{t,s}} = -(1-\tau). \tag{95}$$

Substituting (95) into (94) and rearranging terms yields

$$\frac{\phi_{t,s}(\partial U_1/\partial M_{t,s})/(1+r)^t}{\{\phi_{t,s}(\partial U_1/\partial \pi_{t,s})/(1+r)^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}} = (1-\tau), \tag{96}$$

which is the desired result. Q.E.D.

Note that the marginal rate of substitution between net income and managerial emoluments on the left-hand side of (96) takes into account the cash flow impact on discounted expected collective utility of a change in net income. What theorem VII-10 implies, then, is that the contractor will have equated its subjective internal rate of trade off between managerial emoluments and net income to the externally imposed rate, $1 - \tau$, at which these quantities can be traded off when it is in equilibrium, and also, that this subjective rate of trade off takes into account explicitly the cash flow impact of a change in net income.

This subsection has discussed the optimal operating policies suggested by the airframe builder model (37) developed in section B of this chapter. The next subsection explores the model's implications for the airframe builder's optimal financial policies.

3. Optimal Financial Policies

For the purposes of this subsection the airframe builder's financial policies can be considered to be of two types: (i) issue/redemption policies for bonds and shares and (ii) cash management policies. The former may be regarded collectively as the firm's leverage policy. Under the assumptions stated in section B the firm's dividend policy is taken as given, and so is not considered in this subsection.

Beginning with the bond issue/redemption policy, the amount of bonds outstanding at each date and state, $B_{t,s}$, must satisfy the following necessary condition:

$$\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}\frac{\partial\pi_{t,s}}{\partial B_{t,s}}\left(\frac{1}{1+r}\right)^t+\sum_{s'=1}^{S}\lambda_{7,t,s,s'}-\sum_{s'=1}^{S}\lambda_{7,t+1,s',s}$$
$$+\lambda_{10,t,s}-\left\{\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\right\}\frac{\partial\pi_{t,s}}{\partial B_{t,s}}=0\ . \tag{97}$$

The issues/redemptions at each date and state, $Y_{t,s}$, must satisfy the following necessary condition:

$$\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}\frac{\partial\pi_{t,s}}{\partial Y_{t,s}}\left(\frac{1}{1+r}\right)^t-\sum_{s'=1}^{S}\lambda_{7,t,s,s'}$$
$$+\left\{\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\right\}\left(-\frac{\partial\pi_{t,s}}{\partial Y_{t,s}}\right)-\sum_{s'=1}^{S}\lambda_{11,t,s,s'}=0\ . \tag{98}$$

Solving (98) for $\sum_{s'=1}^{S}\lambda_{7,t,s,s'}$ yields

$$\sum_{s'=1}^{S}\lambda_{7,t,s,s'}=\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}\left(\frac{1}{1+r}\right)^t-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}$$
$$+\lambda_{11,t,s,s'})\frac{\partial\pi_{t,s}}{\partial Y_{t,s}}-\sum_{s'=1}^{S}\lambda_{11,t,s,s'} \tag{99}$$

and substituting (99) into (97) and solving for $\lambda_{10,t,s}$ yields

$$\lambda_{10,t,s}=-\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}\left(\frac{1}{1+r}\right)^t-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times \tag{100}$$

$$\frac{\partial \pi_{t,s}}{\partial B_{t,s}} - \{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \times$$

$$\frac{\partial \pi_{t,s}}{\partial Y_{t,s}} + \sum_{s'=1}^{S} \lambda_{7,t+1,s',s} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'} ,$$

where (32) could be used to obtain expressions for $\partial \pi_{t,s}/\partial B_{t,s}$ and $\partial \pi_{t,s}/\partial Y_{t,s}$ in terms of the partial derivatives of the interest rate function $i_{t,s} = i_{t,s}(B_{t,s}, Y_{t,s})$.

To interpret (100) note that the first term on the right-hand side represents the immediate impact on net income of a change in the amount of debt outstanding, while the second term on the right-hand side represents the immediate impact of a change in the rate of new issues/redemptions, where the two effects are related through (22) and are transmitted through the induced changes in the interest rate given by (23). Moreover, each of these terms is strictly positive since $\partial \pi/\partial B < 0$ and $\partial \pi/\partial Y < 0$, by (24) and (32), and since the expression { } must be strictly positive (as a result of lemma VII-2, which is proved below). The third term on the right-hand side of (100) can be interpreted as the value of an additional dollar of debt in terms of reduced future needs, as can be seen from (38). As such, one would normally expect this sum to be nonnegative. Finally, the last term on the right-hand side of (100) can be interpreted as the direct cash flow impact of an additional dollar of new debt. Collectively, the four terms on the right-hand side of (100), and hence $\lambda_{10,t,s}$ itself, can be interpreted as an equilibrium marginal cost of debt capital, where the marginal cost is adjusted for the impact of increasing the current stock of debt on future debt requirements and also on current cash flow.[54]

Turning to share issue/redemption policy, the number of shares outstanding at each date and state, $n_{t,s}$, must be such that the following necessary condition is satisfied:

$$\sum_{s'=1}^{S} \lambda_{8,t,s,s'} - \sum_{s'=1}^{S} \lambda_{8,t+1,s',s} + d\cdot \sum_{s'=1}^{S} \lambda_{9,t,s,s'} + d\cdot \sum_{s'=1}^{S} \lambda_{11,t,s,s'} = 0 . \tag{101}$$

The issues/redemptions at each date and state, $Z_{t,s}$, must be such that the following necessary condition is satisfied:

$$- \sum_{s'=1}^{S} \lambda_{8,t,s,s'} - v_{t,s}\cdot \sum_{s'=1}^{S} \lambda_{9,t,s,s'} - v_{t,s}\cdot \sum_{s'=1}^{S} \lambda_{11,t,s,s'} = 0 . \tag{102}$$

In addition, total stockholders' equity at each date and state, $E_{t,s}$, must be such that the following necessary condition is satisfied:

$$\sum_{s'=1}^{S} \lambda_{9,t,s,s'} - \sum_{s'=1}^{S} \lambda_{9,t+1,s',s} + \lambda_{10,t,s} = 0 . \tag{103}$$

Solving (102) for $\sum_{s'=1}^{S} \lambda_{8,t,s,s'}$ yields

$$\sum_{s'=1}^{S} \lambda_{8,t,s,s'} = - v_{t,s}\left(\sum_{s'=1}^{S} \lambda_{9,t,s,s'} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'}\right). \tag{104}$$

Substituting (104) into (101) yields

$$(d-v_{t,s})\left(\sum_{s'=1}^{S} \lambda_{9,t,s,s'} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'}\right) - \sum_{s'=1}^{S} \lambda_{8,t+1,s',s} = 0. \tag{105}$$

Solving (105) for $\sum_{s'=1}^{S} \lambda_{9,t,s,s'}$, substituting into (103), and rearranging terms yields

$$\lambda_{10,t,s} = \frac{1}{v_{t,s}-d} \sum_{s'=1}^{S} \lambda_{8,t+1,s',s} + \sum_{s'=1}^{S} \lambda_{9,t+1,s',s} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'} . \tag{106}$$

To interpret (106) note that the first term on the right-hand side can be interpreted as the value of an additional dollar's worth of newly issued shares. To see this recall that, according to (37), each new share is issued at the beginning of the period, so that net receipts for the period are equal to the price less the dividend, $v_{t,s} - d$. Also, since $\lambda_{8,t+1,s',s}$ applies to the new equity issue/share redemption constraint, the effect of dividing $\sum_{s'=1}^{S} \lambda_{8,t+1,s',s}$ by $v_{t,s}-d$ is to convert the value measure in terms of shares into one in terms of a dollar's worth of newly issued shares. The second and third terms on the right-hand side are perfectly analogous to the third and fourth terms, respectively, on the right-hand side of (100). Similar to (100), then, $\lambda_{10,t,s}$ given by (106) can be interpreted as the equilibrium marginal cost of external equity capital, where the marginal cost is adjusted for the impact of increasing the current stock of externally raised equity on future external equity requirements and also on current cash flow.

The equilibrium conditions (100) and (106) lead to the following theorem.

Theorem VII-11

In equilibrium the airframe builder will have adjusted its capital structure so that the marginal cost of debt equals the marginal cost of external equity.

Proof

Equate (100) and (106). Q.E.D.

The importance of theorem VII-11 is that $\lambda_{10,t,s}$ can be interpreted as the airframe builder's marginal cost of capital (from either external source).

In view of the results obtained in the previous subsection, a question arises concerning the relationship between the two costs of capital, $\lambda_{5,t,s}$ and $\lambda_{10,t,s}$, where it should be recalled that each is probability-weighted (by the probability that state s obtains at time t, $\phi_{t,s}$) and discounted (by $(\frac{1}{1+r})^t$) since the units in which each is measured must be consistent with the units in which the objective functional in (37) is measured. As described below, the relationship between the discounted expected values $\sum_{s=1}^{S} \lambda_{10,t,s}$ and $\sum_{s=1}^{S} \lambda_{5,t,s}$ is analogous to the relationship between the interest rate r and the cost of capital $i = rq + (1-\tau)q_0 - \dot{q}$ in the deterministic multi-period model of the firm discussed in chapter three of this thesis.

First, rewrite the expression (62) for the firm's marginal cost of physical capital as

$$\lambda_{5,t,s} = \lambda_{10,t,s} \cdot q_{t,s} + \{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} \quad (107)$$

$$+\lambda_{11,t,s,s'})\}(1-\tau)q_{t,s}\cdot\delta - \{\sum_{s'=1}^{S}\lambda_{11,t,s,s'}q_{t,s}$$

$$-(\frac{1}{1+\delta})\sum_{s'=1}^{S}\lambda_{1,t+1,s',s}\} .$$

Next, summing each side of (60) over s,[55] it follows that

$$\sum_{s=1}^{S}(\frac{1}{1+\delta})\sum_{s'=1}^{S}\lambda_{1,t+1,s',s} = \sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{11,t+1,s's}q_{t+1,s} . \quad (108)$$

Then summing each side of (107) over s and substituting using (108) gives

$$\begin{aligned}\sum_{s=1}^{S}\lambda_{5,t,s} = \sum_{s=1}^{S}\lambda_{10,t,s}\cdot q_{t,s} & \\ + \sum_{s=1}^{S}\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} & \quad (109) \\ +\lambda_{11,t,s,s'})\}(1-\tau)q_{t,s}\cdot\delta & \\ - \sum_{s=1}^{S}(-1)\{\sum_{s'=1}^{S}\lambda_{11,t+1,s',s}q_{t+1,s} - \sum_{s'=1}^{S}\lambda_{11,t,s,s'}q_{t,s}\} &\end{aligned}$$

$$\begin{aligned}= \sum_{s=1}^{S}[\lambda_{10,t,s}\cdot q_{t,s}+\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} & \\ +\lambda_{11,t,s,s'})\}(1-\tau)q_{t,s}\cdot\delta-\{\sum_{s'=1}^{S}\lambda_{11,t,s,s'}q_{t,s} & \quad (110) \\ - \sum_{s'=1}^{S}\lambda_{11,t+1,s',s}q_{t+1,s}\}] &\end{aligned}$$

Comparing (109) with $i = rq + (1-\tau)q\delta - \dot{q}$, the first term in (109) is analogous to rq, with the firm's cost of financial capital, $\lambda_{10,t,s}$, playing a role analogous to the interest rate, r, in the deterministic case. The second and third terms in (109) are analogous to $(1-\tau)q\delta$ and $-\dot{q}$, respectively, in the deterministic case.[56] This analogy is brought out further in theorem VII-12.

(109) characterizes the equilibrium position of the representative airframe builder modeled in (37) with regard to investment. This result is stated formally as the following theorem.

<u>Theorem VII-12</u>

In equilibrium the representative airframe builder modeled in (37) will have carried investment in contractor-furnished capital in each period t up to the point at which the expected marginal value of an additional unit of physical capital at t, $\sum_{s=1}^{S} \lambda_{5,t,s}$, just equals the expected cost of the required financial capital, $\sum_{s=1}^{S} \lambda_{10,t,s} q_{t,s}$, plus the expected cost implied by the increased depreciation (net of tax), $\sum_{s=1}^{S} \{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \times (1-\tau) q_{t,s} \cdot \delta$, less the expected cash flow impact of changing capital goods prices, $\sum_{s=1}^{S} (-1)\{ \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} q_{t+1,s} - \sum_{s'=1}^{S} \lambda_{11,t,s,s'} q_{t,s}\}$.

Rewriting (109) as (110) suggests the following investment criterion for each date t and state s.

<u>Corollary VII-12-1</u>

A sufficient condition for meeting the criterion for optimal investment in contractor-furnished capital at time t in state of nature s is the following: the contractor should expand investment until the point at which the marginal value of an additional unit of physical capital at t in s, $\lambda_{5,t,s}$, just equals the marginal cost of the required financial capital, $\lambda_{10,t,s} \cdot q_{t,s}$, plus the implied cost of depreciation $\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \times (1-\tau) q_{t,s} \delta$, less the expected impact of changing capital

goods prices, $\sum_{s'=1}^{S} \lambda_{11,t,s,s'}q_{t,s} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s}q_{t+1,s'}$
or in equation form,

$$\lambda_{5,t,s} = \lambda_{10,t,s}q_{t,s}+\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}(1-\tau)q_{t,s}\delta - \{\sum_{s'=1}^{S}\lambda_{11,t,s,s'}q_{t,s} - \sum_{s'=1}^{S}\lambda_{11,t+1,s',s}q_{t+1,s}\}. \quad (111)$$

Proof

For given period t, if (111) holds for each possible state of nature s at t, then (110) follows by summing each side of (111) over s. But since (110) is identically equal to (109), the conclusion follows. Q.E.D.

(109) also suggests the following important result.

Corollary VII-12-2

If U_1 is strictly concave; if $\partial^2 i/\partial B^2 > 0$ and $\partial^2 i/\partial B \partial Y > 0$; and if the direct effect of substituting debt for equity in the firm's capital structure,

$$\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times$$

$$[-\frac{\partial^2 \pi_{t,s}}{\partial B^2_{t,s}} - \frac{\partial^2 \pi_{t,s}}{\partial B_{t,s}\partial Y_{t,s}}] + \{\phi_{t,s} \frac{\partial^2 U_1}{\partial \pi_{t,s}^2}(\frac{1}{1+r})^t\}\times$$

$$[-(\frac{\partial \pi_{t,s}}{\partial B_{t,s}})^2 - (\frac{\partial \pi_{t,s}}{\partial B_{t,s}})(\frac{\partial \pi_{t,s}}{\partial Y_{t,s}})]$$

exceeds in absolute value the indirect effect, Σ, which is defined below, then substituting debt for equity in the firm's equilibrium capital structure at time t in state of nature s will cause its cost of financial capital, $\lambda_{10,t,s}$, to increase, and under such circumstances it may be concluded that the investment decision of the representative airframe builder modeled in (37) is not independent of its financial policy choices.

Proof

Consider the effect on $\lambda_{10,t,s}$ of changes dB and dE in the amounts of outstanding debt and equity that are equal in magnitude but opposite in sign. That is, dE = -dB, but where dB > 0 when debt is substituted for equity. If the firm's investment decision were independent of its financial policy choices, then $\lambda_{10,t,s}$ should remain constant as debt is substituted for equity (and vice versa) in the firm's capital structure.

From (100) and (106), the change in the firm's financial cost of capital in response to a change in its capital structure can be expressed as

$$d\lambda_{10,t,s} = -\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}\left(\frac{1}{1+r}\right)^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times \tag{112}$$

$$\frac{\partial^2\pi_{t,s}}{\partial B_{t,s}^2}dB_{t,s} - \{\phi_{t,s}\frac{\partial^2 U_1}{\partial \pi_{t,s}^2}\left(\frac{1}{1+r}\right)^t - \sum_{s'=1}^{S}\left(\frac{d\lambda_{9,t,s,s'}}{d\pi_{t,s}}\right.$$

$$\left.+\frac{d\lambda_{11,t,s,s'}}{d\pi_{t,s}}\right)\}\left(\frac{\partial\pi_{t,s}}{\partial B_{t,s}}\right)^2 dB_{t,s} - \{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}\left(\frac{1}{1+r}\right)^t$$

$$-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial^2\pi_{t,s}}{\partial B_{t,s}\partial Y_{t,s}}dB_{t,s}$$

$$-\{\phi_{t,s}\frac{\partial^2 U_1}{\partial \pi_{t,s}^2}\left(\frac{1}{1+r}\right)^t - \sum_{s'=1}^{S}\left(\frac{d\lambda_{9,t,s,s'}}{d\pi_{t,s}}+\frac{d\lambda_{11,t,s,s'}}{d\pi_{t,s}}\right)\}\times$$

$$(\frac{\partial\pi_{t,s}}{\partial B_{t,s}})(\frac{\partial\pi_{t,s}}{\partial Y_{t,s}})dB_{t,s}+\sum_{s'=1}^{S}\frac{d\lambda_{7,t+1,s',s}}{dB_{t,s}}dB_{t,s}$$

$$+\sum_{s'=1}^{S}\frac{d\lambda_{11,t,s,s'}}{dB_{t,s}}dB_{t,s}-\frac{1}{v_{t,s}-d}\sum_{s'=1}^{S}\frac{d\lambda_{8,t+1,s',s}}{dE_{t,s}}dB_{t,s}$$

$$-\sum_{s'=1}^{S}\frac{d\lambda_{9,t+1,s',s}}{dE_{t,s}}dB_{t,s}-\sum_{s'=1}^{S}\frac{d\lambda_{11,t,s,s'}}{dE_{t,s}}dB_{t,s}$$

$$=\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}(\frac{1}{1+r})^t-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times$$

$$[-\frac{\partial^2\pi_{t,s}}{\partial B_{t,s}^2}-\frac{\partial^2\pi_{t,s}}{\partial B_{t,s}\partial Y_{t,s}}]dB_{t,s}+\{\phi_{t,s}\frac{\partial^2 U_1}{\partial\pi_{t,s}^2}(\frac{1}{1+r})^t\}\times \tag{113}$$

$$[-(\frac{\partial\pi_{t,s}}{\partial B_{t,s}})^2-(\frac{\partial\pi_{t,s}}{\partial B_{t,s}})(\frac{\partial\pi_{t,s}}{\partial Y_{t,s}})]dB_{t,s}+\Sigma ,$$

where Σ in (113) denotes the grand sum of all the terms in (112) that contain rates of change of Lagrange multipliers. But it follows from (24) and the stated assumptions that the first term in (113) is strictly positive since

$$\frac{\partial^2\pi}{\partial B^2}=\frac{\partial}{\partial B}(-i-\frac{\partial i}{\partial B}B)=-2\frac{\partial i}{\partial B}-\frac{\partial^2 i}{\partial B^2}<0$$

$$\frac{\partial^2\pi}{\partial B\partial Y}=\frac{\partial}{\partial B}(-\frac{\partial i}{\partial Y}B)=-\frac{\partial i}{\partial Y}-\frac{\partial^2 i}{\partial B\partial Y}B<0 .$$

The second term in (113) is also strictly positive by (24) and by the assumed strict concavity of U_1. While the sign and magnitude of Σ are indeterminate in general, under the stated assumption that the direct effect outweighs the indirect effect, $d\lambda_{10,t,s}>0$. Thus, by (109), the firm's cost of physical capital, and hence its investment decision, is affected by the change in the firm's capital structure. Q.E.D.

The significance of corollary VII-12-2 is that it states conditions sufficient to ensure that the separability theorems do not apply to the firm modeled in (37).[57] The contractor's investment decision is not, under the conditions stated in the corollary, independent of its financial policies - and in particular, of its choice of capital structure - and there exists an optimum debt-equity ratio, $B_{t,s}/n_{t,s} \cdot v_{t,s}$, that is necessary for discounted expected collective utility to be maximized.

Turning next to the airframe builder's cash management policy, the amount of precautionary and speculative cash balances at time t in state of nature s, $\hat{C}_{t,s}$, must be such that the following necessary conditions are satisfied:

$$-\lambda_{10,t,s} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} + \lambda_{12,t,s} = 0 \qquad (114)$$

$$\lambda_{12,t,s}\hat{C}_{t,s} = 0 \qquad \hat{C}_{t,s} \geq 0 \qquad \lambda_{12,t,s} \geq 0 \qquad (115)$$

It follows from (115) that two cases need to be considered: $\lambda_{12,t,s} > 0$ or $\hat{C}_{t,s} > 0$.

<u>case (i): $\lambda_{12,t,s} > 0$.</u>

In this case the optimal stock of precautionary cash balances at time t in state s is zero since, by (115), $\lambda_{12,t,s} > 0$ implies $\hat{C}_{t,s} = 0$. It follows from (114) that

$$-\lambda_{10,t,s} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s's} < 0 . \qquad (116)$$

This suggests the following theorem:

<u>Theorem VII-13</u>

During time periods t and states of nature s when precautionary cash balances are zero, the airframe builder's commercial output decisions are constrained, and in particular, output levels tend to be restricted, by the impact a change in the level of output of either commercial product would have on the firm's transactions balance and working capital requirements.

Proof

It follows from (18), (20), and (116) that the third term on the left-hand side of equation (76) is strictly negative,

$$-\{\lambda_{10,t,s}-[\sum_{s'=1}^{S}\lambda_{11,t,s,s'}-\sum_{s'=1}^{S}\lambda_{11,t+1,s',s}]\}(\frac{\partial\bar{C}_{t,s}}{\partial_k Q_{t,s}} + \frac{\partial V_{t,s}}{\partial_k Q_{t,s}}) < 0 \ , \tag{117}$$

so that the equilibrium condition that commercial output levels must satisfy is not independent of the transactions balance and working capital requirements associated with an incremental change in one or both commercial output levels. Moreover, it follows from (66) and the standard assumption that the implicit production function F_k is written so that $\partial F_k/\partial_k K^C{}_{t,s} < 0$ and $\partial F_k/\partial_k Q_{t,s} > 0$ that $\lambda_\ell < 0$ and

$$\lambda_\ell(\partial F_k/\partial_k Q_{t,s}) < 0 \ . \tag{118}$$

It follows from (117) and (118) that the sum of the first two terms in (76) is strictly positive,

$$\{\phi_{t,s}\frac{\partial U_1}{\partial R_{j,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} + \{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} > 0 \ . \tag{119}$$

It remains to be shown that (119) implies that output is restricted. First note that if the commercial output decision were independent of the transactions balance and working capital requirements associated with an incremental change in output, (119) would still hold at optimality, though in light of (117), the quantity on the left-hand side of (119) would be smaller in magnitude.[58] Thus, it needs to be shown that the magnitude of the expression on the left-hand side of (119) decreases with increasing output. But this follows directly from (32) and the assumed concavity of U_1 and of $R_{j,t,s}$. Q.E.D

<u>case (ii): $\hat{C}_{t,s} > 0$.</u>

In this case the optimal stock of precautionary cash balances at time t in state s is strictly positive, and by (114), $\lambda_{12,t,s} = 0$. It follows from (114) that the inequalities in (116) and (117) must be replaced by equalities, which leads to the following corollary to the theorem just proved.

<u>Corollary VII-13-1</u>

If precautionary cash balances are strictly positive at time t in state of nature s, then the airframe builder's commercial output decisions, ${}_kQ_{t,s}$, k = R, N, can be made independently of the transactions balance and working capital requirements associated with an incremental change in the level of output of either good.

<u>Proof</u>

Follows directly from (117) with equality in place of the inequality and (76). Q.E.D.

It should be noted that even if the airframe builder need not consider explicitly the transactions balance and working capital requirements in making its commercial output decisions, these output decisions are still not, in general, completely independent of all financial considerations. This is a consequence of the next lemma.

Lemma VII-2

If

$$\frac{\partial U_1/\partial R_{GA,t,s}}{\partial U_1/\partial \pi_{t,s}} \geq (1-\tau)\{\frac{1}{(1-\beta)[\gamma'(C)\cdot C+\gamma(C)]} -1\} \qquad (90)$$

holds when the airframe builder modeled in (37) is in equilibrium, [59] then

$$\sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'}) < 0 \ . \qquad (120)$$

Proof

Follows directly from (91) and (92) demonstrated in the proof of lemma VII-1 and from (46) demonstrated in the proof of theorem VII-2. Q.E.D.

Necessary conditions (114) and (115) also lead to the following important theorem that characterizes the contractor's decision as to whether or not to maintain precautionary cash balances at time t in state of nature s.

Theorem VII-14

For any period t and state of nature s, the equilibrium expected value (in terms of discounted collective utility) of additional cash balances at t in s contingent upon s' at t - 1 less the equilibrium expected value of additional cash balances at t in s contingent upon s' at t + 1,

$$\sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} \qquad (121)$$

will never exceed the firm's equilibrium marginal cost of external capital, $\lambda_{10,t,s}$. Moreover, if the difference of expected values is strictly less than the firm's equilibrium marginal cost of external capital for any date and state (t, s), then the firm's optimal level of precautionary cash balances for that date and state is zero, i.e. $\hat{C}_{t,s} = 0$.

<u>Remark</u>

The expression (121) can be interpreted as the net marginal value of an additional dollar of cash balances at time t in state of nature s, where the net marginal value is computed as a difference of expected values. The first of these, $\sum_{s'=1}^{S} \lambda_{11,t,s,s'}$ is the expected marginal value of additional cash balances at t in s contingent upon s' at t - 1, where the expectation is taken over states in the previous period. The second expectation $\sum_{s'=1}^{S} \lambda_{11,t+1,s',s}$, is interpreted similarly, although the expectation is taken over states in the following period. As indicated by (121), the firm's cash policy is very much dependent on intertemporal considerations.

<u>Proof</u>

The first statement follows directly from (114) and (115), i.e.

$$\sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} \leq \lambda_{10,t,s} \qquad (122)$$

for all t and s.

The second statement also follows directly from (114) and (115) since strict inequality in (122) implies

$\lambda_{12,t,s} > 0$, which implies $\hat{C}_{t,s} = 0$ by (115). Q.E.D.

The importance of theorem VII-14 is that it links the firm's decision whether to maintain precautionary cash balances to the firm's cost of external capital. When the marginal value of cash balances is less than the marginal cost of external capital, precautionary cash balances are zero. If the value of precautionary cash balances should exceed $\lambda_{10,t,s}$ for some values of $\hat{C}_{t,s}$, then the contractor will expand its holdings of precautionary cash balances up to the point at which the marginal value of additional cash balances at time t in state s, which is given by (121), just equals the firm's marginal cost of external capital, $\lambda_{10,t,s}$.

Corollary VII-14-1

If in any period t and state s the contractor will hold nonzero precautionary cash balances in equilibrium, then the marginal value of cash balances will equal the firm's marginal cost of external capital at that date and state.

Proof

It follows from (114) and (115) that $\hat{C}_{t,s} > 0$ implies equality in (122). Q.E.D.

Theorem VII-15

When the airframe builder modeled in (37) is in equilibrium, then for each period t and state s for which the optimal stock of precautionary cash balances is nonzero, the airframe builder will produce more of each commercial output, ${}_kQ_{t,s}$, than a short run profit maximizer would, i.e. $\frac{w_j}{\partial_k Q_{t,s}/\partial_k L^j{}_{t,s}} > \frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}}$, so that marginal production cost exceeds marginal revenue at optimality. For each period t and state s for which the optimal stock of precautionary cash balances is zero, the airframe builder will also produce more of each commercial output than a short run profit maximizer would, although in this case it is marginal production cost adjusted for the marginal impact of a change in output on transactions balance and working capital requirements that exceeds marginal revenue at optimality.

<u>Proof</u>

First note that it follows from lemma VII-1 and theorem VII-2 and from necessary condition (40) that

$$\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} > 0. \quad (123)$$

It is shown below that the use of (123), rather than lemma VII-2, leads to a more general result as it does not require the assumption that (120) holds at optimality.

To prove the first statement, it follows from (114) that

$$- \lambda_{10,t,s} + \sum_{s'=1}^{S} \lambda_{11,t,s,s'} - \sum_{s'=1}^{S} \lambda_{11,t+1,s',s} = 0 . \quad (124)$$

Substituting (124) into (83) yields

$$\frac{(1-\tau)w_j}{\partial_k Q_{t,s}/\partial_k L^j{}_{t,s}} = \frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} + \frac{\{\phi_{t,s} \frac{\partial U_1}{\partial R_{j,t,s}} (\frac{1}{1+r})^t\}(\partial R_{j,t,s}/\partial_k Q_{t,s})}{\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\}}. \quad (125)$$

It follows from (32) that

$$\frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} = (1 - \tau)\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} \ . \tag{126}$$

Substituting (126) into (125) and rearranging terms yields

$$\frac{w_j}{\partial_k Q_{t,s}/\partial_k L^j{}_{t,s}} = \frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}}(1 + \alpha) > \frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}} \ , \tag{127}$$

where

$$\alpha = (\frac{1}{1-\tau})\frac{\{\phi_{t,s}\ \frac{\partial U_1}{\partial R_{j,t,s}}(\frac{1}{1+r})^t\}}{\{\phi_{t,s}\ \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum\limits_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}} > 0 \tag{128}$$

follows from (123). (Note that (128) could have been obtained by invoking lemma VII-2, but at the cost of obtaining a weaker result). Thus the first statement has been proved.

To prove the second statement, it follows from (114) and (115) that

$$-\ \lambda_{10,t,s} + \sum_{s'=1}^{S}\lambda_{11,t,s,s'} - \sum_{s'=1}^{S}\lambda_{11,t+1,s',s} \leq 0 \ . \tag{129}$$

It follows from (18), (20), (123), (129), and the assumed concavity of U_1 that the third term on the right-hand side of (83) is negative. Rewrite (83) as

$$\frac{w_j}{\partial_k Q_{t,s}/\partial_k L^j{}_{t,s}} + \beta = \frac{\partial \pi_{t,s}}{\partial_k Q_{t,s}} + \frac{\{\phi_{t,s}\ \frac{\partial U_1}{\partial R_{j,t,s}}(\frac{1}{1+r})^t\}(\partial R_{j,t,s}/\partial_k Q_{t,s})}{\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum\limits_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}} , \tag{130}$$

where

$$\beta=\left(\frac{1}{1-\tau}\right)\frac{\{\lambda_{10,t,s}-[\sum_{s'=1}^{S}\lambda_{11,t,s,s'}-\sum_{s'=1}^{S}\lambda_{11,t+1,s',s}]\}}{\{\phi_{t,s}\frac{\partial U_1}{\partial\pi_{t,s}}\left(\frac{1}{1+r}\right)^{t}-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}}\times \left(\frac{\partial\overline{C}_{t,s}}{\partial_k Q_{t,s}}+\frac{\partial V_{t,s}}{\partial_k Q_{t,s}}\right)>0 \tag{131}$$

is interpretable as the adjustment to marginal production cost required to take into account the impact of a change in output on transactions balance and working capital requirements. Proceeding from (130) in the same manner as in the proof of the first statement leads to the inequality

$$\frac{w_j}{\partial_k Q_{t,s}/\partial_k L^j{}_{t,s}}+\beta=\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}}(1+\alpha)>\frac{\partial R_{j,t,s}}{\partial_k Q_{t,s}}, \tag{132}$$

where α is again given by (128). This proves the second statement. Q.E.D.

Corollary VII-15-1

The airframe builder modeled in (37) exhibits a revenue bias in the sense that commercial production is carried beyond the short run profit maximizing levels and the application of labor under government contracts is carried beyond short run profit maximizing levels.

Proof

Theorems VII-2 and VII-15. Q.E.D.

The importance of theorem VII-15 is twofold in nature. First, the theorem demonstrates that the airframe builder tends to produce more of each commercial good than a short run profit maximizer would - a result that is consistent with the mode of behavior exhibited by the firm modeled in chapters three, four, and five of this thesis. As indicated in corollary VII-15-1, this behavior is also consistent with behavior exhibited under government contracts.

Second, theorem VII-15 demonstrates the importance of transactions balance and working capital requirements on the airframe builder's commercial output decisions - a result similar to one obtained in chapter four of this thesis and also similar to results obtained from the Vickers model of the firm.[60] More importantly, (131) suggests the general form of the required adjustment to marginal production cost, and in particular, the importance of equity considerations ($\lambda_{9,t,s,s'}$) and balance sheet considerations ($\lambda_{10,t,s}$) as well as considerations as to the direct impact on cash flow ($\lambda_{11,t,s,s'}$ and $\lambda_{11,t+1,s',s}$). Together (130) and (131) indicate the interdependence of operating policies and financial policies.

Theorems VII-13 and VII-15 suggest that the representative airframe builder's commercial output decisions are not, in general, independent of its cash management policy. In addition, since $\lambda_{10,t,s}$ was shown in corollary VII-12-2 to be sensitive to the firm's choice of capital structure (at least under the conditions specified in the corollary), it follows from (83), (114), and (115) that when the optimal level of precautionary cash balances at time t and state s is zero, the representative airframe builder's commercial output decisions are also sensitive to changes in the firm's capital structure:

Corollary VII-15-2

When the optimal level of precautionary cash balances is zero, $\hat{C}_{t,s} = 0$, and when the conditions specified in corollary VII-12-2 are satisfied, the commercial output decisions of the representative airframe builder modeled in (37) are not independent of the firm's choice of capital structure.

4. Summary

This section has explored the implications of the model of the representative airframe builder developed in section B for the representative airframe builder's optimal operating policies and optimal financial policies.

The main results generated in this section include optimality rules for allocating labor to work under government contracts (theorem VII-1 plus corollaries) and to work on commercial production (theorem VII-3 plus corollaries); for allocating contractor-furnished capital to government contracts and commercial production (theorem VII-4 plus corollaries); for combining contractor-furnished capital and government-furnished capital under government contracts (theorem VII-5 plus corollaries); for allocating commercial production (theorem VII-6) and for determining effectiveness/performance levels under government contracts (theorem VII-8), each across dates and states; and for allocating labor to government contracts in accordance with effectiveness/performance goals (theorem VII-9). In addition, rules were developed for establishing the optimal levels of commercial outputs (theorem VII-7) and managerial emoluments (theorem VII-10). It was shown that the representative airframe builder modeled in (37) tends to allocate more labor to each government contract than a short run profit maximizer (theorem VII-2) and also tends to produce more of each of its commercial products than a short run profit maximizer (theorem VII-15), in each case with the intention of achieving a higher level of discounted expected collective utility by sacrificing a portion of maximum possible profit in order to increase total revenue. It was also shown that, when the airframe builder modeled in (37) is in equilibrium, the marginal cost of debt capital equals the marginal cost of external equity capital (theorem VII-11); that the expected marginal value of an additional unit of physical capital at each time t just equals the firm's expected marginal cost of financial capital at t, plus the expected cost (net of tax) implied by the increased depreciation expense at t, less the expected cash flow impact of changing capital goods prices (theorem VII-12); and that financial considerations can affect the airframe builder's commercial output decisions (theorems VII-13 and VII-15 and corollaries VII-12-2 and VII-15-2).

The results obtained in this section concerning the relationship between the representative airframe builder's operating (i.e. output and investment) decisions and its financial policy decisions deserve additional emphasis. It was shown that, in general, the representative airframe builder's investment decision is not independent of its choice of capital structure (corollary VII-12-2); that its commercial output decisions are not, in general, independent of its cash management policy (theorems VII-13 and VII-15);

and that its commercial output decisions are also not, in general, independent of its choice of capital structure (corollary VII-15-2). These results are important because they specify conditions under which the representative airframe builder's operating decisions are not separable from its financial decisions.

Since the model developed in section B was able to exclude progress payments by assuming that cash flow from the government to the contractor matched perfectly the recognition of revenue and fees by the contractor, the results just summarized are subject to that qualification. However, as demonstrated in the next section, the qualitative results obtained in this section are not materially affected when progress payments are incorporated into the model. What does happen is that financial restrictions on the airframe builder's behavior tend to become more severe, thereby affecting the airframe builder's operating policies to an extent greater than that implied in this section.

D. THE POLICY IMPLICATIONS OF PROGRESS PAYMENTS

In the preceding section the optimal operating and financial policies of the representative airframe builder were derived for the special case in which separate contracts are awarded on an annual basis so that progress payments are unnecessary. Under such a procurement policy contractors would be compensated in full for all allowable costs and paid an annual fee, so that the flow of cash from the government to the contractor would match perfectly the recognition of revenue and profit by the contractor.

In this section the model of a representative airframe builder developed in section B is modified to incorporate progress payments. This second version of the model is the more 'realistic' version in that it is more consistent with current government procurement policy. By comparing the optimal operating and financial policies implied by the two versions of the model, conclusions can be drawn concerning the impact on contractor behavior of the policy of granting progress payments to cover some fraction of allowable costs. This is done later in this section.

Progress payments, which are payments to the contractor for work in process and which are calculated as a percentage of costs already incurred, serve an important financial function. Progress payments serve as an important source of short term finance, much like an interest-free loan, without which the contractor would have to increase bank borrowing or issue additional bonds or shares. To incorporate progress payments into the model developed in section B it is assumed that during the course of a production contract the airframe builder is compensated on an annual basis for a proportion ρ of its allowable costs,[61] but the fee is not paid until the entire contract has been completed. The focal point of this section is the imperfect matching of cash inflows and cash outflows under government contracts caused by the government's progress payments policy.

The first subsection modifies the model developed in section B to incorporate progress payments. The second subsection derives the representative airframe builder's optimal operating and financial policies, as implied by the modified model. The third subsection summarizes the impact on the behavior of the representative airframe builder of the current progress payments policy.

1. The Airframe Builder Model with Progress Payments

The introduction of progress payments into the model developed in section B requires several basic modifications of that model. First, progress payments cover only a portion, $0 < \rho < 1$, of allowable costs. Second, the fee is paid only upon completion of the entire contract. Thus, while the contractor typically recognizes the full amount of allowable costs plus a pro rata portion of the fee as annual revenue under ongoing production contracts, the amount of cash received from the government in the form of progress payments is usually a significantly smaller amount, with the differences accruing in accounts receivable until the entire contract has been completed.

The introduction of progress payments into the model requires that greater attention be paid to the time phasing of the various production contracts. In this section it is assumed that all production contracts are of the multiyear variety. For convenience it is assumed that contracts on which the firm intends to bid during the planning period will not (or at least are not expected to) terminate until after the planning horizon t = T. At time zero there are one or more production contracts in force, each with exogenously determined total contract target cost ${}_cC^*_o$, target proportionate fee, ${}_c\alpha_o$, and sharing ratio, ${}_c\beta_o$. For each time t and for each new production contract on which the firm intends to bid at time t there is a target cost, ${}_cC^*_{t,s}$, which is interpreted as in section B as the maximum bid that will secure for the firm contract c in state of nature s (where the states of nature take into account the bids of the firm's competitors). In addition, there is a proportionate target fee ${}_c\alpha_{t,s}$ and a sharing ratio ${}_c\beta_{t,s}$. That is, the values of the production contract parameters ${}_c\alpha_{t,s}$ and ${}_c\beta_{t,s}$ are determined either as of time t = 0 or at the time of bidding, rather than annually as in section B.

Research and development contracts are treated in this section in the same manner in which they were handled in section B. For these contracts total revenue earned is again given by (10) and the fee net of disallowed costs is again given by (11). That is, it is assumed that all research and development contracts are written on an annual basis. Therefore the introduction of progress payments affects only the treatment of production contracts.

Because of progress payments the expression (12) for total revenue earned under a production contract must be modified. In particular, the year of completion must be clearly distinguished from the earlier years of the contract. For an arbitrarily selected pretermination year of a production contract, the contractor recognizes as revenue the amount of allowable costs plus a pro rata portion of the anticipated total contract fee, ${}_c\alpha_o \cdot {}_cC^*_o$ for contracts that were ongoing at t = 0 or ${}_c\alpha_{t,s} \cdot {}_cC^*_{t,s}$ for contracts secured at time t > 0. The pro rata portion is assumed to be figured as the ratio of physical output in the current year, ${}_cQ_{t,s}$, to the total volume of physical output specified in the contract, ${}_cQ^*_o$ for contracts that were ongoing at t = 0 or ${}_cQ^*_{t,s}$ for contracts secured at time t > 0. For a pretermination year total revenue is given by

$$ {}_cR_{t,s} = {}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s} + [{}_cQ_{t,s}/{}_cQ^*_{t,s}]\cdot {}_c\alpha_{t,s}\cdot {}_cC^*_{t,s} , \qquad (133) $$

where ${}_cQ^*_{t,s} \equiv {}_cQ^*_o$, ${}_c\alpha_{t,s} \equiv {}_c\alpha_o$, and ${}_cC^*_{t,s} = {}_cC^*_o$ all hold for contracts that are ongoing at t = 0.

For the termination year (133) needs to be adjusted. Up to and including the termination year the contractor has recognized the total anticipated fee as revenue since

$$ \Sigma[{}_cQ_{t,s}/{}_cQ^*_{t,s}]\cdot {}_c\alpha_{t,s}\cdot {}_cC^*_{t,s} = \frac{{}_c\alpha_{t,s}\cdot {}_cC^*_{t,s}}{{}_cQ^*_{t,s}}\Sigma {}_cQ_{t,s} \equiv {}_c\alpha_{t,s}\cdot {}_cC^*_{t,s} , $$

where the two sums are taken over the years of the contract. But if the accumulated sum of allowable costs deviates from contract target cost, ${}_cC^*_{t,s}$, then the actual fee will deviate from the target fee. Let ${}_c\hat{C}_{t,s}$ denote the accumulated allowable costs at time t in state of nature s, so that

$$ {}_c\hat{C}_{t,s} = {}_c\hat{C}_{t-1,s'} + {}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s} , \qquad (134) $$

where ${}_c\hat{C}_o$ is exogenously determined for those contracts that are ongoing at t = 0. Then the actual fee paid by the government is ${}_c\alpha_{t,s}\cdot {}_cC^*_{t,s} + {}_c\beta_{t,s}({}_cC^*_{t,s} - {}_c\hat{C}_{t,s})$ and total revenue recognized in the termination year is given by

$$ \begin{aligned} {}_cR_{t,s} = {} & {}_c\gamma_{t,s}({}_cC_{t,s})\cdot {}_cC_{t,s} + [{}_cQ_{t,s}/{}_cQ^*_{t,s}]\cdot {}_c\alpha_{t,s}\cdot {}_cC^*_{t,s} \\ & + {}_c\beta_{t,s}({}_cC^*_{t,s} - {}_c\hat{C}_{t,s}) , \end{aligned} \qquad (135) $$

where ${}_cQ^*_{t,s}$ and ${}_c\alpha_{t,s}$ are interpreted as in (133) and where the third term on the right represents the incentive fee adjustment.

Next (133) and (135) are used to determine expressions for $R_{GA,t,s}$ and $R_{GO,t,s}$. In section B the expressions (15) and (16) were determined simply by summing over the contracts in force at time t in state s. But (133) and (135) require that contracts also be distinguished by termination date. Let $C^-_{i,t,s}$ denote the contracts of type i that terminate at time t in state s and let $C^+_{i,t,s}$ denote the contracts of type i that in state s remain ongoing after time t. Note that for $C_{i,t,s}$ defined in section B,

$$C_{i,t,s} = C^-_{i,t,s} \cup C^+_{i,t,s}, \text{ with } C^-_{i,t,s} \cap C^+_{i,t,s} = \phi \ .$$

Also, by definition, $C_{1,t,s} \equiv C^-_{1,t,s}$. With these definitions (15) and (16) become

$$\begin{aligned} R_{GA,t,s} = & \sum_{C_{1,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + {}_c\overline{\pi}_{t,s}] \\ & + \sum_{C^+_{2,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + ({}_cQ_{t,s}/{}_cQ^*{}_{t,s}) \cdot {}_c\alpha_{t,s} \cdot {}_cC^*{}_{t,s}] \\ & + \sum_{C^-_{2,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + ({}_cQ_{t,s}/{}_cQ^*{}_o) \cdot {}_c\alpha_o \cdot {}_cC^*{}_o \\ & \qquad + {}_c\beta_o({}_cC^*{}_o - {}_c\hat{C}_{t,s})] \end{aligned} \tag{136}$$

and

$$\begin{aligned} R_{GO,t,s} = & \sum_{C^+_{3,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + ({}_cQ_{t,s}/{}_cQ^*{}_{t,s}) \cdot {}_c\alpha_{t,s} \cdot {}_cC^*{}_{t,s}] \\ & + \sum_{C^-_{3,t,s}} [{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + ({}_cQ_{t,s}/{}_cQ^*{}_o) \cdot {}_c\alpha_o \cdot {}_cC^*{}_o \\ & \qquad + {}_c\beta_o({}_cC^*{}_0 - {}_c\hat{C}_{t,s})] \end{aligned} \tag{137}$$

where ${}_cQ^*{}_{t,s}$ and ${}_c\alpha_{t,s}$ are interpreted as in (133) for contracts that were ongoing at t = 0 and that are still ongoing at

$t > 0$. The expressions (136) and (137) together with expressions (14) constitute the four revenue arguments of the objective functional of the modified model.

The modifications (136) and (137) to the expressions for $R_{GA,t,s}$ and $R_{GO,t,s}$, respectively, suggest the intertemporal nature of policy decisions involving the allocation of labor and contractor-furnished capital to government contracts. There is an impact in the current period since the current period's revenue is affected. There is also an impact on the total allowable costs under the contract, which is felt in the period the contract terminates. To allow in the model for this future period effect as it concerns contracts that terminate after the planning horizon, $t = T$, the utility function

$$U_3([{}_c\hat{C}_{T,s}]_{C^+_{2,T,s}} ; [{}_c\hat{C}_{T,s}]_{C^+_{3,T,s}}) \qquad (138)$$

is introduced into the objective functional of the modified model.[62]

The only other changes in the model (37) required to accommodate progress payments are modifications of the balance sheet identity (37r) and of the expression for the change in the stock of cash (37s). The introduction of progress payments necessitates the recognition of accounts receivable on the left-hand side of the firm's balance sheet. Let $\bar{R}_{t,s}$ denote the amount of accounts receivable at time t in state of nature s. The new balance sheet is shown in table VII-5, which is just table VII-1 with accounts receivable inserted on the left-hand side. The balance sheet identity becomes:

$$\bar{C}_{t,s}([{}_cQ_{t,s}],{}_RQ_{t,s},{}_NQ_{t,s})+\hat{C}_{t,s}+\bar{R}_{t,s}+q_{t,s}\,\bar{K}^C_{t,s}$$
$$+V_{t,s}([{}_cQ_{t,s}],{}_RQ_{t,s},{}_NQ_{t,s}) = B_{t,s} + E_{t,s}\ . \tag{139}$$

Table VII-5 Representative Airframe Builder's Balance Sheet Including Accounts Receivable

Assets		Liabilities	
Cash	$C_{t,s}$	Debt	$B_{t,s}$
Accts Rec	$\bar{R}_{t,s}$	Equity	$E_{t,s}$
Inventories	$V_{t,s}$		
Fixed Assets	$q_{t,s}\,\bar{K}^C_{t,s}$		
Total Assets	$C_{t,s}+\bar{R}_{t,s}+V_{t,s}+q_{t,s}\,\bar{K}^C_{t,s}$	Total Liabilities and Stockholders' Equity	$B_{t,s}+E_{t,s}$

For convenience it is assumed that revenue earned on sales of commercial products is realized in cash payments during the period in which the revenue is recognized in the income statement. Given the earlier assumption that all research and development contracts have a duration of one year, accounts receivable reflect allowable costs in excess of progress payments plus the accumulated unpaid pro rata portions of anticipated fees under ongoing government production contracts. Accounts receivable tend to increase due to ongoing contracts and tend to decrease due to completed contracts. Thus $\bar{R}_{t,s}$ in (139) satisfies the difference equation,

$$\bar{R}_{t,s}=\bar{R}_{t-1,s}+\sum_{C^+_{2,t,s}}\{(1-\rho)\,{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+({}_cQ_{t,s}/{}_cQ^*_{t,s})\times$$

$${}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}\}+\sum_{C^+_{3,t,s}}\{(1-\rho)\,{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$

$$+({}_cQ_{t,s}/{}_cQ^*_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}\}-\sum_{C^-_{2,t,s}}\{(1-\rho)\,{}_c\hat{C}_{t,s} \qquad (140)$$

$$+{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}\}-\sum_{C^-_{3,t,s}}\{(1-\rho)\,{}_c\hat{C}_{t,s}+{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}\}\ ,$$

where $\hat{C}$ given by (134) represents accumulated allowable costs. According to (140) accounts receivable at time t in state of nature s are equal to their value in the previous period plus increases associated with ongoing aerospace and non-aerospace production contracts less amounts associated with aerospace and non-aerospace production contracts terminating in year t and state of nature s.

The final modification of (37) required by the introduction of progress payments is the development of a new expression for the stock of cash, $C_{t,s}$, that takes into account the imperfect matching of cost and revenue flows under government contracts. The representative air-frame builder's statement of sources and uses of cash modified to take into account progress payments is provided in table VII-6. The main difference between table VII-6 and table VII-3 lies in the treatment of revenue earned on sales to the government, only a portion of which is realized as cash receipts (i.e. progress payments) under ongoing contracts. In both tables it is assumed that expenses are paid in cash in the period in which they are incurred. As a consequence, total cash generated by operations will tend to be more volatile in table VII-6 than in table VII-3

Table VII-6 Representative Airframe Builder's Statement of Sources and Uses of Cash Allowing for Progress Payments

Sources of cash:
From operations:
Sales revenue

Commercial $R_{CA,t,s}+R_{CO,t,s}$

Government $\sum_{C_{1,t,s}} \{{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+{}_c\bar{\pi}_{t,s}\}+\sum_{C^+_{2,t,s}\cup C^+_{3,t,s}} \rho\cdot{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$

$+\sum_{C^-_{2,t,s}\cup C^-_{3,t,s}} \{\rho\,{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+(1-\rho)\,{}_c\hat{C}_{t,s}+{}_c\alpha_o\;{}_cC^*_o$

$+{}_c\beta_o[{}_cC^*_o - {}_c\hat{C}_{t,s}]\}$

Total expenses and taxes $(1-\tau)\{\sum_j w_j L^j_{t,s}+M_{t,s}+q_{t,s}\cdot\delta\cdot\bar{K}^C_{t,s}+i_{t,s}\cdot B_{t,s}\}+\tau\sum_i R_{i,t,s}$

Adjustment for noncash outlay $q_{t,s}\cdot\delta\cdot\bar{K}^C_{t,s}$

Cash outflow for expenses $(1-\tau)\{\sum_j w_j L^j_{t,s}+M_{t,s}+i_{t,s}\cdot B_{t,s}\}+\tau\{\sum_i R_{i,t,s}-q_{t,s}\cdot\delta\cdot\bar{K}^C_{t,s}\}$

$$+\tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^{C}_{t,s}-\tau(R_{GA,t,s}+R_{GOt,s,})$$

$$+C^{\Sigma}_{1,t,s}\{{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}+{}_{c}\overline{\pi}_{t,s}\}$$

$$+\sum_{C^{+}_{2,t,s}\cup C^{+}_{3,t,s}}\rho\cdot{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}$$

$$+\sum_{C^{-}_{2,t,s}\cup C^{-}_{3,t,s}}\{\rho\cdot{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}$$

$$+(1-\rho){}_{c}\hat{C}_{t,s}+{}_{c}\alpha_{o}\cdot{}_{c}C^{*}_{o}+{}_{c}\beta_{o}[{}_{c}C^{*}_{o}-{}_{c}\hat{C}_{t,s}]\}$$

From other sources:

New debt issues $\quad Y_{t,s}$

New equity issues $\quad Z_{t,s}\cdot v_{t,s}$

Total cash generated

$$(1-\tau)\{R_{CA,t,s}+R_{CO,t,s}-\sum_{j}w_{j}L^{j}_{t,s}-M_{t,s}-i_{t,s}\cdot B_{t,s}\}+\tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^{C}_{t,s}$$

$$-\tau(R_{GA,t,s}+R_{GO,t,s})+C^{\Sigma}_{1,t,s}\{{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}+{}_{c}\overline{\pi}_{t,s}\}$$

$$+\sum_{C^{+}_{2,t,s}\cup C^{+}_{3,t,s}}\rho\cdot{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}$$

$$+\sum_{C^{-}_{2,t,s}\cup C^{-}_{3,t,s}}\{\rho\cdot{}_{c}\gamma_{t,s}({}_{c}C_{t,s})\cdot{}_{c}C_{t,s}+(1-\rho){}_{c}\hat{C}_{t,s}+{}_{c}\alpha_{o}\cdot{}_{c}C^{*}_{o}$$

$$+{}_{c}\beta_{o}[{}_{c}C^{*}_{o}-{}_{c}\hat{C}_{t,s}]\}+Y_{t,s}+Z_{t,s}\cdot v_{t,s}$$

Table VII-6 (concd)

Uses of cash:	
To pay dividends	$d \cdot n_{t,s}$
To purchase capital goods	$q_{t,s} \cdot I_{t,s}$
To increase inventories	$V_{t,s} - V_{t-1,s}$'
Total cash applied	$d \cdot n_{t,s} + q_{t,s} \cdot I_{t,s} + V_{t,s} - V_{t-1,s}$'

Increase (decrease) in stock of cash

$$C_{t,s} - C_{t-1,s}' \equiv (1-\tau)\{R_{CA,t,s} + R_{CO,t,s} - \sum_j w_j L^j{}_{t,s} - M_{t,s}$$

$$-i_{t,s} \cdot B_{t,s}\} + \tau \cdot q_{t,s} \cdot \delta \cdot \overline{K}^C{}_{t,s} - \tau(R_{GA,t,s} + R_{GO,t,s})$$

$$+ C^{\Sigma}_{1,t,s}\{{}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + \overline{\pi}_{t,s}\}$$

$$+ C^{+}_{2,t,s} \overset{\Sigma}{\cup} C^{+}_{3,t,s} \; \rho \cdot {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s}$$

$$+ C^{-}_{2,t,s} \overset{\Sigma}{\cup} C^{-}_{3,t,s} \; \{\rho \cdot {}_c\gamma_{t,s}({}_cC_{t,s}) \cdot {}_cC_{t,s} + (1-\rho)\, {}_c\hat{C}_{t,s}$$

$$+ {}_c\alpha_o \; {}_cC^*{}_o + {}_c\beta_o[{}_cC^*{}_o - {}_c\hat{C}_{t,s}]\} + Y_{t,s} + Z_{t,s} \cdot v_{t,s} - d \cdot n_{t,s}$$

$$-q_{t,s} \cdot I_{t,s} - V_{t,s} + V_{t-1,s}'$$

depending on the time phasing of government production contracts. The desired expression for the stock of cash at time t in state of nature s is obtained by rewriting the expression for the increase (decrease) in the stock of cash provided in table VII-6.

It should be noted that the stock of cash at time t in state of nature s, $C_{t,s}$, can be reexpressed in terms of net income at time t in state of nature s, $\pi_{t,s}$, which will prove helpful in modifying (38). From (32), (137), and (139) it follows that

$$\pi_{t,s}=(1-\tau)\{\sum_i R_{i,t,s}-\sum_j w_j L^j_{t,s}-M_{t,s}-q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}-i_{t,s}B_{t,s}\}$$

$$=(1-\tau)\{R_{CA,t,s}+R_{CO,t,s}-\sum_j w_j L^j_{t,s}-M_{t,s}-i_{t,s}B_{t,s}\}$$

$$+\tau\cdot q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}-\tau(R_{GA,t,s}+R_{GO,t,s})-q_{t,s}\delta\cdot\overline{K}^C_{t,s}$$

$$+\sum_{C_{1,t,s}}\{{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+{}_c\overline{\pi}_{t,s}\}$$

$$+\sum_{C^+_{2,t,s}\cup C^+_{3,t,s}}\rho\cdot{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$

$$+\sum_{C^-_{2,t,s}\cup C^-_{3,t,s}}\{\rho\cdot{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+{}_c\beta_o({}_cC^*_o-{}_c\hat{C}_{t,s})\}$$

$$+\sum_{C^+_{2,t,s}\cup C^+_{3,t,s}}\{(1-\rho){}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$

$$+({}_cQ_{t,s}/{}_cQ^*_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}\}$$

$$+\sum_{C^-_{2,t,s}\cup C^-_{3,t,s}}\{(1-\rho){}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$

$$+({}_cQ_{t,s}/{}_cQ^*_o)\cdot{}_c\alpha_o\cdot{}_cC^*_o\}$$

and hence that

$$C_{t,s}=C_{t-1,s'}+\pi_{t,s}+q_{t,s}\cdot\delta\cdot\overline{K}^C_{t,s}-\sum_{C^+_{2,t,s}\cup C^+_{3,t,s}}\{(1-\rho){}_c\gamma_{t,s}({}_cC_{t,s})\times \qquad (141)$$

$${}_cC_{t,s}+({}_cQ_{t,s}/{}_cQ^*_{t,s})\cdot{}_c\alpha_{t,s}\;{}_cC^*_{t,s}\}$$

$$+\sum_{C^-_{2,t,s}\cup C^-_{3,t,s}}\{(1-\rho)[{}_c\hat{C}_{t,s}-{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}]$$

$$+{}_c\alpha_o\cdot{}_cC^*_o[1-{}_cQ_{t,s}/{}_cQ^*_o]\}+Y_{t,s}+Z_{t,s}\cdot v_{t,s}-d\cdot n_{t,s}-q_{t,s}\times$$

$$I_{t,s}-V_{t,s}+V_{t-1,s}\,.$$

Using (134) and (137)-(141) to modify (37), the model of the representative military airframe builder adjusted to account for progress payments can be formulated as the following stochastic optimal control problem:

$$\begin{aligned}
\text{maximize }\{&\sum_{t=1}^{T}E_s[U_1(R_{GA,t,s};R_{GO,t,s};R_{CA,t,s};R_{CO,t,s};\pi_{t,s};\\
&[{}_c\hat{E}_{t,s}];M_{t,s})](\tfrac{1}{1+r})^t+E_s[U_2(\overline{K}^C_{T,s},\hat{B}_{T,s})](\tfrac{1}{1+r})^T\\
&+E_s[U_3([{}_c\hat{C}_{T,s}]_{C^+_{2,T,s}};[{}_c\hat{C}_{T,s}]_{C^+_{3,T,s}})](\tfrac{1}{1+r})^T
\end{aligned} \tag{142}$$

$$\begin{aligned}
\text{subject to } R_{GA,t,s}=&\sum_{C_{1,t,s}}[{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+{}_c\overline{\pi}_{t,s}]\\
&+\sum_{C^+_{2,t,s}}[{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}\\
&+({}_cQ_{t,s}/{}_cQ^*_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}]\\
&+\sum_{C^-_{2,t,s}}[{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+({}_cQ_{t,s}/{}_cQ^*_o)\times\\
&{}_c\alpha_o\cdot{}_cC^*_o+{}_c\beta_o({}_cC^*_o-{}_c\hat{C}_{t,s})]\\
R_{GO,t,s}=&\sum_{C^+_{3,t,s}}[{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}\\
&+({}_cQ_{t,s}/{}_cQ^*_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*_{t,s}]\\
&+\sum_{C^-_{3,t,s}}[{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}+({}_cQ_{t,s}/{}_cQ^*_o)\times\\
&{}_c\alpha_o\cdot{}_cC^*_o+{}_c\beta_o({}_cC^*_o-{}_c\hat{C}_{t,s})]
\end{aligned}$$

$$
{}_cC_{t,s}=w_M\cdot{}_cL^{M}_{t,s}+w_A\cdot{}_cL^{A}_{t,s}+w_E\cdot{}_cL^{E}_{t,s}+(\bar{i}_{t,s}+\delta)\times
q_{t,s}\cdot{}_cK^{C}_{t,s}
$$

$$
{}_c\hat{C}_{t,s}={}_c\hat{C}_{t-1,s'}+{}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}
$$

$$
R_{CA,t,s}({}_RQ_{t,s})={}_RQ_{t,s}\cdot{}_Rp_{t,s}({}_RQ_{t,s})
$$

$$
R_{CO,t,s}({}_NQ_{t,s})={}_NQ_{t,s}\cdot{}_Np_{t,s}({}_NQ_{t,s})
$$

$$
\begin{aligned}
\pi_{t,s}=(1-\tau)[&R_{GA,t,s}+R_{GO,t,s}+R_{CA,t,s}+R_{CO,t,s}\\
&-w_M\{U\{C^{\Sigma}_{i,t,s}\}{}_cL^{M}_{t,s}+{}_RL^{M}_{t,s}+{}_NL^{M}_{t,s}\}\\
&-w_A\{U\{C^{\Sigma}_{i,t,s}\}{}_cL^{A}_{t,s}+{}_RL^{A}_{t,s}+{}_NL^{A}_{t,s}\}\\
&-w_E\{U\{C^{\Sigma}_{i,t,s}\}{}_cL^{E}_{t,s}+{}_RL^{E}_{t,s}+{}_NL^{E}_{t,s}\}\\
&-M_{t,s}-q_{t,s}\cdot\delta\cdot\bar{K}^{C}_{t,s}-i_{t,s}(B_{t,s},Y_{t,s})\cdot B_{t,s}]
\end{aligned}
$$

$$
\begin{aligned}
\hat{B}_{T,s}=C_{2,T,s}\overset{\Sigma}{\cup}C_{3,T,s}&({}_cR_{T,s}/{}_cQ_{T,s})\times\\
&{}_c\hat{B}_{T,s}[{}_cR_{T,s}/{}_cQ_{T,s},{}_c\hat{E}_{T,s}]
\end{aligned}
$$

$$
\bar{K}^{C}_{t,s}=\left(\frac{1}{1+\delta}\right)\bar{K}^{C}_{t-1,s'}+\left(\frac{1}{1+\delta}\right)I_{t,s}
$$

$$
\begin{aligned}
F_c([{}_cQ_{t,s}];[{}_c\hat{E}_{t,s}];[{}_cL^{M}_{t,s}];[{}_cL^{A}_{t,s}];[{}_cL^{E}_{t,s}];&\\
[{}_cK^{C}_{t,s}];[{}_cK^{G}_{t,s}]) = 0&
\end{aligned}
$$

$$
F_R([{}_RQ_{t,s}];[{}_RL^{M}_{t,s}];[{}_RL^{A}_{t,s}];[{}_RL^{E}_{t,s}];[{}_RK^{C}_{t,s}]) = 0
$$

$$
F_N([{}_NQ_{t,s}];[{}_NL^{M}_{t,s}];[{}_NL^{A}_{t,s}];[{}_NL^{E}_{t,s}];[{}_NK^{C}_{t,s}]) = 0
$$

$$
U\{C^{\Sigma}_{i,t,s}\}{}_cK^{C}_{t,s}+{}_RK^{C}_{t,s}+{}_NK^{C}_{t,s}\le\bar{K}^{C}_{t,s}
$$

$$U\{C^{\Sigma}_{i,t,s}\}{}_cK^G{}_{t,s} \le \overline{K}^G{}_{t,s}$$

$$B_{t,s}=B_{t-1,s'}+Y_{t,s}$$

$$n_{t,s}=n_{t-1,s'}+Z_{t,s}$$

$$E_{t,s}=E_{t-1,s'}+Z_{t,s}\cdot v_{t,s}+\pi_{t,s}-d\cdot n_{t,s}$$

$$\overline{C}_{t,s}([{}_cQ_{t,s}],{}_RQ_{t,s},{}_NQ_{t,s})+\hat{C}_{t,s}+\overline{R}_{t,s}+q_{t,s}\cdot\overline{K}^C{}_{t,s}$$
$$+V_{t,s}([{}_cQ_{t,s}],{}_RQ_{t,s},{}_NQ_{t,s})=B_{t,s}+E_{t,s}$$

$$\overline{R}_{t,s}=\overline{R}_{t-1,s'}+\sum_{C^+_{2,t,s}}\{(1-\rho)_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$
$$+({}_cQ_{t,s}/{}_cQ^*{}_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*{}_{t,s}\}$$
$$+\sum_{C^+_{3,t,s}}\{(1-\rho)_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$
$$+({}_cQ_{t,s}/{}_cQ^*{}_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*{}_{t,s}\}$$
$$-\sum_{C^-_{2,t,s}}\{(1-\rho)_c\hat{C}_{t,s}+{}_c\alpha_{t,s}\cdot{}_cC^*{}_{t,s}\}$$
$$-\sum_{C^-_{3,t,s}}\{(1-\rho)_c\hat{C}_{t,s}+{}_c\alpha_{t,s}\cdot{}_cC^*{}_{t,s}\}\ ,$$

$$C_{t,s}=C_{t-1,s'}+\pi_{t,s}+q_{t,s}\cdot\delta\cdot\overline{K}^C{}_{t,s}$$
$$-\sum_{C^+_{2,t,s}\cup C^+_{3,t,s}}\{(1-\rho)_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s}$$
$$+({}_cQ_{t,s}/{}_cQ^*{}_{t,s})\cdot{}_c\alpha_{t,s}\cdot{}_cC^*{}_{t,s}\}$$
$$+\sum_{C^-_{2,t,s}\cup C^-_{3,t,s}}\{(1-\rho)\cdot[{}_c\hat{C}_{t,s}-{}_c\gamma_{t,s}({}_cC_{t,s})\times$$
$${}_cC_{t,s}]+{}_c\alpha_o\cdot{}_cC^*{}_o[1-{}_cQ_{t,s}/{}_cQ^*{}_o]\}+Y_{t,s}+Z_{t,s}\times$$
$$v_{t,s}-d\cdot n_{t,s}-q_{t,s}\cdot I_{t,s}-V_{t,s}+V_{t-1,s'}$$

along with the same nonnegativity constraints as in (37).

The boundary conditions on the first order difference equations are analogous to those in chapter four and so are left unstated. The decision variables and the exogenous variables are again those listed in table VII-4, with the addition of the government procurement policy parameter ρ. The reader is reminded that a complete list of the symbols used in this chapter is provided in the appendix at the end of this chapter.

2. Optimal Operating and Financial Policies

The purpose of this subsection is to characterize the optimal operating and financial policies of the representative airframe builder, as implied by the model (142). Since (142) was developed by modifying (37) to incorporate progress payments, this characterization is carried out by modifying appropriately the necessary conditions discussed in section C. In the next subsection the impact of progress payments is evaluated by summarizing the analytical results presented in this subsection.

To obtain the characterization of the optimal solution to (142) define the following generalized Lagrangian:

$$
\begin{aligned}
L_\lambda = \sum_{t=1}^{T} \sum_{s=1}^{S} \phi_{t,s} \cdot U_1[& R_{GA,t,s}({}_cL^M_{t,s}, {}_cL^A_{t,s}, {}_cL^E_{t,s}, {}_cK^C_{t,s}, \qquad (143) \\
& [{}_c\hat{C}_{t,s}]_{\bar{C}_{2,t,s}}); R_{GO,t,s}({}_cL^M_{t,s}, {}_cL^A_{t,s}, {}_cL^E_{t,s}, {}_cK^C_{t,s}, \\
& [{}_c\hat{C}_{t,s}]_{\bar{C}_{3,t,s}}); R_{CA,t,s}({}_RQ_{t,s}); R_{CO,t,s}({}_NQ_{t,s}); \pi_{t,s}({}_cL^M_{t,s}, \\
& {}_cL^A_{t,s}, {}_cL^E_{t,s}, {}_RL^M_{t,s}, {}_RL^A_{t,s}, {}_RL^E_{t,s}, {}_NL^M_{t,s}, {}_NL^A_{t,s}, \\
& {}_NL^E_{t,s}, {}_cK^C_{t,s}, [{}_c\hat{C}_{t,s}]_{\bar{C}_{2,t,s}}, [{}_c\hat{C}_{t,s}]_{\bar{C}_{3,t,s}}, {}_RQ_{t,s},
\end{aligned}
$$

$$ {}_{N}Q_{t,s}, M_{t,s}, \overline{K}^{C}_{t,s}, B_{t,s}, Y_{t,s}); [{}_{c}\hat{E}_{t,s}]; M_{t,s}](\frac{1}{1+r})^{t} $$

$$ + \sum_{s=1}^{S} \phi_{T,s} \cdot U_{2}[K_{T,s}; \hat{B}_{T,s}({}_{c}L^{M}_{T,s}, {}_{c}L^{A}_{T,s}, {}_{c}L^{E}_{T,s}, {}_{c}K^{C}_{T,s}, $$

$$ {}_{c}\hat{E}_{T,s})](\frac{1}{1+r})^{T} $$

$$ + \sum_{s=1}^{S} \phi_{T,s} \, U_{3}([{}_{c}\hat{C}_{T,s}]_{C^{+}_{2,T,s}} ; [{}_{c}\hat{C}_{T,s}]_{C^{+}_{3,T,s}})(\frac{1}{1+r})^{T} $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{1,t,s,s'}[\overline{K}^{C}_{t,s} - (\frac{1}{1+\delta})\overline{K}^{C}_{t-1,s'} - (\frac{1}{1-\delta}) I_{t,s}] $$

$$ + \sum_{c=1}^{C_{1}+C_{2}+C_{3}} \lambda_{2,c}\{F_{c}([{}_{c}Q_{t,s}]; [{}_{c}\hat{E}_{t,s}]; [{}_{c}L^{M}_{t,s}]; [{}_{c}L^{A}_{t,s}]; $$

$$ [{}_{c}L^{E}_{t,s}]; [{}_{c}K^{C}_{t,s}]; [{}_{c}K^{G}_{t,s}])\} $$

$$ +\lambda_{3}\{F_{R}([{}_{R}Q_{t,s}]; [{}_{R}L^{M}_{t,s}]; [{}_{R}L^{A}_{t,s}]; [{}_{R}L^{E}_{t,s}]; [{}_{R}K^{C}_{t,s}])\} $$

$$ +\lambda_{4}\{F_{N}([{}_{N}Q_{t,s}]; [{}_{N}L^{M}_{t,s}]; [{}_{N}L^{A}_{t,s}]; [{}_{N}L^{E}_{t,s}]; [{}_{N}K^{C}_{t,s}])\} $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \lambda_{5,t,s}[\overline{K}^{C}_{t,s} - \sum_{U\{C_{i,t,s}\}} {}_{c}K^{C}_{t,s} - {}_{R}K^{C}_{t,s} - {}_{N}K^{C}_{t,s}] $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \lambda_{6,t,s}[\overline{K}^{G}_{t,s} - \sum_{U\{C_{i,t,s}\}} {}_{c}K^{G}_{t,s}] $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{7,t,s,s'}[B_{t,s} - B_{t-1,s'} - Y_{t,s}] $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{8,t,s,s'}[n_{t,s} - n_{t-1,s'} - Z_{t,s}] $$

$$ + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{s'=1}^{S} \lambda_{9,t,s,s'}[E_{t,s} - E_{t-1,s'} - Z_{t,s} \cdot v_{t,s} - \pi_{t,s}({}_{c}L^{M}_{t,s}, $$

$$ {}_{c}L^{A}_{t,s}, {}_{c}L^{E}_{t,s}, {}_{R}L^{M}_{t,s}, {}_{R}L^{A}_{t,s}, {}_{R}L^{E}_{t,s}, {}_{N}L^{M}_{t,s}, {}_{N}L^{A}_{t,s}, $$

$$
{}_{N}L^{E}_{t,s},\ {}_{c}K^{C}_{t,s},\ [{}_{c}\hat{C}_{t,s}]_{C^{-}_{2,t,s}},\ [{}_{c}\hat{C}_{t,s}]_{C^{-}_{3,t,s}},
$$

$$
{}_{R}Q_{t,s},\ {}_{N}Q_{t,s},\ M_{t,s},\ \bar{K}^{C}_{t,s},\ B_{t,s},\ Y_{t,s})+d\cdot n_{t,s}]
$$

$$
+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{10,t,s}[B_{t,s}+E_{t,s}-\bar{C}_{t,s}({}_{R}Q_{t,s},{}_{N}Q_{t,s})-\hat{C}_{t,s}
$$

$$
-\bar{R}_{t,s}-q_{t,s}\bar{K}^{C}_{t,s}-V_{t,s}({}_{R}Q_{t,s},{}_{N}Q_{t,s})]
$$

$$
+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{11,t,s,s'}[\bar{C}_{t,s}({}_{R}Q_{t,s},{}_{N}Q_{t,s})+\hat{C}_{t,s}
$$

$$
-\bar{C}_{t-1,s'}({}_{R}Q_{t-1,s'},{}_{N}Q_{t-1,s'})-\hat{C}_{t-1,s'}-\pi_{t,s}({}_{c}L^{M}_{t,s},
$$

$$
{}_{c}L^{A}_{t,s},{}_{c}L^{E}_{t,s},{}_{R}L^{M}_{t,s},{}_{R}L^{A}_{t,s},{}_{R}L^{E}_{t,s},{}_{N}L^{M}_{t,s},{}_{N}L^{A}_{t,s}
$$

$$
{}_{N}L^{E}_{t,s},{}_{c}K^{C}_{t,s},[{}_{c}\hat{C}_{t,s}]_{C^{-}_{2,t,s}},[{}_{c}\hat{C}_{t,s}]_{C^{-}_{3,t,s}},{}_{R}Q_{t,s},
$$

$$
{}_{N}Q_{t,s},M_{t,s},\bar{K}^{C}_{t,s},B_{t,s},Y_{t,s})-q_{t,s}\cdot\delta\cdot\bar{K}^{C}_{t,s}
$$

$$
+\sum_{C^{+}_{2,t,s}\cup C^{+}_{3,t,s}}\{(1-\rho)\,{}_{c}C_{t,s}({}_{c}L^{M}_{t,s},{}_{c}L^{A}_{t,s},{}_{c}L^{E}_{t,s},
$$

$$
{}_{c}K^{C}_{t,s})\}
$$

$$
-\sum_{C^{-}_{2,t,s}\cup C^{-}_{3,t,s}}\{(1-\rho)[{}_{c}\hat{C}_{t,s}-{}_{c}C_{t,s}({}_{c}L^{M}_{t,s},{}_{c}L^{A}_{t,s},
$$

$$
{}_{c}L^{E}_{t,s},{}_{c}K^{C}_{t,s})]\}-Y_{t,s}-Z_{t,s}\cdot v_{t,s}+d\cdot n_{t,s}+q_{t,s}\cdot I_{t,s}
$$

$$
+V_{t,s}-V_{t-1,s'}]+\sum_{t=1}^{T}\sum_{s=1}^{S}\lambda_{12,t,s}\hat{C}_{t,s}
$$

$$
+\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{s'=1}^{S}\lambda_{13,t,s,s'}[\bar{R}_{t,s}-\bar{R}_{t-1,s'}
$$

$$
-\sum_{C^{+}_{2,t,s}\cup C^{+}_{3,t,s}}\{(1-\rho)\,{}_{c}C_{t,s}({}_{c}L^{M}_{t,s},{}_{c}L^{A}_{t,s},{}_{c}L^{E}_{t,s},{}_{c}K^{C}_{t,s})\}
$$

$$+\sum_{C^-_{2,t,s} \cup C^-_{3,t,s}} \{(1-\rho)[{}_c\hat{C}_{t,s} - {}_cC_{t,s}({}_cL^M_{t,s}, {}_cL^A_{t,s},$$

$${}_cL^E_{t,s}, {}_cK^C_{t,s})]\}] \ ,$$

where the first difference equation indicating the change in accounts receivable has been incorporated last in the generalized Lagrangian in order that the Lagrange multipliers λ_1 through λ_{12} in (143) correspond to the same constraints as in the generalized Lagrangian (38). As in the case of (37) and (38), only the signs of $\lambda_{5,t,s}$, $\lambda_{6,t,s}$, and $\lambda_{12,t,s}$ - all must be nonnegative at optimality for all t and s - can be discerned without further study of the necessary conditions.

In comparing (142) with (37) it can be seen that the revenue functions $R_{GA,t,s}$ and $R_{GO,t,s}$, and therefore, the net income function, $\pi_{t,s}$, have been modified, and in particular, that the airframe builder's allocation of labor and contractor-furnished capital to aerospace government production contracts and to non-aerospace government contracts in one year has an impact on revenue earned, net income earned, and cash flow in the year the contract terminates. Thus necessary conditions (40) and (65) must be modified to take into account the intertemporal revenue, net income, and

and cash flow effects attributable to the policy of granting progress payments. Since the analysis is similar for labor and contractor-furnished capital and is also similar for government aerospace production contracts and non-aerospace government contracts, the discussion below can deal with the allocation of labor to government aerospace production contracts without loss of generality. Thus, the analysis for the allocation of labor to government contracts is set out in full, and then the analogy between the allocation of labor and the allocation of contractor-furnished capital is exploited to summarize the important implications of the government's progress payments policy for the allocation of contractor-furnished capital to government contracts.

To discuss these intertemporal effects connected with any particular contract c at time t and state of nature s, three cases need to be distinguished:

- the contract is terminating at time t

- the contract is ongoing at time t and will also be ongoing at time t = T

- the contract is ongoing at time t but will terminate at time $t' \leq T$

These three cases are considered in order below.

In the first case, time t is the year of termination. To abstract from the backlog effect, which was considered in the previous section, assume $t < T$. By differentiating (143) it is found that the optimal labor allocation ${}_cL^j_{t,s}$ must satisfy the following necessary condition:

$$\phi_{t,s}\{\frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}} + \frac{\partial R_{GA,t,s}}{\partial_c \hat{C}_{t,s}}\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}})$$

$$+ \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} + \frac{\partial \pi_{t,s}}{\partial_c \hat{C}_{t,s}}\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}})\}(\frac{1}{1+r})^t$$

$$+ \lambda_{2,c}\frac{\partial F_c}{\partial_c L^j_{t,s}} - \sum_{s'=1}^{S}\lambda_{9,t,s,s'}(\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} + \frac{\partial \pi_{t,s}}{\partial_c \hat{C}_{t,s}}\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}}) \tag{144}$$

$$- \sum_{s'=1}^{S}\lambda_{11,t,s,s'}\{\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} + \frac{\partial \pi_{t,s}}{\partial_c \hat{C}_{t,s}}\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}} + (1-\rho)[\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}}$$

$$- \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}]\} + \sum_{s'=1}^{S}\lambda_{13,t,s,s'}\{(1-\rho)[\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}} - \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}]\} = 0 .$$

Rearranging terms in (144) to facilitate a comparison with (40) yields

$$\{\phi_{t,s}\frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}}$$

$$+\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}}$$

$$+ \lambda_{2,c}\frac{\partial F_c}{\partial_c L^j_{t,s}} \tag{145}$$

$$+\{\phi_{t,s}\frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{GA,t,s}}{\partial_c \hat{C}_{t,s}}\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}}$$

$$+\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \times$$

$$\frac{\partial \pi_{t,s}}{\partial {}_c\hat{C}_{t,s}} \frac{\partial {}_c\hat{C}_{t,s}}{\partial {}_cL^j_{t,s}} -(1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \times$$

$$[\frac{\partial {}_c\hat{C}_{t,s}}{\partial {}_cL^j_{t,s}} - \frac{\partial {}_cC_{t,s}}{\partial {}_cL^j_{t,s}}] = 0 \ .$$

To interpret (145) note that the first three terms on the left-hand side of (145) are identical to the left-hand side of (40). In addition there are three other terms on the left-hand side of (145). The first two of these

$$\{ \quad \} \frac{\partial R}{\partial \hat{C}} \frac{\partial \hat{C}}{\partial L} \text{ and } \{ \quad \} \frac{\partial \pi}{\partial \hat{C}} \frac{\partial \hat{C}}{\partial L} \qquad (146)$$

reflect the indirect impact on current period revenue and net income, respectively, of a change in the current allocation of labor (of type j) to contract c, where this indirect impact is transmitted through the change in total accumulated allowable contract costs, ${}_c\hat{C}_{t,s}$. The last term on the left-hand side of (145) reflects the cash flow impact of a change in the labor allocation, ${}_cL^j_{t,s}$. Note that it follows from (9) and (134) that the difference $[\frac{\partial \hat{C}}{\partial L} - \frac{\partial C}{\partial L}] \leq 0$, so that the sign of the last term on the left-hand side of (145) is determined by the sign of the sum of Lagrange multiplier differences

$$\sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \ . \qquad (147)$$

In general the sign of (147) is indeterminate, being dependent on the time phasing of government contracts, and in particular, on the differential cash flow impact of contract termination (when the contractor receives cash amounting to allowable costs for the current year, plus ρ times the sum of previous years' allowable costs under the contract, plus the fee) and nontermination (when only ρ times the current year's allowable costs are received in cash). The significance of the sign of (147) is indicated below in corollary VII-16-1.

In the second case, the production contract c, which is ongoing at time t, will not terminate until after the planning horizon t = T. The optimal allocation ${}_cL^j_{t,s}$ must satisfy the following necessary condition:

$$\begin{aligned}
&\left\{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}} \left(\frac{1}{1+r}\right)^t\right\} \frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}} \\
&\quad + \left\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} \left(\frac{1}{1+r}\right)^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\right\} \frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}} \\
&\quad + \lambda_{2,c} \frac{\partial F_c}{\partial_c L^j_{t,s}} \\
&\quad + \sum_{s=1}^{S} \phi_{T,s} \frac{\partial U_3}{\partial_c \hat{C}_{T,s}} \frac{\partial \hat{C}_{T,s}}{\partial_c L^j_{t,s}} + (1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \times \\
&\quad \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}} = 0 ,
\end{aligned} \tag{148}$$

where the sum of Lagrange multiplier differences (147) once again has indeterminate sign. While (148) is similar in form to (145) - the first three terms on the left-hand side are identical and the remaining terms reflect the intertemporal cash flow impact of progress payments - the nature of the impact of progress payments is different in the two cases. In (148) there are two effects, the first interpretable as the expected marginal impact on expected collective utility of the amount of accumulated allowable costs under the contract as of the planning horizon, and the second interpretable as the cash flow impact of the proportion $(1 - \rho)$ of allowable costs that are not received as cash but that must be added to accounts receivable.

In the third case, the production contract c, which is ongoing at time t, terminates at some time $t' \leq T$. The optimal allocation ${}_cL^j_{t,s}$ must satisfy the following necessary condition:

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}} (\frac{1}{1+r})^t\} \frac{\partial R_{GA,t,s}}{\partial {}_cL^j_{t,s}}$$

$$+\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \frac{\partial \pi_{t,s}}{\partial {}_cL^j_{t,s}}$$

$$+ \lambda_{2,c} \frac{\partial F_c}{\partial {}_cL^j_{t,s}}$$

$$+ \sum_{s=1}^{S} \{\phi_{t',s} \frac{\partial U_1}{\partial R_{GA,t',s}} (\frac{1}{1+r})^{t'}\} \frac{\partial R_{GA,t',s}}{\partial {}_c\hat{C}_{t',s}} \frac{\partial {}_c\hat{C}_{t',s}}{\partial {}_cL^j_{t,s}}$$

$$+ \sum_{s=1}^{S} \{\phi_{t',s} \frac{\partial U_1}{\partial \pi_{t',s}} (\frac{1}{1+r})^{t'} - \sum_{s'=1}^{S} (\lambda_{9,t',s,s'} + \lambda_{11,t's,s,'})\} \times \tag{149}$$

$$\frac{\partial \pi_{t',s}}{\partial {}_c\hat{C}_{t',s}} \frac{\partial {}_c\hat{C}_{t',s}}{\partial {}_cL^j_{t,s}}$$

$$+(1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \frac{\partial {}_cC_{t,s}}{\partial {}_cL^j_{t,s}}$$

$$-(1-\rho) \sum_{s=1}^{S} \sum_{s'=1}^{S} (\lambda_{11,t',s,s'} - \lambda_{13,t',s,s'}) \frac{\partial {}_c\hat{C}_{t',s}}{\partial {}_cL^j_{t,s}} = 0 .$$

(149) is similar in form to (145) and (148), though there are now four terms required to convey the intertemporal impact of progress payments. In (149) the two terms

$$\sum_{s=1}^{S} \{ \quad \} \frac{\partial R}{\partial \hat{C}} \frac{\partial \hat{C}}{\partial L} \text{ and } \sum_{s=1}^{S} \{ \quad \} \frac{\partial \pi}{\partial \hat{C}} \frac{\partial \hat{C}}{\partial L} \tag{150}$$

are analogous to (146) in that they reflect the indirect impact on the revenue and net income, respectively, of the period in which the contract terminates of a change in the current allocation of labor (of type j) to contract c, where this indirect impact is once again transmitted through the change in total accumulated allowable contract costs, ${}_c\hat{C}_{t',s}$. What distinguishes (150) from (146) is the period in which the contract terminates - the current period in the latter and some future time period in the former. The last two terms in (149) are collectively analogous to the last term in (145), with the fact that the period in which the contract terminates differs from the current period in (149) being responsible for the two terms, rather than one, that indicate the separate period-specific cash flow impacts of progress payments.

To summarize briefly, then, the necessary conditions (145), (148), and (149) arose out of the need to consider explicitly the impact of progress payments on the allocation of labor (and in view of the comments made at the beginning of this subsection, contractor-furnished capital also). The three necessary conditions are all of the same general form

$$\{\phi_{t,s}\frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{GA,t,s}}{\partial_c L^j_{t,s}} \tag{151}$$

$$+\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_c L^j_{t,s}}$$

$$+ \lambda_{2,c}\frac{\partial F_c}{\partial_c L^j_{t,s}} + \theta = 0 ,$$

where the first three terms on the left-hand side are identical to the left-hand side of (40) and where θ is the sum of two or more terms that reflect the intertemporal cash flow impact of progress payments.

The comparison of (145), (148), and (149) with (40) leads to the following theorem.

Theorem VII-16

If the utility function U_1 is strictly concave with respect to each of its arguments $R_{GA,t,s}$, $R_{GO,t,s}$, and $\pi_{t,s}$, then labor usage under government contracts is directly related to the value of θ. If $\theta < 0$ in (151), then the effect of the present policy of progress payments is to reduce the contractor's labor usage under government contracts. If $\theta > 0$, then the effect is to increase the contractor's labor usage under government contracts.

Proof

If $\theta < 0$, then the left-hand side of (40) is strictly positive. Since it was shown in the proof of corollary VII-2-1 that (50) holds under the assumptions of this theorem, it follows that labor usage is restricted when $\theta < 0$. When $\theta > 0$ the left-hand side of (40) is strictly negative. Since (50) still holds, it follows that labor usage is increased when $\theta > 0$. Thus, labor usage under government contracts is directly related to the value of θ. Q.E.D.

Examination of the model incorporating progress payments (142) suggests a possible interpretation of theorem VII-16. During a period in which there are many ongoing government production contracts, but only a very small number, or possibly even none, terminating, the combined effect of the cash outflows in excess of cash inflows may be sufficient to threaten (or possibly even to precipitate) a cash flow crisis for the firm. Mathematically such a crisis would correspond to $\theta < 0$ and restricted labor usage by the firm. In the opposite case, many contracts terminating would correspond to $\theta > 0$. The variable θ corresponds, then, to a particular type of variation in the state of the firm's financial environment - a type of variation attributable to the system of progress payments and explainable on the basis of the imperfect matching of cash outflows and cash inflows that is a direct result of the policy concerning progress payments.

The significance of the sign of θ is brought out more clearly in the following corollary to theorem VII-16, which links the sign of (147) to the sign of one of the terms that comprise θ in (145), (148), and (149).

<u>Corollary VII-16-1</u>

For any government production contract that is terminating at time t, $-(1-\rho)\sum_{s'=1}^{S}(\lambda_{11,t,s,s'}-\lambda_{13,t,s,s'})\left[\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}} - \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}\right]$ in (145) is positive, zero, or negative as (147) is positive, zero, or negative, respectively. If instead, the contract is ongoing at t and will also be ongoing at $t = T$, then $(1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}$ in (148) is positive, zero, or negative as (147) is positive, zero, or negative, respectively; and in addition, $\theta < 0$ when (147) is negative, if $\partial U_3/\partial \hat{C} < 0$. If instead, the contract is ongoing at t but will terminate at $t' \leq T$, then the immediate cash flow impact $(1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}$ in (149) is positive, zero, or negative as (147) is positive, zero, or negative, respectively, whereas the future cash flow impact for each possible state s at t' $-(1-\rho) \sum_{s=1}^{S} \sum_{s'=1}^{S} (\lambda_{11,t',s,s'} - \lambda_{13,t',s,s'}) \frac{\partial_c \hat{C}_{t',s}}{\partial_c L^j_{t,s}}$ is negative, zero, or positive as $\sum_{s'=1}^{S} (\lambda_{11,t',s,s'} - \lambda_{13,t',s,s'})$ in (149) is positive, zero, or negative, respectively.

<u>Proof</u>

The first statement follows directly from $0 < \rho < 1$ and $[\frac{\partial_c \hat{C}_{t,s}}{\partial_c L^j_{t,s}} - \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}] < 0$. The second statement follows directly from $0< \rho <1$ and $\partial_c C_{t,s}/\partial_c L^j_{t,s} > 0$ and from the fact that $\theta \equiv \sum_{s=1}^{S} \phi_{T,s} \frac{\partial U_3}{\partial \hat{C}_{T,s}} \cdot \frac{\partial \hat{C}_{T,s}}{\partial_c L^j_{t,s}} + (1-\rho) \sum_{s'=1}^{S} (\lambda_{11,t,s,s'} - \lambda_{13,t,s,s'}) \frac{\partial_c C_{t,s}}{\partial_c L^j_{t,s}}$, where the first term is strictly negative when $\partial U_3/\partial \hat{C}_{T,s} < 0$. The third statement follows directly from $0 < \rho < 1$ and $\partial_c C_{t,s}/\partial_c L^j_{t,s} > 0$ and $\partial_c \hat{C}_{t',s}/\partial_c L^j_{t,s} > 0$. Q.E.D.

The significance of corollary VII-16-1 is that the sign of (147) plays an important role in determining the sign of the 'intertemporal cash flow impact of progress payments' term, θ, in (151). Theorem VII-16 and corollary VII-16-1 suggest that when (147) is negative for any date t and state s, the cash flow impact of a change in the labor allocation, ${}_cL^j{}_{t,s}$, i.e. for that date and state, is negative, tending to cause $\theta < 0$ and thereby tending to reduce the contractor's labor usage under government contracts. However, as the last term on the left-hand side of (149) suggests, this effect is partially muted by the positive impact on termination year cash flow. Finally, when (147) is positive for any date t and state s, the effects just noted are reversed, i.e. the immediate impact is positive, tending to cause $\theta > 0$, and the termination year impact is negative. In view of the comments made following the proof of theorem VII-16, it can be suggested that (147) tends to be negative for those dates and states when relatively few contracts terminate and tends to be positive for those dates and states when relatively many contracts terminate.[63]

The foregoing has dealt exclusively with the contractor's allocation of labor to government contracts. Similar results could be obtained for the allocation of contractor-furnished capital, and in particular, necessary conditions analogous to (145), (148), and (149) could be developed for the extension of (65). But rather than repeat this analysis, the following summary necessary condition for contractor-furnished capital, which is analogous to (151),

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{GA,t,s}}{\partial {}_cK^C{}_{t,s}} \qquad (15$$

$$+\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial {}_cK^C{}_{t,s}}$$

$$+ \lambda_{2,c} \frac{\partial F_c}{\partial_c K^C_{t,s}} - \lambda_{5,t,s} + \theta = 0 \, ,$$

where the first four terms on the left-hand side are identical to the left-hand side of (65) when k = GA and where θ represents the sum of two or more terms as in (151), is used to determine the important implications of the government's progress payments policy for the contractor's allocation of its own capital to government contracts.

<u>Theorem VII-17</u>

When $\theta > 0$ in (152), the effect of the government's progress payments policy on contractor investment behavior is to cause the contractor to substitute contractor-furnished capital for government-furnished capital. When $\theta < 0$, the effect is reversed, causing the contractor to substitute government-furnished capital for contractor-furnished capital. When $\theta = 0$, the impact of the government's progress payments policy is neutral with regard to contractor investment behavior.

<u>Proof</u>

Solving (72) and (152) for $\lambda_{2,c}$, equating the resulting expressions, and rearranging terms leads to the following modification of (73):

$$- \frac{\partial_c K^G_{t,s}}{\partial_c K^C_{t,s}} = \frac{\lambda_{5,t,s} - \{ \ \} \partial R_{GA,t,s} / \partial_c K^C_{t,s} - \{ \ \} \partial \pi_{t,s} / \partial_c K^C_{t,s} - \theta}{\lambda_{6,t,s}} \, , \tag{153}$$

where the terms in braces are the same as those in (152). When $\theta > 0$ the equilibrium marginal rate of technical substitution between government-furnished capital and contractor-furnished capital expressed by (153) diminishes, causing contractor-furnished capital to be substituted for government-furnished capital, as illustrated in figure VII-1.[64] When $\theta < 0$ the reverse occurs since the marginal rate of technical substitution expressed by (153) increases. When $\theta = 0$ (153) becomes identical to (73) with $k = GA$. Q.E.D.

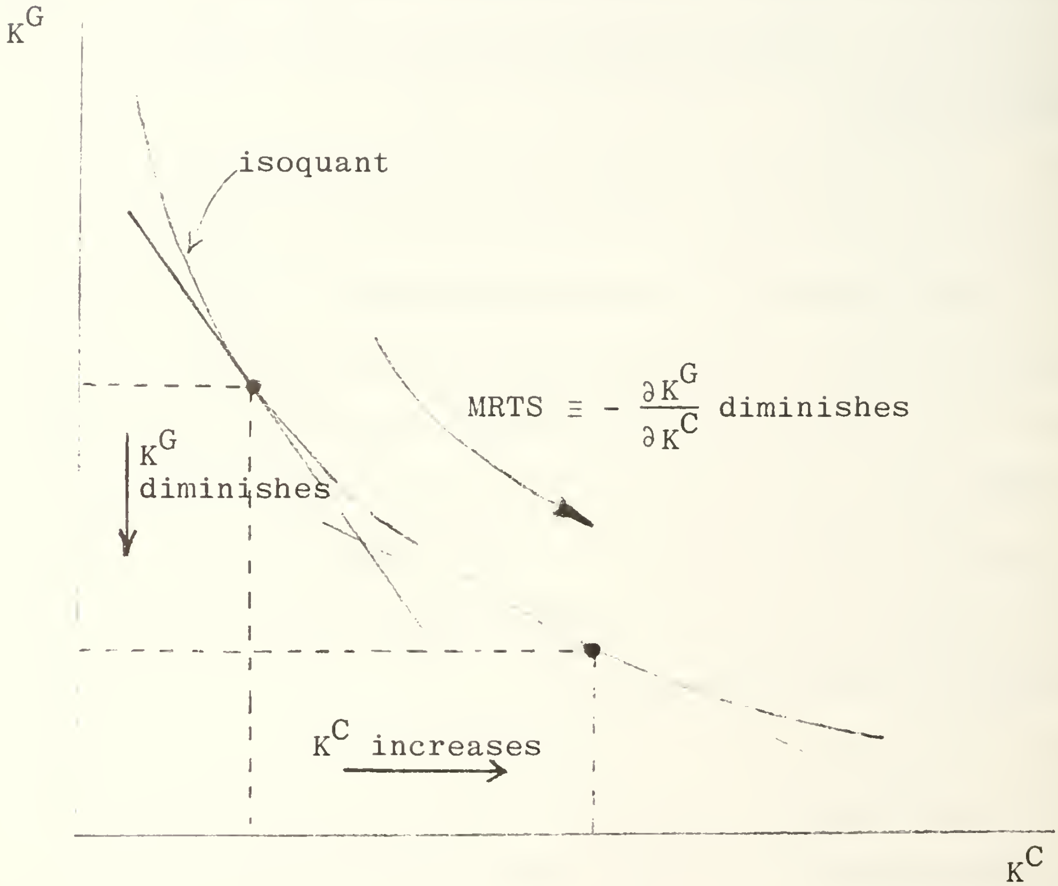

Figure VII-1 Diminishing Marginal Rate of Technical Substitution

Corollary VII-17-1

When $\theta > 0$ in (152) the government's progress payments policy tends to decrease the equilibrium net marginal cost of contractor-furnished capital allocated to each government contract,

$$\lambda_{5,t,s} - \{ \quad \} \partial R_{GA,t,s}/\partial_c K^C_{t,s} - \{ \quad \} \partial \pi_{t,s}/\partial_c K^C_{t,s} - \theta \ . \qquad (154)$$

When $\theta < 0$ in (152) the equilibrium net marginal cost is increased. When $\theta = 0$ the net marginal cost is unaffected.

Proof

Follows directly by comparing (154) with (67) for different values of θ. Q.E.D

The importance of theorem VII-17 and corollary VII-17-1 is that they demonstrate the impact the government's progress payments policy can have on contractor investment behavior. When $\theta > 0$ and cash flow is relatively strong due to the cash receipts associated with contract completion, the cost of capital adjusted for the implicit cost of cash utilized for investment purposes rather than for alternative uses is, in effect, diminished. As a result, the contractor is induced to invest in plant and equipment and to substitute contractor-furnished capital for government-furnished capital. This may account in part for contractor willingness during certain periods to purchase government-owned plant and equipment. When $\theta < 0$ and cash flow is relatively weak, the above effects are reversed since the implicit cost of cash used for investment purposes has risen.

In addition to the just discussed effects of the government's progress payments policy on the representative airframe builder's operating decisions, there is an important financial impact due to the required growth in total accounts receivable in the years prior to the year of contract completion. To highlight this effect, it was assumed in formulating (142) that commercial sales are matched perfectly by cash flows, i.e. that accounts receivable are due solely to the government's progress payments policy.

Differentiating (143) with respect to $\overline{R}_{t,s}$, setting the resulting expression equal to zero, and solving for $\lambda_{10,t,s}$ leads to the following necessary condition:

$$\lambda_{10,t,s} = \sum_{s'=1}^{S} \lambda_{13,t,s,s'} - \sum_{s'=1}^{S} \lambda_{13,t+1,s',s} \qquad (155)$$

As discussed above, $\lambda_{10,t,s}$ can be interpreted as the contractor's marginal cost of money capital. The right-hand side of (155), which is analogous to (121), can be interpreted as the marginal value of accounts receivable, which is expressed as the difference between the expected value of additional accounts receivable at t in s contingent upon s' at t - 1 less the expected value of additional accounts receivable at t in s contingent upon s' at t + 1. This interpretation of (155) leads to the following theorem.

Theorem VII-18

When the representative airframe builder modeled in (142) is in equilibrium, the marginal value of accounts receivable at each date t and state s will equal the firm's marginal cost of money capital.

The significance of theorem VII-18 is that accounts receivable represent a form of trade credit extended by the contractor to the government. The financial resources tied up in accounts receivable involve an opportunity cost to the firm that is measured by $\lambda_{10,t,s}$. In order that the amount of accounts receivable be optimal from the standpoint of the airframe builder, the marginal value of accounts receivable (in terms of discounted expected collective utility) must equal the marginal cost of the financial resources used for that purpose. Put somewhat differently, if $\lambda_{10,t,s}$ were actually the interest rate the firm had to pay to borrow funds, then extending trade credit of $\bar{R}_{t,s}$ to the government would involve an interest cost of $\lambda_{10,t,s} \cdot \bar{R}_{t,s}$ per period. The important point to be made here is that the government's progress payments policy implicitly involves the extension of trade credit to the government by the contractor and that such trade credit involves a cost that must be borne by the contractor.

Theorem VII-18 together with theorem VII-11 and corollary VII-13-1 lead to the following result.

<u>Corollary VII-18-1</u>

When the representative airframe builder modeled in (142) is in equilibrium, if it is optimal for the contractor to hold nonzero precautionary cash balances, then the marginal value of cash balances will equal the marginal value of accounts receivable; the marginal value of each of these alternative uses of funds will equal the firm's marginal cost of money capital, $\lambda_{10,t,s}$; and this marginal cost of money capital will be the same for both debt and equity.

Corollary VII-18-1 is a characterization of balance sheet equilibrium, since as seen previously, the marginal cost of money capital, $\lambda_{10,t,s}$, plays a crucial role in the contractor's investment decision. Thus, $\lambda_{10,t,s}$ measures the marginal cost of funds obtained from the sources listed on the right-hand side of the firm's balance sheet in table VII-5 and is numerically equal to the marginal value of those funds applied in the uses listed on the left-hand side of the balance sheet in table VII-5.

The necessary conditions (145), (148), and (149); the analogous modifications of (65) for the allocation of contractor-furnished capital to government contracts; and the necessary condition (155) for accounts receivable; are the only modifications needed to the necessary conditions presented in section C. The new necessary conditions do necessitate minor modifications in the rules, such as (41), for the optimal allocation of inputs under government contracts. These can be handled simply by redefining the net marginal values of the labor and capital inputs to incorporate θ - that is, to redefine net marginal value to reflect the intertemporal cash flow impact of progress payments discussed above.

3. Summary: The Impact of Progress Payments

In this section the model of the representative airframe builder formulated in section B was modified to incorporate progress payments. This modification required several changes in order that the model adequately reflect the intertemporal nature of the revenue-, net income-, and cash-flow related effects associated with the current progress payments policy. These changes resulted in terms having to be added to the necessary conditions stated in section C for the allocation of inputs to government contracts.

The main result of this section is that the current progress payments policy can have a significant cash flow impact on the behavior of the airframe builder, depending on the time phasing of government contracts (theorems VII-16 and VII-17). In years prior to the year of contract completion the contractor receives some portion ($\rho < 1$) of allowable costs in the form of cash. The balance, along with the fee (if any) earned under the contract, is received during the year the contract is completed. Thus, cash outflows and cash inflows are imperfectly matched. As a consequence, a large number of ongoing production contracts coupled with zero (or some small number of) contract completions/terminations tend to cause the contractor to reduce its allocations of labor and of its own capital to government contracts, while a relatively large number of contract completions/terminations would, by improving the firm's cash position, tend to have the opposite effect.

This section has considered just one aspect of government procurement policy. Several others are discussed in the next section.

E. ADDITIONAL PROCUREMENT POLICY ISSUES

The purpose of this section is to apply the model of the representative airframe builder that was developed in sections B and D of this chapter to attempt to explain the hoarding of labor, of which the airframe builders have often been accused, and to suggest some important implications of the design-to-cost policy and also of the recent procurement policy changes resulting from the Profit '76 study. In discussing these procurement policy issues it is the author's intention to demonstrate the applicability of the airframe builder model formulated in this chapter to these issues and to indicate that the model could be employed fruitfully in further research involving government procurement policy.

1. Labor Hoarding

As discussed in chapter six of this thesis,[65] it has been suggested by Peck and Scherer, as well as by others, that the major airframe builders have at times hoarded engineers, scientists, and administrative personnel, and that this hoarding has manifested itself in the assignment of these personnel to routine jobs not requiring their skills. The purpose of this subsection is to offer two possible explanations for this sort of behavior, one of which might explain 'permanent' hoarding and the other of which might explain 'temporary' or 'cyclical' hoarding. The former is most easily dealt with, and so is discussed first.

According to theorem VII-2, the representative airframe builder modeled in (37) employs labor of each type under each government contract beyond the short run profit maximizing level. To the extent that the amounts of administrative labor on the one hand and engineering and scientific labor on the other exceed their respective profit maximizing levels and to the extent that these 'excess' amounts are allocated to jobs that do not require such skills, a government auditor could reasonably interpret this sort of behavior as 'hoarding'. But such behavior is not the result of management's reluctance to lay off workers - i.e. its desire to hoard. Rather, the observed behavior is the result of management's willingness to sacrifice some potential profit in order to increase total revenue and expected collective utility. Moreover, since the contractor modeled in (37) will always allocate more of each type of labor to each contract than a short run profit maximizer, there is what may appear to be 'permanent hoarding' of labor by the contractor. Thus, one possible explanation for the hypothesized hoarding of labor is the nature of the firm's objectives, which can lead it to hire labor beyond profit maximizing levels.

The foregoing explanation of hoarding brings out an important issue associated with government contracting. For any given weapons system (e.g. plane or missile) already in production, it is in the government's interest to promote cost minimization, since ceteris paribus a lower unit cost tends to lead to an increase in the number of units that can be procured (this is true when, for example, there is a fixed total budget for a particular program). If contractors were interested in maximizing their fee on each FPI or CPIF contract, then their interests would also be served by cost minimization. Thus, cost minimization would be a mutual objective. However, if contractors attach some positive weight to total revenue and to managerial emoluments, as in (37) and in (142), then cost minimization is no longer a mutual objective. Under such circumstances, and as long as the airframe builders pursue alternative objectives while the government strives for cost minimization, it is likely that government auditors looking for evidence that costs are not being minimized would be able to gather such evidence. The central issue here is not whether or not contractors are technically efficient, but rather, whether their objectives are consistent with the government's (i.e. cost minimization) - and the analysis presented in sections C and D of this chapter suggests they are not.

The second explanation, which suggest that the hypothesized hoarding of labor - or at least a major component of such hoarding - is of a temporary or cyclical nature, is based on the existence of imperfections in the markets for administrators, engineers, and scientists. As discussed in chapter six, the major airframe builders are particularly concerned about providing stable employment opportunities for their skilled engineers and scientists as well as for their skilled administrative personnel (division managers, their staffs, etc.). One reason for this concern is the high cost of a hire-and-fire policy since laid off engineers and scientists may be hired by another airframe builder and hence be unavailable when a new program is won. To locate and hire new managers, engineers, and scientists can thus involve high costs of search in addition to the cost of training/retraining (of which at least some minimum amount must be done in order to familiarize newly hired or rehired personnel with the program on which they will work).

In the extreme case, the firm may react to these costs by setting manpower levels $\bar{L}^A$ for administrative labor and $\bar{L}^E$ for engineering and scientific labor that are to remain constant throughout the planning period. In terms of the airframe builder model (37),[66] this policy can be incorporated into the model by formulating the constraints (7), where $\bar{L}^A$ and $\bar{L}^E$ are treated as exogenously determined. Then modifying the generalized Lagrangian (38) by appending to it the terms

$$+ \lambda_{14,t,s}(\bar{L}^A - \sum_{U\{C_{i,t,s}\}} {}_cL^A_{t,s} - {}_RL^A_{t,s} - {}_NL^A_{t,s}) \quad (156)$$

and

$$+ \lambda_{15,t,s}(\bar{L}^E - \sum_{U\{C_{i,t,s}\}} {}_cL^E_{t,s} - {}_RL^E_{t,s} - {}_NL^E_{t,s}) \quad (157)$$

and differentiating with respect to ${}_cL^j_{t,s}$, j = A or E, leads to the following necessary condition:

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}} (\frac{1}{1+r})^t\} \frac{\partial R_{GA,t,s}}{\partial {}_cL^j_{t,s}}$$
$$+\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}} (\frac{1}{1+r})^t - \sum_{s'=1}^{S} (\lambda_{9,t,s,s'} + \lambda_{11,t,s,s'})\} \frac{\partial \pi_{t,s}}{\partial {}_cL^j_{t,s}} \quad (158)$$

$$+ \lambda_{2,c} \frac{\partial F_c}{\partial_c L^j_{t,s}} - \lambda_{j'} = 0 ,$$

where (j, j') = (A, 14) or (E, 15). While in general it is impossible to determine the signs of $\lambda_{14,t,s}$ and $\lambda_{15,t,s}$ since (7) are equality constraints, the following lemma suggests that, by characterizing the firm's attitude toward its levels of administrative labor and engineering and scientific labor at any time t and state of nature s as either 'would hire one or more workers in the absence of the constraint' or 'would lay off one or more workers in the absence of the constraint', it is possible to determine the signs of $\lambda_{14,t,s}$ and $\lambda_{15,t,s}$ during these (alternating) periods and to use this information to explain labor hoarding as a cyclical phenomenon.

<u>Lemma VII-3</u>

For time periods t and states of nature s for which the contractor would hire one or more administrative (or scientific and engineering) workers in the absence of the constraints (7), $\lambda_{14,t,s}$ (or $\lambda_{15,t,s}$) ≥ 0. For time periods t and states of nature s for which the contractor would lay off one or more administrative (or scientific and engineering) workers in the absence of the constraints (7), $\lambda_{14,t,s}$ (or $\lambda_{15,t,s}$) ≤ 0.

<u>Remark</u>

In the first case there is a scarcity of labor within the firm and $\lambda_{14,t,s}$ (or $\lambda_{15,t,s}$) acts as an internally imposed tax, rationing the use of whichever type(s) of labor is (are) in short supply, by deflating the net marginal value of labor,

$$\{\quad\}\frac{\partial R}{\partial L} + \{\quad\}\frac{\partial \pi}{\partial L} - \lambda \ . \qquad (159)$$

In the second case there is a surplus of labor within the firm and $\lambda_{14,t,s}$ (or $\lambda_{15,t,s}$) acts as an internally imposed subsidy by inflating the net marginal value of labor given by (159) in order to ensure that all available labor is allocated (and that the constraint is satisfied).

Proof of lemma VII-3

Since (156) and (157) are identical in form, the proof is the same for both types of labor. So consider $\lambda_{14,t,s}$. In the first case, the effective constraint on administrative labor is

$$\overset{\Sigma}{U}\{C_{i,t,s}\}{}_{C}L^{A}_{t,s} + {}_{R}L^{A}_{t,s} + {}_{N}L^{A}_{t,s} \leq \bar{L}^{A} , \qquad (160)$$

from which it follows that $\lambda_{14,t,s} \geq 0$. In the second case, the inequality in (160) is reversed and so is the sign of $\lambda_{14,t,s}$. Q.E.D.

The significance of lemma VII-3 is that the Lagrange multipliers $\lambda_{14,t,s}$ and $\lambda_{15,t,s}$ serve as indicators of the state of the firm's labor requirements. A positive value indicates a scarcity. All available labor of that type has been allocated to those positions in which they are of the greatest relative value to the firm due to the rationing role played by $\lambda > 0$ in (159). A negative value indicates the opposite situation. There is a surplus and $\lambda < 0$ in (159) acts as a subsidy. This case corresponds, of course, to the temporary hoarding of administrative and/or scientific and engineering labor. Moreover, this hoarding is of a cyclical character as the firm finds itself alternating between periods of scarcity and plenty (and $\lambda_{14,t,s}$ and $\lambda_{15,t,s}$ alternating between positive and negative values - though not necessarily in synchronization).

The second of the two explanations of labor hoarding is, in the opinion of this writer, the more likely cause of the hoarding of scientists and engineers during the periods between major programs. During such periods labor hoarding would be a rational response on the part of the contractor provided the prorated cost per period (in terms of expected collective utility) attributable to severance, search, and rehiring/retraining per unit of (scientific and engineering) labor exceeded λ, the amount of the subsidy. The main point is that 'labor hoarding' might, in spite of the negative connotation the terms carries, constitute a rational response on the part of the contractor to its cyclical need for a specialized class of labor, the market for which is imperfect.

2. Design-to-Cost

The second major procurement policy issue that is discussed in this section concerns design-to-cost,[67] which is an attempt by the government to redesign the weapons acquisition process in such a way that the massive cost overruns experienced in recent years can be avoided in the future. The purpose of this subsection is to suggest one method of altering the current procurement policy that would reduce, and possibly even eliminate, the cost bias exhibited by the representative airframe builder modeled in (37).

As discussed in section B of this chapter, for each government contract c at time t and state of nature s, there is a target cost ${}_cC^*_{t,s}$, an actual cost ${}_cC_{t,s}$, and a function ${}_c\gamma_{t,s}({}_cC_{t,s})$, which expresses the proportion of actual costs that are allowable and which satisfies (9). Suppose that the government were to establish ${}_cC^*_{t,s}$ as the absolute cost ceiling for contract c at time t and state s.[68] This could be accomplished within the model (37) by defining

$$ {}_c\gamma_{t,s}({}_cC_{t,s}) = \begin{cases} 1, & \text{if } C \leq C^* \\ C^*/C, & \text{if } C > C^* \end{cases} \qquad (161) $$

for in that case total allowable costs are equal to ${}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s} = {}_cC_{t,s}$, if $C \leq C^*$, and are equal to ${}_c\gamma_{t,s}({}_cC_{t,s})\cdot{}_cC_{t,s} = (C^*/C)\cdot C = C^*$, if $C > C^*$. From (161) it follows that

$$ \left.\begin{aligned} \frac{d\gamma}{dC} &= \begin{cases} 0, & \text{if } C \leq C^* \\ -C^*/C^2, & \text{if } C > C^* \end{cases} \\ \frac{d^2\gamma}{dC^2} &= \begin{cases} 0, & \text{if } C \leq C^* \\ 2C^*/C^3, & \text{if } C > C^* \end{cases} \end{aligned}\right\} \qquad (162) $$

With $\gamma(C)$ defined by (161), it follows from (8), (15), and (162) that for an aerospace research and development contract,

$$ \frac{\partial R}{\partial L^j} = [\gamma(C)+\gamma'(C)\cdot C]\frac{\partial C}{\partial L^j} = \begin{cases} w^j, & \text{if } C \leq C^* \\ 0, & \text{if } C > C^* \end{cases} \qquad (163) $$

Similarly, for an aerospace or a non-aerospace production contract,

$$\frac{\partial R}{\partial L^j} = \begin{cases} (1-\beta)w^j, & \text{if } C \leq C^* \\ 0, & \text{if } C > C^*. \end{cases} \qquad (164)$$

It also follows from (8), (15), (32), and (162) that for an aerospace research and development contract,

$$\frac{\partial \pi}{\partial L^j} = (1-\tau)\{[\gamma(C)+\gamma'(C)\cdot C]\frac{\partial C}{\partial L^j} - \frac{\partial C}{\partial L^j}\} = \begin{cases} 0, & \text{if } C \leq C^* \\ -(1-\tau)w^j, & \text{if } C > C^*. \end{cases} \qquad (16$$

Similarly, for an aerospace or a non-aerospace production contract,

$$\frac{\partial \pi}{\partial L^j} = \begin{cases} -\beta(1-\tau)w^j, & \text{if } C \leq C^* \\ -(1-\tau)w^j, & \text{if } C > C^* \end{cases} \qquad (166)$$

(163)-(166) lead to the following theorem.

<u>Theorem VII-19</u>

For the representative airframe builder modeled in (37), the cost level ${}_cC_{t,s} = {}_cC^*_{t,s}$ is optimal under government aerospace research and development contracts and the optimal cost level under government production contracts is some cost level ${}_cC_{t,s} \leq {}_cC^*_{t,s}$. That is, in neither case is it optimal for the contractor to overrun ${}_cC^*_{t,s}$.

<u>Proof</u>

In the case of research and development contracts, it follows from (163) and (165) that $\partial R/\partial L^j > 0$ and $\partial\pi/\partial L^j = 0$, if $C \leq C^*$ but that

$$\partial R/\partial L^j = 0 \text{ and } \partial\pi/\partial L^j < 0, \text{ if } C > C^* . \qquad (167)$$

Thus, when $C < C^*$ the contractor can increase revenue without suffering a loss of net income by increasing L^j. Thus, expected collective utility must increase with L^j when $C < C^*$. But when $C > C^*$ increases in L^j reduce net income without increasing revenue, so that expected collective utility falls as L^j increases. Thus, the labor usage level L^j that gives $C = C^*$ is optimal.

In the case of production contracts, it follows from (164) and (166) that (167) holds, so that under government production contracts it is again not optimal to use labor beyond the level L^j for which $C = C^*$. However, since

$$\partial R/\partial L > 0 \text{ and } \partial \pi/\partial L < 0, \text{ if } C \leq C^*,$$

$C = C^*$ is not necessarily optimal. The optimal level of labor usage, which by the foregoing must lead to $C \leq C^*$, is that which satisfies the necessary condition (40), which may be written as

$$(1-\beta)w^j\{\quad\}-(1-\tau)\beta w^j\{\quad\}+\lambda_{2,c}\frac{\partial F_c}{\partial L^j} = 0 \ . \qquad \text{Q.E.D.}$$

The significance of theorem VII-19 is that it demonstrates the possibility of designing government procurement policy to prevent overruns. Whether the government would want to be as restrictive with regard to cost allowability as (161) implies, particularly in an era of high inflation, is a much more difficult issue to resolve. The main point is that if the government were to implement an effective design-to-cost policy - one coupled with tight restrictions regarding the allowability of costs exceeding the contract's target cost - the problem of persistent cost overruns could be ameliorated.[69]

3. Profit '76

The last major procurement policy issue discussed in this section concerns the policy changes resulting from the recently completed Profit '76 study.[70] The purpose of this subsection is to explore the implications of two of these changes: (i) the allowability of interest expense and (ii) the introduction of capital investment as one of the determinants of the contract fee, ${}_c\pi_{t,s}$.

As a result of the Profit '76 study, interest expense is now an allowable cost under government contracts. It was hoped by the Profit '76 study team that making interest expense an allowable cost would encourage contractors to substitute contractor-furnished capital for government-furnished capital for work done under government contracts.[71] The following theorem states that the representative airframe builder modeled in (37) would react as expected to the policy change making interest expense an allowable cost.

Theorem VII-20

The representative airframe builder modeled in (37) would react to the making of interest expense an allowable cost under government contracts by substituting contractor-furnished capital for government-furnished capital.

Proof

As before, without loss of generality assume the government contract is of the aerospace production variety. It follows from (8) and (15) that

$$\frac{\partial R_{GA,t,s}}{\partial_c K^C_{t,s}} = (1-\beta)[\gamma(C) + \gamma'(C)\cdot C](\bar{i}_{t,s} + \delta)q_{t,s}$$
$$> (1-\beta)[\gamma(C) + \gamma'(C)\cdot C]\cdot\delta\cdot q_{t,s} \; . \tag{168}$$

It also follows from (8), (15), and (32) that

$$\frac{\partial \pi_{t,s}}{\partial_c K^C_{t,s}} = (1-\tau)\{(1-\beta)[\gamma(C) + \gamma'(C)\cdot C](\bar{i}_{t,s} + \delta)q_{t,s}\}$$
$$> (1-\tau)\{(1-\beta)[\gamma(C) + \gamma'(C)\cdot C]\cdot\delta\cdot q_{t,s}\} \; , \tag{169}$$

where it is assumed in (168) and (169) that the overall stock of contractor-furnished capital, $\bar{K}^C_{t,s}$, remains fixed[72] and where the inequalities in (168) and (169) follow from $0< \tau < 1$ and the fact that $(1-\beta)[\gamma(C) + \gamma'(C)\times C]\bar{i}_{t,s}\cdot q_{t,s} > 0$. Thus, making interest expense allowable has the effect of increasing both $\partial R_{GA,t,s}/\partial_c K^C_{t,s}$ and $\partial\pi_{t,s}/\partial_c K^C_{t,s}$. Since $\partial U_1/\partial R_{GA,t,s} > 0$ by assumption, it follows that the product

$$\{\phi_{t,s} \frac{\partial U_1}{\partial R_{GA,t,s}}(\frac{1}{1+r})^t\}\frac{\partial R_{GA,t,s}}{\partial_c K^C_{t,s}} \; ,$$

which appears as the second term in the numerator of the expression (73) for the marginal rate of technical substitution between government-furnished capital and contractor-furnished capital, must have increased as a result of making interest expense an allowable cost. Similarly, it follows from theorem VII-2 that $\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}$ must be strictly positive at optimality. Hence the product

$$\{\phi_{t,s} \frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial_c K^C_{t,s}}$$

which appears as the third term in the numerator of (73), also must have increased as a result of making interest expense an allowable cost. Since $\lambda_{5,t,s}$ and $\lambda_{6,t,s}$ are unaffected, it follows that the marginal rate of technical substitution diminishes, causing contractor-furnished capital to be substituted for government-furnished capital, as illustrated above in figure VII-1. Q.E.D.

The significance of theorem VII-20 is that making interest expense an allowable cost will have the desired effect on the behavior of the airframe builder modeled in (37). This result is an extension of corollary VII-5-1 because it makes clear how the allowability of interest expense affects contractor behavior. The following corollary contains the further obvious extension, namely, that increasing the cost of money rate $\bar{i}_{t,s}$ will induce a further substitution of contractor-furnished capital for government-furnished capital.

<u>Corollary VII-20-1</u>

Increasing the cost of money rate $\bar{i}_{t,s}$ with which the imputed interest expense under government contracts is computed will have the effect of encouraging contractors to substitute contractor-furnished capital for government-furnished capital.

<u>Proof</u>

It follows from (8), (15), (16), and (32) that

$$\frac{d}{d\bar{i}}\left(\frac{\partial R}{\partial K}\right) > 0 \text{ and } \frac{d}{d\bar{i}}\left(\frac{\partial \pi}{\partial K}\right) > 0 \ ,$$

which imply by (73) that

$$\frac{d}{d\bar{I}}\left(-\frac{\partial_c K^G}{\partial_c K^C}\right) < 0 \ .$$

Q.E.D.

The second important policy change resulting from the Profit '76 study discussed in this section concerns the addition of contractor investment in facilities and equipment to the list of factors that are considered in negotiating the contractor's fee. The purpose of this change was to provide a direct incentive to contractors to modernize their facilities and to purchase new labor-saving equipment.[73] As proved in the next theorem, this policy change will have the desired impact on the behavior of the airframe builder modeled in (37).

<u>Theorem VII-21</u>

By making the fee negotiated for each government contract a function of contractor investment in facilities and equipment, and hence, by making net income a function of investment, $\pi_{t,s} = \pi_{t,s}(I_{t,s})$,[74] the government will stimulate the representative airframe builder modeled in (37) to increase total investment (or decrease total disinvestment) during each period t and state of nature s.

<u>Proof</u>

Proceeding as in the development of the necessary condition (62) for optimal investment, both (58) and (59) must be satisfied simultaneously. Making the contractor's net income directly dependent on its level of investment requires that (59) be modified. The new necessary condition is

$$\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\frac{\partial \pi_{t,s}}{\partial I_{t,s}}$$

$$+(\sum_{s'=1}^{S}\lambda_{1,t,s,s'})(-\frac{1}{1+\delta})+(\sum_{s'=1}^{S}\lambda_{11,t,s,s'})(q_{t,s}) = 0. \tag{170}$$

Combining (170) and (58) and rearranging terms yields

$$\lambda_{5,t,s} + (1+\delta)\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t - \sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}\times$$

$$\frac{\partial \pi_{t,s}}{\partial I_{t,s}}=\{\phi_{t,s}\frac{\partial U_1}{\partial \pi_{t,s}}(\frac{1}{1+r})^t-\sum_{s'=1}^{S}(\lambda_{9,t,s,s'}+\lambda_{11,t,s,s'})\}(1-\tau)\times \tag{171}$$

$$q_{t,s}\cdot\delta+(\frac{1}{1+\delta})\sum_{s'=1}^{S}\lambda_{1,t+1,s',s}+\lambda_{10,t,s}\cdot q_{t,s}-(\sum_{s'=1}^{S}\lambda_{11,t,s,s'})q_{t,s},$$

where the right-hand side is the same as in (62). But since $(1+\delta)\{\quad\}\frac{\partial\pi}{\partial I}$ on the left-hand side of (171) is strictly positive, it follows that the adoption of contractor investment as a determinant of its fee has the effect of increasing the marginal value of each unit of capital without affecting marginal cost, thereby inducing an increase in contractor investment, as illustrated in figure VII-2. Q.E.D.

The significance of theorem VII-21 is that it indicates that making the contractor's fee dependent on its level of investment will produce the desired effect by augmenting the marginal value of capital. That is, contractor investment still has an indirect effect on expected collective utility, which is measured by $\lambda_{5,t,s}$, and in addition, a direct effect on net income and expected

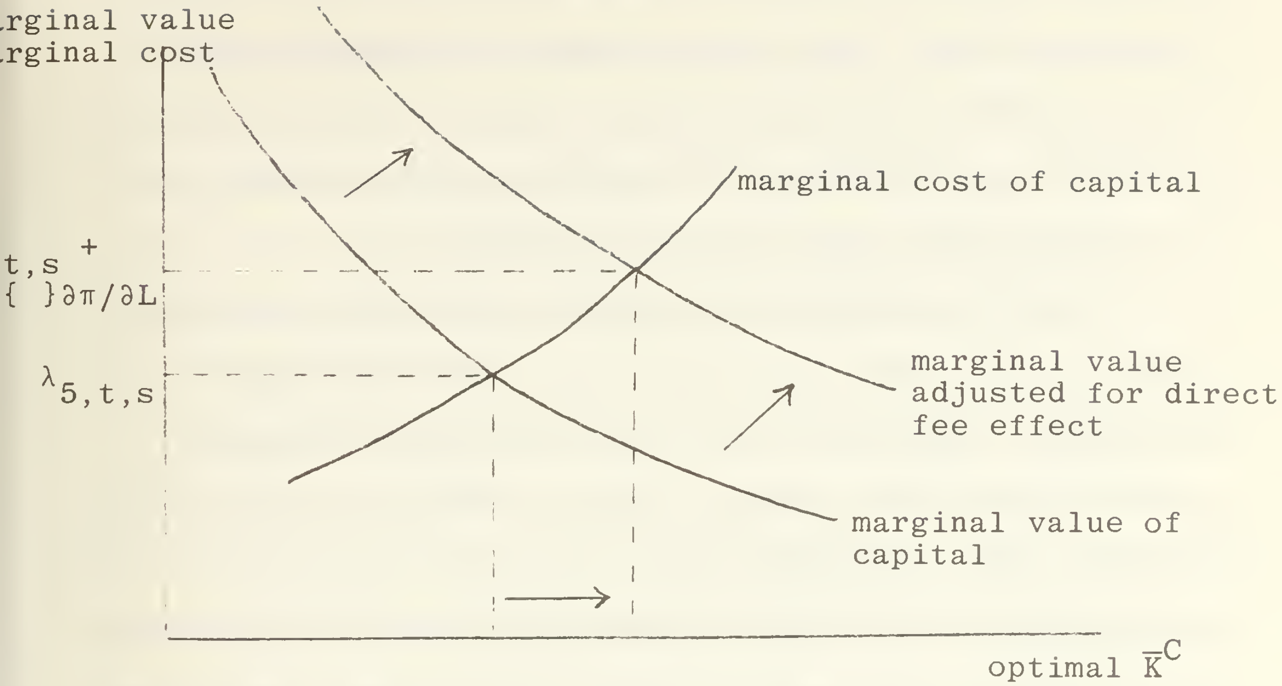

Figure VII-2 Increase in the Optimal Stock of Contractor-Furnished Capital Due to π(I) Relationship

collective utility, which is represented by the term $(1+\delta)\{\ \}\frac{\partial\pi}{\partial I}$ in (171). Given any initial capital stock, the effect of the policy change under discussion is to increase the optimal size of the capital stock $\overline{K}^C$, as illustrated in figure VII-2, by causing the marginal value of capital curve to shift to the right. It should be noted that if contractor-furnished capital is in surplus, i.e. if there is strict inequality in (6) and $\lambda_{5,t,s} = 0$, the policy change under discussion causes the contractor to reduce disinvestment, or possibly even to engage in net investment when disinvestment would otherwise be called for. On the one hand, such investment could have the positive effect of improving the quality of the firm's capital stock, while on the other hand, it could also have the negative effect of exacerbating the problem of excess capacity. Hence, the incentive structure needs to be carefully designed so as to ensure that the quality-quantity trade offs made by the various contractors are optimal from the government's standpoint.

This subsection has used the representative airframe builder model developed earlier in this chapter to demonstrate that two of the important changes resulting from the Profit '76 study - making interest expense an allowable cost and making contractor investment in facilities and equipment one of the determinants of the contractor's fee - will have the desired impact on contractor behavior.

F. CHAPTER SUMMARY

In this chapter a model of a representative airframe builder was developed in two stages. At the first stage, the model was formulated under the assumption that the recognition of revenue and fee by the contractor is perfectly matched by a flow of cash from the government to the contractor. This was done in section B, and the implications of the model concerning the optimal operating and financial policies of the representative airframe builder were explored in section C.

At the second stage of the model's development, progress payments, or cash payments to the contractor amounting to some fixed proportion $\rho < 1$ of allowable costs under each ongoing production contract, were incorporated into the model. This was done in section D, where it was also found that the progress payments policy produces cash flow effects that tend to force the firm to alter its labor usage policy (theorem VII-16) and its investment behavior (theorem VII-17). In particular, where cash flow is restricted, so too is labor usage under government contracts, and where cash flow is enhanced, so too is labor usage.

The model was used in section E to explore several additional procurement policy issues. Two explanations were offered for the often cited hoarding of administrative, scientific, and engineering labor by defense contractors, and it was suggested that such behavior may in fact be a rational response by contractors to their cyclical needs for labor. In addition, it was suggested how the treatment of the allowability of costs could be handled so as to eliminate, or at least reduce the likelihood of, cost overruns, and it was shown that the representative airframe builder would react in the expected manner to the Profit '76 study's policies of making interest expense allowable (theorem VII-20) and of making contract fees sensitive to contractor investment (theorem VII-21).

The representative airframe builder model formulated in this chapter is an extension of the author's basic theoretical model, which is developed and analyzed in chapters three through five of this thesis. The optimality rules established in this chapter (theorems VII-1, VII-3, VII-4, VII-6, VII-7, VII-10, VII-11, and VII-18) are similar to rules developed in chapter four. As in chapters three through five, it was found that the representative airframe builder tends to use more labor and produce more output than a short run profit maximizer (theorems VII-2 and VII-15). As in chapter four, it was found that the airframe builder's optimal operating policies are not, in general, independent of financial considerations (theorems VII-13 and VII-15 and corollaries VII-12-2 and VII-15-2).

CHAPTER SEVEN FOOTNOTES

1. Stekler, op. cit.; Carroll, op. cit.; and Ridder and Heinz, op. cit.

2. Peck and Scherer, op. cit.; Scherer, The Weapons Acquisition Process: Economic Incentives, op. cit.; and Fox, op. cit.

3. Scherer, The Theory of Contractual Incentives for Cost Reduction, op. cit.; McCall, The Simple Economics of Incentive Contracting, op. cit.; and Baron, Incentive Contracts and Competitive Bidding, op. cit.

4. In Scherer's model the firm is assumed to maximize the expected value of its contract profit. Scherer, The Theory of Contractual Incentives for Cost Reduction, op. cit., p. 261. Later in the paper Scherer suggests how to generalize the basic model to take into account contractor risk aversion. The generalization involves a managerial utility function that has the following three arguments: the expected value of long run profits, the probability of an outright loss on the contract, and the disagreeableness of cost reduction actions. Ibid., p. 276. In McCall's model the firm is assumed to maximize expected profits. McCall, The Simple Economics of Incentive Contracting, op. cit., p. 839. In Baron's model the firm is assumed to maximize the expected utility of total wealth. Baron, Incentive Contracts and Competitive Bidding, op. cit., p. 386.

5. Gorgol, op. cit.

6. C.R. Jones, "A Representative Defense Contractor: Model Specification I," technical report (Naval Postgraduate School; Monterey, Calif.; June 1975).

7. Ibid., p. 9.

8 Scherer has provided empirical support for the assumption that defense contractors are risk averse. Scherer, The Theory of Contractual Incentives for Cost Reduction, op. cit., pp. 273-276. Baron and Jones also assume risk aversion. Baron, Incentive Contracts and Competitive Bidding, op. cit., and Jones, op. cit.

9. Department of Defense Directive 5000.28, "Design to Cost" (May 23, 1975).

10. Profit '76 Summary Report (U.S. Government Printing Office; Washington, D.C.; December 7, 1976).

11. Jones, op. cit., p. 16, makes the same assumption.

12. Ibid.

13. Research and development contracts could also be distinguished from test and evaluation contracts. Ibid., pp. 13-20. For convenience, the two are treated collectively. In the opinion of this writer, the differences between manufacturing contracts on the one hand and research and development contracts and test and evaluation contracts on the other are significant enough as to preclude treating all three types collectively.

14. See sections E and F of chapter four for a discussion of the meaning of 'complete' and 'incomplete' markets for contingent claims for outputs.

15. The variable ${}_c\hat{E}$, where the subscript c denotes the contract, measures the overall effectiveness of the weapons system produced under contract c, and in addition, the firm's overall performance (e.g. in meeting delivery schedules under the contract). Thus, ${}_c\hat{E}$ is best regarded as a contract-specific index of effectiveness/performance. In theory at least, such an index could be constructed as a composite index, with the scales adopted in constructing each of the component indexes based on provisions of the contract (e.g. penalties for deviating from the delivery schedule), and with the composite index formed by weighting the component indexes according to the contractor's perception of how the government would weight them.

16. Note that incorporating K(T) also allows for growth as an objective. However, due to the substantial excess capacity in the industry, the two indicators of growth could move in opposite directions, i.e. sales and backlogs could be increasing at the same time that capacity was decreasing.

17. As argued below, (1) may be interpreted as the embodiment of the goals and objectives of the firm as formulated each year at the beginning of the planning process described in chapter six. In the model r is assumed to be an exogenously determined constant. In actuality, r would be selected by the firm's planners at time t = 0. It should be noted that, since planning is repeated in an annual cycle, r could change in a real time sense (i.e. from one plan to the next). But within each planning cycle, and hence within the model, it is reasonable, in the opinion of this writer, to treat r as a constant.

18. Similar treatments have been given defense contractor productive resources in _ibid._ and in Peck and Scherer _op. cit._

19. The historical importance of the two classes of capital are discussed in _ibid._, ch. 6.

20. _Defense Procurement Circular No. 76-3_, _op. cit._

21. Jones, _op. cit._, pp. 16, 22.

22. Jones treats performance/effectiveness in a similar manner, although he allows explicitly for multiple performance/effectiveness criteria. _Ibid._

23. Jones defines contract-specific production functions similarly. _Ibid._

24. Note that in (5), as in (4), for any time t and any state of nature s, the output level at t in s, ${}_kQ_{t,s}$ (k = R or N) is defined implicitly as a function of other possible output levels at t in the other S - 1 possible states that might obtain at t, as well as of the possible output levels at other dates and states (t', s), t ≠ t'. That is, the technological relationship between inputs and outputs is defined implicitly over the entire time period $0 \leq t \leq T$ and over all possible states that might obtain at any time t, $0 \leq t \leq T$. For given t, $0 \leq t \leq T$, and for any two states s and s' that might obtain at time t, the marginal rate of transformation

$$-\frac{\partial\, {}_kQ_{t,s'}}{\partial\, {}_kQ_{t,s}} = \frac{\partial F_k / \partial\, {}_kQ_{t,s}}{\partial F_k / \ {}_kQ_{t,s'}}$$

can be interpreted as the instantaneous rate at which a unit of output contingent upon state s occurring could be traded off for a unit of output contingent upon state s' occurring, where this rate of trade off is conditioned by the alternative productive techniques that will be available in these two states, as well as by input and output levels in previous periods, and where all input levels and all other output levels are, by definition, held fixed.

25. Another way of formulating the constraints (7) is to specify a slack variable for each to stand for the allocation of labor to 'special' projects during periods (and states of nature) when the rate of production slackens. (7) does not allow for this device, but rather, assumes that all available administrative labor and engineering and scientific labor are allocated to ongoing government contracts or to commercial production.

26. _Defense Procurement Circular No. 76-3_, _op. cit._, p.12.

27. _Ibid._, p. 30.

28. Note that since $\bar{i}_{t,s}$ is a money rate of interest, total interest expense must be computed by multiplying $\bar{i}_{t,s}$ by the value of fixed assets, $q_{t,s} \cdot {}_cK^C{}_{t,s}$

29. O.E. Williamson, The Economics of Defense Contracting: Incentives and Performance, op. cit.; S. Sheffrin and R. Spady, "The Persistence of Cost Overruns," Journal of Economic Issues (vol. 10; no. 2; June 1976), pp. 404-415; M.R. Dohan, "Cost Maximization and Buyer Dependence on Seller Provided Information," Journal of Economic Issues (vol. 10; no. 2; June 1976), pp. 430-452; and L.J. Dumas, "Payment Functions and the Productive Efficiency of Military Industrial Firms," Journal of Economic Issues (vol. 10; no. 2; June 1976), pp. 454-474.

30. It is assumed here that final bidding on major programs does not take place until production is ready to begin. When bidding takes place before the end of the development process, then for the purposes of this paper, subsequent development contracts are treated as production contracts. The line of demarcation between 'research and development contracts' and 'production contracts' in this paper is the contract for which final bidding for the entire program takes place. Because of the learning curve effect discussed in chapter six, once such bidding has taken place the government is virtually obligated to sole source its purchases from that point in time onward, with the consequence that all subsequent contracts are determined under conditions of bilateral monopoly.

31. This is the type most frequently adopted, although later production contracts may be of the firm-fixed-price type. Defense Procurement Circular No. 76-3, op. cit., p. 12.

32. That is, the government reimburses the contractor for a proportion $1 - {}_c\beta_{t,s}$ of costs in excess of the target cost and the contractor 'returns' to the government the same proportion of cost savings in the event that actual cost is less than ${}_cC^*{}_{t,s}$. Normally, $0 < {}_c\beta_{t,s} < .3$.

33. Scherer, The Theory of Contractual Incentives for Cost Reduction, op. cit., p. 258.

34. Cost floors can be ignored, since seldom have major programs had cost underruns in recent years. It should be noted that if cost ceilings are ignored altogether, then the contract is really of the cost-plus-incentive-fee (CPIF) type. In addition to the use of ${}_c\gamma_{t,s}$, a second possible justification for ignoring explicit cost ceilings is the fact that

the government has shown a willingness in recent years to modify contracts so that contractors can be compensated for overruns.

35. Since ${}_c\alpha_{t,s}$ and ${}_c\beta_{t,s}$ will actually depend on the government's bargaining posture, variability in ${}_c\alpha_{t,s}$ and ${}_c\beta_{t,s}$ in response to the contractor-government interaction is allowed for in the definition of the states of nature.

36. As a check, note that ${}_cR_{t,s} - {}_c\pi_{t,s} \equiv {}_cC_{t,s}$, as it should.

37. Note that nothing requires that $i_{t,s} = \bar{i}_{t,s}$, since the latter is the result of current government procurement policy while the former reflects the state of the financial markets.

38. The use of the prefix 'quasi' is intended to denote the fact that the price-quantity relationship modeled in (35) is dependent implicitly on prices and quantities from previous periods, which makes it different from the usual demand function.

39. Where it is assumed that all decision variables but $\hat{C}_{t,s}$ and all state variables are strictly positive all along their time-state trajectories. The reason for giving precautionary balances special treatment is discussed below.

40. See section B of chapter two of this thesis.

41. Note that because of lemma VII-2 it is possible to state a more general result that permits the states at times t and t - 1 to differ. By holding the state of nature fixed in corollary VII-1-2 it is the author's intention to focus on the 'pure' intertemporal nature of the allocation.

42. Peck and Scherer, op. cit.; Fox, op. cit.; Suarez, op. cit.; Sheffrin and Spady, op. cit.; Dohan, op. cit.; and Dumas, op. cit.

43. It is shown below in lemma VII-1 that $\lambda_{2,c} < 0$ at optimality when F_c is defined so that $\partial F_c/\partial {}_cL^j{}_{t,s} < 0$. By the assumed concavity of F_c, $\partial^2 F_c/\partial({}_cL^j{}_{t,s})^2 < 0$.

44. Note that it follows from the derivation of (47) that $\partial^2 R/\partial(L^j)^2 < 0$.

45. Recent procurement policy changes have worked in the opposite direction. For example, past performance has been deleted from the list of weighted guidelines that are used to establish the target fee. Defense Procurement Circular No. 76-3, op. cit., p. ii.

46. This is an important qualification because one of the major reasons given for dropping past performance from the list of weighted guidelines was the lack of an objective measure of past contractor performance.

47. Note that if $\lambda_{5,t,s} = 0$, then from (66), either $\lambda_{\ell} = 0$ or $\partial F_k/\partial_k K^C_{t,s} = 0$. In general, $\lambda_{\ell} \neq 0$. Since the marginal productivity of capital is given by

$$\frac{\partial Q}{\partial K} = - \frac{\partial F/\partial K^C}{\partial F/\partial Q} ,$$

$\partial F/\partial K = 0$ implies that the marginal productivity of capital is zero at optimality. If it is assumed that $\partial F/\partial K^C < 0$ for all K^C, then it can be inferred that $\lambda_{5,t,s} > 0$ and $\lambda_{\ell} < 0$ at optimality.

48. As in the case of $\lambda_{5,t,s}$, it is possible, though unlikely, that $\lambda_{6,t,s} = 0$ at optimality. In general, $\lambda_{2,c} < 0$ (as shown below in lemma VII-1), so that

$$\lambda_{6,t,s} = 0 \Rightarrow \partial Q/\partial K^G = 0$$

by the same line of argument employed in footnote 47. If it is assumed that $\partial F/\partial K^G < 0$ for all values of K^G, then it may be inferred that, in general, $\lambda_{6,t,s} > 0$ at optimality. The special case $\lambda_{6,t,s} = 0$ is considered in corollary VII-5-2.

49. See footnote 24 for an interpretation of this particular marginal rate of transformation.

50. Henderson and Quandt, op. cit., p. 95.

51. Note that it follows from (9) that $\gamma(C) \leq 1$ and $\gamma'(C) \leq 0$, and hence, that $\gamma'(C)\cdot C + \gamma(C) \leq 1$. Since $0 < \beta < 1$ it follows that the expression within braces on the right-hand side of (90) is always strictly positive. Therefore, it is not assured that (90) must hold for all t and s, although it is argued in the next footnote that one would expect (90) to hold normally.

52. (91) is demonstrated as follows. From (8) and (15),

$$\frac{\partial U_1}{\partial R_{GA,t,s}} = (1-\beta)[\gamma'(C)\cdot C + \gamma(C)]w_j , \qquad (*)$$

and from (8) and (32),

$$\frac{\partial U_1}{\partial \pi_{t,s}} = (1-\tau)\{(1-\beta)[\gamma'(C)\cdot C + \gamma(C)]w_j - w_j\} . \qquad (**)$$

Then

$$\{\phi_{t,s} \frac{\partial U}{\partial R} \frac{\partial R}{\partial L} + \frac{\partial U}{\partial \pi} \frac{\partial \pi}{\partial L}\} (\frac{1}{1+r})^t \geq 0$$

$$\Longleftrightarrow \frac{\partial U}{\partial R} \frac{\partial R}{\partial L} + \frac{\partial U}{\partial \pi} \frac{\partial \pi}{\partial L} \geq 0$$

$$\Longleftrightarrow \frac{\partial U}{\partial R}\{(1-\beta)[\gamma'(C)\cdot C + \gamma(C)]w_j\}$$

$$+ \frac{\partial U}{\partial \pi}\{(1-\tau)[(1-\beta)[\gamma'(C)\cdot C + \gamma(C)]w_j - w_j]\} \geq 0$$

$$\Longleftrightarrow \frac{\partial U/\partial R}{\partial U/\partial \pi} \geq (1-\tau)\{\frac{1}{(1-\beta)[\gamma'(C)\cdot C + \gamma(C)]} -1\} ,$$

where the first implication follows from $\phi_{t,s} > 0$ and $r > 0$; where the second follows by substitution of (*) and (**); and where the third follows by dividing through by

$$(\partial U/\partial \pi)(1-\beta)[\gamma'(C)\cdot C + \gamma(C)]w_j > 0.$$

(91) implies that (90) would be expected to hold normally. That is, one would expect that for the airframe builder modeled in (37), further increases in labor usage under any government contract would lead to increases in expected collective utility, but that the firm was constrained from doing so by the potential balance sheet and cash flow impacts of such increases.

53. This interpretation follows from the steps carried out in footnote 52 - i.e. the right-hand side of (90) is identically equal to

$$\frac{\partial \pi/\partial L}{\partial R/\partial L} .$$

54. That is, a change in the debt level has an immediate impact on cash flow since the firm's bond interest obligations are altered. A change in the rate of issues/redemptions also has an immediate cash impact as new debt issues are a source of cash while redemptions use up cash. In addition, a change in the level of debt outstanding in the current period will, given the firm's debt requirements next period, affect next period's required issues/redemptions decision, which in turn will affect next period's average interest rate $i = i(B, Y)$. This impact is measured by $-\sum_{s'=1}^{S} \lambda_{7,t+1,s',s}$ in (100).

55. This is necessary because the impact of changing capital goods prices on the optimum size of the firm's capital stock is dependent on both the state of nature at time t and the state of nature at time t + 1. The change in capital goods prices,

as it affects the optimum size of the capital stock, is thus computed below as a mathematical expectation with respect to the states s that might obtain at t and the states s' that might obtain at t + 1.

56. As noted in section E of chapter four, the Lagrange multiplier for the balance sheet identity, in this case $\lambda_{10,t,s}$, is measured in utility units. By dividing each side of (109) and (110) through by the coefficient { } of the $(1-\tau)q_{t,s}\cdot\delta$ term in (110), this units problem can be avoided.

57. These separability theorems are discussed in sections I and K of chapter two of this thesis.

58. Just how much smaller depends on the magnitude of $\partial^2 F_k/\partial({}_kQ_{t,s})^2$, which reflects indirectly the change in input requirements (since the constraint $F_k[\ ] = 0$ must be satisfied in equilibrium). These secondary effects are ignored, and hence the qualification 'tends to' in the statement of the theorem.

59. The interpretation of (90) is discussed above in the remark following the proof of lemma VII-1. Also, note that if (90) fails to hold at optimality, the sign of the sum on the left-hand side of (120) becomes indeterminate.

60. See section I of chapter two of this thesis.

61. Typically $\rho = 0.8$. Actually, the constant ρ constitutes an upper bound on the proportionate flow of cash from the government to the contractor. For example, if the government is dissatisfied with the item under production or with the rate of production, it can pay a proportion smaller than ρ. The model developed in this section abstracts from this complication.

62. Note that the effect on discounted expected collective utility of cumulative allowable costs that is embodied in U_3 is related to, though is not identical to, the backlog effect embodied in U_2. The latter is sensitive only to allowable costs in the terminal period and involves the quantity of units that are planned to be sold to the government beyond the planning horizon. The cumulative allowable costs are of interest primarily for their financial and cash flow impacts at the time the fee is determined (again, beyond the planning horizon).

63. The foregoing statements suggest conditions that would be sufficient to determine the sign of θ in (151). They also suggest the basis for an empirical investigation of the cash flow impact of the government's progress payments policy. For example,

one would expect to find systematic variations in commercial receivables and payables and in borrowing requirements over time depending on the relative cash-generating and cash-absorbing impact of government contracts.

64. Note that the shape of the isoquant in the figure implies that contractor-furnished capital and government-furnished capital are not perfect substitutes. This is consistent with the form of the production function (4), which contains these two types of capital as separate arguments.

65. See subsection 5 in section E of chapter six, and see also footnote 180 of chapter six.

66. The discussion could also proceed in terms of the modified model (142), although care would have to be taken to specify the date of contract termination. The use of (37) avoids this complication (without biasing the results).

67. <u>Department of Defense Directive 5000.28</u>, "Design to Cost," <u>op. cit.</u>

68. Strictly speaking, design-to-cost is concerned with unit cost, and only indirectly with total contract cost. However, since a given order quantity ${}_cQ_{t,s}$ and a given unit cost imply a particular total contract (target) cost ${}_cC^*_{t,s}$, it is the opinion of this writer that dealing directly with total contract cost is at least consistent in spirit with the design-to-cost policy and should not bias seriously the results obtained in this subsection.

69. A particularly important possibility from which this subsection has abstracted is the possibility of bankruptcy, which, in view of Lockheed's difficulties under the C-5A contract and Grumman's difficulties under the F-14 contract, would have to be considered carefully before the more restrictive attitude toward overruns discussed in this subsection could actually be implemented. This subsection was concerned with an 'if ... then' type of analysis, rather than with trying to recommend a specific course for government procurement policy.

70. <u>Profit '76 Summary Report</u>, <u>op. cit.</u>

71. <u>Ibid.</u>

72. Given the excess capacity in the aerospace industry, this is not an unreasonable assumption. Moreover, specific incentives to encourage investment in contractor-furnished capital are considered later in this subsection.

73. *Ibid.*

74. It follows from the change in procurement policy that functions of the form ${}_c\pi_{t,s} = {}_c\pi_{t,s}({}_cI_{t,s})$, where ${}_cI_{t,s}$ denotes contract-specific investment, can be defined. It is assumed here for convenience that these contract-specific functions can be used to define net income as a function of overall investment, $\pi_{t,s} = \pi_{t,s}(I_{t,s})$, where $\partial\pi_{t,s}/\partial I_{t,s} > 0$.

APPENDIX TO CHAPTER SEVEN

DEFINITION OF SYMBOLS

I. Roman[1]

t	index denoting the time period, where $t = 0, 1, \ldots, T$, with $t = 0$ denoting the initial period (in which the values of all relevant quantities are assumed to be known with certainty) and with $t = T$ denoting the terminal period
s	index denoting the state of nature, where $s = 1, \ldots, S$
t,s	ordered pair denoting the state of nature s at time t
$B_{t,s}$	the total quantity of debt outstanding (loosely, 'bonds')[2]
$\hat{B}_{T,s}$	total contract backlogs (as of the terminal period)
${}_c\hat{B}_{T,s}$	contract-specific backlog function, expressing total contract order backlog as a function of unit cost and weapons system effectiveness/contract performance in the terminal period
$C_{t,s}$	total cash (listed in the balance sheet)
$\hat{C}_{t,s}$	total precautionary cash balances
$\bar{C}_{t,s}$	total transactions cash balances

$C_{1,t,s}$ set of government contracts for aerospace research and development ongoing at t in s

$C_{2,t,s}$ set of government contracts for aerospace manufacturing ongoing at t in s

$C_{3,t,s}$ set of government contracts for non-aerospace manufacturing ongoing at t in s

$C^{+}_{i,t,s}$ denotes the set of government contracts of type i, i = 1, 2, 3, that remain ongoing after t in s

$C^{-}_{i,t,s}$ denotes the set of government contracts of type i, i = 1, 2, 3, that terminate at t in s

${}_cC_{t,s}$ actual cost under government contract c

${}_cC^*_{t,s}$ target cost for government contract c won at t in s

${}_cC^*_o$ target cost for government contracts ongoing at t = 0

${}_c\hat{C}_{t,s}$ accumulated allowable costs as of t and s under government contract c

d dividend per share (exogenously determined)

$D_{t,s}$ total dividends paid

$e_{t,s}$ retained earnings

E_s expectations operator

$E_{t,s}$ book value of total equity

${}_KE_{t,s}$ book value of contributed capital

${}_RE_{t,s}$ book value of accumulated retained earnings

$_c\hat{E}_{t,s}$ — weapons system effectiveness/contract performance under government contract c

F_c — government contract-specific production function

F_N — production function for commercial non-aerospace product

F_R — production function for commercial aerospace product

$i_{t,s}$ — average interest rate on debt

$\bar{i}_{t,s}$ — cost of money rate (exogenously determined)

$I_{t,s}$ — total investment in capital (measured in physical units)

$\bar{K}^C_{t,s}$ — total stock of contractor-furnished capital

$\bar{K}^G_{t,s}$ — total stock of government-furnished capital

$_cK^C_{t,s}$ — quantity of contractor-furnished capital employed under government contract c

$_NK^C_{t,s}$ — quantity of contractor-furnished capital employed in commercial non-aerospace production

$_RK^C_{t,s}$ — quantity of contractor-furnished capital employed in commercial aerospace production

$_cK^G_{t,s}$ — quantity of government-furnished capital employed under government contract c

$\bar{L}^A$ — exogenously determined constant level of administrative labor

$\bar{L}^E$ — exogenously determined constant level of engineering and scientific labor

${}_{c}L^{A}_{t,s}$, ${}_{N}L^{A}_{t,s}$, ${}_{R}L^{A}_{t,s}$	amounts of administrative labor employed under government contract c, in commercial non-aerospace production N, and in commercial aerospace production R, respectively
${}_{c}L^{E}_{t,s}$, ${}_{N}L^{E}_{t,s}$, ${}_{R}L^{E}_{t,s}$	amounts of engineering and scientific labor employed under government contract c, in commercial non-aerospace production N, and in commercial aerospace production R, respectively
${}_{c}L^{M}_{t,s}$, ${}_{N}L^{M}_{t,s}$, ${}_{R}L^{M}_{t,s}$	amounts of manufacturing labor employed under government contract c, in commercial non-aerospace production N, and in commercial aerospace production R, respectively
$M_{t,s}$	total managerial emoluments paid
$n_{t,s}$	total number of equity shares outstanding
${}_{N}p_{t,s}$	price of non-aerospace commercial product (expressed as a function of ${}_{N}Q_{t,s}$)
${}_{R}p_{t,s}$	price of aerospace commercial product (expressed as a function of ${}_{R}Q_{t,s}$)
$q_{t,s}$	price per unit of (physical) capital
${}_{c}Q_{t,s}$	output (per period) under government contract c
${}_{c}Q^{*}_{t,s}$	total volume of physical output specified in government contract c won at t in s
${}_{c}Q^{*}_{o}$	total volume of physical output specified in government contract c ongoing at t = 0
${}_{N}Q_{t,s}$	output (per period) of non-aerospace commercial product
${}_{R}Q_{t,s}$	output (per period) of aerospace commercial product

r discount rate (exogenously determined and taken to be constant throughout the planning period)

$\bar{R}_{t,s}$ total accounts receivable

$R_{GA,t,s}$ total revenue earned on aerospace sales to the government

$R_{GO,t,s}$ total revenue earned on non-aerospace sales to the government

$R_{CA,t,s}$ total revenue earned on commercial aerospace sales

$R_{CO,t,s}$ total revenue earned on commercial non-aerospace sales

${}_cR_{t,s}$ total revenue earned under government contract c

$\left.\begin{array}{l} U_1 \\ U_2 \\ U_3 \end{array}\right\}$ collective utility functions

$v_{t,s}$ stock market value of a share of stock

$V_{t,s}$ value of inventories (recorded in the balance sheet)

$\left.\begin{array}{l} w_A \\ w_E \\ w_M \end{array}\right\}$ unit prices of administrative labor A, engineering and scientific labor E, and manufacturing labor M, respectively

$Y_{t,s}$ new issues/redemptions of debt

$Z_{t,s}$ new issues/redemptions of shares of stock

II. Greek

${}_c\alpha_{t,s}$	ratio of target fee to target cost under government contract c won at t in s
${}_c\alpha_o$	ratio of target fee to target cost under government contract c ongoing at t = 0
${}_c\beta_{t,s}$	the contractor's share of cost overruns and underruns under government contract c won at t in s
${}_c\beta_o$	the contractor's share of cost overruns and underruns under government contract c ongoing at t = 0
${}_c\gamma_{t,s}$	proportion of costs incurred that are judged allowable under government contract c
δ	constant percentage rate of physical depreciation
$\phi_{t,s}$	the probability that state of nature s will obtain at time t
$\pi_{t,s}$	net income (after tax)
${}_c\pi_{t,s}$	actual fee earned under government contract c
${}_c\bar{\pi}_{t,s}$	target fee under government contract c
ρ	proportion of allowable costs in any period under an ongoing government contract that is granted as progress payments (actual $\rho \leq 0.8$)
τ	constant tax rate
$\lambda_{1,t,s,s'}$	value of an additional unit of (physical) capital at time t in state s contingent upon the transition to s at t from s' at t - 1

$\lambda_{2,c}$	Lagrange multipliers associated with government contract-specific production functions
λ_3	Lagrange multiplier associated with the production function for the commercial aerospace product
λ_4	Lagrange multiplier associated with the production function for the commercial non-aerospace product
$\lambda_{5,t,s}$	value of an additional unit of contractor-furnished capital ($\lambda_{5,t,s} \geq 0, \forall t, \forall s$)
$\lambda_{6,t,s}$	value of an additional unit of government-furnished capital ($\lambda_{6,t,s} \geq 0, \forall t, \forall s$)
$\lambda_{7,t,s,s'}$	value of a change in the amount of debt outstanding at time t in state s contingent upon the transition to s at t from s' at t - 1
$\lambda_{8,t,s,s'}$	value of a change in the number of shares of stock outstanding at t in state s contingent upon the transition to s at t from s' at t - 1
$\lambda_{9,t,s,s'}$	value of a change in the book value of total equity at t in state s contingent upon the transition to s at t from s' at t - 1
$\lambda_{10,t,s}$	Lagrange multiplier associated with the balance sheet identity (interpreted as the firm's marginal cost of financial capital)
$\lambda_{11,t,s,s'}$	value of an additional dollar of cash balances at t in state s contingent upon the transition to s at t from s' at t - 1
$\lambda_{12,t,s}$	measure of the impact of the nonnegativity constraint on precautionary cash balances ($\lambda_{12,t,s} \geq 0, \forall t, \forall s$)

$\lambda_{13,t,s,s'}$ value of an additional dollar of accounts receivable at t in state s contingent upon the transition to s at t from s' at t - 1

$\lambda_{14,t,s}$ value of an additional unit of administrative labor

$\lambda_{15,t,s}$ value of an additional unit of engineering and scientific labor

1. With the exception of t and s, the symbols drawn from the roman alphabet are listed in alphabetical order

2. Wherever the pair of subscripts t,s appears the statement 'at time t in state of nature s' should be added to the definition of the symbol. In the interest of brevity this is not done for each symbol in the table bearing these subscripts.

VIII. SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

A. SUMMARY AND CONCLUSIONS

At the end of an undertaking such as this it is important to take stock. This is done in this chapter by first, indicating what sparked the author's interest in the theory of the firm; then reviewing the research results reported herein and summarizing the major contributions of the thesis; and lastly, suggesting what might lie ahead.

During the period 1971-1973 the author studied at the University of Cambridge, where he worked under Ajit Singh and attended two series of lectures given by Robin Marris. The latter, whose managerial model of the firm is presented in section G in chapter two of this thesis, and the former, whose empirical studies of the role of takeovers were also discussed in section G, indicated to the author the limitations of existing models of the firm and stimulated the author's interest in developing more 'realistic' models of the firm that could be used to explore a wider range of issues than those for which the models that had appeared in the literature were suitable. Even before leaving Cambridge it had become apparent to the author that the formulation of more meaningful models of the firm would likely entail the use of more sophisticated mathematical techniques than those that had previously been employed.

In selecting the specific research topic for this thesis it was the author's intention to utilize the technique of optimal control theory in order to extend the theory of the firm by developing a stochastic multiperiod model of the firm and by using the model to study the operating decisions and financial decisions - and the relationship between them - made by the firm in the face of uncertainty in general and systematic demand shifts - e.g. the business cycle - in particular. It was also the author's intention to study the behavior of a representative firm in a specific industry - the industry selected was the U.S. airframe industry - in the context of that industry's institutional and regulatory milieu. The purpose of such an analysis would be to formulate a model that incorporated these industry-specific phenomena and to use the model to study analytically the behavior of the firm under alternative institutional and regulatory settings.

1. The General Literature and Its Patterns

The initial phase of the research effort involved a survey of the literature in order to identify clearly the state of the art in modeling the firm and its behavior. In order to make sense out of the multiplicity of models that had been formulated, a taxonomy was developed and the most significant of the previous contributions to the theory of the firm were classified according to their principal economic and mathematical attributes. The results are presented in chapter two and a convenient summary of the results is provided in tables II-34 and II-35.

The principal economic attributes are: the nature of the firm's objective (traditional/managerial/behavioral/modern traditional); the treatment of financial considerations (subsumed/internal only/external - debt only/external - equity only/external - both debt and equity); the existence of uncertainty (or lack of it); the consideration (or lack of it) of disequilibrium issues; and the treatment of time (single period or multiperiod). The principal mathematical attributes are: the nature of the optimization (static or dynamic); the existence (or lack of it) of nonlinearities; the nature of the constraints; and the solution technique employed. The survey revealed the following lacunae in modeling (and understanding the behavior of) a firm making operating and financial decisions in an uncertain world:

- Due at least in part to the conceptual difficulties associated with specifying the appropriate objective for the traditional type firm operating within a stochastic multiperiod environment,[1] there were no models that considered the behavior of the firm within a context that was both multiperiod and stochastic in nature.
- There were no models that permitted an interaction between phenomena external to the firm and phenomena internal to it.
- The models classified firms into those in which the shareholders' goals always predominated (traditional and modern traditional); those in which the firm's professional manager's goals always predominated (managerial); and those in which goal setting was assumed to take place through some internal bargaining process, with the consequence that specific goals at any point in time were ill-defined

a stable relationship (over time) between the goals of both shareholders and managers.

2. The Basic Model and Significant Results

Thus it was decided to develop a model of the firm with the following attributes:

- The model would explicitly allow for the objectives of shareholders and managers - and a stable relationship between these sets of objectives over time - within a single collective utility function.
- The model would integrate factors relevant to the firm's operating policy decisions and those relevant to its financial policy decisions into a stochastic multiperiod model in order to permit conclusions concerning the relationship between these two sets of policies to be drawn.
- The model would explicitly recognize the interaction between phenomena external to the firm and phenomena internal to it in order to permit systematic changes in the behavior of the firm in response to shifts in demand, such as those caused by the business cycle, to be studied.
- The uncertainty version of the integrated production-finance model would employ the time-state-preference approach to modeling uncertainty.
- The uncertainty version of the integrated production-finance model would be formulated as a stochastic optimal control problem and its optimal solution characterized using appropriate mathematical techniques.

The specification of the model possessing the aforementioned attributes permitted the following results to be obtained:

- The traditional, managerial, and behavioral views of the firm were synthesized and conditions under which traditional objectives would rank ahead of managerial objectives were distinguished from conditions under which managerial objectives would rank ahead of traditional objectives (chapter three).
- The extent of managerial discretion, as indicated by whether or not the profit constraint was binding, was shown to have a significant effect on the behavior of the firm (chapter three).
- It was shown that factors external to the firm could play an important role in determining the firm's choice of operating policies, and in particular, that the behavior of the firm over time could vary systematically between the traditional and managerial modes due to shifts in demand, such as those attributable to the business cycle (chapter three).
- The firm's optimal operating (i.e. output, input usage, and investment) policies and optimal financial (i.e. cash management, leverage, and dividend) policies, and the relationships between these two sets of policies, were derived (chapter four).
- It was shown that, except under special circumstances (which were specified), the firm's output decision is not independent of its cash management policy, and also that, except under special circumstances (which

were also specified), the firm's cost of capital, and hence its investment decision, is not independent of its leverage policy (chapter four).

- Given the lack of separability, it was shown how a planning algorithm not requiring separability could be devised for the multidivision firm (chapter five).
- It was also shown how the firm's degree of organizational slack can vary systematically in response to external pressures brought on by changes in demand over the business cycle (chapter five).
- Internal resource allocation rules were derived for the decentralized multidivision firm and the possible implications for internal control and X-efficiency when division managers are able to exercise discretion were derived (chapter five).
- It was further shown throughout the model's development that mathematical programming theory could be invoked in order to develop economically meaningful interpretations of the Lagrange multipliers (chapters three through five).

3. The Representative Airframe Builder Model

The study of the U.S. airframe industry proceeded from a review of the relevant literature that revealed few previous attempts to model a defense contractor - with the main exceptions being the contract bidding literature,[2] in which just one aspect of contractor behavior was modeled, and the models due to Gorgol[3] and Jones,[4] which, as noted in chapter seven, were concerned mainly with model formulation, rather than with obtaining analytical results. In order to obtain a better appreciation for the institutional constraints the airframe builders face and how these constraints affect their long term and short term planning, interviews were conducted at the nine major airframe builders,[5] which resulted in the description of their internal planning processes presented in chapter six.

Based on the work of Jones and the aforementioned interviews, a model of the representative airframe builder incorporating the following distinguishing features was formulated:

- The objective function reflects the principal objectives of these firms, and in particular, their interest in diversification, which is modeled by specifying four sales objectives: aerospace sales to the government, non-aerospace sales to the government, commercial aerospace sales, and commercial non-aerospace sales.
- The model integrates those factors relevant to the airframe builder's operating decisions and those factors relevant to its financial decisions into an analytical model.
- The model is of the stochastic multiperiod variety and draws heavily on the author's basic theoretical model formulated in chapter four.
- The model incorporates the government's principal procurement policy parameters and explicitly takes into account the government's progress payments policy.
- The model is formulated as a stochastic optimal control problem, with uncertainty modeled using the time-state-preference framework.

The representative airframe builder model is used in chapter seven to characterize the airframe builder's optimal operating policy decisions and optimal financial policy decisions and to explore how these two sets of policy decisions are related. As in chapter three, it was found that the representative airframe builder tends to produce more commercial output than a short run profit maximizer. In addition, it was found that the representative airframe builder tends to allocate more labor to each government contract than a short run profit maximizer. As in chapter four, it was found that the representative airframe builder's optimal operating policies are not, in general, independent of financial considerations.

The representative airframe builder model was also utilized in chapter seven to study the impact of alternative government regulatory policies, with the following results:

- It was shown that the progress payments policy produces cash flow effects that tend to cause the airframe builder to alter its labor usage policy, and in particular, that labor usage under government contracts (beyond short run profit maximizing levels) varies inversely with the degree of cash flow stringency attributable to the time phasing of government contracts.
- The model was used to provide two explanations for the often cited hoarding of skilled labor by defense contractors.
- It was shown that the representative airframe builder would react in the expected manner to the Profit '76 study's recommended policies of making interest expense

allowable and of making contractor fees sensitive to contractor investment.

In the analysis of the airframe builder's policy decisions the optimality criteria of the short run profit maximizer were used for comparison. This was done for two reasons. First, the Armed Services Procurement Regulation (ASPR) assumes that contractors are motivated to maximize short run profit.[6] Second, short run profit maximization (i.e. the traditional model discussed in chapter two), which leads to Pareto optimality in the context of a perfectly competitive economy,[7] has been adopted by economists as the bench mark in the analysis of alternative modes of behavior on the part of the firm. How close the behavior of actual firms comes to short run profit maximization is an empirical question, and several studies dealing with this issue were discussed in chapter two. This point is an important one because, as the literature on the theory of second best makes clear,[8] if large firms outside the airframe industry depart from the ideal of short run profit maximization, as the basic theoretical model developed in chapters three through five suggests, then forcing the airframe builders to adopt short run profit maximizing policies may lead to a decrease, rather than an increase, in social welfare. More important, in view of ASPR's assumption concerning contractor motivation, if the model formulated in chapter seven of this thesis could be validated empirically, then a basic conflict would be shown to exist between actual contractor behavior and the type of contractor behavior considered optimal by the government.

The model formulated in chapter seven and its use in studying several procurement policy questions should be viewed, then, as a general discussion of public policy regulatory analysis, and not as a call for further immediate reforms in U.S. government procurement policy. Hard policy recommendations must await careful empirical research. It is the author's hope that the model of the representative airframe builder formulated in chapter seven will serve as an important first step toward a fuller, more comprehensive and scholarly analysis of the difficult issues involved in setting a procurement policy.

B. SUGGESTIONS FOR FURTHER RESEARCH

During the course of the development of both the basic model and the representative airframe builder model alternative approaches were eschewed in favor of those reported in the thesis. Several of these alternative paths are reasonable directions for further research. In addition, the specific directions taken in the author's research can be pursued further, and this further evolution of the basic model and these further refinements of the airframe builder model are discussed below.

1. Extensions of and Empirical Tests of the Basic Model

The basic theoretical model presented in chapters three through five placed the firm in a partial equilibrium setting. The analysis of the (expected) collective utility maximizer focused on the individual firm, its behavior over time under uncertainty, its choice of optimal operating policies and optimal financial policies, and its internal allocation of productive resources. Further research could explore the interactions between firms of the type modeled in chapters three through five and the implications of such interactions for the behavior of the individual firm. Such research would be welcome because, as noted in section D of chapter one, the bulk of the research within the theory of the firm has dealt with the firm in isolation. By imbedding the firm within alternative market structures, the importance of market structure as it affects the behavior of the firm, for example the quality of goods produced,[9] could be studied.

At an empirical level, the basic theoretical model yields the following empirically testable propositions:

- The behavior of the typical large firm varies systematically over the business cycle, being more consistent with the managerial models during the upswing and more consistent with the traditional models during the downswing.
- An increase in the tax rate on corporate profits will tend to cause the firm to increase managerial emoluments at the expense of dividends and to substitute debt for equity in its capital structure.
- The typical large firm's payment of managerial emoluments and hiring of (nonproductive) staff will vary systematically over the business cycle, both increasing relative to other variables during the upswing and decreasing relative to other variables during the downswing.

- The level of relative X-efficiency within the typical large firm will vary directly with the degree of centralization i.e. the more centralized the firm, the greater the relative degree of X-efficiency. Put somewhat differently, the relatively higher the extent of decentralized decision-making, the relatively greater the degree of organizational slack (nonproductive administrative labor within each division as a percentage of the division's total labor force).

2. Extensions of and Empirical Tests of the Representative Airframe Builder Model

The second main direction for further research concerns the further development of the model of the representative airframe builder. For example, the model presented in chapter seven abstracted from the Renegotiation Board and its impact on the firm's behavior.[10] Recently Agapos claims to have shown that the Renegotiation Board has had a positive effect on pre-tax profits.[11] By reformulating the model presented in chapter seven to include the Renegotiation Board the role of that board in regulating defense contractors' profits could be studied.

The representative airframe builder model could also be extended in the same manner as suggested above for the basic theoretical model to study the interactions among airframe builders, particularly during the bidding process. Such a model might also prove useful in studying the contractor-subcontractor relationship and the extent to which the subcontractor's relationship to the contractor parallels the contractor's relationship to the government.

The representative airframe builder model formulated and analyzed in chapter seven abstracted from the determinants of the government's demand for aircraft. It was assumed that the government's demand for aircraft could be expressed in the form of a bivariate quasi-demand function, which was specified in section B.[12] Additional research could be directed toward illuminating the determinants and form of the actual demand relationship.

At an empirical level, the model of the representative airframe builder yields the following empirically testable propositions:

- The airframe builder systematically allocates labor to government contracts and sets commercial output beyond the levels consistent with short run profit maximization.
- The airframe builder's tendency to allocate inputs to work under government contracts beyond short run profit maximizing levels is directly related to the cash flow impact of the number of contracts being completed/terminated.
- The procurement policy changes resulting from the Profit '76 study will induce contractors to substitute contractor-furnished capital for government-furnished capital and will induce contractors to increase their investment in plant and equipment (i.e. such investment will be greater than it would have been without the changes).

It is this writer's hope that a careful reading of this thesis will stimulate further research into the areas discussed above.

CHAPTER EIGHT FOOTNOTES

1. See subsection 4 in section K of chapter two.

2. See footnote 3 of chapter seven for references.

3. Gorgol, op. cit.

4. Jones, op. cit.

5. The nine firms at which interviews were conducted by the author are listed in footnote 1 of chapter six.

6. Defense Procurement Circular No. 76-3, op. cit., p. 3.

7. For a mathematical exposition see Henderson and Quandt, op. cit., ch. 7, or Takayama, op. cit., pp. 185-201.

8. R.G. Lipsey and K. Lancaster, "The General Theory of Second Best," Review of Economic Studies (vol. 24; no. 63; January 1956), pp. 11-32, and E.J. Mishan, "Second Thoughts on Second Best," Oxford Economic Papers (new series, vol. 14; no. 3; October 1962), pp. 205-217.

9. A recent study that explores the relationship between market structure and the durability of capital goods is A. Raviv and E. Zemel, "Durability of Capital Goods: Taxes and Market Structure," Econometrica (vol. 45; no. 3; April 1977), pp. 703-717.

10. The Renegotiation Board is included in the model specified by Jones. Jones, op. cit.

11. A.M. Agapos, Government-Industry and Defense: Economics and Administration (University of Alabama Press; Alabama; 1975).

12. See footnote 38 in chapter seven.

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